Contents

Part II

Categorical Logic

Chapter 1

Categorical Statements

1.1 Quantified Categorical Statements

We saw that a statement was a unit of language that could be true or false. In this chapter and the next we are going to look at a particular kind of statement, called a quantified categorical statement, and begin to develop a formal theory of how to create arguments using these statements. This kind of logic is generally called "categorical" or "Aristotelian" logic, because it was originally invented by the great logician and philosopher Aristotle in the fourth century BCE. This kind of logic dominated the European and Islamic worlds for 20 centuries afterward, and was expanded in all kinds of fascinating ways, some of which we will look at here.

Consider the following propositions:

(a) All dogs are mammals.

(b) Most physicists are male.

(c) Few teachers are rock climbers.

(d) No dogs are cats.

(e) Some Americans are doctors.

(f) Some adults are not logicians.

(g) Thirty percent of Canadians speak French.

(h) One chair is missing.

These are all examples of quantified categorical statements. A QUANTIFIED CATEGORICAL STATEMENT is a statement that makes a claim about a certain quantity of the members of a class or group. (Sometimes we will just call these "categorical statements") Statement (a), for example, is about the class of dogs and the class of mammals. These statements make no mention of any particular members of the categories or classes or types they are about. The propositions are also *quantified* in that they state *how many* of the things in one class are also members of the other. For instance, statement (b) talks about *most* physicists, while statement (c) talks about *few* teachers.

Categorical statements can be broken down into four parts: the quantifier, the subject term, the predicate term, and the copula. The QUANTIFIER is the part of a categorical sentence that specifies a portion of a class. It is the "how many" term. The quantifiers in the sentences above are all, most, few, no, some, thirty percent, and one. Notice that the "no" in sentence (d) counts as a quantifier, the same way zero counts as a number. The subject and predicate terms are the two classes the statement talks about. The SUBJECT CLASS is the first class mentioned in a quantified categorical statement, and the PREDICATE CLASS is the second. In sentence (e), for instance, the subject class is the class of Americans and the predicate class is the class of doctors. The COPULA is simply the form of the verb "to be" that links subject and predicate. Notice that the quantifier is always referring to the subject. The statement "Thirty percent of Canadians speak French" is saying something about a portion of Canadians, not about a portion of French speakers.

Sentence (g) is a little different than the others. In sentence (g) the subject is the class of Canadians and the predicate is the class of people who speak French. That's not quite the way it is written, however. There is no explicit copula, and instead of giving a noun phrase for the predicate term, like "people who speak French," it has a verb phrase, "speak French." If you are asked to identify the copula and predicate for a sentence like this, you should say that the copula is implicit and transform the verb phrase into a noun phrase. You would do something similar for sentence (h): the subject term is "chair," and the predicate term is "things that are missing." We will go into more detail about these issues in Section 1.3.

In the previous chapter we noted that formal logic achieves content neutrality by replacing some or all of the ordinary words in a statement with symbols. For categorical logic, we are only going to be making one such substitution. Sometimes we will replace the classes referred to in a quantified categorical statement with capital letters that act as variables. Typically we will use the letter S when referring to the class in the subject term and P when referring to the predicate term, although sometimes more letters will be needed. Thus the sentence "Some Americans are doctors," above, will sometimes become "Some S are P." The sentence "No dogs are cats" will sometimes become"No S is P."

1.2 Quantity, Quality, Distribution, and Venn Diagrams

Ordinary English contains all kinds of quantifiers, including the counting numbers themselves. In this chapter and the next, however, we are only going to deal with two quantifiers: "all," and "some." We are restricting ourselves to the quantifiers "all" and "some" because they are the ones that can easily be combined to create valid arguments using the system of logic that was invented

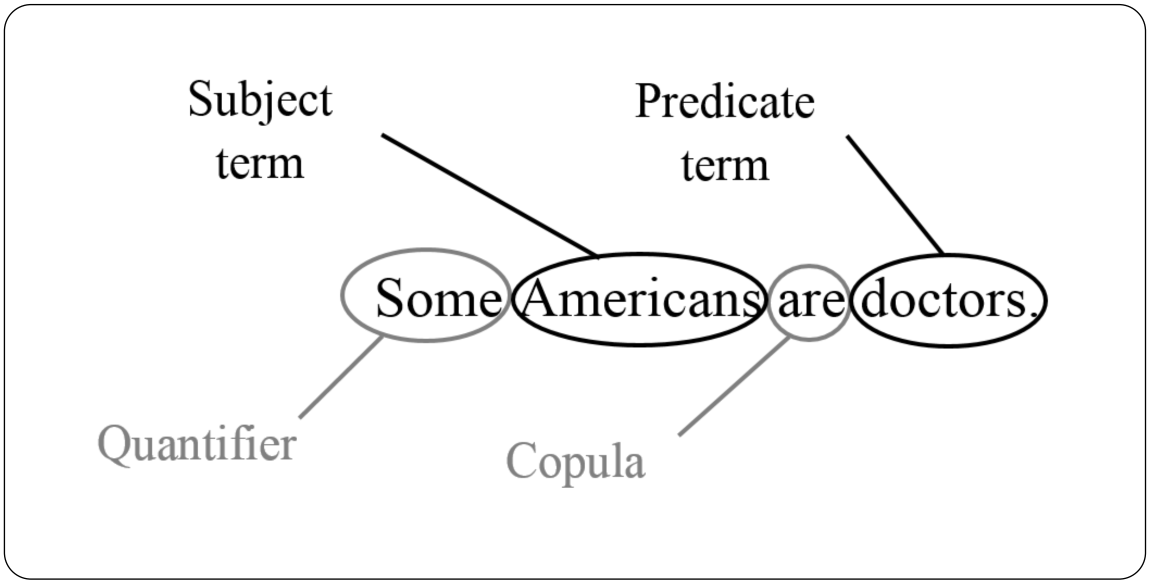


Figure 1.1: Parts of a quantified categorical statement.

by Aristotle.

The quantifier used in a statement is said to give the QUANTITY of the statement. Statements with the quantifier "All" are said to be "UNIVERSAL" and those with the quantifier "some" are said to be "PARTICULAR."

Here "some" will just mean "at least one." So, "some people in the room are standing" will be true even if there is only one person standing. Also, because "some" means "at least one," it is compatible with "all" statements. If I say "some people in the room are standing" it might actually be that *all* people in the room are standing, because if all people are standing, then at least one person is standing. This can sound a little weird, because in ordinary circumstances, you wouldn't bother to point out that something applies to some members of a class when, in fact, it applies to all of them. It sounds odd to say "*some* dogs are mammals," when in fact they *all* are. Nevertheless, when "some" means "at least one" it is perfectly true that some dogs are mammals.

In addition to talking about the quantity of statements, we will talk about their QUALITY. The quality of a statement refers to whether the statement is negated. Statements that include the words "no" or "not" are NEGATIVE, and other statements are AFFIRMATIVE. Combining quantity and quality gives us four basic types of quantified categorical statements, which we call the STATEMENT MOODS or just "moods." The four moods are labeled with the letters A, E, I, and O. Statements that are universal and affirmative are MOOD-A STATEMENTS. Statements that are universal and negative are MOOD-E STATEMENTS. Particular and affirmative statements are MOOD-I STATEMENTS, and particular and negative statements are MOOD-O STATEMENTS. (See Table 1.1.)

Mood	Form	Example
A	All S are P	All dogs are mammals.
E	No S are P	No dogs are reptiles.
I	Some S are P	Some birds can fly.
O	Some S are not P	Some birds cannot fly.

Table 1.1: The four moods of a categorical statement

Aristotle didn't actually use those letters to name the kinds of categorical propositions. His later followers writing in Latin came up with the idea. They remembered the labels because the "A" and the "I" were in the Latin word "**a**ff**i**rmo," ("I affirm") and the "E" and the "O" were in the Latin word "n**e**g**o**" ("I deny").

The DISTRIBUTION of a categorical statement refers to how the statement describes its subject and predicate class. A term in a sentence is said to be distributed if a claim is being made about the whole class. In the sentence "All dogs are mammals," the subject class, dogs, is distributed, because the quantifier "All" refers to the subject. The sentence is asserting that every dog out there is a mammal. On the other hand, the predicate class, mammals, is not distributed, because the sentence isn't making a claim about all the mammals. We can infer that at least some of them are dogs, but we can't infer that all of them are dogs. So in mood-A statements, only the subject is distributed.

On the other hand, in an I sentence like "Some birds can fly" the subject is not distributed. The quantifier "some" refers to the subject, and indicates that we are not saying something about all of that subject. We also aren't saying anything about all flying things, either. So in mood-I statements, neither subject nor predicate is distributed.

Even though the quantifier always refers to the subject, the predicate class can be distributed as well. This happens when the statement is negative. The sentence "No dogs are reptiles" is making a claim about all dogs: they are all not reptiles. It is also making a claim about all reptiles: they are all not dogs. So mood-E statements distribute both subject and predicate. Finally, negative particular statements (mood-O) have only the predicate class distributed. The statement "some birds cannot fly" does not say anything about all birds. It does, however say something about all flying things: the class of all flying things excludes some birds.

The quantity, quality, and distribution of the four forms of a categorical statement are given in Table 1.2. The general rule to remember here is that universal statements distribute the subject, and negative statements distribute the predicate.

In 1880 English logician John Venn published two essays on the use of diagrams with circles to represent categorical propositions (Venn 1880a, 1880b). Venn noted that the best use of such diagrams so far had come from the brilliant Swiss mathematician Leonhard Euler, but they still had many problems, which Venn felt could be solved by bringing in some ideas about logic from his fellow English logician George Boole. Although Venn only claimed to be building on the long

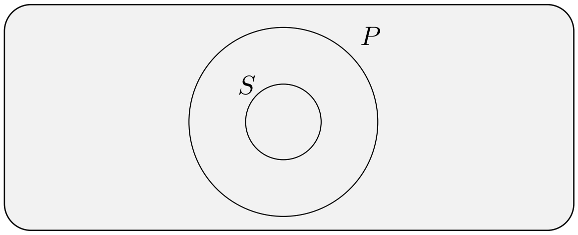


Figure 1.2: Euler Circles

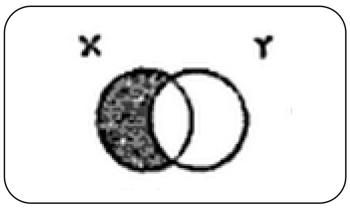


Figure 1.3: Venn's original diagram for an mood-A statement (Venn 1880a).

logical tradition he traced, since his time these kinds of circle diagrams have been known as Venn diagrams.

In this section we are going to learn to use Venn diagrams to represent our four basic types of categorical statement. Later in this chapter, we will find them useful in evaluating arguments. Let us start with a statement in mood A: "All S are P." We are going to use one circle to represent S and another to represent P. There are a couple of different ways we could draw the circles if we wanted to represent "All S are P." One option would be to draw the circle for S entirely inside the circle for P, as in Figure 1.2

It is clear from Figure 1.2 that all S are in fact P. And outside of college logic classes, you may have seen people use a diagram like this to represent a situation where one group is a subclass of another. You may have even seen people call concentric circles like this a Venn diagram. But Venn did not think we should put one circle entirely inside the other if we just want to represent

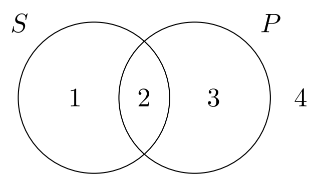
Mood	Form	Quantity	Quality	Terms Distributed
A	All S are P	Universal	Affirmative	S
E	No S are P	Universal	Negative	S and P
I	Some S are P	Particular	Affirmative	None
O	Some S are not P	Particular	Negative	P

Table 1.2: Quantity, quality, and distribution.

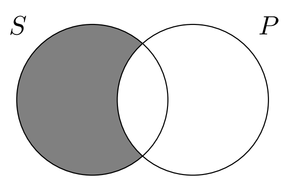
"All S is P." Technically speaking Figure 1.2 shows Euler circles.

Venn pointed out that the circles in Figure 1.2 don't just say that "All S are P." They also says that "All P are S" is false. But we don't necessarily know that if we have only asserted "All S are P." The statement "All S are P" leaves it open whether the S circle should be smaller than or the same size as the P circle.

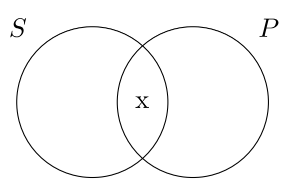
Venn suggested that to represent just the content of a single proposition, we should always begin by drawing partially overlapping circles. This means that we always have spaces available to represent the four possible ways the terms can combine:



Area 1 represents things that are S but not P; area 2, things that are S and P; area 3, things that are just P; and area 4 represents things that are neither S nor P. We can then mark up these areas to indicate whether something is there or could be there. We shade a region of the diagram to represent the claim that nothing can exist in that region. For instance, if we say "All S are P," we are asserting that nothing can exist that is in the S circle unless it is also in the P circle. So we shade out the part of the S circle that doesn't overlap with P.



If we want to say that something does exist in a region, we put an "x" in it. This is the diagram for "Some S are P":



If a region of a Venn diagram is blank, if it is neither shaded nor has an x in it, it could go either way. Maybe such things exist, maybe they do not.

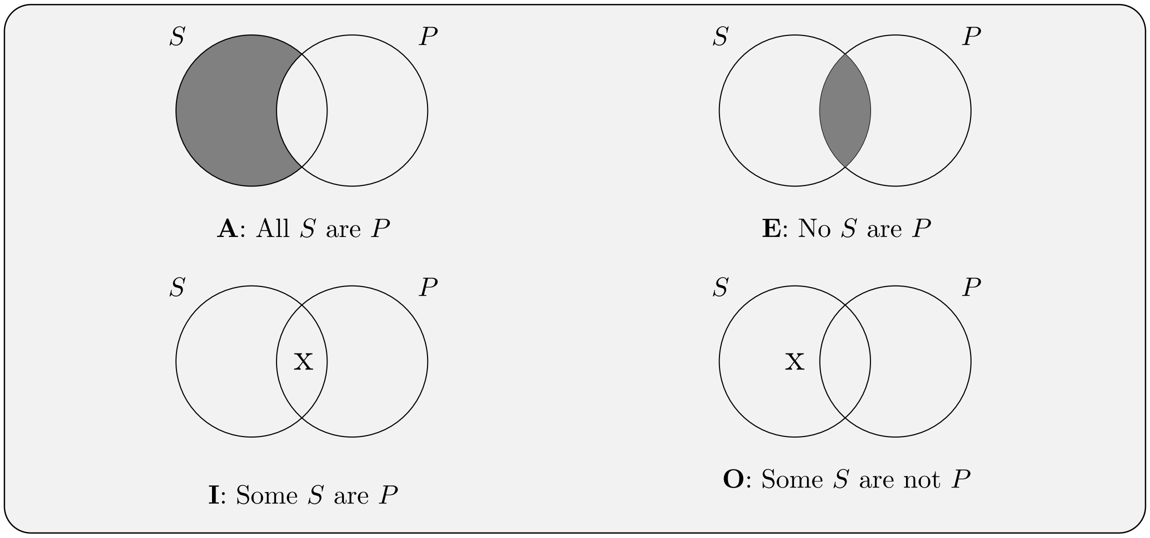
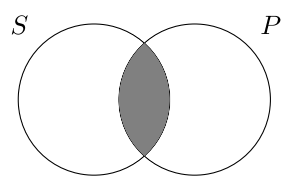


Figure 1.4: Venn Diagrams for the Four Basic Forms of a Categorical Statement

The Venn diagrams for all four basic forms of categorical statements are in Figure 1.4. Notice that when we draw diagrams for the two universal forms, A and E, we do not draw any x's. For these forms we are only ruling out possibilities, not asserting that things actually exist. This is part of what Venn learned from Boole, and we will see its importance in Section **??**.

Finally, notice that so far, we have only been talking about categorical statements involving the variables S and P. Sometimes, though, we will want to represent statements in regular English. To do this, we will include a dictionary saying what the variables S and P represent in this case. For instance, this is the diagram for "No dogs are reptiles."



S: Dogs
P: Reptiles

1.3 Transforming English into Logically Structured English

Because the four basic forms are stated using variables, they have a great deal of generality. We can expand on that generality by showing how many different kinds of English sentences can be transformed into sentences in our four basic forms. We already touched on this a little in section 1.1, when we look at sentences like "Thirty percent of Canadians speak French." There we saw that the predicate was not explicitly a class. We needed to change "speak French" to "people who speak French." In this section, we are going to expand on that to show how ordinary English sentences can be transformed into something we will call "logically structured English." LOGICALLY STRUCTURED ENGLISH is English that has been put into a standardized form that allows us to see its logical structure more clearly and removes ambiguity. Doing this is a step towards the creation of formal languages, which we will start doing soon.

Transforming English sentences into logically structured English is fundamentally a matter of understanding the meaning of the English sentence and then finding the logically structured English statements with the same or similar meaning. Sometimes this will require judgment calls. English, like any natural language, is fraught with ambiguity. One of our goals with logically structured English is to reduce the amount of ambiguity. Clarifying ambiguous sentences will always require making judgments that can be questioned. Things will only get harder when we start using full blown formal languages, which are supposed to be completely free of ambiguity.

To transform a quantified categorical statement into logically structured English, we have to put all of its elements in a fixed order and be sure they are all of the right type. All statements must begin with the quantifies "All" or "Some" or the negated quantifier "No." Next comes the subject term, which must be a plural noun, a noun phrase, or a variable that stands for any plural noun or noun phrase. Then comes the copula "are" or the negated copula "are not." Last is the predicate term, which must also be a plural noun or noun phrase. We also specify that you can only say "are not" with the quantifier "some," that way the universal negative statement is always phrased "No S are P," not "All S are not P." Taken together, these criteria define the STANDARD FORM FOR A CATEGORICAL STATEMENT in logically structured English.

The subsections below identify different kinds of changes you might need to make to put a statement into logically structured English. Sometimes translating a sentence will require using multiple changes.

Nonstandard Verbs

In section 1.1 we saw that "Some Canadians speak French" has a verb phrase "speaks French" instead of a copula and a plural noun phrase. To transform these sentences into logically structured English, you need to add the copula and turn all the terms into plural nouns or plural noun phrases.

Below are some examples

English	Logically Structured English
No cats bark.	No cats are animals that bark.
All birds can fly.	All birds are animals that can fly.
Some thoughts should be left unsaid.	Some thoughts are things that should be left unsaid.

Adding a plural noun phrase means you have to come up with some category, like “people” or “animals.” When in doubt, you can always use the most general category, “things.”

Implicit Noun Phrases

Sometimes you just have an adjective for the predicate, and you need to turn it into a noun, as in the examples below.

English	Logically Structured English
Some roses are red.	Some roses are red flowers.
Football players are strong.	All football players are strong persons.
Some names are hurtful.	Some names are hurtful things.

Again, you will have to come up with a category for the predicate, and when it doubt, you can just use “things.”

Unexpressed Quantifiers

Sometimes categorical generalizations come without an explicit quantifier, which you need to add.

English	Logically Structured English
Boots are footwear.	All boots are footwear.
Giraffes are tall.	All giraffes are tall things.
A dog is not a cat.	No dogs are cats.
A lion is a fierce creature.	All lions are fierce creatures.

Notice that in the second sentence we had to make two changes, adding both the words “All” and “things.”

In the last two sentences, the indefinite article “a” is being used to create a kind of generic sentence. Not all sentences using the indefinite article work this way. If a story begins “A man is walking down the street,” it is not talking about all men generically. It is introducing some specific man. For this kind of statement, see the subsection on singular propositions. You will have to use your good judgment and understanding of context to know how the indefinite article is being used.

Nonstandard Quantifiers

English has many alternate ways of saying "all" and "some." You need to change these when translating to logically structured English.

English	Logically Structured English
Every day is a blessing.	All days are blessings.
Whatever is a dog is not a cat.	No dogs are cats.
Not a single dog is a cat.	No dogs are cats.
There are Americans that are doctors.	Some Americans are doctors.
Someone in America is a doctor.	Some Americans are doctors.
At least a few Americans are doctors.	Some Americans are doctors.
Not everyone who is an adult is a logician.	Some adults are not logicians.
Most people with a PhD in psychology are female.	Some people with a PhD in psychology are female.
Among the things that Sylvia inherited was a large mirror	Some things that Sylvia inherited were large mirrors

Notice in the last case we are losing quite a bit of information when we transform the sentence into logically structured English. "Most" means more that fifty percent, while "some" could be any percentage less than a hundred. This is simply a price we have to pay in creating a standard logical form. As we will see when we move to constructing artificial languages in later chapters, no logical language has the expressive richness of a natural language.

Singular Propositions

Aristotle treated sentences about individual things, like specific people, differently than either general or particular categorical statements. A statement like "Socrates is mortal," for Aristotle, was neither A, E, I, nor O. We can expand the power of logically structured English by bringing these kind of singular propositions into our system of categorical propositions. Using phrases like "All things identical to..." we can turn singular terms into general ones.

English	Logically Structured English
Socrates is mortal.	All persons identical with Socrates are mortal.
The Empire State Building is tall.	All things identical to The Empire State Building are tall things.
Ludwig was not happy.	No persons identical with Ludwig are happy.
A man is walking down the street.	Some men are things that are walking down the street.

Adverbs and Pronouns

In English we use specific adverbs like "everywhere" and "always" to create quantified statements about place and time. We can transform these into logically structured English by talking about "all places" or "all times" and things like that. English also has specific pronouns for quantified statements about people or things, such as "everyone" or "nothing."

English	Logically Structured English
"Whenever you need me, I'll be there." – Michael Jackson	All times that you need me are times that I will be there.
"We are never, ever, ever getting back together." – Taylor Swift	No times are times when we will get back together.
"Whoever fights with monsters should be careful lest he thereby become a monster." –Friedrich Nietzsche	All persons who fight with monsters are persons who should be careful lest they become a monster.
"What does not destroy me, makes me stronger." –Friedrich Nietzsche	All things that do not destroy me are things that make me stronger.

Conditional Statements

A conditional is a statement of the form "If . . . then . . . ." They will become a big focus of our attention starting in Chapter *Sentential Logic* when we begin introducing modern formal languages. They are not given special treatment in the Aristotelian tradition, however. Instead, where we can, we just treat them as categorical generalizations:

English	Logically Structured English
If something is a cat, then it is a feline.	All cats are feline.
If something is a dog, then it's not a cat.	No dogs are cats.

Exclusive Propositions

Phrases like "only," "none but," or "none except" are used in English to create exclusive propositions. They are applied to the predicate term and exclude everything but the predicate term from the subject term.

English	Logically Structured English
Only people over 21 may drink.	All people who drink are over 21.

English	Logically Structured English
No one, except those with a ticket, may enter the theater.	All people who enter the theater have a ticket.
None but the strong survive.	All people who survive are strong people.

The important thing to see here is that words like "only" are actually modifying the predicate, and not the subject. So when you translate them into logically structured English, the order of the words often gets turned around. In "only people over 21 may drink" the predicate is actually "people over 21."

"The Only"

Sentences with "The only" are a little different than sentences that just have "only" in them. The sentence "Humans are the only animals that talk on cell phones" should be translated as "All animals who talk on cell phones are humans." In this sentence, "the only" introduces the subject, rather than the predicate.

English	Logically Structured English
Humans are the only animals who talk on cell phones.	All animals who talk on cell phones are human.
Shrews are the only venomous mammal in North America.	All venomous mammals in North America are shrews.

1.4 Conversion, Obversion, and Contraposition

Now that we have shown the wide range of statements that can be represented in our four standard logical forms A, E, I, and O, it is time to begin constructing arguments with them. The arguments we are going to look at are sometimes called "immediate inferences" because they only have one premise. We are going to learn to identify some valid forms of these one-premise arguments by looking at ways you can transform a true sentence that maintain its truth value. For instance, "No dogs are reptiles" and "No reptiles are dogs" have the same truth value and basically mean the same thing. On the other hand if you change "All dogs are mammals" into "All mammals are dogs" you turn a true sentence into a false one. In this section we are going to look at three ways of transforming categorical statements—conversion, obversion, and contraposition—and use Venn diagrams to determine whether these transformations also lead to a change in truth value. From there we can identify valid argument forms.

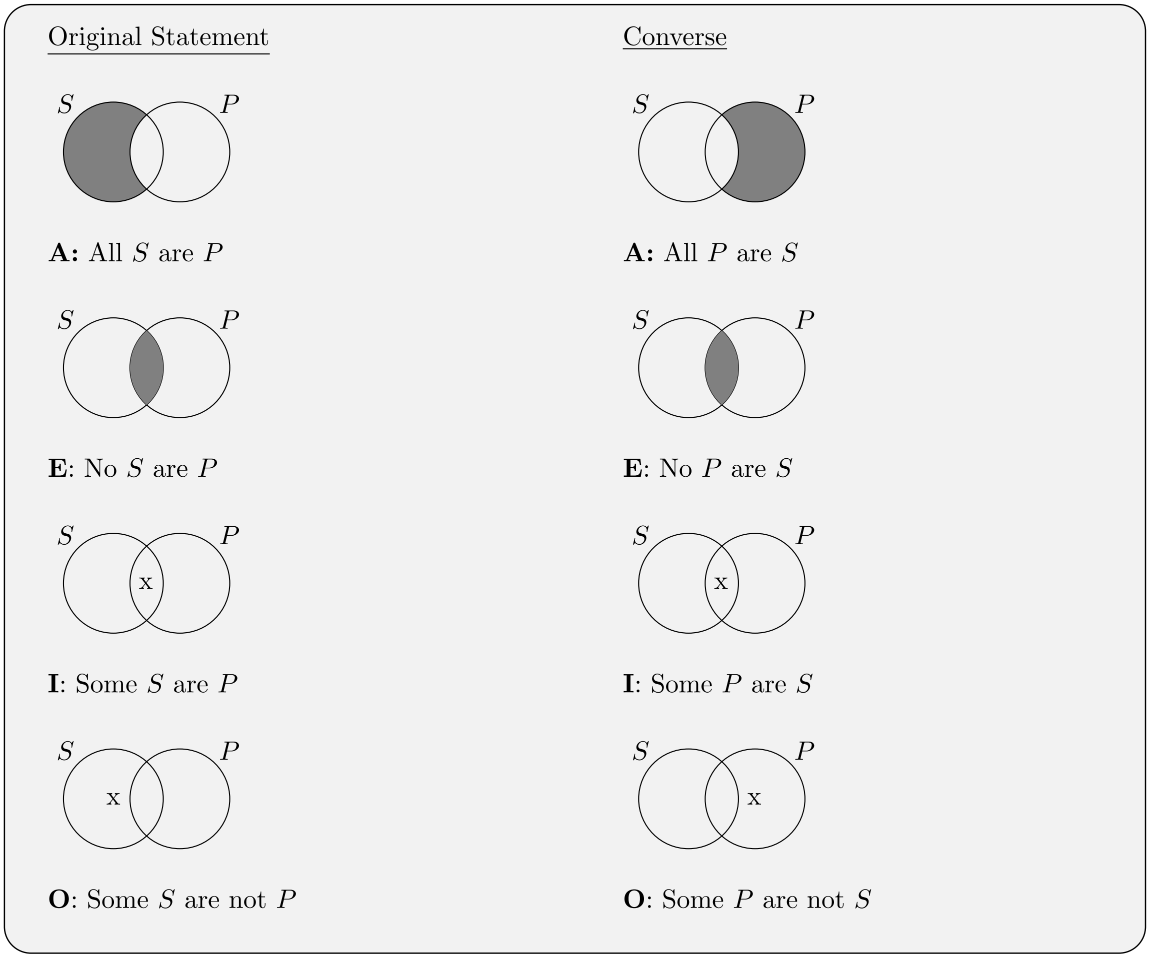


Figure 1.5: Conversions of the Four Basic Forms

Conversion

The two examples in the last paragraph are examples of conversion. CONVERSION is the process of transforming a categorical statement by switching the subject and the predicate. When you convert a statement, it keeps its form—an A statement remains an A statement, an E statement remains an E statement—however it might change its truth value. The Venn diagrams in Figure 1.5 illustrate this.

As you can see, the Venn diagram for the converse of an E statement is exactly the same as the original E statement, and likewise for I statements. This means that the two statements are logically equivalent. Recall that two statements are logically equivalent if they always have the same truth value. (See page **??**). In this case, that means that if an E statement is true, then its converse is also true, and if an E statement is false, then its converse is also false. For instance,

P. No S are P.	P. Some S are P.
C. No P are S.	C. Some P are S.

Figure 1.6: Valid Arguments by Conversion

"No dogs are reptiles" is true, and so is "No reptiles are dogs." On the other hand "No dogs are mammals" is false, and so is "No mammals are dogs."

Likewise, if an I statement is true, its converse is true, and if an I statement is false, than its converse is false. "Some dogs are pets" is true, and so is "Some pets are dogs." On the other hand "Some dogs can fly" is false and so is "Some flying things are dogs."

The converses of A and O statements are not so illuminating. As you can see from the Venn diagrams, these statements are not identical to their converses. They also don't contradict their converses. If we know that an A or O statement is true, we still don't know anything about their converses. We say their truth value is undetermined.

Because E and I statements are logically equivalent to their converses, we can use them to construct valid arguments. Recall from *The Basics of Evaluating Argument* (page **??**) that an argument is valid if it is impossible for its conclusion to be false whenever its premises are true. Because E and I are logically equivalent to their converses, the two argument forms in Figure 1.6 are valid.

Notice that these are argument forms, with variables in the place of the key terms. This means that these arguments will be valid no matter what; S and P could be people, or squirrels, or the Gross Domestic Product of industrialized nations, or anything, and the arguments are still valid. While these particular argument forms may seem trivial and obvious, we are beginning to see some of the power of formal logic here. We have uncovered a very general truth about the nature of validity with these two argument forms.

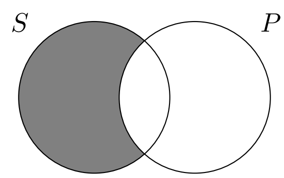
The truth value of the converses of A and O statements, on the other hand, are undetermined by the truth value of the original statements. This means we cannot construct valid arguments from them. Imagine you have an argument with an A or O statement as its premise and the converse of that statement as the conclusion. Even if the premise is true, we know nothing about the truth of the conclusion. So there are no valid argument forms to be found here.

Obversion

Obversion is a more complex process. To understand what an obverse is, we first need to define the complement of a class. The COMPLEMENT of a class is everything that is not in the class. So the complement of the class of dogs is everything that is not a dog, including not just cats, but battleships, pop songs, and black holes. In English we can easily create a name for the complement of any class using the prefix "non-". So the complement of the class of dogs is the class of non-dogs. We will use complements in defining both obversion and contraposition.

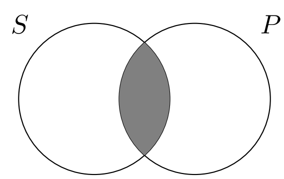
The OBVERSION of a categorical proposition is a new proposition created by changing the quality of the original proposition and switching its predicate to its complement. Obversion is thus a two step process. Take, again, the proposition "All dogs are mammals." For step 1, we change its quality, in this case going from affirmative to negative. That gives us "No dogs are mammals." For step 2, we take the complement of the predicate. The predicate in this case is "mammals" so the complement is "non-mammals." That gives us the obverse "No dogs are non-mammals."

We can map this process out using Venn diagrams. Let's start with an A statement.



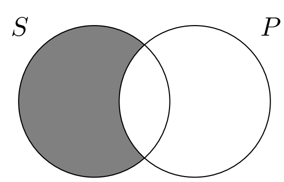
A: All S are P.

Changing the quality turns it into an E statement.



E: No S are P.

Now what happens when we take the complement of P? That means we will shade in all the parts of S that are non-P, which puts us back where we started.



E: No S are non-P.

The final statement is logically equivalent to the original A statement. It has the same form as an E statement, but because we have changed the predicate, it is not logically equivalent to an A statement. As you can see from Figure 1.7 this is true for all four forms of categorical statement. This in turn gives us four valid argument forms, which are shown in Figure 1.8

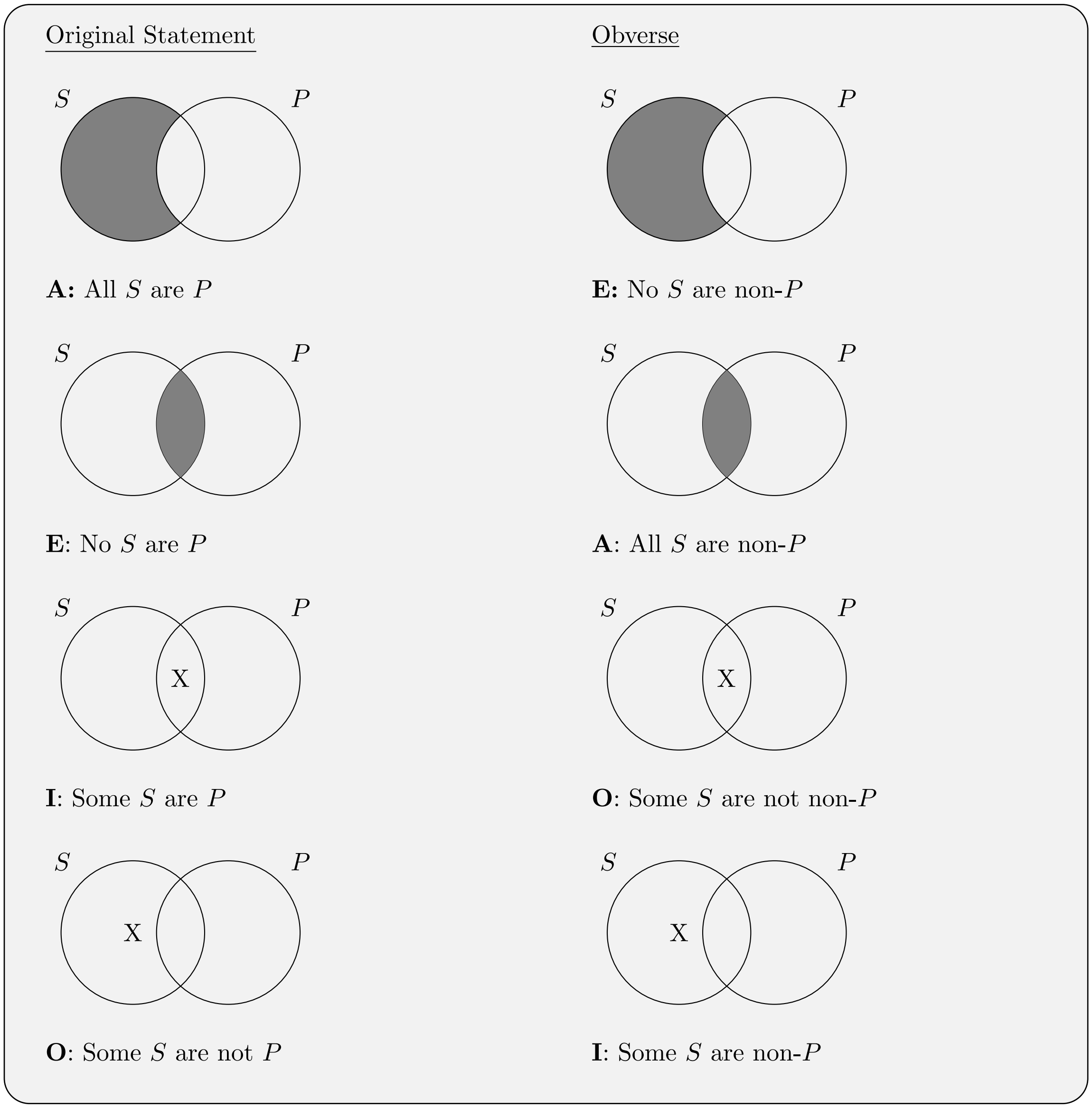
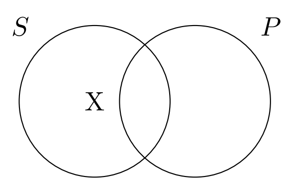


Figure 1.7: Obversions of the Four Basic Forms

P_1: All S are P.	P_1: No S are P.
C: No S are non-P.	C: All S are non-P.
P_1: Some S are P.	P_1: Some S are not P.
C: Some S are not non-P.	C: Some S are non-P.

Figure 1.8: Valid argument forms by obversion

One further note on complements. We don't just use complements to describe sentences that come out of obversion and contraposition. We can also perform these operations on statements that already have complements in them. Consider the sentence "Some S are non-P." This is its Venn diagram.



Some S are non-P

How would we take the obverse of this statement? Step 1 is to change the quality, making it "Some S are not non-P." Now how do we take the complement of the predicate? We could write "non-non-P," but if we think about it for a second, we'd realize that this is the same thing as P. So we can just write "Some S is not P." This is logically equivalent to the original statement, which is what we wanted.

Taking the converse of "Some S are non-P" also takes a moment of thought. We are supposed to reverse subject and predicate. But does that mean that the "non-" moves to the subject position along with the "P"? Or does the "non-" now attach to the S? We saw that E and I statements kept their truth value after conversion, and we want this to still be true when the statements start out referring to the complement of some class. This means that the "non-" has to travel with the predicate, because "Some S are non-P" will always have the same truth value as "Some non-P are S." Another way of thinking about this is that the "non-" is part of the name of the class that forms the predicate of "Some S are non-P." The statement is making a claim about a class, and that class happens to be defined as the complement of another class. So, the bottom line is when you take the converse of a statement where one of the terms is a complement, move the "non-" with that term.

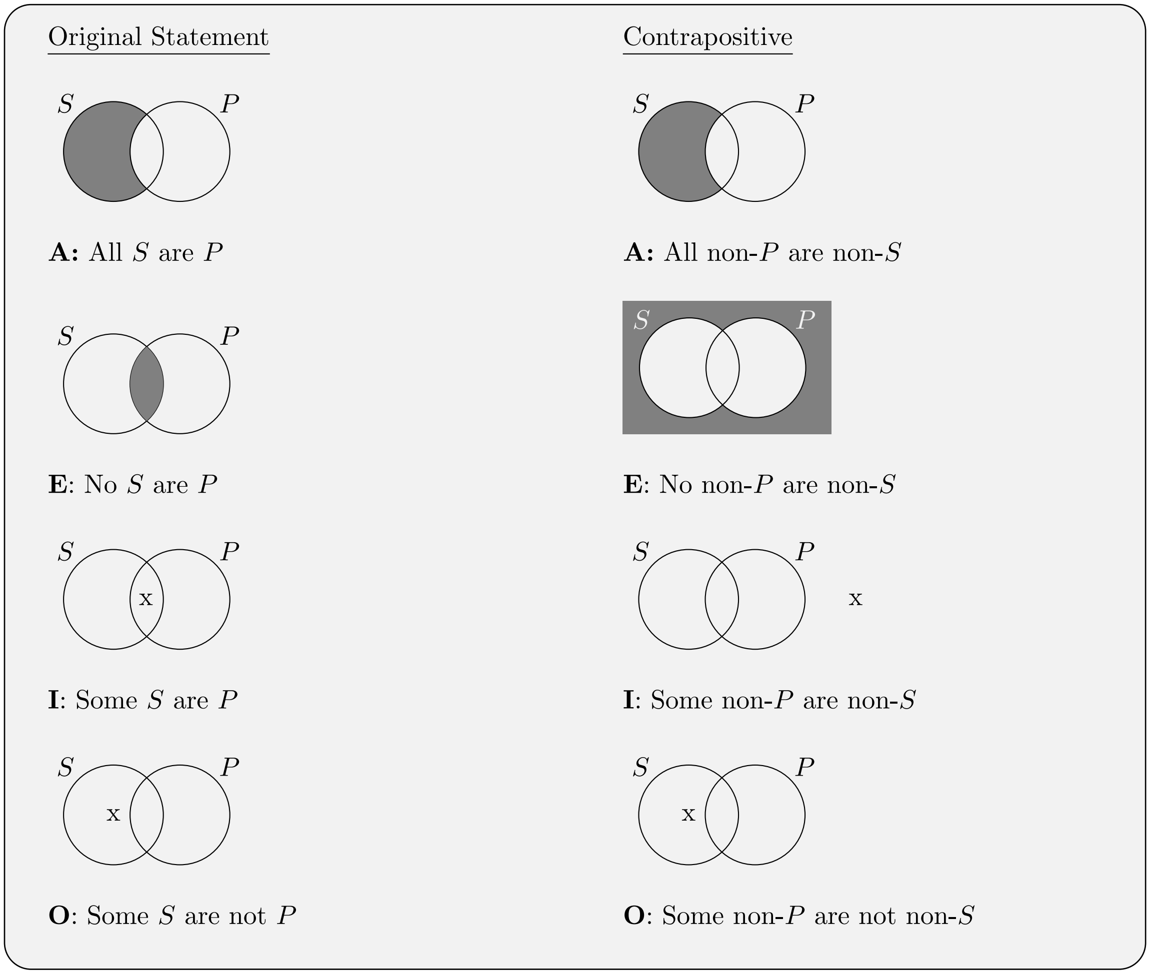


Figure 1.9: Contrapositions the Four Basic Forms

Contraposition

CONTRAPOSITION is a two-step process, like obversion, but it doesn't always lead to results that are logically equivalent to the original sentence. The contrapositive of a categorical sentence is the sentence that results from reversing subject and predicate and then replacing them with their complements. Thus "All S are P" becomes "All non-P are non-S."

Figure 1.9 shows the corresponding Venn diagrams. In this case, the shading around the outside of the two circles in the contraposed form of E is meant to indicate that nothing can lie outside the two circles. Everything must be S or P or both. Like conversion, applying contraposition to two of the forms gives us statements that are logically equivalent to the original. This time, though, it is forms A and O that come through the process without changing their truth value.

P_1: All S are P.	P_1: Some S are not P
C: All non-P are non-S.	C: Some non-P are not non-S.

Figure 1.10: Valid argument forms from contraposition

This then gives us two valid argument forms, shown in Figure 1.10. If you have an argument with an A or O statement as its premise and the contraposition of that statement as the conclusion, you know it must be valid. Whenever the premise is true, the conclusion must be true, because the two statements are logically equivalent. On the other hand, if you had an E or an I statement as the premise, the truth of the conclusion is undetermined, so these arguments would not be valid.

Evaluating Short Arguments

So far we have seen eight valid forms of argument with one premise: two arguments that are valid by conversion, four that are valid by obversion, and two that are valid by contraposition. As we said, short arguments like these are sometimes called "immediate inferences," because your brain just flits automatically from the truth of the premises to the truth of the conclusion. Now that we have identified these valid forms of inference, we can use this knowledge to see whether some of the arguments we encounter in ordinary language are valid. We can now tell in a few cases if our brain is right to flit so seamlessly from the premise to the conclusion.

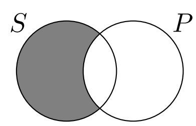
In the real world, the inferences we make are messy and hard to classify. Much of the complexity of this issue is tackled in the parts of the complete version of this text that cover critical thinking. Right now we are just going to deal with a limited subset of inferences: immediate inferences that might be based on conversion, obversion, or contraposition. Let's start start with the uncontroversial premise "All dogs are mammals." Can we infer from this that all non-mammals are non-dogs? In canonical form, the argument would look like this.

P_1: All dogs are mammals

C: All non-mammals are non-dogs.

Evaluating an immediate inference like this is a four step process. First, identify the subject and predicate classes. Second, draw the Venn diagram for the premise. Third, see if the Venn diagram shows that the conclusion must be true. If it must be, then the argument is valid. Finally, if the argument is valid, identify the process that makes it valid. (You can skip this step if the argument is invalid.)

For the argument above, the result of the first two steps would look like this:



S: Dogs
P: Mammals

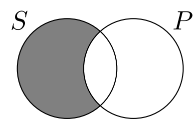
The Venn diagram for the premise shades out the possibility that there are dogs that aren't mammals. For step three, we ask, does this mean the conclusion must be true? In this case, it does. The same shading implies that everything that is not a mammal must also not be a dog. In fact, the Venn diagram for the premise and the Venn diagram for the conclusion are the same. So the argument is valid. This means that we must go on to step four and identify the process that makes it valid. In this case, the conclusion is created by reversing subject and predicate and taking their complements, which means that this is a valid argument by contraposition.

Now, remember what it means for an argument to be valid. As we said on page **??**, an argument is valid if it is impossible for the premises to be true and the conclusion false. This means that we can have a valid argument with false premises, so long as it is the case that *if* the premises were true, the conclusion would have to be true. So if the argument above is valid, then so is this one:

P_1: All dogs are reptiles.

C: All non-reptiles are non-dogs.

The premise is now false: all dogs are not reptiles. However, *if* all dogs were reptiles, then it would also have to be true that all non-reptiles are non-dogs. The Venn diagram works the same way.



S: Dogs
P: Reptiles

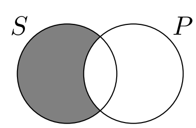
The Venn diagram for the premise still matches the Venn diagram for the conclusion. Only the labels have changed. The fact that this argument form remains true even with a false premise is just a variation on a theme we saw in Figure **??** when we saw a valid argument (with false premises) for the conclusion "Socrates is a carrot." So arguments by transposition, just like any argument, can be valid even if they have false premises. The same is true for arguments by conversion and obversion.

Arguments like these can also be invalid, even if they have true premises and a true conclusion. Remember that A statements are not logically equivalent to their converse. So this is an invalid argument with a true premise and a false conclusion:

P_1: All dogs are mammals.

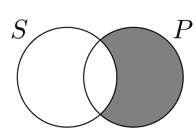
C: All mammals are dogs.

Our Venn diagram test shows that this is invalid. Steps one and two give us this for the premise:



S: Dogs
P: Mammals

But this is the Venn diagram for the conclusion:



S: Dogs
P: Mammals

This is an argument by conversion on an mood-A statement, which is invalid. The argument remains invalid, even if we substitute in a predicate where the conclusion happens to be true. For instance this argument is invalid.

P_1: All dogs are *Canis familiaris*.

C: All *Canis familiaris* are dogs.

The Venn diagrams for the premise and conclusion of this argument will be just like the ones for the previous argument, just with different labels. So even though the argument has a true premise and a true conclusion, it is still invalid, because it is possible for an argument of this form to have a true premise and a false conclusion. This is an unreliable argument form that just happened, in this instance, not to lead to a false conclusion. This again is just a variation on a theme we saw in *The Basics of Evaluating Argument*, when we saw an invalid argument for the conclusion that Paris was in France.

Key Terms

Affirmative

Complement

Contradictories

Contraposition

Contraries

Converse

Copula

Distribution

Existential import

Logically structured English

Mood-A statement

Mood-E statement

Mood-I statement

Mood-O statement

Negative

Obverse

Particular

Predicate class

Quality

Quantified categorical statement

Quantifier

Quantity

Square of opposition

Standard form categorical statement

Subalternation

Subcontraries

Subject class

Truth value

Universal

Vacuous truth

Venn diagram

Chapter 2

Categorical Syllogisms

2.1 Standard Form, Mood, and Figure

So far we have just been looking at very short arguments using categorical statements. The arguments just had one premise and a conclusion that was often logically equivalent to the premise. For most of the history of logic in the West, however, the focus has been on arguments that are a step more complicated called CATEGORICAL SYLLOGISMS. A categorical syllogism is a two-premise argument composed of categorical statements. Aristotle began the study of this kind of argument in his book the *Prior Analytics* (c.350 BCE/1984b). This work was refined over the centuries by many thinkers in the Pagan, Christian, Jewish, and Islamic traditions until it reached the form it is in today.

There are actually all kinds of two-premise arguments using categorical statements, but Aristotle only looked at arguments where each statement is in one of the moods A, E, I, or O. The arguments also had to have exactly three terms, arranged so that any two pairs of statements will share one term. Let's call a categorical syllogism that fits this more narrow description an ARISTOTELIAN SYLLOGISM Here is a typical Aristotelian syllogism using only mood-A sentences:

P_1: All mammals are vertebrates.
P_2: All dogs are mammals.

C: All dogs are vertebrates.

Notice how the statements in this argument overlap each other. Each statement shares a term with the other two. Premise 2 shares its subject term with the conclusion and its predicate with Premise 1. Thus there are only three terms spread across the three statements. Aristotle dubbed these the major, middle, and minor premises, but there was initially some confusion about how to define them. In the 6th century, the Christian philosopher John Philoponus, drawing on the work of his pagan teacher Ammonius, decided to arbitrarily designate the MAJOR TERM as the predicate of the conclusion, the MINOR TERM as the subject of the conclusion, and the MIDDLE TERM as the one term of the Aristotelian syllogism that does not appear in the conclusion. So in the argument above, the major term is "vertebrate," the middle term is "mammal," and the minor term is "dog." We can also define the MAJOR PREMISE as the one premise in an Aristotelian syllogism that names the major term, and the MINOR PREMISE as the one premise that names the minor term. So in the argument above, Premise 1 is the major premise and Premise 2 is the minor premise.

With these definitions in place, we can now define the STANDARD FORM FOR AN ARISTOTELIAN SYLLOGISM in logically structured English. Recall that in Section 1.2, we started standardizing our language into something we called "logically structured English" in order to remove ambiguity and to make its logical structure clear. The first step was to define the standard form for a categorical statement, which we did on page 9. Now we do the same thing for an Aristotelian syllogism. We say that an Aristotelian syllogism is in standard form for logically structured English if and only if these criteria have been met: (1) all of the individual statements are in standard form, (2) each instance of a term is in the same format and is used in the same sense, (3) the major premise appears first, followed by the minor premise, and then the conclusion.

Once we standardize things this way, we can actually catalog every possible form of an Aristotelian syllogism. To begin with, each of the three statements can take one of four forms: A, E, I, or O. This gives us $4 \times 4 \times 4$, or 64 possibilities. These 64 possibilities are called the SYLLOGISM MOOD, and we designate it just by writing the three letters of the moods of the statements that make it up. So the mood of the argument on page 24 is simply AAA.

In addition to varying the kind of statements we use in an Aristotelian syllogism, we can also vary the placement of the major, middle, and minor terms. There are four ways we can arrange them that fit the definition of an Aristotelian syllogism in standard form, shown in Table 2.1. Here S stands for the major term, P for the minor term, and M for the middle. The thing to pay attention to is the placement of the middle terms. In figure 1, the middle terms form a line slanting down to the right. In figure 2, the middle terms are both pushed over to the right. In figure 3, they are pushed to the left, and in figure 4, they slant in the opposite direction from figure 1.

The combination of 64 moods and 4 figures gives us a total of 256 possible Aristotelian syllogisms. We can name them by simply giving their mood and figure. So this is OAO-3:

P_1: Some M are not P.
P_2: All M are S.

C: Some S are not P.

Syllogism OAO-3 is a valid argument. We will be able to prove this with Venn diagrams in the next section. For now just read it over and try to see intuitively why it is valid. Most of the 256 possible syllogisms, however, are not valid. In fact, most of them, like IIE-2, are quite obviously invalid:

$$\begin{array}{ll} P_1: & \mathbf{M} \quad P \\ P_2: & S \quad \mathbf{M} \\ \hline C: & S \quad P \end{array}$$

Figure 1

$$\begin{array}{ll} P_1: & P \quad \mathbf{M} \\ P_2: & S \quad \mathbf{M} \\ \hline C: & S \quad P \end{array}$$

Figure 2

$$\begin{array}{ll} P_1: & \mathbf{M} \quad P \\ P_2: & \mathbf{M} \quad S \\ \hline C: & S \quad P \end{array}$$

Figure 3

$$\begin{array}{ll} P_1: & P \quad \mathbf{M} \\ P_2: & \mathbf{M} \quad S \\ \hline C: & S \quad P \end{array}$$

Figure 4

Table 2.1: The four figures of the Aristotelian syllogism

$$\begin{array}{rl} P_1: & \text{Some } P \text{ are } M. \\ P_2: & \text{Some } S \text{ are } M. \\ \hline C: & \text{No } S \text{ are } P. \end{array}$$

Given an Aristotelian syllogism in ordinary English, we can transform it into standard form in logically structured English and identify its mood and figure. Consider the following:

> No geckos are cats. I know this because all geckos are lizards, but cats aren't lizards.

The first step is to identify the conclusion, using the basic skills you acquired back in *What is Logic?*. In this case, you can see that "because" is a premise indicator word, so the statement before it, "No geckos are cats," must be the conclusion.

Step two is to identify the major, middle, and minor terms. Remember that the major term is the predicate of the conclusion, and the minor term is the subject. So here the major term is "cats," the minor term is "geckos." The leftover term, "lizards," must then be the middle term.

We show that we have identified the major, middle, and minor terms by writing a TRANSLATION KEY. A translation key is just a list that assigns English phrases or sentences to variable names. For categorical syllogism, this means matching the English phrases for the terms with the variables S, M, and P.

S**:** Geckos
M**:** Lizards
P**:** Cats

Step three is to write the argument in canonical form using variables for the terms. The last statement, "cats aren't lizards," is the major premise, because it has the major term in it. We need to change it to standard form, however, before we substitute in the variables. So first we change it to "No cats are lizards." Then we write "No S are M." For the minor premise and the conclusion we can just substitute in the variables, so we get this:

P_1: No S are M.
P_2: All P are M.

C: No S are P.

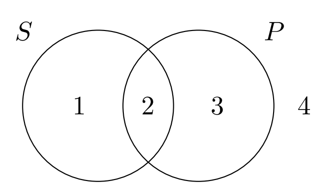
Step four is to identify mood and figure. We can see that this is figure 2, because the middle term is in the predicate of both premises. Looking at the form of the sentences tells us that this is EAE.

2.2 Testing Validity

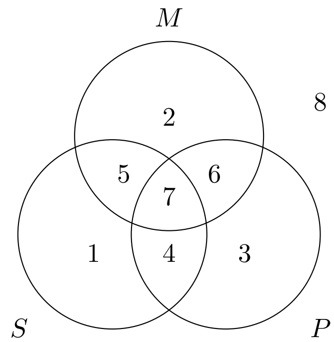
We have seen that there are 256 possible categorical arguments that fit Aristotle's requirements. Most of them are not valid, and as you probably saw in the exercises, many don't even make sense. In this section, we will learn to use Venn diagrams to sort the good arguments from the bad. The method we will use will simply be an extension of what we did in the last chapter, except with three circles instead of two.

Venn Diagrams for Single Propositions

In the previous chapter, we drew Venn diagrams with two circles for arguments that had had two terms. The circles partially overlapped, giving us four areas, each of which represented a way an individual could relate to the two classes. So area 1 represented things that were S but not P, etc.



Now that we are considering arguments with three terms, we will need to draw three circles, and they need to overlap in a way that will let us represent the eight possible ways an individual can be inside or outside these three classes.

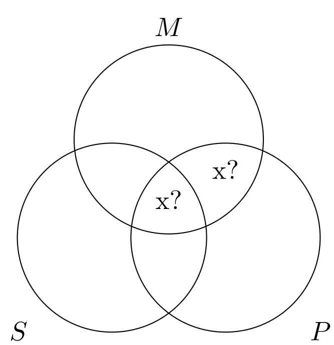


So in this diagram, area 1 represents the things that are S but not M or P, area 2 represents the things that are M but not S or P, etc.

As before, we represent universal statements by filling in the area that the statement says cannot be occupied. The only difference is that now there are more possibilities. So, for instance, there are now four mood-A propositions that can occur in the two premises. The major premise can either be "All P are M" or "All M are P," and the minor premise can be either "All S are M" or "All M are S." The Venn diagrams for those four sentences are given in the top two rows of Figure 2.1.

Similarly, there are four mood-E propositions that can occur in the premises of an Aristotelian syllogism: "No P are M," "No M are P," "No S are M," and "No M are S." And again, we diagram these by shading out overlap between the two relevant circles. In this case, however, the first two statements are equivalent by conversion (see page 1.4), as are the second two. Thus we only have two diagrams to worry about. See the bottom of Figure 2.1

Particular propositions are a bit trickier. Consider the statement "Some M are P." With a two circle diagram, you would just put an x in the overlap between the M circle and the P circle. But with the three circle diagram, there are now two places we can put it. It can go in either area 6 or area 7:



The solution here will be to put the x on the boundary between areas 6 and 7, to represent the

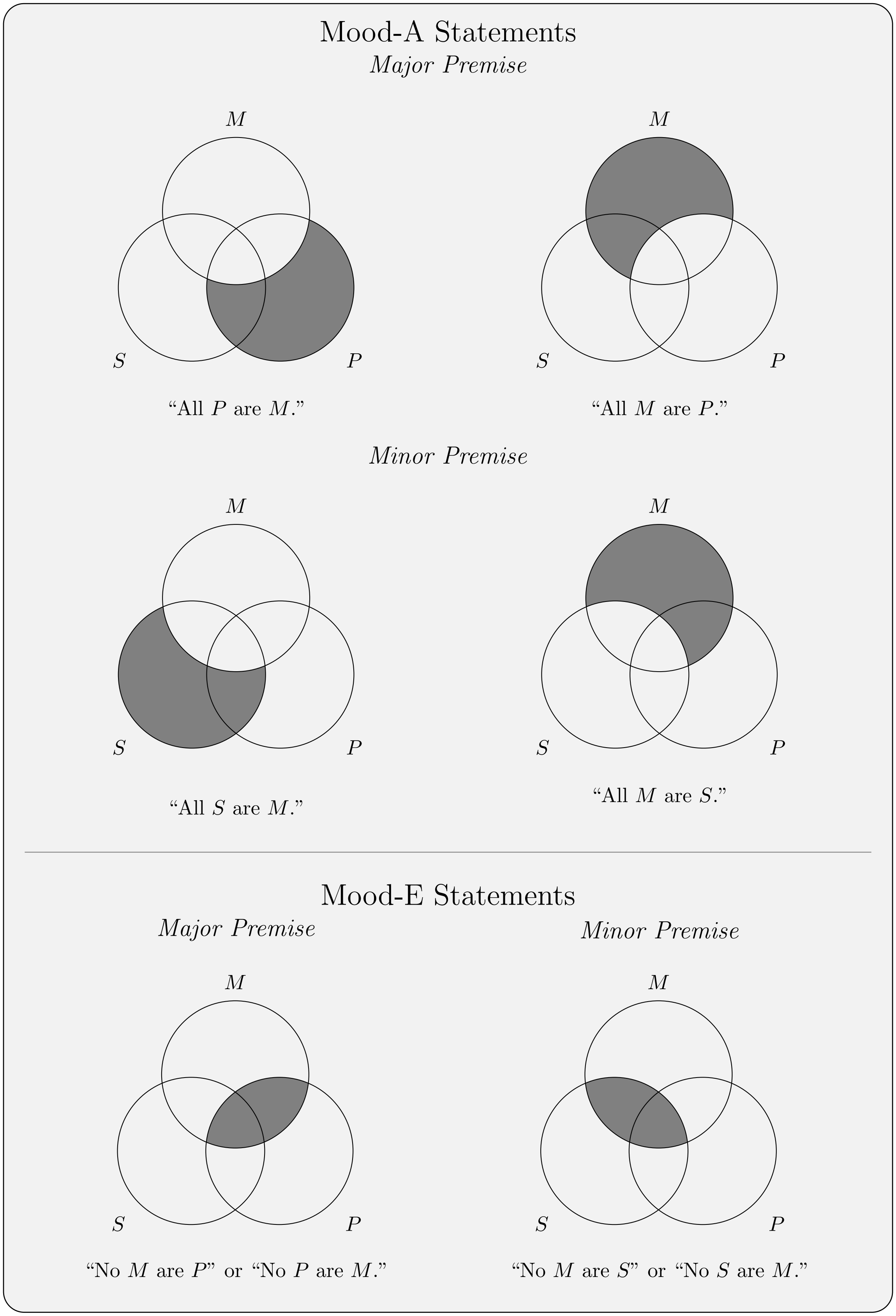
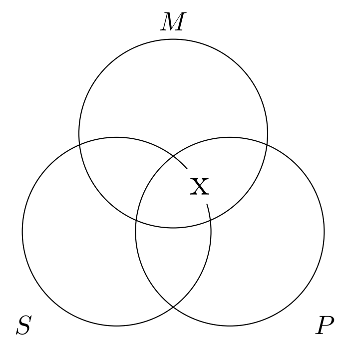
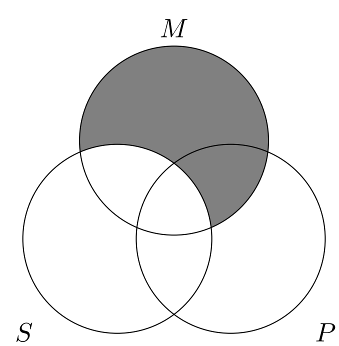


Figure 2.1: Venn diagrams for the eight universal statements that can occur in the premises.

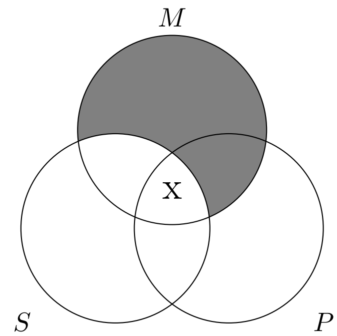
fact that it could go in either location.



Sometimes, however, you won't have to draw the x on a border between two areas, because you will already know that one of those areas can't be occupied. Suppose, for instance, that you want to diagram "Some M are P," but you already know that all M are S. You would diagram "All M are S" like this:



Then, when it comes time to add the x for "Some M are P," you know that it has to go in the exact center of the diagram:



The Venn diagrams for the particular premises that can appear in Aristotelian syllogisms are given in Figure 2.2. The figure assumes that you are just representing the individual premises, and don't know any other premises that would shade some regions out. Again, some of these premises are equivalent by conversion, and thus share a Venn diagram.

Venn Diagrams for Full Syllogisms

In the last chapter, we used Venn diagrams to evaluate arguments with single premises. It turned out that when those arguments were valid, the conclusion was logically equivalent to the premise, so they had the exact same Venn diagram. This time we have two premises to diagram, and the conclusion won't be logically equivalent to either of them. Nevertheless we will find that for valid arguments, once we have diagrammed the two premises, we will also have diagrammed the conclusion.

First we need to specify a rule about the order to diagram the premises in: if one of the premises is universal and the other is particular, diagram the universal one first. This will allow us to narrow down the area where we need to put the x from the particular premise, as in the example above where we diagrammed "Some M are P" assuming that we already knew that all M are S.

Let's start with a simple example, an argument with the form AAA-1.

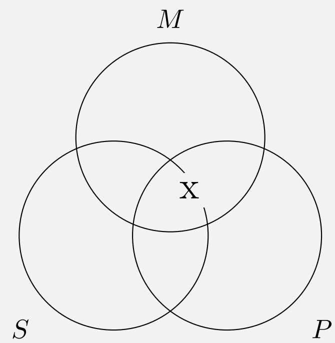
P_1: All M are P.
P_2: All S are M.

C: All S are P.

Since both premises are universal, it doesn't matter what order we do them in. Let's do the major premise first. The major premise has us shade out the parts of the M circle that don't overlap the P circle, like this:

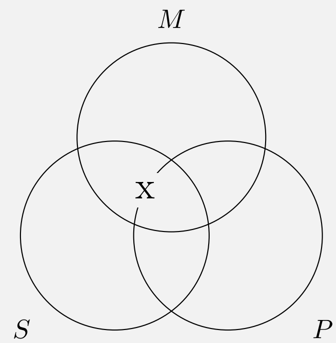
Mood-I Statements

Major Premise



"Some M are P" or "Some P are M."

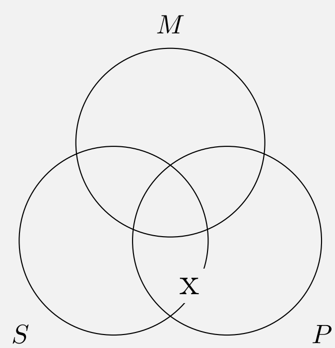
Minor Premise



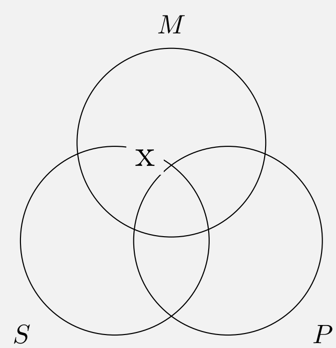
"Some M are S" or "Some S are M."

Mood-O Statements

Major Premise

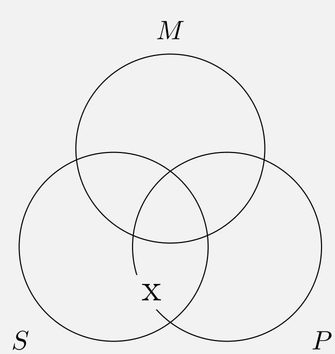


"Some P are not M."

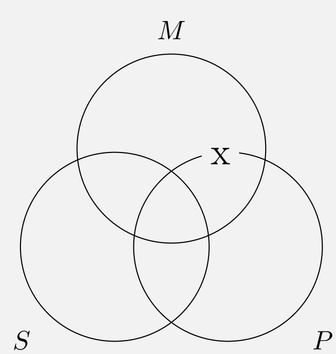


"Some M are not P."

Minor Premise

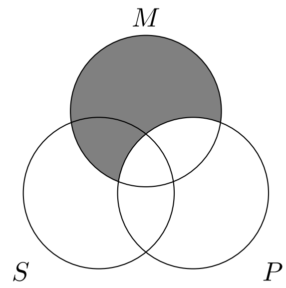


"Some S are not M."

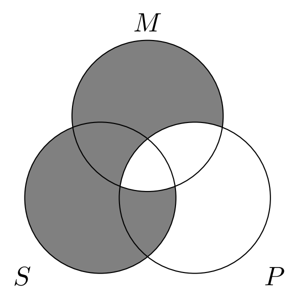


"Some M are not S."

Figure 2.2: Venn diagrams for the eight particular statements that can occur in the premises.



The second premise, on the other hand, tells us that there is nothing in the S circle that isn't also in the M circle. We put that together with the first diagram, and we get this:



From this we can see that the conclusion must be true. All S are P, because the only space left in S is the area in the exact center, area 7.

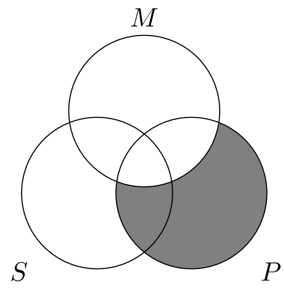
Now let's look at an argument that is invalid. One of the interesting things about the syllogism AAA-1 is that if you change the figure, it ceases to be valid. Consider AAA-2.

P_1: All P are M.

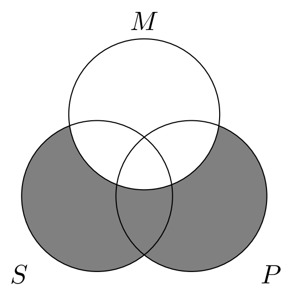
P_2: All S are M.

C: All S are P.

Again, both premises are universal, so we can do them in any order, so we will do the major premise first. This time, the major premise tells us to shade out the part of P that does not overlap M.



The second premise adds the idea that all S are M, which we diagram like this:



Now we ask if the diagram of the two premises also shows that the conclusion is true. Here the conclusion is that all S are P. If this diagram had made this true, we would have shaded out all the parts of S that do not overlap P. But we haven't done that. It is still possible for something to be in area 5. Therefore this argument is invalid.

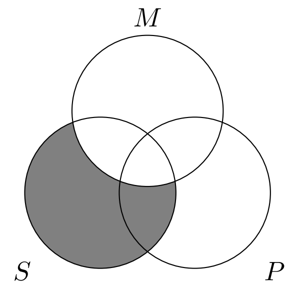
Now let's try an argument with a particular statement in the premises. Consider the argument IAI-1:

P_1: Some M are P.

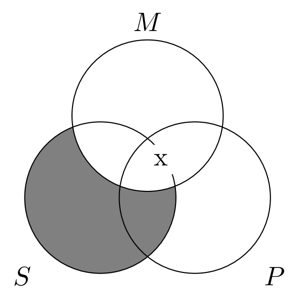
P_2: All S are M.

C: Some S are P.

Here, the second premise is universal, while the first is particular, so we begin by diagramming the universal premise.



Then we diagram the particular premise "Some M are P." This tells us that something is in the overlap between M and P, but it doesn't tell us whether that thing is in the exact center of the diagram or in the area for things that are M and P but not S. Therefore, we place the x on the border between these two areas.



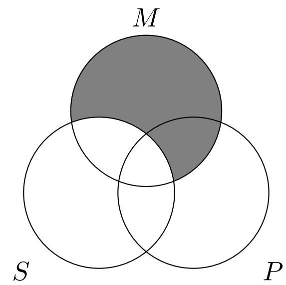
Now we can see that the argument is not valid. The conclusion asserts that something is in the overlap between S and P. But the x we drew does not necessarily represent an object that exists in that overlap. There is something out there that could be in area 7, but it could just as easily be in area 6. The second premise doesn't help us, because it just rules out the existence of objects in areas 1 and 4.

For a final example, let's look at a case of a valid argument with a particular statement in the premises. If we simply change the figure of the argument in the last example from 1 to 3, we get a valid argument. This is the argument IAI-3:

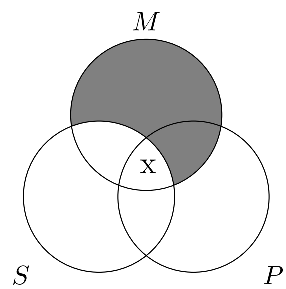
P_1: Some M are P.
P_2: All M are S.

C: Some S are P.

Again, we begin with the universal premise. This time it tells us to shade out part of the M circle.



But now we fill in the parts of M that don't overlap with S, we have to put the x in the exact center of the diagram.



And now this time we see that "Some S are P" has to be true based on the premises, because the X has to be in area 7. So this argument is valid.

Using this method, we can show that 15 of the 256 possible syllogisms are valid. Remember, however, that the Venn diagram method uses Boolean assumptions about existential import. If you make other assumptions about existential import, you will allow more valid syllogisms, as we will see in the next section. The additional syllogisms we will be able to prove valid in the next section will be said to have CONDITIONAL VALIDITY because they are valid on the condition that the objects talked about in the universal statements actually exist. The 15 syllogisms that we can prove valid using the Venn diagram method have UNCONDITIONAL VALIDITY. These syllogisms are given in Table 2.2.

The names on Table 2.2 come from the Christian part of the Aristotelian tradition, where thinkers were writing in Latin. Students in that part of the tradition learned the valid forms by giving each one a female name. The vowels in the name represented the mood of the syllogism. So B**a**rb**ara** has the mood AAA, Fr**esiso**n has the mood EIO, etc. The consonants in each name were also significant: they related to a process the Aristotelians were interested in called reduction, where arguments in the later figures were shown to be equivalent to arguments in the first figure, which was taken to be more self-evident. We won't worry about reduction in this textbook, however. The names of the valid syllogisms were often worked into a mnemonic poem. The oldest known version of the poem appears in a late 13th century book called *Introduction to Logic* by

Figure 1	**Figure 2**	**Figure 3**	**Figure 4**
Barbara (AAA)	Camestres (AEE)	Disamis (IAI)	Calemes (AEE)
Celarent (EAE)	Cesare (EAE)	Bocardo (OAO)	Dimatis (IAI)
Ferio (EIO)	Festino (EIO)	Ferison (EIO)	Fresison (EIO)
Darii (AII)	Baroco (AOO)	Datisi (AII)	

Table 2.2: The 15 unconditionally valid syllogisms.

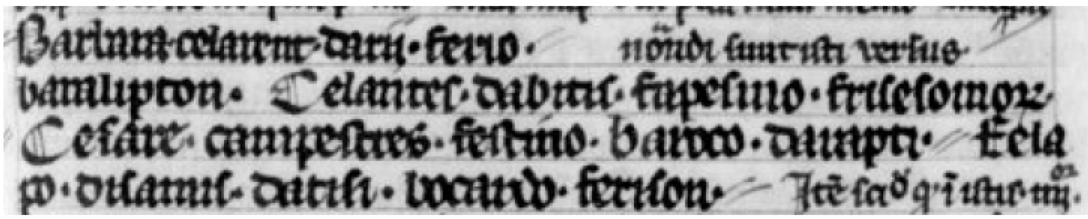


Figure 2.3: The oldest surviving version of the "Barbara, Celarent..." poem, from William of Sherwood (c. 1275/1966)

William of Sherwood (Sherwood c. 1275/1966). Figure 2.3 is an image of the oldest surviving manuscript of the poem, digitized by the Bibliothque Nationale de France.

The columns in Table 2.2 represent the four figures. Syllogisms with the same mood also appear in the same row. So the EIO sisters—Ferio, Festino, Ferison, and Fresison—fill up row 3. Camestres and Calemes share row 1; Celarent and Cesare share row 2; and Darii and Datisi share row 4.

2.3 Existential Import and Conditionally Valid Forms

In the last section, we mentioned that you can prove more syllogisms valid if you make different assumptions about existential import. Recall that a statement has existential import if, when you assert the statement, you are also asserting the existence of the things the statement talks about. So if you interpret a mood-A statement as having existential import, it not only asserts "All S is P," it also asserts "S exists." Thus the mood-A statement "All unicorns have one horn" is false, if it is taken to have existential import, because unicorns do not exist. It is probably true, however, if you do not imagine the statement as having existential import. If anything is true of unicorns, it is that they would have one horn if they existed.

We saw in Section **??** that before Boole, Aristotelian thinkers had all sorts of opinions about existential import, or, as they put it, whether a term "supposits." This generally led them to recognize additional syllogism forms as valid. You can see this pretty quickly if you just remember the traditional square of opposition. The traditional square allowed for many more valid immediate inferences than the modern square. It stands to reason that traditional ideas about existential import will also allow for more valid syllogisms.

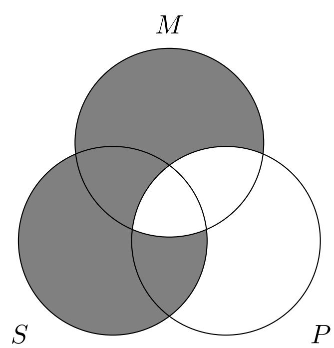
Our system of Venn diagrams can't represent all of the alternative ideas about existential import. For instance, it has no way of representing Ockham's belief that mood-O statements do *not* have existential import. Nevertheless, it would be nice if we could expand our system of Venn diagrams to show that some syllogisms are valid if you make additional assumptions about existence.

Consider the argument Barbari (AAI-1).

P$_1$: All M are P.
P$_2$: All S are M.

C: Some S are P.

You won't find this argument in the list of unconditionally valid forms in Table 2.2. This is because under Boolean assumptions about existence it is not valid. The Venn diagram, which follows Boolean assumptions, shows this.



This is essentially the same argument as Barbara, but the mood-A statement in the conclusion has been replaced by a mood-I statement. We can see from the diagram that the mood-A statement "All S are P" is true. There is no place to put an S other than in the overlap with P. But we don't actually know the mood-I statement "Some S is P," because we haven't drawn an x in that spot. Really, all we have shown is that *if* an S existed, it would be P.

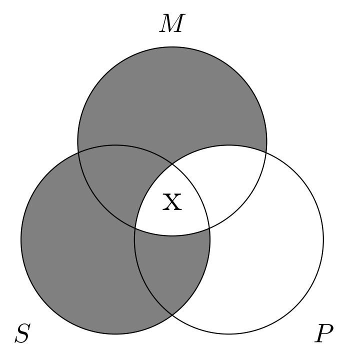
But by the traditional square of opposition we know that the mood-I statement is true. The traditional square, unlike the modern one, allows us to infer the truth of a particular statement given the truth of its corresponding universal statement. This is because the traditional square assumes that the universal statement has existential import. It is really two statements, "All S is P" and "Some S exists."

Because the mood-A statement is actually two statements on the traditional interpretation, we can represent it simply by adding an additional line to our argument. It is always legitimate to change an argument by making additional assumptions. The new argument won't have the exact same impact on the audience as the old argument. The audience will now have to accept an additional premise, but in this case all we are doing is making explicit an assumption that the Aristotelian audience was making anyway. The expanded argument will look like this:

P_1: All M are P.
P_2: All S are M.
P_3: Some S exists.*

C: Some S are P

Here the asterisk indicates that we are looking at an implicit premise that has been made explicit. Now that we have an extra premise, we can add it to our Venn diagram. Since there is only one place for the S to be, we know where to put our x.



In this argument S is what we call the "critical term." The critical term is the term that names things that must exist in order for a conditionally valid argument to be actually valid. In this argument, the critical term was S, but sometimes it will be M or P.

We have used Venn diagrams to show that Barbari is valid once you include the additional premise. Using this method we can identify nine more forms, on top of the previous 15, that are valid if we add the right existence assumptions (Table 2.4)

Thus we now have an expanded method for evaluating arguments using Venn diagrams. To evaluate an argument, we first use a Venn diagram to determine whether it is unconditionally valid. If it is, then we are done. If it is not, then we see if adding an existence assumption can make it conditionally valid. If we can add such an assumption, add it to the list of premises and put an x in the relevant part of the Venn diagram. If we cannot make the argument valid by including additional existence assumptions, we say it is completely invalid.

	Figure 1	Figure 2	Figure 3	Figure 4	Condition
Unconditional	Barbara (AAA)	Camestres (AEE)	Disamis (IAI)	Calemes (AEE)	
	Celarent (EAE)	Cesare (EAE)	Bocardo (OAO)	Dimatis (IAI)	
	Ferio (EIO)	Festino (EIO)	Ferison (EIO)	Fresison (EIO)	
	Darii (AII)	Baroco (AOO)	Datisi (AII)		
Conditional	Barbari (AAI)	Camestros (AEO)		Calemos (AEO)	S exists
	Celaront (EAO)	Cesaro (EAO)			S exists
			Felapton (EAO)	Fesapo (EAO)	M exists
			Darapti (AAI)		M exists
				Bamalip (AAI)	P exists

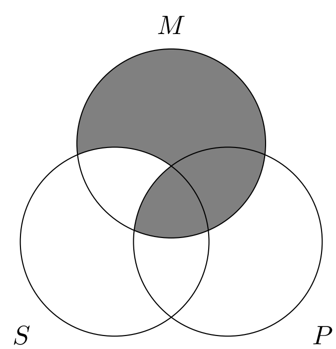
Table 2.4: All 24 Valid Syllogisms

Let's run through a couple examples. Consider the argument EAO-3.

P_1: No M are P.
P_2: All M are S.

C: Some S are not P.

First we use the regular Venn digram method to see whether the argument is unconditionally valid.



We can see from this that the argument is not valid. The conclusion says that some S are not P, but we can't tell that from this diagram. There are three possible ways something could be S, and we don't know if any of them are occupied.

Simply adding the premise S exists won't help us, because we don't know whether to put the x in the overlap between S and M, the overlap between S and P, or in the area that is just S. Of course, we would want to put it in the overlap between S and M, because that would mean that there is an S that is not P. However, we can't justifying doing this simply based on the premise that S exists.

The premise that P exists will definitely not help us. The P would either go in the overlap between S and P or in the area that is only P. Neither of these would show "Some S is not P."

The premise "M exists" does the trick, however. If an M exists, it has to also be S but not P. And this is sufficient so show that some S is not P. We can then add this additional premise to the argument to make it valid.

P_1: No M are P.
P_2: All M are S.
P_3: M exists.*

C: Some S are not P.

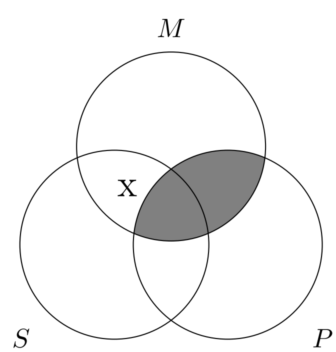
Checking it against Table 2.4, we see that we were right: this is a conditionally valid argument named Felapton.

Now consider the argument EII-3:

P_1: No M are P.
P_2: Some M are S.

C: Some S are P.

First we need to see if it is unconditionally valid. So we draw the Venn diagram.



The conclusion says that some S are P, but we obviously don't know this from the diagram above. There is no x in the overlap between S and P. Part of that region is shaded out, but the rest could go either way.

What about conditional validity? Can we add an existence assumption that would make this valid? Well, the x we have already drawn lets us know that both S and M exist, so it won't help to add those premises. What about adding P? That won't help either. We could add the premise "P exists" but we wouldn't know whether that P is in the overlap between S and P or in the area to the right, which is just P.

Therefore this argument is invalid. And when we check the argument against Table 2.4, we see that it is not present.

2.4 Validity and the Counterexample Method

Except for a brief discussion of logically structured English in section 2.1, so far, we have only been evaluating arguments that use variables for the subject, middle, and predicate terms. Now, we will be looking in detail at issues that come up when we try to evaluate categorical arguments that come up in ordinary English. In this section we will consider how your knowledge of the real world terms discussed in an argument can distract you from evaluating the form of the argument itself. In this section we will consider difficulties in putting ordinary language arguments into logically structured English, so they can be analyzed using the techniques we have learned.

Let's go back again to the definition of validity. (It is always good for beginning student to reinforce their understanding of validity). We did this in the last chapter in Section 1.4, and we are doing it again now. Validity is a fundamental concept in logic that can be confusing. A valid argument is not necessarily one with true premises or a true conclusion. An argument is valid if the premises *would* make the conclusion true *if* the premises were true.

This means that, as we have seen before, there can be valid arguments with false conclusions. Consider this argument:

> No reptiles are chihuahuas. But all dogs are reptiles. Therefore, no dogs are chihuahuas.

This seems silly, if only because the conclusion is false. We know that some dogs are chihuahuas. But the argument is still valid. In fact, it shares a form with an argument that makes perfect sense:

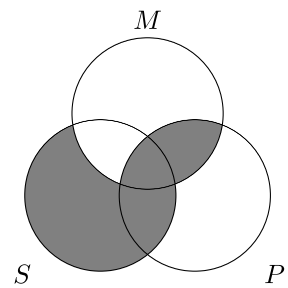
> No reptiles are dogs, but all chameleons are reptiles. Therefore, no dogs are chameleons.

Both of these arguments have the form Celarent:

P_1: No M are P.
P_2: All S are M.

C: No S are P.

This form is valid, whether the subject and predicate term are dogs and chameleons, or dogs and chihuahuas, which you can see from this Venn diagram.



This means you can't assume an argument is invalid because it has a false conclusion. The reverse is also true. You can't assume an argument is valid just because it has a true conclusion. Consider this

> All cats are animals, and some animals are dogs. Therefore no dogs are cats.

Makes sense, right? Everything is true. But the argument isn't valid. The premises aren't making the conclusion true. Other arguments with the same form have true premises and a false conclusion. Like this one.

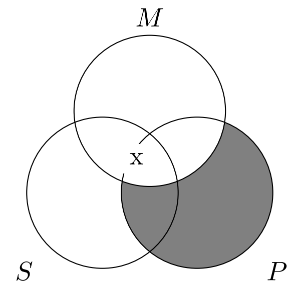
> All chihuahuas are animals, and some animals are dogs. Therefore, no dogs are chihuahuas.

Really, the arguments in both these passages have the same form: AEE-IV:

P_1: All P are M.
P_2: Some M are S.

C: No S are P.

This in an invalid form, and it remains invalid whether the P stands for cats or chihuahuas. You can see this in the Venn diagram:



All these examples bring out an important fact about the kind of logic we are doing in this chapter and the last one: this is *formal* logic. As we discussed, formal logic is a way of making our investigation content neutral. By using variables for terms instead of noun phrases in English we can show that certain ways of arguing are good or bad regardless of the topic being argued about. Parts This method will be extended in Part **??**, when we introduce the full formal language SL

The examples above also show us another way of proving that an argument given in English is invalid, called the counterexample method. As we have just seen, if you are given an argument in English with, say, false premises and a false conclusion, you cannot determine immediately whether the argument is valid. However, we can look at arguments that have the same form, and use them to see whether the argument is valid. If we can find an argument that has the exact same form as a given argument but has true premises and a false conclusion, then we know the argument is invalid. We just did that with the AEE-IV argument above. We were given an argument with true premises and a true conclusion involving cats, dogs, and animals. We were able to show this argument invalid by finding an argument with the same form that has true premises and a false conclusion, this time involving chihuahuas, dogs, and animals.

More precisely, we can define the COUNTEREXAMPLE METHOD as a method for determining if an argument with ordinary English words for terms is valid, where one consistently substitutes other English terms for the terms in the given argument to see if one can find an argument with the same form that has true premises and a false conclusion. Let's run through a couple more examples to see how this works.

First consider this argument in English:

> All tablet computers are computers. We know this because a computer is a kind of machine, and some machines are not tablet computers.

Every statement in this argument is true, but it doesn't seem right. The premises don't really relate to the conclusion. That means you can probably find an argument with the same form that has true premises and a false conclusion. Let's start by putting the argument in canonical form. Notice that the English passage had the conclusion first.

P_1: All computers are machines.

P_2: Some machines are not tablet computers.

C: All tablet computers are computers.

Let's find substitutes for "machines," "computers," and "tablet computers" that will give us true premises and a false conclusion. It is easiest to work with really common sense categories, like "dog" and "cat." It is also easiest to start with a false conclusion and then try to find a middle term that will give you true premises. "All dogs are cats" is a nice false conclusion to start with:

P_1: All cats are M.
P_2: Some M are not dogs.

C: All dogs are cats.

So what can we substitute for M (which used to be "machines") that will make P_1 and P_2 true? "Animals" works fine.

P_1: All cats are animals.
P_2: Some animals are not dogs.

C: All dogs are cats.

There you have it: a counterexample that shows the argument invalid. Let's try another one.

> Some diseases are viral, therefore some things caused by bacteria are not things that are caused by viruses, because all diseases are bacterial.

This will take a bit more unpacking. You can see from the indicator words that the conclusion is in the middle. We also have to fix "viral" and "things that are caused by viruses" so they match, and the same is true for "bacterial" and "things that are caused by bacteria." Once we have the sentences in canonical form, the argument will look like this:

P_1: Some diseases are things caused by viruses.
P_2: All diseases are things causes by bacteria.

C: Some things caused by bacteria are not things caused by viruses.

Once you manage to think through the complicated wording here, you can see that P_1 and the conclusion are true. Some diseases come from viruses, and not everything that comes from a bacteria comes from a virus. But P_2 is false. All diseases are not caused by bacteria. In fact, P_1 contradicts P_2. But none of this is enough to show the argument is invalid. To do that, we need to find an argument with the same form that has true premises and a false conclusion.

Let's go back to the simple categories: "dogs," "animals," etc. We need a false conclusion. Let's go with "Some dogs are not animals."

P_1: Some M are dogs.
P_2: All M are animals.

C: Some dogs are not animals.

We need a middle term that will make the premises true. It needs to be a class that is more general than "dogs" but more narrow than "animals." "Mammals" is a good standby here.

P_1: Some mammals are dogs.
P_2: All mammals are animals.

C: Some dogs are not animals.

And again we have it, a counterexample to the given syllogism.

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Chapter 3

Glossary

Affirmative The quality of a statement without a “not” or a “no.” 4

Aristotelian syllogism A categorical syllogism where each statement is in one of the moods A, E, I, or O, and which has exactly three terms, arranged so that any two pairs of statements will share one term. 24

Categorical syllogism An argument with two premises composed of categorical statements. 24

Complement The class of everything that is not in a given class. 15

Conditional validity A kind if validity that Aristotelian syllogisms have if they are valid only given the assumption that the objects named by its terms actually exist. 36

Contraposition The process of transforming a categorical statement by reversing subject and predicate and replacing them with their complements. 19

Conversion The process of changing a sentence by reversing the subject and predicate. 14

Copula The form of the verb “to be” that links subject and predicate. 3

Counterexample method A method for determining whether an argument with ordinary English words for terms is valid. One consistently substitutes other English terms for the terms in the given argument to see whether one can find an argument with the same form that has true premises and a false conclusion. 44

Critical term the term that names things that must exist in order for a conditionally valid argument to be actually valid. 39

Distribution A property of the terms of a categorical statement that is present when the statement makes a claim about the whole term. 5

Logically structured English English that has been regimented into a standard form to make its logical structure clear and to remove ambiguity. A stepping stone to full-fledged formal languages. 9

Major premise The one premise in an Aristotelian syllogism that names the major term. 25

Major term The term that is used as the predicate of the conclusion of an Aristotelian syllogism. 25

Middle term The one term in an Aristotelian syllogism that does not appear in the conclusion. 25

Minor premise The one premise in an Aristotelian syllogism that names the minor term. 25

Minor term The term that is used as the subject of the conclusion of an Aristotelian syllogism. 25

Mood-A statement A quantified categorical statement of the form "All S are P." 4

Mood-E statement A quantified categorical statement of the form "No S are P." 4

Mood-I statement A quantified categorical statement of the form "Some S are P." 4

Mood-O statement A quantified categorical statement of the form "Some S are not P." 4

Negative The quality of a statement containing a "not" or "no." 4

Obversion The process of transforming a categorical statement by changing its quality and replacing the predicate with its complement. 16

Particular The quantity of a statement that uses the quantifier "some." 4

Predicate class The second class named in a quantified categorical statement. 3

Quality The status of a categorical statement as affirmative or negative. 4

Quantified categorical statement A statement that makes a claim about a certain quantity of the members of a class or group. 2

Quantifier The part of a categorical sentence that specifies a portion of a class. 3

Quantity The portion of the subject class described by a categorical statement. Generally "some" or "none." 4

Standard form for a categorical statement A categorical statement that has been put into logically structured English, with the following elements in the following order: (1) The quantifiers "all," "some," or "no"; (2) the subject term; (3) the copula "are" or "are not"; and (4) the predicate term. 9

Standard form for an Aristotelian syllogism An Aristotelian syllogism that has been put into logically structured English with the following criteria: (1) all of the individual statements are in standard form, (2) each instance of a term is in the same format and is used in the same sense, and (3) the major premise appears first, followed by the minor premise, and then the conclusion. 25

Statement mood The classification of a categorical statement based on its quantity and quality. 4

Subject class The first class named in a quantified categorical statement. 3

Syllogism mood The classification of an Aristotelian syllogism based on the moods of statements it contains. The mood is designated simply by listing the three letters for the moods of the statements in the argument, such as AAA, EAE, AII, etc. 25

Translation key A list that assigns English phrases or sentences to variable names. 26

Unconditional validity A kind of validity that an Aristotelian syllogism has regardless of whether the objects named by its terms actually exist. 36

Universal The quantity of a statement that uses the quantifier "all." 4

Venn diagram A diagram that represents categorical statements using circles that stand for classes. 6