Presentation for the FES Project

Option Analysis With Advanced Artificial Intelligence Techniques

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March 29, 2024

NATIONAL RESEARCH
UNIVERSITY

Abstract

What is the problem?

We want to come up with a universal, applicable, efficient, scalable, and open-source method for estimating (and forecasting) option prices. It turns out that AI tools seem to be sufficient for such purposes.

Why is it relevant?

Options are popular instruments. However, they are generally considered as advanced investment tools (high risks and uncertainty). Therefore, precise, reliable, open, and available to public methods will be beneficial both for students, companies, and traders.

What we propose?

The classic way to solve this problem is to apply well-known analytic solutions (such as Black-Scholes formula and its extensions). However, this family of methods has several limitations. To eliminate most of them, we will switch to using AI tools.

Benchmark (Black-Scholes model)

The Black-Scholes formula is used to calculate the theoretical price of European-style call options. It is given by:

$$C(S_0, t) = N(d_1)S_0 - N(d_2)Xe^{-r(T-t)}. \quad (1)$$

$$d_1 = \frac{\log\left(\frac{S_0}{X}\right) + \left(r + \frac{\sigma^2}{2}\right)(T-t)}{\sigma\sqrt{T-t}}. \quad (2)$$

$$d_2 = d_1 - \sigma\sqrt{T-t}. \quad (3)$$

S_0 - current price of the underlying asset, $N(x)$ - cumulative distribution function of the standard normal distribution, X - strike price of the option, r - risk-free interest rate, T - time to expiration, t - current time, σ - volatility of the underlying asset.

Derivation (p. 1)

Dynamics of stocks are given as a Geometric Brownian motion:

$$dS_t = \mu S_t dt + \sigma S_t dZ_t. \tag{4}$$

Change in the price of an option $f(t, S_t)$ is described by Ito's Lemma:

$$df = (\frac{\partial f}{\partial t} + \mu S_t \frac{\partial f}{\partial S_t} + \frac{1}{2}\sigma^2 \frac{\partial^2 f}{\partial {S_t}^2})dt + \sigma S_t \frac{\partial f}{\partial S_t} dZ_t. \tag{5}$$

In order to eliminate risk, we apply the principle of dynamic hedging to construct a portfolio with deterministic dynamics:

$$\Pi = -f(t, S_t) + \frac{\partial f}{\partial S_t} S_t \Rightarrow d\Pi = (-\frac{\partial f}{\partial t} - \frac{1}{2}\sigma^2 S_t^2 \frac{\partial^2 f}{\partial {S_t}^2})dt. \tag{6}$$

Derivation (p. 2)

Since portfolio is deterministic, it must grow exactly at a risk-free rate, therefore, $d\Pi = r\Pi dt$. Combining equations and plugging everything into:

$$rf = \frac{\partial f}{\partial t} + rS_t\frac{\partial f}{\partial S_t} + \frac{1}{2}\sigma^2 S_t^2\frac{\partial^2 f}{\partial {S_t}^2}. \tag{7}$$

We obtained the Black-Scholes PDE. The Black-Scholes formula is a solution to this equation given the following boundaries:

- We never execute the option when the underlying asset costs nothing, therefore, $f(t, 0) = 0$.
- At maturity (T), the asset will cost S_T, and the strike price remains being X. If the deal is profitable ($S_T > X$), we will exercise the option, and the profit will be exactly $S_T - X$. Otherwise, we will not exercise the option, so its value will be 0. Hence, $f(T, S_T) = max(S_T - X, 0)$.
- If $S_t \to +\infty$, the probability of exercising the option approaches 1.

Significance of Black-Scholes model

Historical significance

Model brought a new (systematic, clear, and rather easy to implement) quantitative approach to pricing options. People developed option trading strategies (using the principle of dynamic hedging) and started doing more research related to options.

The way it is applied today

While the Black-Scholes model is unlikely to be used in its original form, the modern approach to options trading is based on the principle of dynamic hedging (at least partially). Moreover, Black-Scholes model is used to calculate implied volatility.

Extensions of the model

Several "more realistic"models were created (e.g. Heston, Jump-Diffusion). Moreover, researchers continue to offer various modifications of the model. However, the main attention turns to the use of neural networks.

Assumptions of the Black-Scholes model

- Stock prices follow a random walk. So, prices of financial assets cannot be predicted with consistent accuracy.
- Stock prices are log normally distributed.
- Stock returns are normally distributed.
- Volatility is constant over time.
- Stocks do not pay dividends.
- Risk-free interest rate does not fluctuate over time.
- Options can only be exercised at expiration.
- Markets are frictionless (no transaction costs).
- There are no arbitrage opportunities in the market.

Extensions of Black-Scholes

The assumption of constant volatility has been a subject of criticism. There were invented methods that model volatility as a function of time. One example is the family of stochastic volatility models.

Heston model:

$$dS_t = rS_t dt + \sqrt{V_t} S_t dZ_{1t}. \tag{8}$$

$$dV_t = k(\theta - V_t)dt + \sigma \sqrt{V_t} dZ_{2t}. \tag{9}$$

V_t is the volatility of an asset price, while σ is the volatility of the volatility. The Heston model does not have analytic solutions, and is thus solved numerically (for example, using Monte Carlo simulation).

Another possible approach is to use GARCH, where volatility is a deterministic function, which is estimated using historical data and parameters.

Assumptions we make

- The data for modeling is of high quality and is large enough.
- We do not put prior distributions on the evolvement of prices of underlying assets and returns on them.
- Risk-free is constant. In the process of modeling, the rate on US treasuries (which we consider risk-free) is unlikely to change.
- Companies do not pay dividends (since we do not have data representing dividends).
- Market can be inefficient, there may be transactions costs, and the short-selling opportunities are likely to be restricted.
- Volatility is not constant.
- Options are not required to be European-style. However, we analyze only call options.
- Additional features (besides the ones used in Black-Scholes model and its extensions) affect the prices of options and should be included in the analysis.

Why using AI-tools is justified

Limitations of the classical methods:

- 1. Rely on many assumptions.
- 2. Do not fully take into account the characteristics of individual markets and companies.
- 3. Cannot take certain features as an input (for example, categorical).
- 4. Work slow on inference.
- 5. Are not appropriate for forecasting.

With implicit modeling we can:

- 1. Estimate prices of all types of options with far less assumptions.
- 2. Learn directly from data (cannot learn irrelevant things).
- 3. Use unlimited number of features.
- 4. Perform hypothesis testing.
- 5. Approximate an arbitrary function (by Universal Approximation Theorem).

Justifications and restrictions of the method

Justifications

- 1. Intuitive: our assumptions imply that there cannot be an analytic or a numerical solution constructed from the differential equation.
- 2. Theoretical: by Universal Approximation Theorem, neural networks can estimate any continuous function. Linear Regression provides us with BLUE estimates.
- 3. Practical: approach is applicable for big data, high frequency trading, and forecasting.

Restrictions

- We face both time and resource limits in terms of acquiring data. It does not mean our data is bad, but there always can be better.
- We may not choose the best available model (since there are too many options). Moreover, each model has both advantages and disadvantages (assumptions, work speed, tendency to overfit).

What researchers found out before us

- Neural networks perform better (more accurately) than the Black-Scholes model on the same set of features.
- Neural networks make predictions faster (compared to classical option pricing methods) on new data.
- It is useful to use neural networks that work with sequences.
- Transformer architecture outperforms other (less complex) neural network architectures.

Benefits of our research

- We will perform a large number of experiments (to assess models, speed, and quality).
- We are interested not only in obtaining high-quality models, but also in the meaningful economic interpretation of the results.
- We will use a significant number of additional features and also test hypotheses.
- We do an open research (public data, available code, reproducible results).

Hypotheses

- Artificial intelligence methods will be faster and more accurate compared to the explicit models.
- Additional features will be useful (models will pay attention to them).
- We can get a high-quality forecasting model without taking into account the price of the underlying asset.
- The model improves as the architecture becomes more complex.

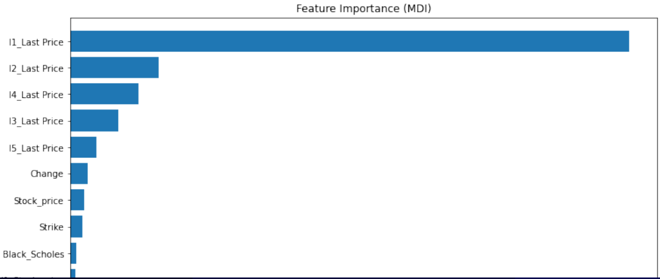
Results achieved so far (p.1)

- A script has been written that collects fresh data on options for 35 US companies.
- The dataset is updated every day and has a panel data structure (as of today there are around 900 000).
- Over 15 additional features were created (lags, market capitalization).
- A comparison of Black-Scholes formula with Linear Regression model on the same features was made (proof of the concept).
- A comparison of a Linear Regression on all features and on the features used by the Black-Scholes model was made.
- A Gradient Boosting model was used to determine the most important features.
- Pipelines for using other models (Random Forest, MLP) were created.

Results achieved so far (p.2)

Model	MSE	MAE	MAPE
LinReg (unrestricted)	37.49	1.94	4.21
LinReg (restricted)	12364	58.4	115.19
CatBoost (unrestricted)	43.83	2.32	3.4
CatBoost (restricted)	561	10.33	10.82

Таблица: Comparison of models



Plans for the future

- Obtain approximately 1 000 000 observations by the beginning of April.
- Conduct a fair comparison of analytic solutions with artificial intelligence methods.
- Select the best model (interpretability, quality metrics, speed).
- Test the hypotheses.
- Make meaningful conclusions and economically justify the results obtained.

Thank you for your attention!