1 Lotka - Volterra model

A - prey
B - predators

$A \longrightarrow 2A$ reproduction
$A + B \longrightarrow 2B$ B feeds A
$B \longrightarrow \emptyset$ B dies

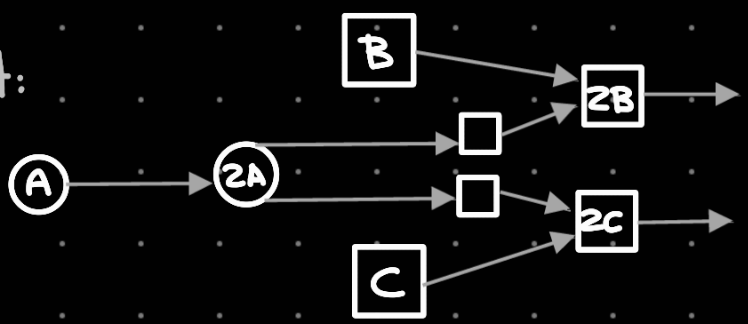
Let's add species C that feeds on A:

$A \longrightarrow 2A$ (reproduction)
$A + B \longrightarrow 2B$ (interaction)
$A + C \longrightarrow 2C$ (interaction)
$B \longrightarrow \emptyset$, $C \longrightarrow \emptyset$ (degradation)

2 Petri Net formalism

Initial state : $\mathcal{M} = \begin{pmatrix} A \\ B \\ C \end{pmatrix} = \begin{pmatrix} 120 \\ 20 \\ 50 \end{pmatrix}$

Petri net:



Transitions	Pre			Post			
	A	B	C	A	B	C	Species
Reproduction	1			2			
AB interaction	1	1			2		
AC interaction	1		1			2	
C degradation			1			0	
B degradation		1			0		

$P = \begin{pmatrix} A \\ B \\ C \end{pmatrix}$; $T = \begin{pmatrix} \text{Reproduction} \\ \text{AB interaction} \\ \text{AC interaction} \\ \text{C degradation} \\ \text{B degradation} \end{pmatrix}$; $\mathcal{M} = \begin{pmatrix} 120 \\ 20 \\ 50 \end{pmatrix} = \begin{pmatrix} A \\ B \\ C \end{pmatrix}$

$$N = (P, T, Pre, Post, \mathcal{M}) \quad Pre = \begin{pmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}, \; Post = \begin{pmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

3 Stoichiometry

$$\underbrace{A}_{\text{reaction matrix}} = Post - Pre = \begin{pmatrix} 1 & 0 & 0 \\ -1 & 1 & 0 \\ -1 & 0 & 1 \\ 0 & 0 & -1 \\ 0 & -1 & 0 \end{pmatrix}, \quad \underbrace{S}_{\text{stoichiometry matrix}} = A^T = \begin{pmatrix} 1 & -1 & -1 & 0 & 0 \\ 0 & 1 & 0 & 0 & -1 \\ 0 & 0 & 1 & -1 & 0 \end{pmatrix}$$

4 State of the system

$$\mathcal{M}_{new} = \mathcal{M} + S \cdot r$$

$$r = \begin{pmatrix} 2 \\ 4 \\ 1 \\ 0 \\ 8 \end{pmatrix} \qquad \mathcal{M}_{new} = \begin{pmatrix} 120 \\ 20 \\ 50 \end{pmatrix} + \begin{pmatrix} 1 & -1 & -1 & 0 & 0 \\ 0 & 1 & 0 & 0 & -1 \\ 0 & 0 & 1 & -1 & 0 \end{pmatrix} \begin{pmatrix} 2 \\ 4 \\ 1 \\ 0 \\ 8 \end{pmatrix}$$

$$\mathcal{M}_{new} = \begin{pmatrix} 120 \\ 20 \\ 50 \end{pmatrix} + \begin{pmatrix} -3 \\ -4 \\ 1 \end{pmatrix} = \begin{pmatrix} 117 \\ 16 \\ 49 \end{pmatrix}$$