PB groupoids & bundle gerbes

Alfonso Garmendia CRM Barcelona

1) Lie 2-groups & examples

2) PB groupoids

(3) Bundle gerbes & PB groupoids

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2) PB groupoids

3) Bundle gerbes & PB groupoids

Lie 2-aroup Homomorphisms

G (1)

Lie groups

1) Lie 2-groups & examples

2) PB groupoids

(3) Bundle gerbes & PB groupoids

Lie 2-group

Homomorphisms

G(1)

G(2)

PB groupoid

PD morphisms

G(2)

Pb morphisms

Ale groups

Principal bundles

1) Lie 2-groups & examples

2) PB groupoids

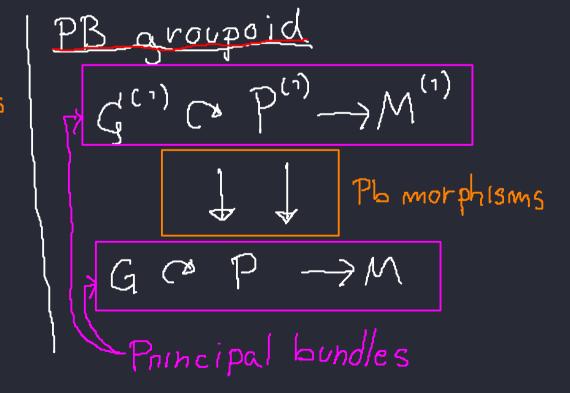
3) Bundle gerbes & PB groupoids

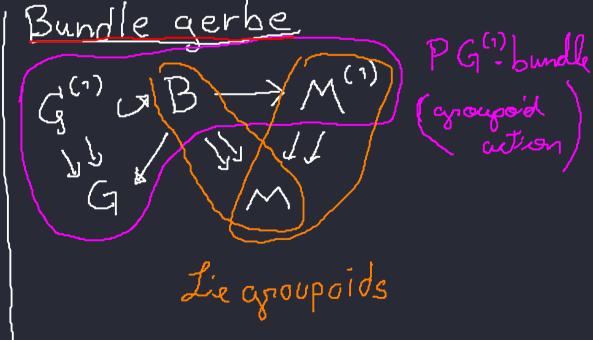
Lie 2-aroup

Homomorphisms

G(1):

Jie groups









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1) Lie 2-Groups de groups

Homomorphis

1 Lie 2-Groups de groups

Homomorphis =) Algebra nesult G(1) ~ Ker(5) NG H x G -> G(1) the majo $(h,g) \mapsto h \cdot N(g)$ is a Lie group isomorphism

1 Lie 2-Groups de groups

Homomorphis =) Algebra nesult G(1) ~ Ker(5) NG H × G -> G(1) the majo $(h,g) \mapsto h \cdot N(g)$ group isomorphism is a Lie Call H:=Ker(5) is a Lie group d=tly: H-> G is homomrf Hag hog=d(n)·g => H×G=3G Lie grd

Action grad

1) Lie 2-Groups de groups

Homomorphism => Algebra result G(1) ~ Ker(5) NG the map HxG -> d(1) $(h,g) \mapsto h \cdot N(g)$ is a Lie 2 group isomorphism (Lie group & Lie gropoid iso) Call H:-Ker(5) is a Lie group d=tly: H-> G is homomrf Hag hog=d(h)og=> H×G=3GLie grd Action appd

Prop Any Lie 2-grp 1s equivalent to a "Cnossed Module" ie. a) d: H -> G a homomrf b) CIG -> Aut (H) (consulty om's led) s.t. & geG hett h'ett d Cgh = 9(dh) 9-7 & Cdh h' = hh'h-7

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G (1) 3 Galielarp For a Crossed Mod (G,H,d,C) G 3 G a Lie Zarip (H×G, N) = (G,·) => H=Ken(s) dath H > G The action appd of Hin Gisalie 2-grp Guker(s) is a Crossed Module

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a) d: H -> G a homomrf
b) C, G -> Aut (H) (usually omited)
s.t. & ge G heth h'eth

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1 Pair gnoupoid G×G3G

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ken (5) = G × {e} ~ G N:G → G×G; g → (9,9)

$$ke_{1}(s): G \times \{e\} \sim G$$
 $N:G \rightarrow G \times G; g \mapsto (g,g)$

- · GxG~GxG (h,g)+> (h,e)·(9,9)=(h9,9)
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- 3 H normal subcroup of G HXG3G
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 - 5) TG 2 9x G is a Lie 2- group
 - 6 Pin(V) -> O(V), O(V) -> Aut (Pin(V))
 is a crossed module
 - Pin(v) x O(v) 3 O(v) is a Lie 2-group



2) PB groupoids

G (1) (2) P(1) -> M(1)

A Lie 2-group action

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G(1) (2) (1) -> M(1)

A Lie 2-group action

Def A Lie 2-group action is a Lie grp act

(H&G) × P(1) a, p(1)

ou"is a Lie grand maph i.e.

G"(2P") -> M")

A Lie 2-group action

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° a" is a Lie grad maph i.e.

This implies that

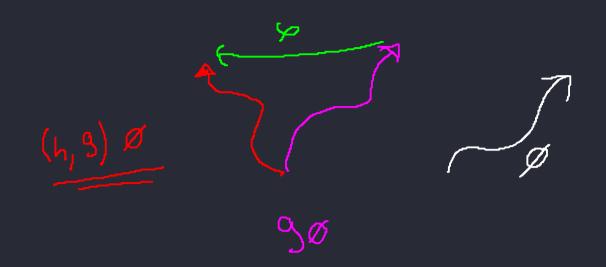
- G= {e}x Gacts by Lie grpd mph
- * the map $Y: H \times P \rightarrow P^{(r)}$, $(h_i P) \mapsto (h_i e) \cdot N(P)$

Def A Lie 2-group action is a Lie grp act

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Prox if (HxG) (AP is

free and proper

Prox is a Lie grad

over P/G



Given GaP M a principal bundle a) The identity grpds generate a PBgrpd b) Pair grpds G²a P⁽²⁾ >M⁽¹⁾ is a PB grpd

Given GaP Ma principal bundle

a) The identity arpds generate a PBarpa

b) Pair grpds G²aP⁽²⁾>M⁽¹⁾ is a PB grpd

c) Given M(1) = M any Lie grad denote P(1) = T - 1 M(1) = P_T x_t M(1) x_T P

=> (GxG)GP(7) -> M(1) is a PB grpd

Examples Given GaP M a principal bundle a) The identity appds generate a PBarpa b) Pair arpos G2aP(2) M(1) is a PB grpd C) Given M⁽¹⁾ = M any Lie grpd denote P⁽¹⁾ = T⁻¹M⁽¹⁾ = P_Tx_tM⁽¹⁾x_TP => (GxG)GP(7) -> M(1) is a PB grpd and its derivative TGUAp > Am is a Bgrpd This Last one is related to Air con and

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Examples Given GaP Ma principal bundle a) The identity grpds generate a PBgrpd b) Pair grpds GZaP(2)-M(1) is a PB grpd c) Given M⁽¹⁾ = M any Lie grpd denote P⁽¹⁾ = T⁻¹M⁽¹⁾ = P_Tx_tM⁽¹⁾x_TP => (GxG)GP(7) -> M(1) is a PB grpd and its derivative TGMAP Am is a Barpa

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d) given any surjective vector bundle map W=>E over a mfd M → V= Ker(ø) is a v.b. take P=Fnames(V) -1) M => short exact segence IR ×P≈ T*V → T*W→TE

Examples Given GaP M a principal bundle a) The identity grpds generate a PBgrpd b) Pair grpds G2aP(2)-M(1) is a PB grpd C) Given M(1) 3 M any Lie grad denote P(1) = T M(1) = Prxt M(1) x P > (G×G) GP(7) -> M(1) is a PB grpd and its derivative TGUAP > Am is a PBgrpd

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d) given any surjective vector bundle map W=>E over a mfd M → V= Ker(ø) is a v.b. take $P = F_names(V) \xrightarrow{\pi} M$ Short exact segence RxP≈ T*V → T*W→T*E



(3) Bundle gerbes & PB grps

- A bundle gerbeis
- HXG (A) B(1) -> M(1) (Anoupoid-action)
 bundle

 Grand mrph

 - st. Y KIEHAG bie B => (K1*b1)0(K2*b2)= (K1·K2)*(b1/b2)

$$S(9,b) = (9,5(b))$$
 $t(9,b) = (9,6),t(b))$

$$(h,9)(9,b) = (99,(c_{16},9^{-1}5^{-1}h,16))*b)$$

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Inverse funday

· grpd mrph

$$S(9,b) = (9,5(b))$$
 $t(9,b) = (9,6),t(b))$

$$(h,9)(9,b) = (99,((p_{(b)})^{-1}g^{-1}h,f(b))*b)$$

Baqueires a grouperd str P(1)/G

Inverse funday

IT works for morphisms

10 g

1 Lie 2-Groups $G^{(1)} \Rightarrow G \Rightarrow G^{(1)} \Rightarrow G$ =) Algebra nesult G(1) ≈ Ker(5) NG

Call H:=Ker(s) is a Lie group d=tln: H -> G is homomrf H ca G h·g = d(h)·g > H×G 3 G Lie grd the map H×G -> d(1) Action

 $(h, g) \mapsto h \cdot N(g)$ is a Lie 2-group isomorphism (Lie group & Lie gropoid isso)

 $G \times G \ni G$

· G > G ~ G * G

ken (5) = G × {e} ~ G

 $N:G \to G \times G_{\mathfrak{Z}} \quad \mathfrak{Z} \longmapsto (\mathfrak{Z},\mathfrak{Z})$

 $(h, \S) \mapsto (h, e) \cdot (\S, \S) = (h\S, \S)$

G×G is the pain anoupoid

Prop Any Lie 2-grp is equivalent to a "Cnossed Module", i.e. a) $d: H \rightarrow G$ a homomrf b) C : G > Aut (H) (corrally omited) s.t. & ge G hett h'ett d Cgh = 9(dh) 97 & Cdh h' = hh' h-1

For a Crossed Mod (G,H,d,C) Proof (1) G a Le Zarp (H×G, n) = (G,·) => H=Ken(s)

d=t|4:H→G the action appd of Hin G is a Lie 2-grp Guker(s) is a Crossed Module

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Eamples 3 H nomal subgroup & G HXG3G 1) Pair gnoupoid

4) given a morph G > Gl(V) the conoviral map $V \xrightarrow{d} G, V \rightarrow e_G$ induce a crossed module so $V \times G \not\ni G$ is a Lie 2-group

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6 Pin(v) \xrightarrow{P} O(v), $O(v) \rightarrow Aut(Pin(v))$ · G>G is action groupoid of Gong is a crossed module Pin(v) × O(v)] O(v) is a Lie 2-group

thank you!

(2) PB anoupoids $G^{(1)}$ $\hookrightarrow P^{(1)} \rightarrow M^{(1)}$

Def A Lie 2-group action is a Lie grp act (HAG) × P(1) a P(1) s.t. "a" is a Lie grad maph ie. This implies that

• G = {e}xG acts by Lie grpd mph

The map $\gamma: H \times P \to P^{(r)}$, $(h_i P) \mapsto (h_i e) \cdot N(P)$ salisfies (h, 9) \$ = \((h, 9tc \(\pi)) \((\frac{9}{3} \pi)

> Proop if (HNG)(1) is free and proper → P(1)(Hag) is a Lie grpd

over P/G

 $(h_1, 9) \emptyset$

Examples

Given GGP M a principal bundle a) The identity grpds generate a PBgrpd b) Pair grpds G²a P⁽²⁾>M⁽¹⁾ is a PB grpd c) Given M(1) 3 M any Lie grpd denote P(1) = T-1 M(1) = Prxt M(1) x p

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R^k×Gl(K) → T*W → E Glus of P ---> M

(3) Bundle gerbes & PB gross

A bundle gerbe is

[A bund

Huguaxo -> May a PB $\begin{array}{cccc}
\downarrow \downarrow & \downarrow \downarrow & \downarrow \downarrow & \text{arpd} \\
G & G & G \times M & \to M
\end{array}$ S(9,6)=(9,56) t(9,6)=(9,6)=(9,6) (h,g)*(3',b)=(33',(c/16)3'-3'-1,16)*b)

Inverse funday H&G Ly P(1) -2 M(1) $G \mapsto G \times M \longrightarrow M$

⇒P⁽¹⁾ ~ G×Ker(Prgo5) Liggeo
B

→ Baqueires a groupoid str P 1/G P(1) & G xB

IT works for morphisms Mon (P(1)) = (HOG) equivariant Magos

Mor(B) = H ×6 equivariant Maps