RSAConference2019
San Francisco | March 4–8 | Moscone Center

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SESSION ID: CRYP-F02

Revisiting the Secret Hiding Assumption Used in Verifiable (Outsourced) Computation

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Agenda of My Presentation

① Background Information

② Attack Strategy

③ Analysis for the Decisional Secret Hiding Assumption

④ Privacy Analysis for Atallah-Frikken Protocols

⑤ Experimental Verification

Outline

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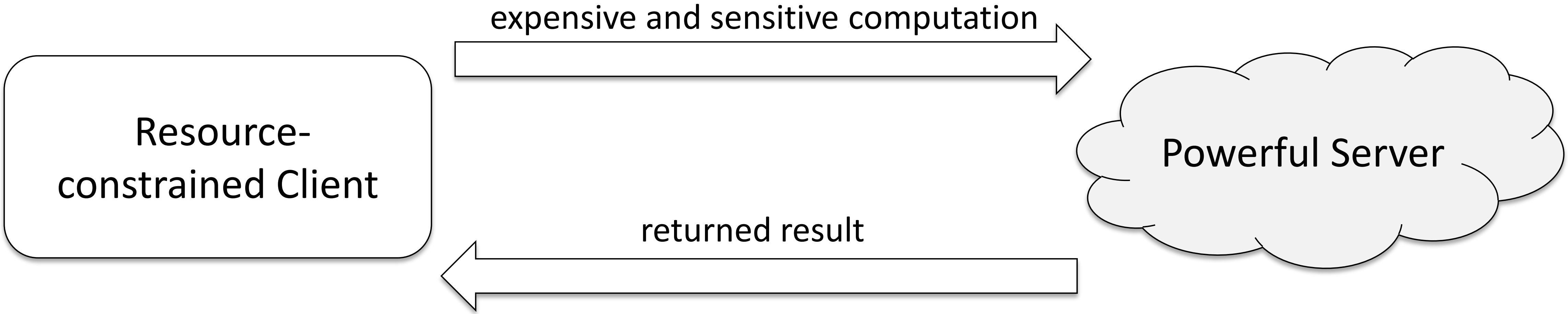
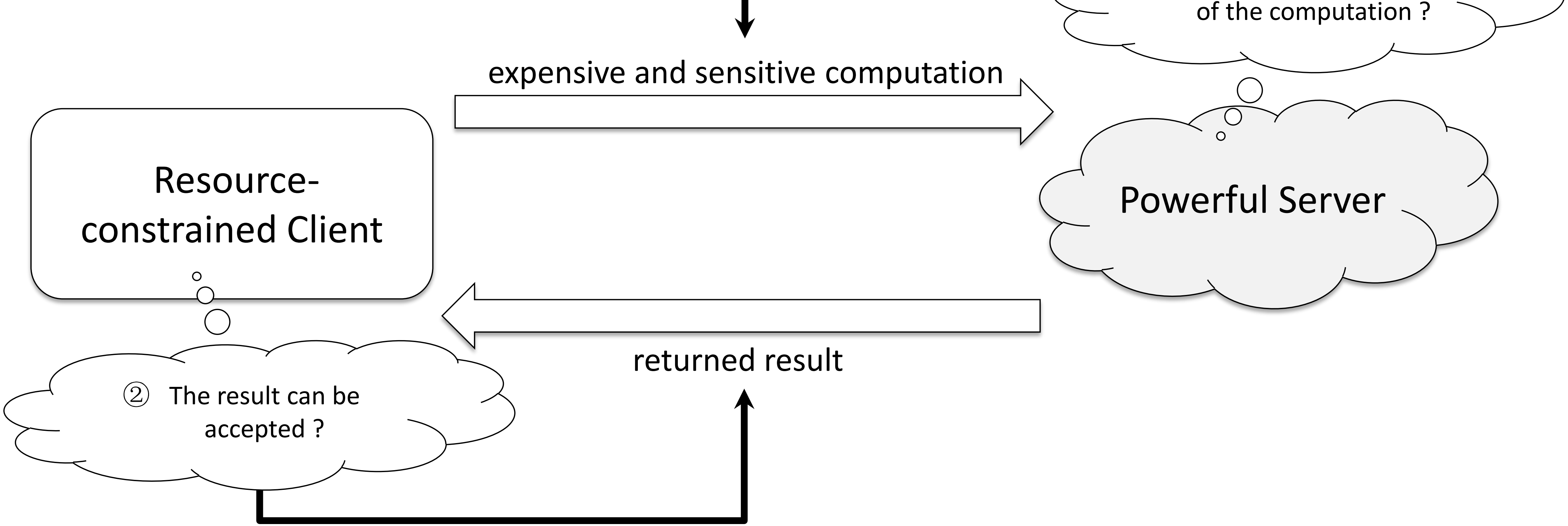
Background Information

- The general scenario about the reasonable outsourcing computation:



Background Information

The considered Privacy-preserving Verifiable (outsourced) Computation (PVC):



Background Information

- Atallah and Frikken proposed a new hardness assumption called the Secret Hiding assumption (SH) at ACM AsiaCCS 2010 [Atallah & Frikken'10]

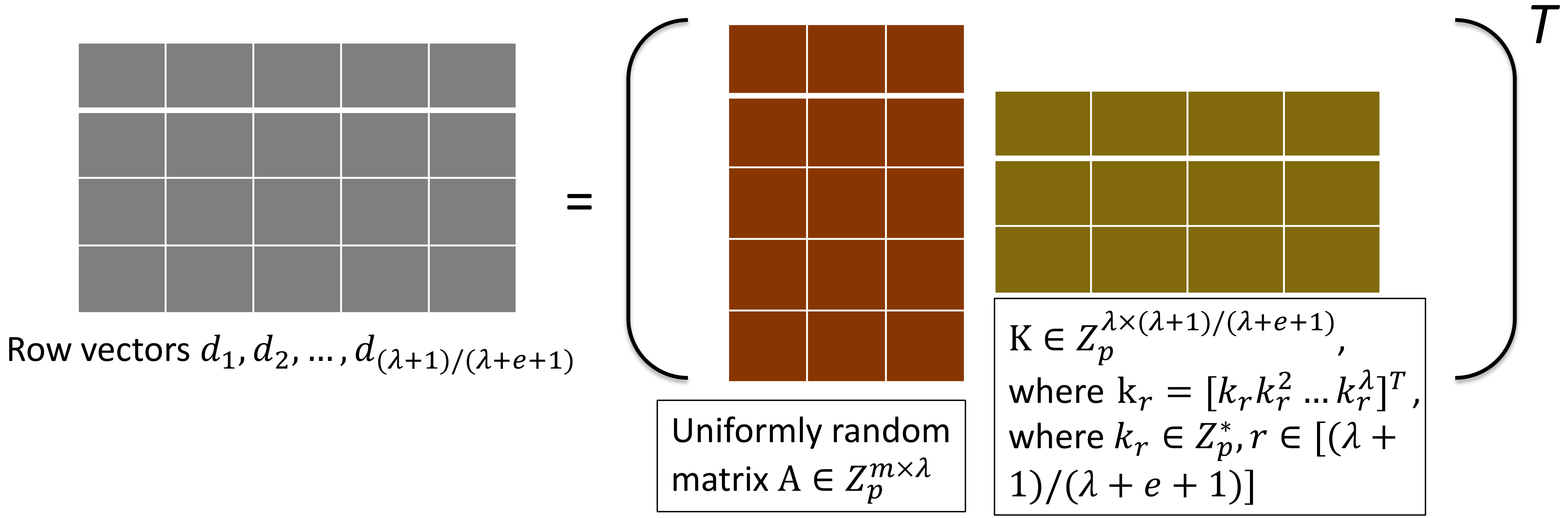
- Two concrete versions:
  - ✓ Weak SH assumption (WSH)
  - ✓ Strong SH assumption (SSH)

Background Information

- What is the WSH/SSH assumption ?

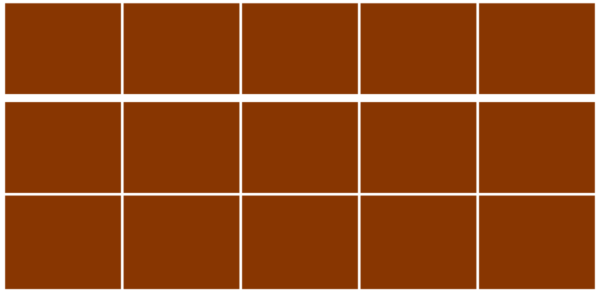
What is the WSH/SSH distribution ?

1



Background Information

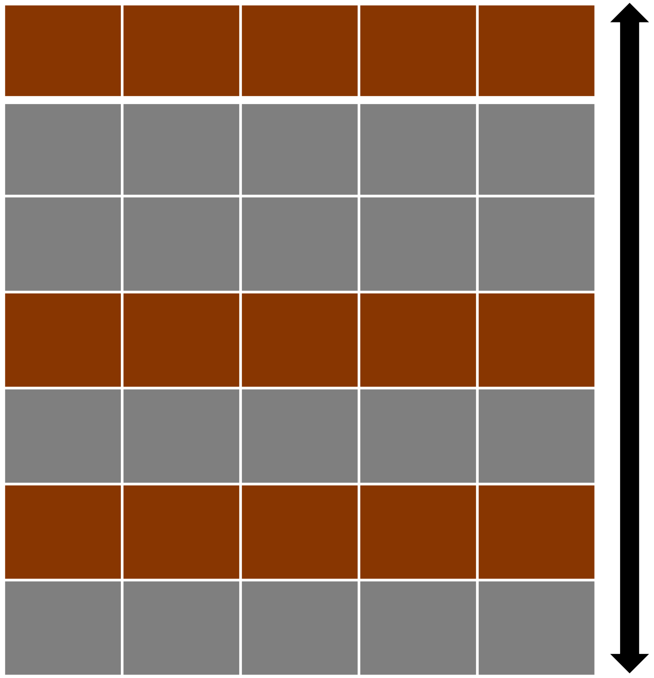
2



Choose some row vectors $u_1, u_2, \dots, u_{\lambda/(\lambda+e+1)}$ uniformly at random, where $u_r \in Z_p^m$

3

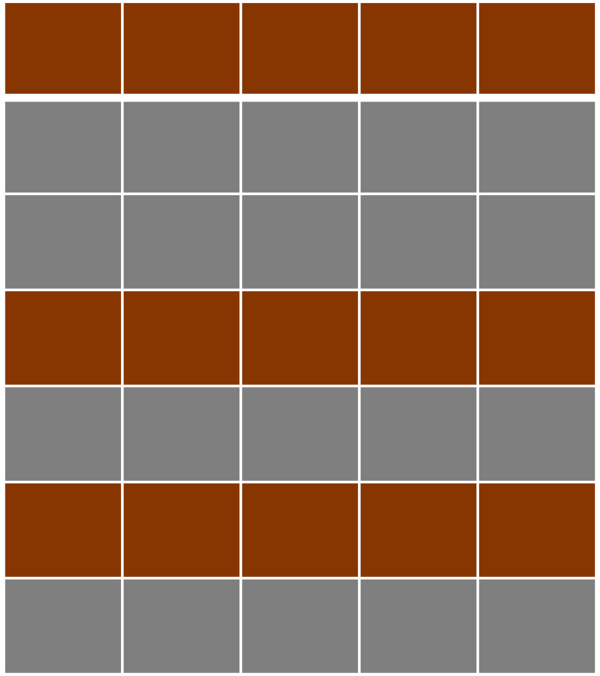
R =



Combine $d_1, d_2, \dots, d_{(\lambda+1)/(\lambda+e+1)}$ with $u_1, u_2, \dots, u_{\lambda/(\lambda+e+1)}$ to generate an $n \times m$ matrix R, and permute the rows of R, where $n \in \{2\lambda + 1, 2\lambda + 2e + 2\}$

Background Information

R =



Decision: Does R look random ?

Search: given R, find $k_1, k_2, \ldots, k_{(\lambda+1)/(\lambda+e+1)}$ or A

NOTE:

① The decisional-WSH is the same as the decisional-SSH

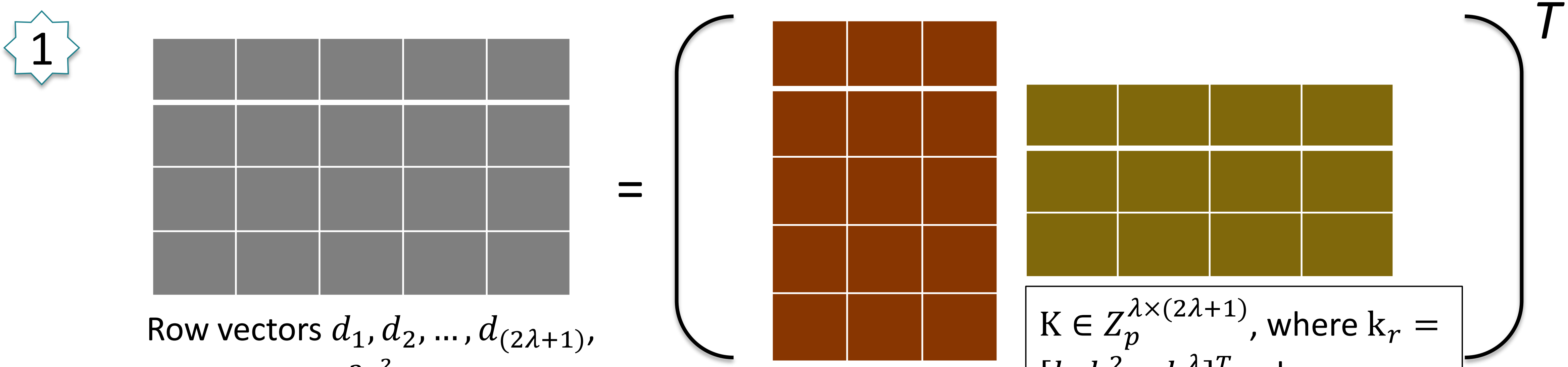
② The search-WSH is to find $k_1, k_2, \ldots, k_{(\lambda+1)}$ or A. The search-SSH is to find $k_1, k_2, \ldots, k_{(\lambda+e+1)}$ or A

WSH/SSH Assumption:

No polynomial-time adversary can solve the decisional and search WSH/SSH problem

Background Information

- Atallah and Frikken proposed some WSH/SSH-based PVC protocols for matrix multiplication
- The idea of the PVC protocols:



Row vectors $d_1, d_2, \ldots, d_{(2\lambda+1)}$, where $d_r \in Z_p^{2v^2}$

Uniformly random matrix $\mathrm{A} \in Z_p^{2v^2 \times \lambda}$

$\mathrm{K} \in Z_p^{\lambda \times (2\lambda+1)}$, where $\mathrm{k}_r = [k_r k_r^2 \ldots k_r^\lambda]^T$, where $k_r \in Z_p^*, r \in [2\lambda+1]$

Background Information

2 For two $v \times v$ matrices M_1 and M_2, use each vector d_r to mask them, and generate $2\lambda + 1$ matrix pairs $(\mathrm{C}_1(k_1), \mathrm{C}_2(k_1)), (\mathrm{C}_1(k_2), \mathrm{C}_2(k_2)), \ldots (\mathrm{C}_1(k_{2\lambda+1}), \mathrm{C}_2(k_{2\lambda+1}))$

$$\mathrm{C}_1(k_r)||\mathrm{C}_2(k_r) = d_r + \mathrm{M}_1||\mathrm{M}_2$$

3 *The Two-Server Case:* Choose 2λ $v \times v$ uniformly random matrices $\mathrm{B}_1, \mathrm{B}_2, \ldots, \mathrm{B}_{2\lambda}$ to create λ pairs $(\mathrm{B}_1, \mathrm{B}_2), \ldots, (\mathrm{B}_{2\lambda-1}, \mathrm{B}_{2\lambda})$. Send λ pairs $(\mathrm{C}_1(k_1), \mathrm{C}_2(k_1)), (\mathrm{C}_1(k_2), \mathrm{C}_2(k_2)), \ldots (\mathrm{C}_1(k_\lambda), \mathrm{C}_2(k_\lambda))$ to the first server. Combine $(\mathrm{B}_1, \mathrm{B}_2), \ldots, (\mathrm{B}_{2\lambda-1}, \mathrm{B}_{2\lambda})$ with $(\mathrm{C}_1(k_{\lambda+1}), \mathrm{C}_2(k_{\lambda+1})), \ldots, (\mathrm{C}_1(k_{2\lambda+1}), \mathrm{C}_2(k_{2\lambda+1}))$ to generate $2\lambda + 1$ matrix pairs and permute these matrix pairs. Send the $2\lambda + 1$ permuted matrix pairs to the second server

The Single-Server Case: Choose $(4\lambda + 2)$ $v \times v$ uniformly random matrices $\mathrm{B}_1, \mathrm{B}_2, \ldots, \mathrm{B}_{4\lambda+2}$ to create $2\lambda + 1$ pairs $(\mathrm{B}_1, \mathrm{B}_2), \ldots, (\mathrm{B}_{4\lambda+1}, \mathrm{B}_{4\lambda+2})$. Combine $(\mathrm{B}_1, \mathrm{B}_2), \ldots, (\mathrm{B}_{4\lambda+1}, \mathrm{B}_{4\lambda+2})$ with $(\mathrm{C}_1(k_1), \mathrm{C}_2(k_1)), \ldots (\mathrm{C}_1(k_{2\lambda+1}), \mathrm{C}_2(k_{2\lambda+1}))$ to generate $4\lambda + 2$ matrix pairs and permute these matrix pairs. Send the $4\lambda + 2$ permuted matrix pairs to a server

Background Information

4 *The Two-Server Case:* Send back the products of all matrix pairs computed by the two servers. Choose some products corresponding to M_1 and M_2, and interpolate these products to find the real result of $\mathrm{M}_1\mathrm{M}_2$

The Single-Server Case: Send back the products of all matrix pairs computed by a server. Choose some products corresponding to M_1 and M_2, and interpolate these products to find the real result of $\mathrm{M}_1\mathrm{M}_2$

Theorems:

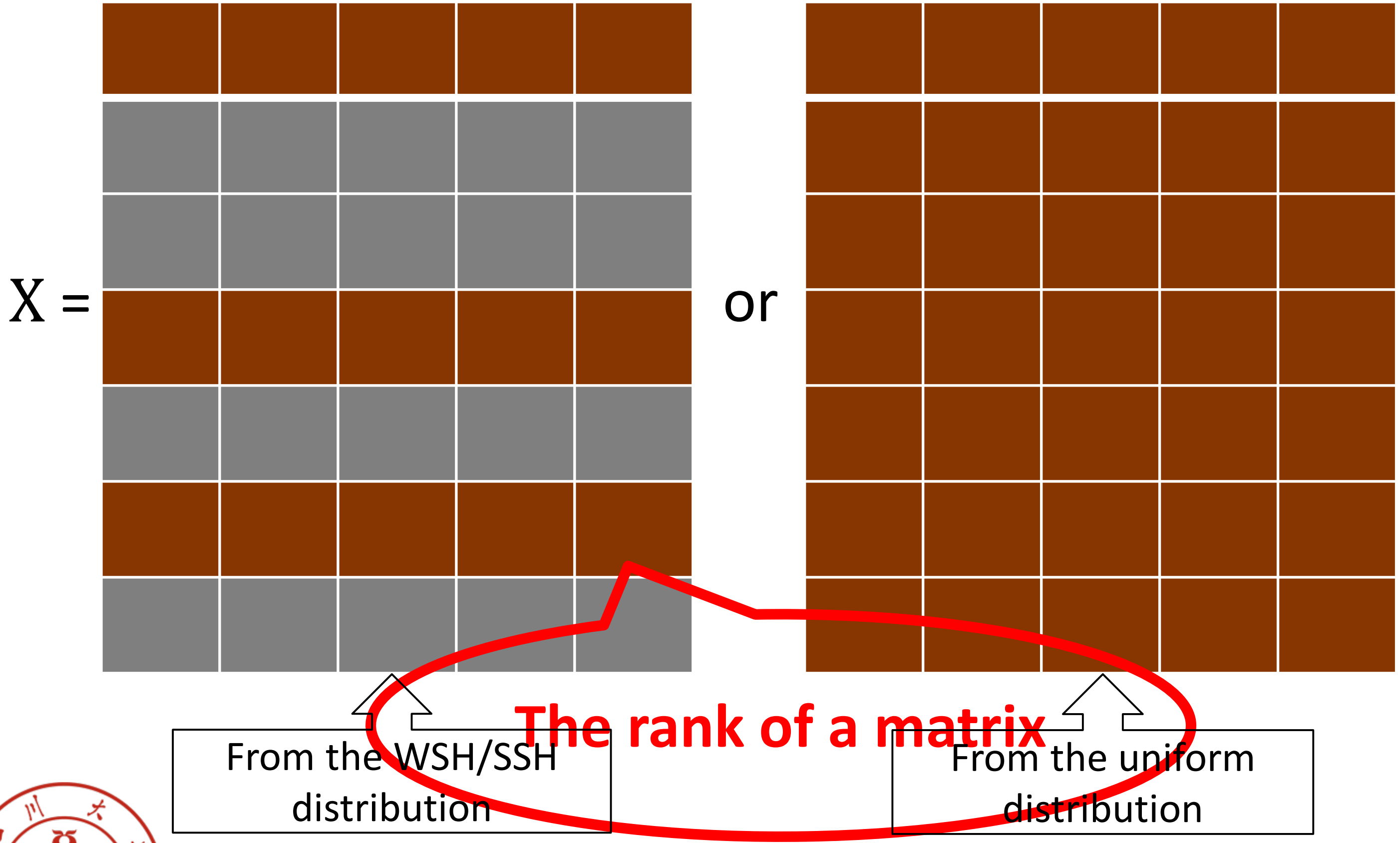
The Two-Server Case: Assume that the two servers do not collude and the decisional-WSH assumption holds. Then, the PVC protocol for matrix multiplication is private

The Single-Server Case: Assume that the decisional-SSH assumption holds. Then, the PVC protocol for matrix multiplication is private

Outline

Attack Strategy

- For a matrix X from either the WSH/SSH distribution or uniformly random, how to evaluate it using some special factor ?



Strategy Overview:

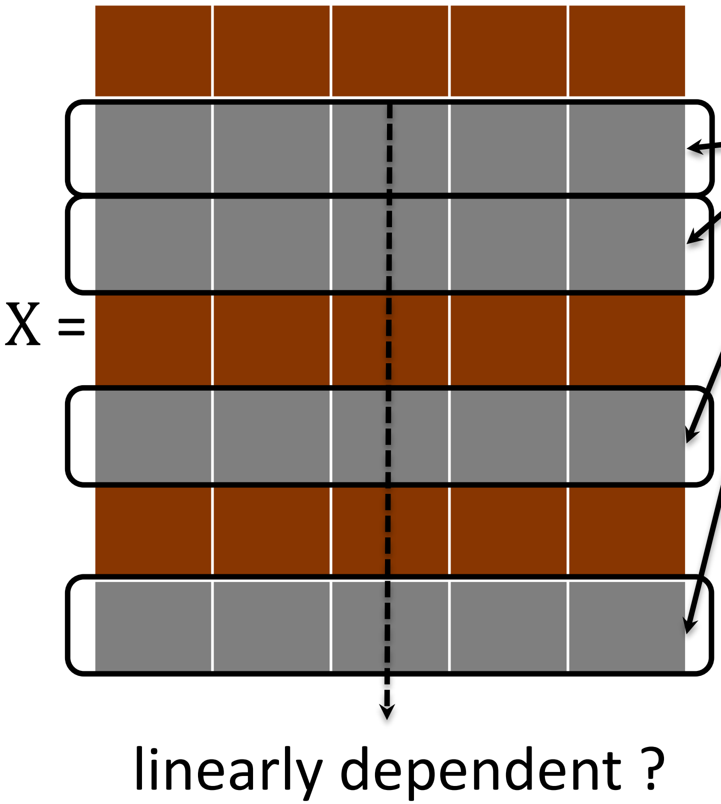
① Compute the rank of X

② Check whether the rank of X is below some value or not below this value

If the rank of X is below some value, X is sampled from the WSH/SSH distribution; otherwise, X is sampled from the uniform distribution

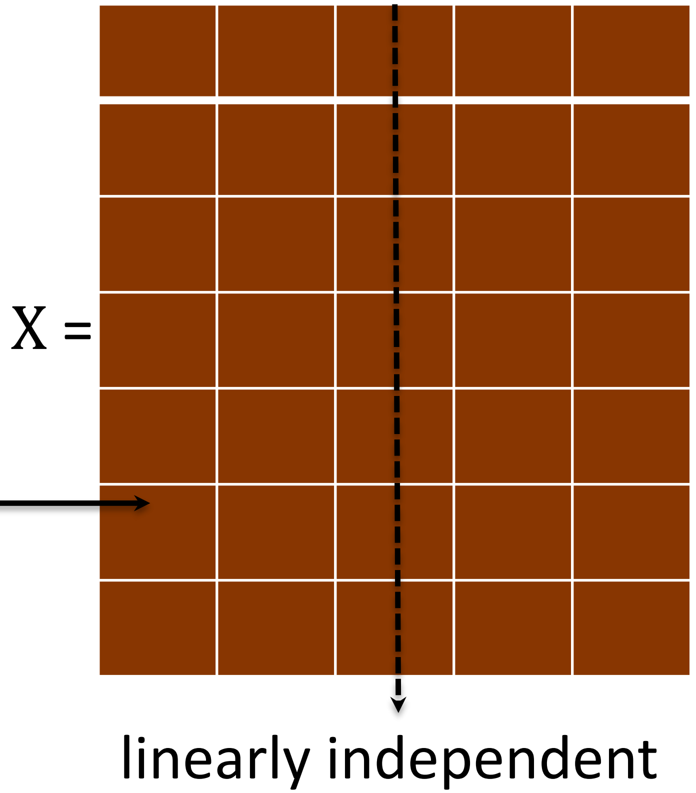
Attack Strategy

- Why the rank-based analysis works?



① *Fact 1:* If some row vectors in X are linearly dependent, all the row vectors of X are linearly dependent

② *Fact 2:* For a matrix X sampled from the uniform distribution, with high probability, all the row vectors of X are linearly independent



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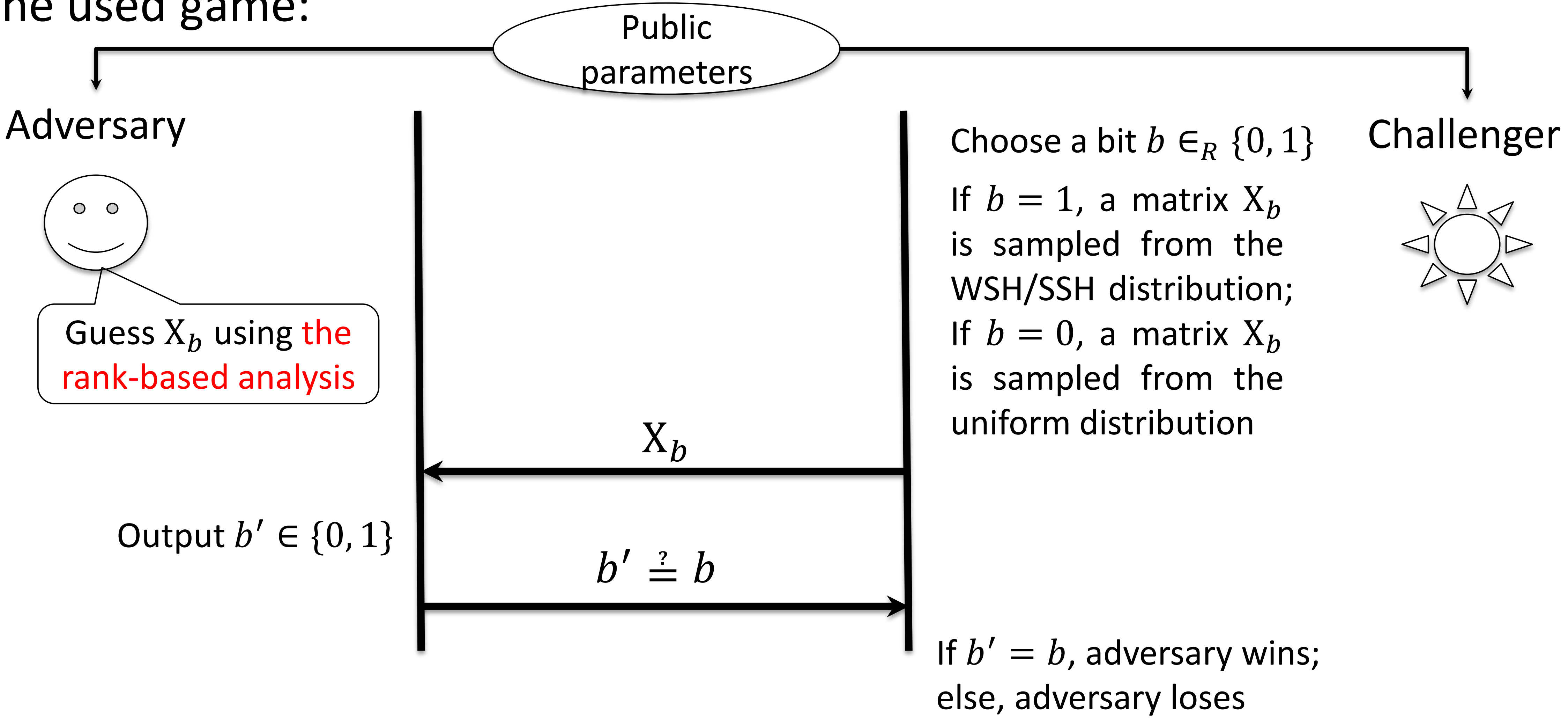
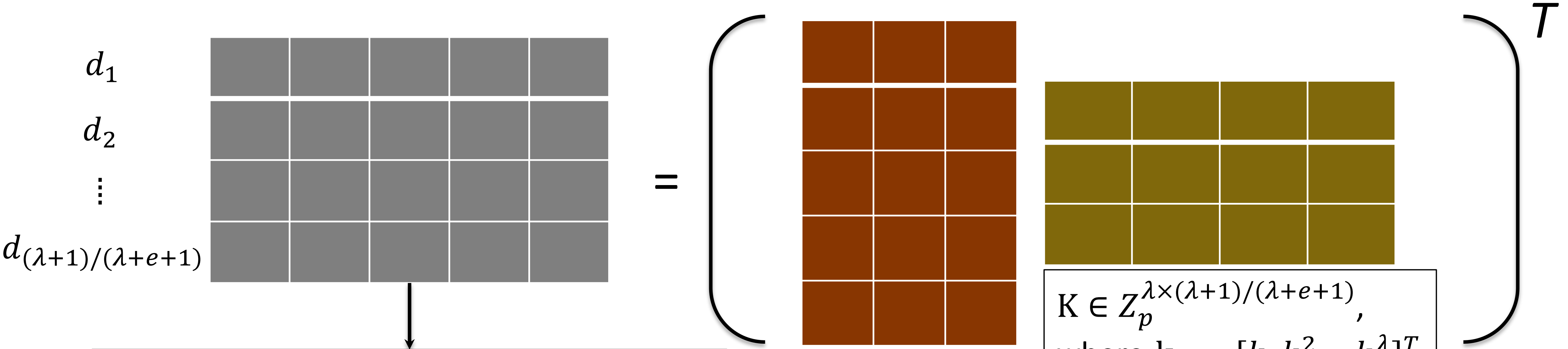
Analysis for the Decisional Secret Hiding Assumption

- The used game:



Analysis for the Decisional Secret Hiding Assumption

A significant motivation:



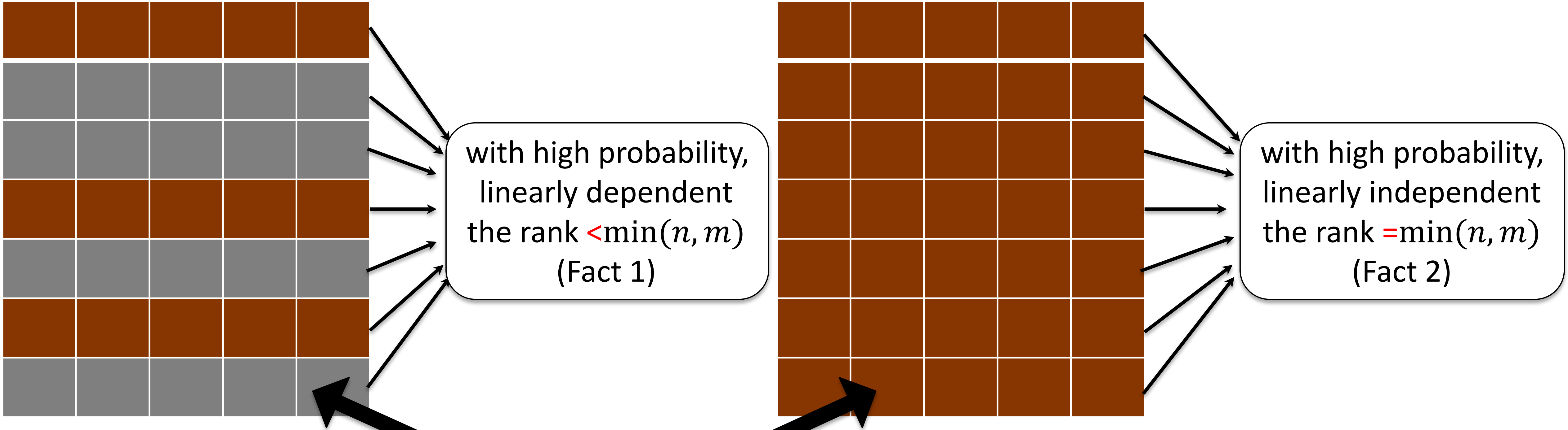
These row vectors $d_1, d_2, \dots, d_{(\lambda+1)/(\lambda+e+1)}$ are linearly dependent

Uniformly random matrix $A \in Z_p^{m\times\lambda}$

$K \in Z_p^{\lambda\times(\lambda+1)/(\lambda+e+1)}$, where $k_r = [k_r k_r^2 \dots k_r^{\lambda}]^T$, where $k_r \in Z_p^*, r \in [(\lambda + 1)/(\lambda + e + 1)]$

Analysis for the Decisional Secret Hiding Assumption

For the $n \times m$ matrix X_b:



Adversary can distinguish the samples from these two distributions with high probability!

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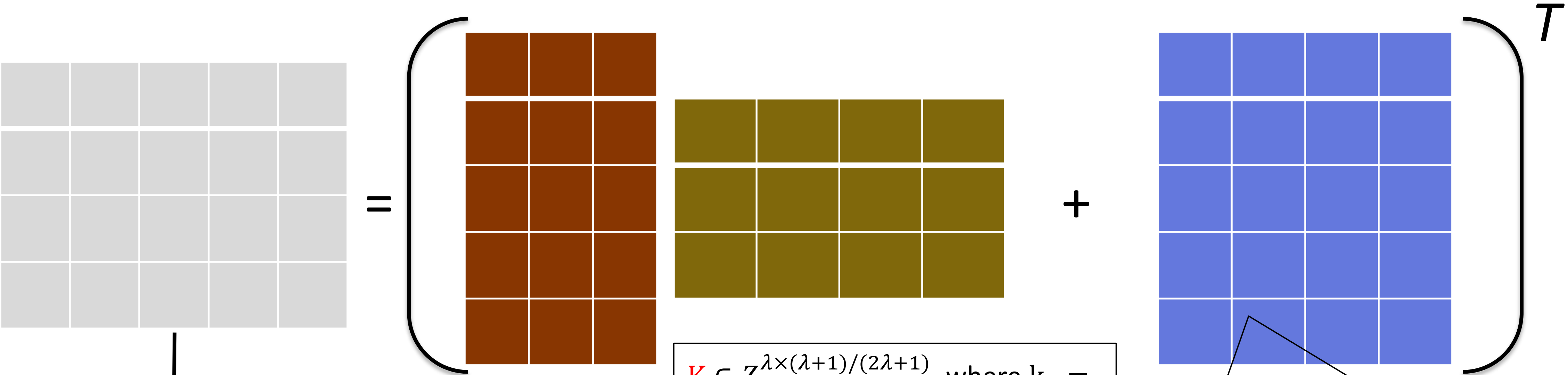
Privacy Analysis for Atallah-Frikken Protocols

A significant motivation:



Uniformly random matrix $\color{red}{A} \in Z_p^{2v^2 \times \lambda}$

$\color{red}{K} \in Z_p^{\lambda \times (\lambda+1)/(2\lambda+1)}$, where $k_r = [k_r k_r^2 \ldots k_r^\lambda]^T$, where $k_r \in Z_p^*, r \in [(\lambda+1)/(2\lambda+1)]$

① $2v^2 \times (\lambda+1)/(2\lambda+1)$ matrix $\color{red}{S} = [s_1, s_1, \ldots s_1]$, where $s_1 \leftarrow_R Z_p^{2v^2}$

② $2v^2 \times (\lambda+1)/(2\lambda+1)$ zero matrix $\color{red}{Z}$

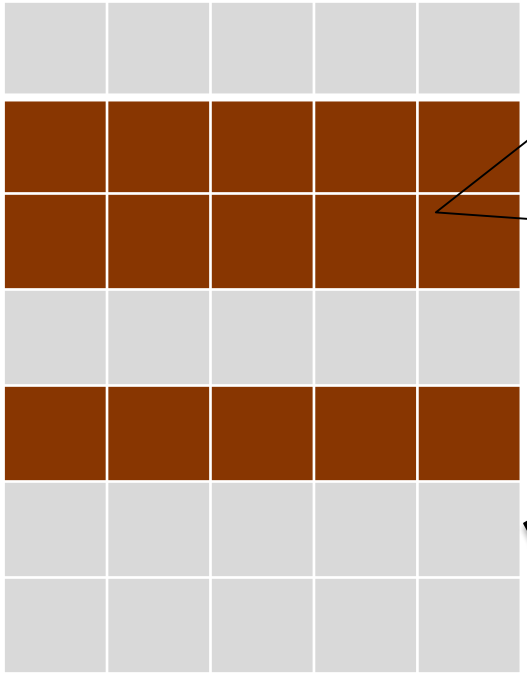
① The rank = $\lambda + 1$

② The rank < $\lambda + 1$

Privacy Analysis for Atallah-Frikken Protocols

For a COA-adversary (COA: Ciphertext-Only Attack):

- ✓ choose a matrix pair $(\mathrm{M}_{1(0)}, \mathrm{M}_{2(0)}) \leftarrow_R Z_p^{v\times v} \times Z_p^{v\times v}$ and a zero matrix pair $(\mathrm{M}_{1(1)}, \mathrm{M}_{2(1)})$
- ✓ use the rank-based analysis for the guess

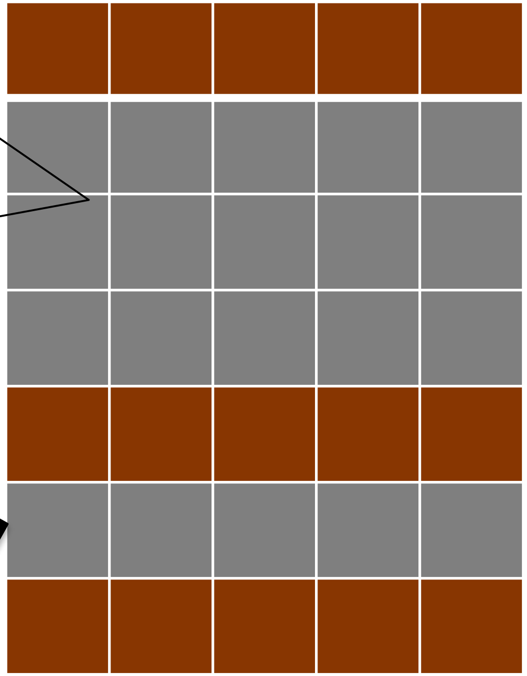


E_0

The rank = $(2\lambda + 1)/(3\lambda + 2)$. Then, this matrix involves all rows of $(\mathrm{AK} + \mathrm{S})^T$. This implies that the matrix pair is $(\mathrm{M}_{1(0)}, \mathrm{M}_{2(0)})$

distinguishable

The rank < $(2\lambda + 1)/(3\lambda + 2)$. Then, this matrix involves all rows of $(\mathrm{AK} + \mathrm{Z})^T$. This implies that the matrix pair is $(\mathrm{M}_{1(1)}, \mathrm{M}_{2(1)})$



E_1

with high probability, the COA-adversary can win the game by checking the rank of the combined ciphertext matrix $\stackrel{?}{=} (2\lambda + 1)/(3\lambda + 2)$

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Experimental Verification

- Hardware and Software:
  - ✓ Lenovo ThinkStation (Intel(R) Xeon(R) E5-2620, 24 hyperthreaded cores at 2.00GHz, 8GB RAM at 2.00GHz)
  - ✓ Windows (Windows 7, x64_64)
  - ✓ NTL library version 10.5.0
- Parameters Choice:
  - ✓ $\lambda \in \{80,\ 128,\ 192,\ 256\}$
  - ✓ $e = h = \lambda,\ \ n \in \{2\lambda + 1,\ 2\lambda + 2e + 2\},\ \ m \in \{2\lambda + 1, 3\lambda + 1, 4\lambda + 2\},$ $p > 4\lambda + 2$
- Result:
  - ✓ Adversary's advantage
  - ✓ Cost

Experimental Verification

The experimental results confirm:

- ✓ Adversary can efficiently break the decisional WSH/SSH assumption with high advantage (i.e., adv.=0.5)
- ✓ COA-adversary can efficiently break the privacy of Atallah-Frikken PVC Protocols with high advantage (i.e., adv.=0.5)

Breaking the decisional WSH/SSH assumption

Our work

Breaking the privacy of Atallah-Frikken PVC Protocols

Breaking the decisional WSH/SSH assumption

Breaking the search variant of the WSH/SSH assumption

Summary

- Break the decisional WSH/SSH assumption
- Break the privacy of Atallah-Frikken PVC Protocols for matrix multiplication
- Give some experimental results to support the theoretical argument

Thank you !! Any question ?

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