Campus Ciudad de México School of Engineering and Sciences Department of Mechatronics	**TE2019 Digital Signal Processing Laboratory** Student's 1 name and ID: Alfredo Zhu Chen - A01651980 Student's 2 name and ID: Luis Arturo Dan Fong - A016506720 Student's 3 name and ID: Miguel Alejandro Cruz Banda - A01656527 Lab # 7 Transfer function and analysis of discrete-time LTI systems Date: 23/04/2020

1 INTRODUCTION

The transfer function *H(z)* is a property of the system that characterizes how the system modifies the input sequence to produce the output sequence. It is an intrinsic property of the system, it does not depend on the type or nature of the input or excitation, and it does not provide information on the physical characteristics of the system, in fact, systems with different structures, dimensions or physical distributions can have the same transfer function [1].

A discrete-time system is stable if, subjected to a signal of limited amplitude, it responds with an output signal that is also limited. A discrete-time system that is excited by a bounded sequence $|u(k)| < c$has BIBO stability if its impulse response converges towards 0 for $k \to \infty$. It is also stable when all poles of the transfer function lie inside the unit circle in the z-plane [2].

By knowing the step response of a system, the evolution of its outputs can be studied by having Heaviside functions as input. Information is also provided on the stability of the system and its ability to reach a steady state when starting from another. It is important to know how a system reacts to an unforeseen input, because large and rapid deviations from the steady state can have extreme effects on the components and parts of the system that depend on that component in the long term [2].

An impulse response is the reaction of any dynamic system in response to some external change. Impulse response describes the reaction of the system as a function of time or as a function of some other independent variable that describes the dynamic behavior of the system [2].

For LTI systems, the frequency response can be viewed as applying the transfer function of the system to a purely imaginary numerical argument representing the frequency of the sinusoidal drive. It is the quantitative measure of the output spectrum and a measure of magnitude and phase of the output as a function of frequency, in comparison to the input [3].

OBJECTIVES

1. Describe the value of the transfer function as a tool for the study and analysis of linear time-invariant systems
2. Apply the z-transform and the transfer function as a tool for the characterization of linear time-invariant systems

2 MATERIALS & METHODS

The materials used in this laboratory were:

- 1 PC or laptop with at least 4GB RAM and and 5kB(approximate size of three scripts) of space available
- MATLAB R2019a or newer versions

2.1 Experiment 1

In order to obtain the z-transform of (1) it was necessary to use the z-transform pair (2).

$$h[n] = n(0.4)^{n} u[n] \tag{1}$$

$$na^{n} u(n) \Leftrightarrow \frac{az^{-1}}{(1-az^{-1})^{2}} \tag{2}$$

After applying (2) on function (1) the resulting transfer function *H(z)* of the system is (3). Since the coefficients of the numerator and denominator are needed, it is developed as (4)

$$H(z) = \frac{10z}{(5z-2)^{2}} \tag{3}$$

$$H(z) = \frac{10z}{25z^{2} - 20z+4} \tag{4}$$

Having the coefficients from (4), it was necessary to define three variables in MATLAB for the coefficients of the numerator, coefficients of denominator and sample period in equations (5), (6) and (7) respectively.

$$num = [10, 0] \tag{5}$$

$$den = [25, -20, 4] \tag{6}$$

$$Ts = 0.1 \tag{7}$$

Then, the transfer function of equation (4), defined as a variable *H_z* in the program,was declared by using the MATLAB function *(8)* in which (5), (6) and (7) were used as parameters.

$$H\_z = tf(num, den, Ts) \tag{8}$$

It was necessary to obtain the poles and zeros of the transfer function (8), therefore MATLAB functions *pole(sys)* and *zero(sys)* were used as (9) and (10) in the MATLAB code.

$$poles = pole(H\_z) \tag{9}$$

$$zeros = zero(H\_z) \tag{10}$$

Once the poles and zeros were defined and obtained, the MATLAB function *pzmap(sys)* was helpful to plot the poles and zeros in a unitary circle, which the argument *sys* is the variable of the transfer function *H_z* for this case.

To compute the step response $s(n)$ of the system it was necessary to use MATLAB function (11) and plot it using *stem(Y)* with the step response *n* as the parameter.

$$s = dstep(num,\ den) \tag{11}$$

In order to compute the impulse response $h[n]$ of the system, the MATLAB function (12) was required, while function *stem(Y)* was used for plotting the impulse response with impulse response *h* as parameter.

$$h = dimpulse(num,\ den) \tag{12}$$

Given the input signal (13), the variable *n* was defined as the interval from 0 to 30, then the input signal was defined using *n.* To compute the response of the system to this input signal, MATLAB function (14) was used with the coefficients (5) and (6) and the input signal (13) as parameters. Later, *stem(Y)* was used to plot the response of the system with response of the system *r* as parameter.

$$x[n] = (0.9)^{n} u[n] \quad 0 \leq n \leq 30 \tag{13}$$

$$r = dslsim(num,\ den, x) \tag{14}$$

For computing and plotting the frequency response of the system, it was necessary to use the MATLAB function *freqz(num,den)* with the coefficients (5) and (6). If there is not any variable assigned to this function, it would automatically give the amplitude in dB and phase plots of the frequency response.

2.2 Experiment 2

To obtain the transfer function *H(z)* given equation (15), it was necessary to use the z transform pair (16).

$$h[n] = (0.9)^{n} u[n] \tag{15}$$

$$a^{n} u(n) \Leftrightarrow \frac{1}{1-az^{-1}} \tag{16}$$

After applying (16) on function (15) the resulting transfer function *H(z)* of the system is (17).

$$H(z) = \frac{1}{1 - 0.9z^{-1}} = \frac{z}{z - 0.9} \tag{17}$$

From (17), it is possible to see the coefficients of the numerator and denominator, so that led to

$$num = [1, 0] \tag{18}$$

$$den = [1, -\ 0.9] \tag{19}$$

In order to compute and plot the zeros and poles of the transfer function, step response *s(n)*, impulse response *h(n)* and frequency response of the system, the same method and functions from section 2.1 were used, but using arguments (18) and (19).

2.3 Experiment 3

For obtaining an opposite type of filter in comparison to the one found in section 2.2, it was proposed to use the opposite sign of the values for the zeros and poles on transfer function (17). Since there is a zero on 0 and a pole on 0.9 from (19) and (20), the new coefficients of the transfer function to obtain a zero on 0 and a pole on -0.9 are (18) and (21).

$$den = [1, +\ 0.9] \tag{20}$$

Once having the new coefficients, the process was exactly as the one made in section 2.2, with the difference that equations (20) substitute (19).

3 RESULTS AND DISCUSSION

3.1 Experiment 1

The following figure 3.1.1 shows the Pole-Zero Map of the transfer function (4). From (4) it is possible to notice that there should be one zero and two poles according to the power degree of the numerator and denominator respectively. In this figure, there is a zero(masked as a circle) located at 0 and a double pole(marked as a cross) located at 0.4. It is worth to also mention that the system is BIBO stable, as the double pole is inside the unitary circle.

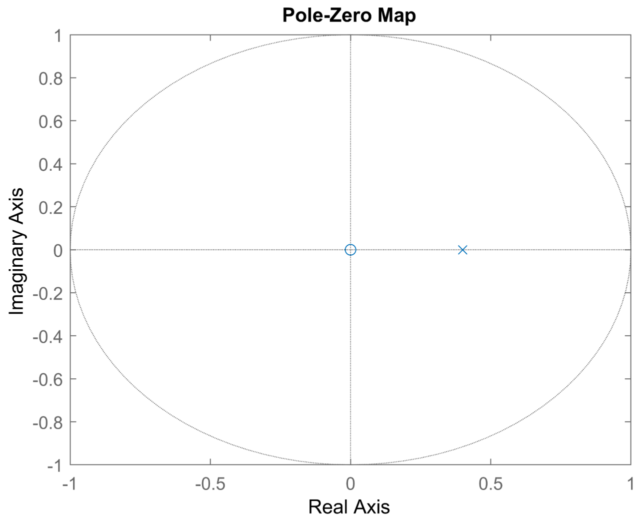

Figure 3.1.1 zeros and poles of the transfer function (4)

The first graph in Figure 3.1.2 shows the response of the system to the unit step sequence *u(n)*. From the beginning of the response, the amplitude increases progressively and smoothly until it stabilizes at a value a bit higher than 1.

The second graph is the response of the system to the unit impulse sequence. The response starts with 0, then it increases immediately to 0.4 and later it decays slowly until it stabilizes reaching to a value of 0 again. Comparing this response with the previous one, both of them tend to be stable at some value at the end, but the impulse response has a quick change at the beginning that could cause an abrupt change of the system while the other one changes more smoothly to the stable value.

The third graph shows the response of the system to the input signal (13). It is observed how the exponential function with base below than 1 is altered by the response, it increases in a very short time and then it gradually decays.

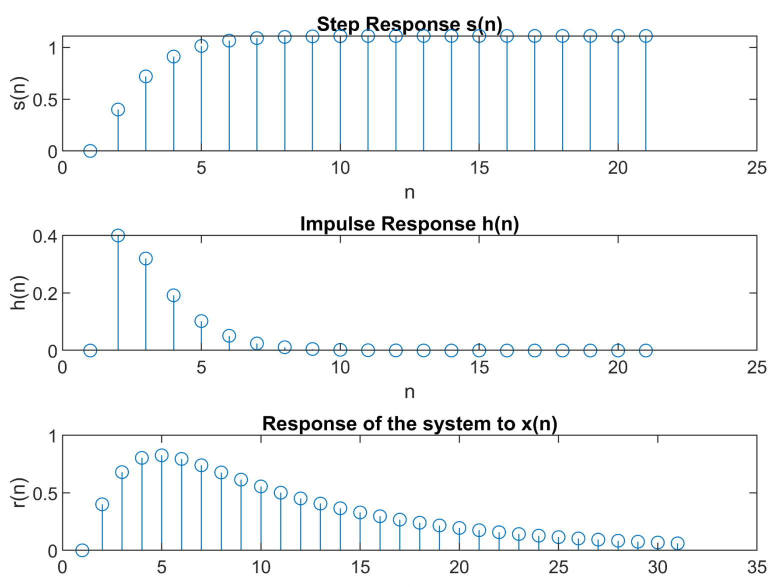

Figure 3.1.2 Different responses of the system

The following figure 3.1.3 illustrates the frequency response of the system. The first graph shows the magnitude in dB and the second one the phase in degrees within the range of the normalized frequency. Relative high levels of magnitude in dB are concentrated at low frequencies and the attenuation is higher as the frequency increases, so it is possible to say that it acts as a low pass filter for the system. There is not any gaining or amplification of the system as there is no magnitude above 0dB.

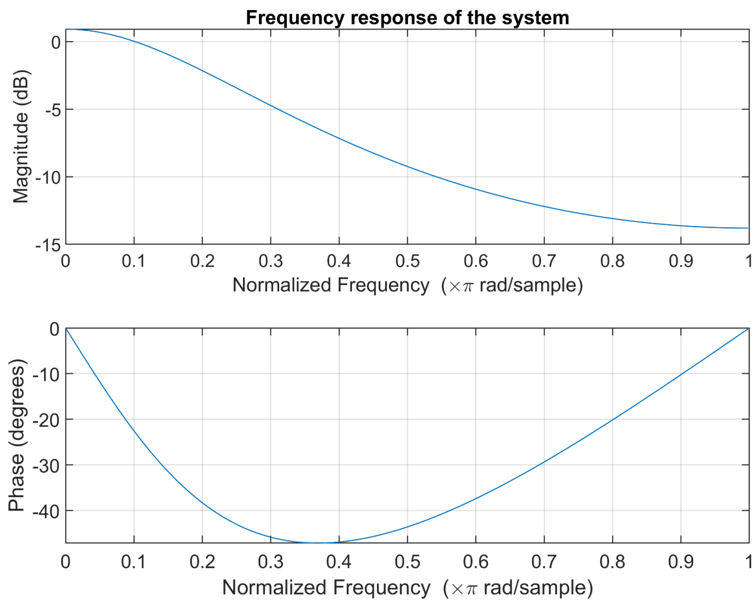

Figure 3.1.3 Frequency response of the system

3.2 Experiment 2

The Pole-Zero map is shown in the figure 3.2.1. From the transfer function (17) it is expected to have one pole and one zero because both the numerator and denominator have a power order of one. The zero is located at 0(marked as a circle) and the pole at 0.9(marked as a cross). Since the pole is inside the unitary circle, this is a BIBO stable system.

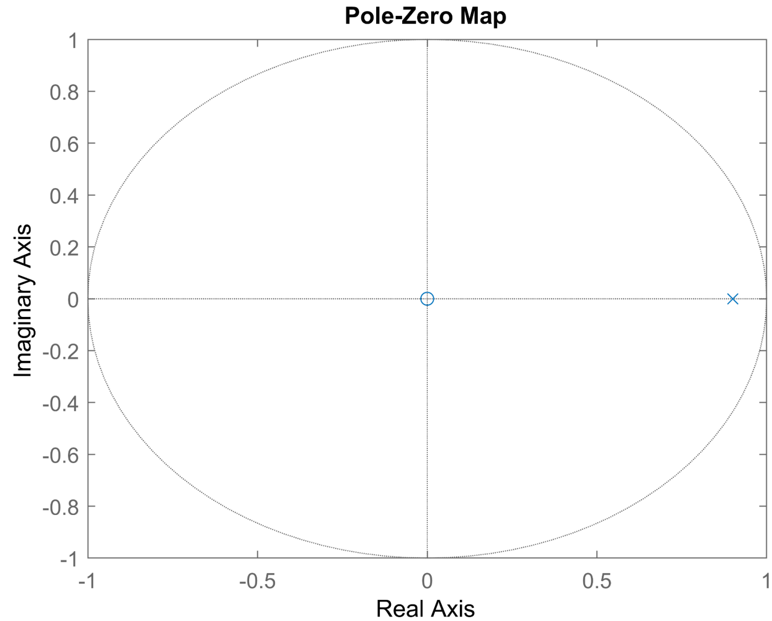

Figure 3.2.1 zeros and poles of the transfer function (17)

The step and impulse response of the system is shown in figure 3.2.2. According to the step response in the first graph, the amplitude increases gradually until it reaches a stable value at 10 approximately. In the case of the impulse response, the amplitude starts at 1 and starts decaying slowly approximating to a value of 0. Both of them stabilize at some value as n increases.

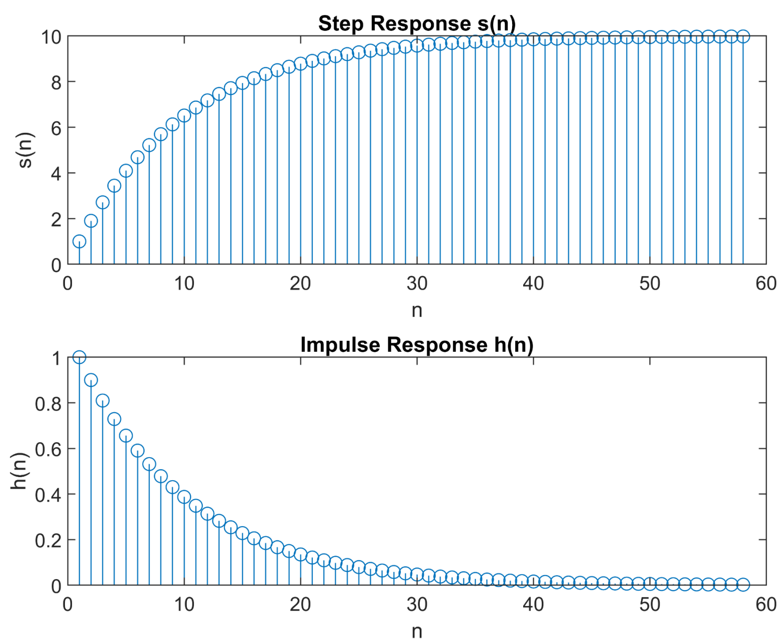

Figure 3.2.2 step and impulse response of the system

As it is shown in figure 3.2.3, the frequency response of the system is plotted, which there is the magnitude in dB in the graph on the top and the phase on the bottom. Low frequencies tend to have higher magnitude values than in higher frequencies, so it is possible to say that this is a low pass filter. According to the values of the magnitude, the filter has a gain of 20 dB starting from the beginning(DC component) and it decreases as the frequency increases.

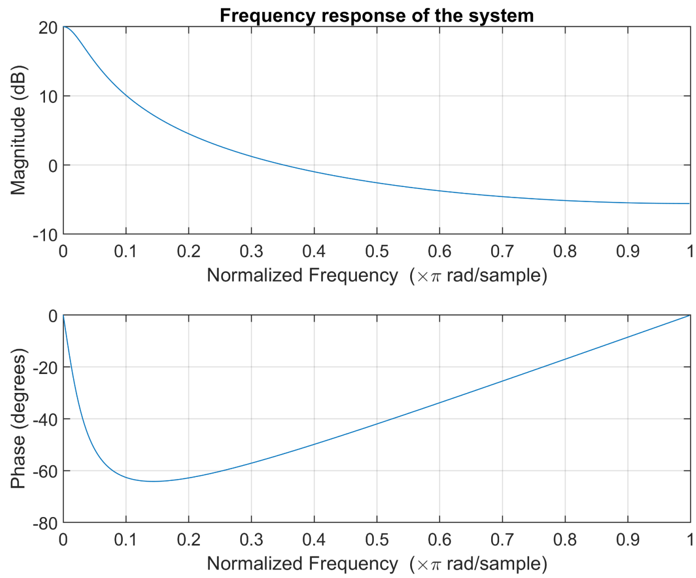

Figure 3.2.3 Frequency response of the system

3.3 Experiment 3

The next figure 3.3.1 is the Pole-Zero Map by reflecting the poles and zeros locations of the Pole-Zero Map from figure 3.2.1. So there is one zero at 0(marked as a circle) and one pole at -0.9(marked as a cross). On one hand, a zero in 3.2.1 was located at 0, so after reflecting it remains at the same point; on the other hand, the reflection of the pole located at 0.9 from figure 3.2.1 becomes -0.9. Since the pole still remains inside the unitary circle, the system is said to be also BIBO stable.

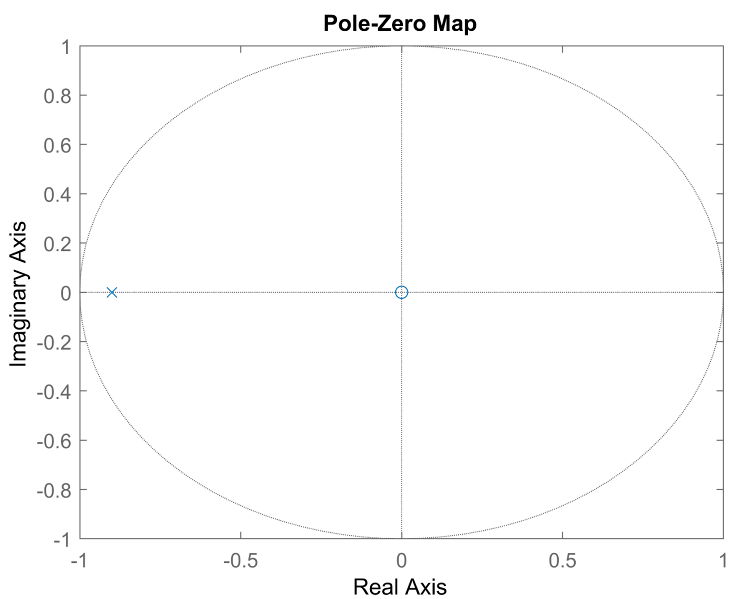

Figure 3.3.1 Reflecting the zeros and poles of the transfer function (17)

As it can be seen from the figure 3.3.2, the step and impulse response changes compared to the one obtained from section 3.2. The amplitude of step response oscillates between 0 and 1, and it stabilizes at 0.5 as *n* increases. The impulse response in this case oscillates between -1 and 1, it changes the polarity at each time and it stabilizes at a value of 0 as *n* increases.

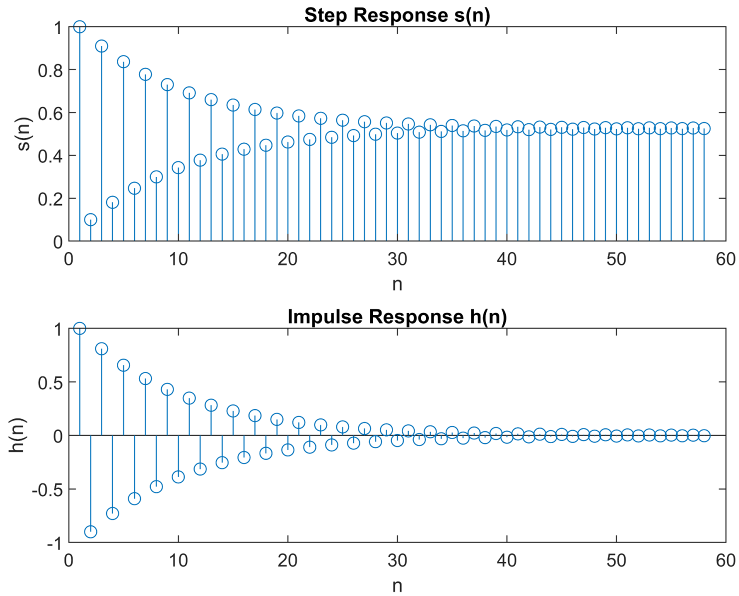

Figure 3.3.2 step and impulse response of the system after reflecting the poles and zeros of transfer function (17)

The following figure 3.3.3 shows the frequency response of the system by reflecting the locations of the poles and zeros. It is noticeable that now relatively high values of the magnitude in dB are concentrated at higher frequencies, so the low pass filter from section 3.2 becomes a high pass filter by reflecting the poles and zeros in the map. This behaviour is important to consider if there is the necessity to obtain the opposite type of a filter.

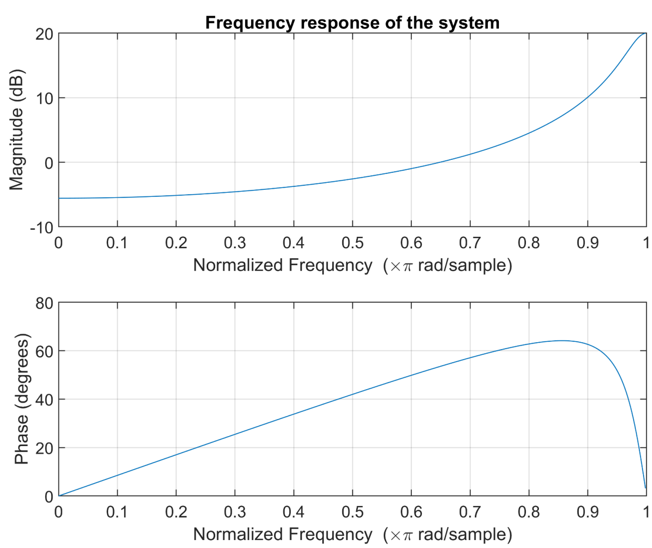

Figure 3.3.3 frequency response of the system after reflecting the poles and zeros of transfer function (17)

4 CONCLUSION

In conclusion, the importance of the usage of z-Transform and transfer function for the analysis and characterization of LTI systems as well as the relevance of MATLAB as a fundamental tool for engineers is shown throughout the report. Likewise, the Z-transform has an important role for an easier analysis of the step, impulse and frequency response of a LTI system, while showing the transfer function of a system. Finally, it is also worth to notice the effects of different responses of the systems, for instance, the frequency response of the system can give us other important information about the frequency content of the signal which time domain does not give us.

5 REFERENCES

[1] Stanley, W. D., Dougherty, G. R., Dougherty, R., & Saunders, H. (1988). Digital signal processing.
[2] Ogata, K. (1996). *Sistemas de control en tiempo discreto*. Pearson educación.
[3] Dennis L. Feucht (1990). *Handbook of Analog Circuit Design*. Elsevier Science. p. 192. ISBN 978-1-4832-5938-3.

6 APPENDIX

1. Experiment 1

```
% Lab 7.1
%%Clearing the environment
clc
clear all
close all

%% 1.1 Determine the transfer function H(z)of the system if h(n)=n(0.9.^n);
num=[10,0]; %coefficients in descending powers of numerator
den=[25,-20,4]; %coefficients in descending powers of denominator
Ts=0.1; %Sampling period
H_z=tf(num,den,Ts) %transfer function
%% 1.2 Compute and plot the zeros and the poles ofthe transfer function.
poles=pole(H_z)
zeros=zero(H_z)
pzmap(H_z)  %plot of the poles and zeros
%% 1.3 Compute and plot the step response s[n]of the system
s=dstep(num,den) %step response
figure
subplot(3,1,1)
stem(s) %plot
xlabel("n")
ylabel("s(n)")
title("Step Response s(n)")
%% 1.4 Compute and plot the impulse response h(n]of the system
h=dimpulse(num,den) %impulse response
subplot(3,1,2)
stem(h) %plot
xlabel("n")
ylabel("h(n)")
title("Impulse Response h(n)")
%% 1.5 Compute and plot the response of the system to the input signal
n=0:30; %time interval of input signal
x=0.9.^n; %input signal
r=dlsim(num,den,x) %response to the input signal
subplot(3,1,3)
stem(r) %plot
xlabel("n")
```

```
ylabel("r(n)")
title("Response of the system to x(n)")
%% 1.6 Compute and plot the frequency response of the system
figure
freqz(num,den) %%without variable assignment, the plot is obtained
title("Frequency response of the system")
```

2. Experiment 2

```
% Lab 7.2
%%Clearing the environment
clc
clear all
close all

%% 1.1 Determine the transfer function H(z)of the system.
num=[1,0];  %Coefficients of the numerator
den=[1,-0.9]; %Coefficients of the denominator
Ts=0.1; %Sample period
H_z=tf(num,den,Ts) %transfer function
%% 1.2 Compute and plot the zeros and the poles of the transfer function.
poles=pole(H_z)
zeros=zero(H_z)
pzmap(H_z) %Pole-Zero Map
%% 1.3 Compute and plot the step response s[n]of the system
s=dstep(num,den) %step response
figure
subplot(2,1,1)
stem(s) %plot
xlabel("n")
ylabel("s(n)")
title("Step Response s(n)")
%% 1.4 Compute and plot the impulse response h(n]of the system
h=dimpulse(num,den) %impulse response
subplot(2,1,2)
stem(h) %plot
xlabel("n")
ylabel("h(n)")
title("Impulse Response h(n)")
%% 1.5 Compute and plot the frequency response of the system
figure
freqz(num,den)  %without variable assignment, the plot is obtained
title("Frequency response of the system")
```

3. Experiment 3

```
% Lab 7.3
%%Clearing the environment
clc
clear all
close all
%% 1.1 Determine the transfer function H(z)of the system.
num=[1,0]   %Coefficients of the numerator
den=[1,0.9]; %Coefficients of the denominator
Ts=0.1; %Sample period
H_z=tf(num,den,Ts) %transfer function
%% 1.2 Compute and plot the zeros and the poles of the transfer function.
```

```
poles=pole(H_z)
zeros=zero(H_z)
pzmap(H_z) %Pole-Zero Map
%% 1.3 Compute and plot the step response s[n]of the system
s=dstep(num,den) %step response
figure
subplot(2,1,1)
stem(s) %plot
xlabel("n")
ylabel("s(n)")
title("Step Response s(n)")
%% 1.4 Compute and plot the impulse response h(n]of the system
h=dimpulse(num,den) %impulse response
subplot(2,1,2)
stem(h) %plot
xlabel("n")
ylabel("h(n)")
title("Impulse Response h(n)")
%% 1.5 Compute and plot the frequency response of the system
figure
freqz(num,den) %without variable assignment, the plot is obtained
title("Frequency response of the system")
```