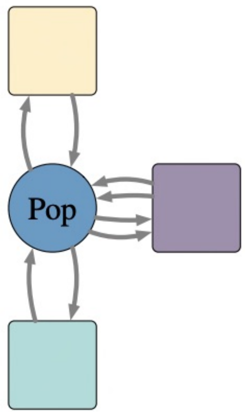
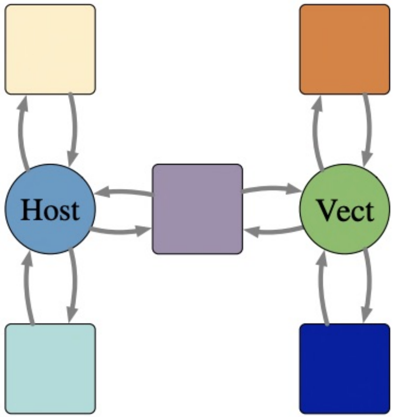
0. Typed Petri nets

A Petri net can represent a domain-specific type system



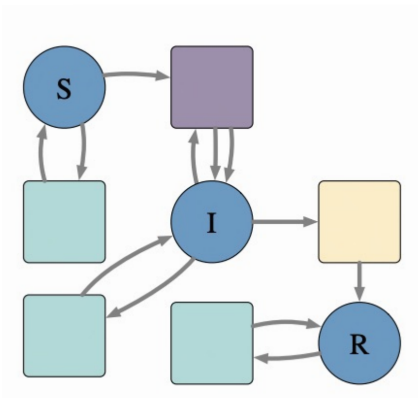
type system for
infectious diseases



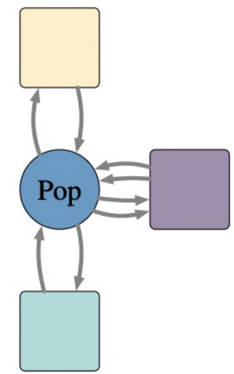
type system for
vector-borne diseases

0. Typed Petri nets

A **typed Petri net** is a $\mathcal{C}$-set homomorphism $P \to P_{type}$



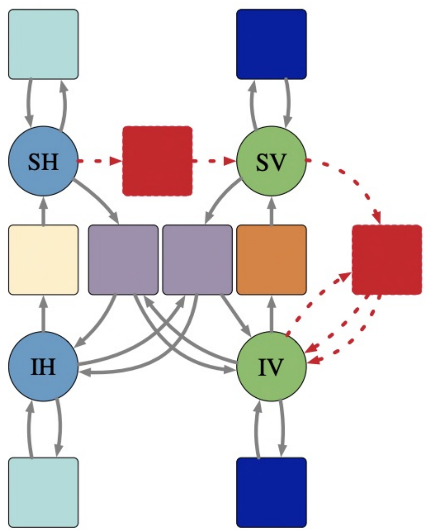
$$\longrightarrow$$

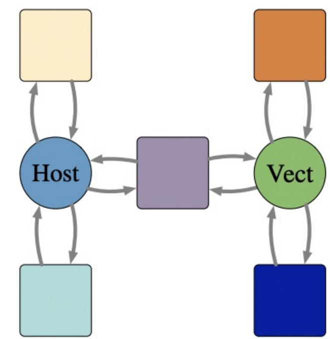


$P_{\mathbf{type}}$ = type system for infectious diseases

0. Typed Petri nets

A typed Petri net is a $\mathcal{C}$-set homomorphism $P \to P_{type}$

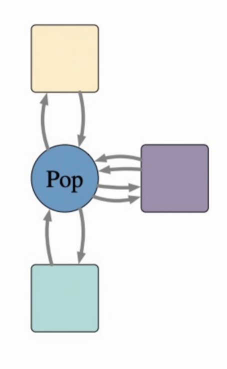




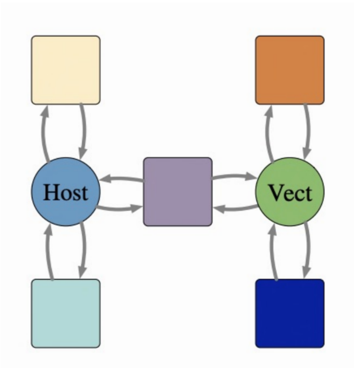
P_{type} = type system for vector-borne diseases

0. Typed Petri nets

Advantages:



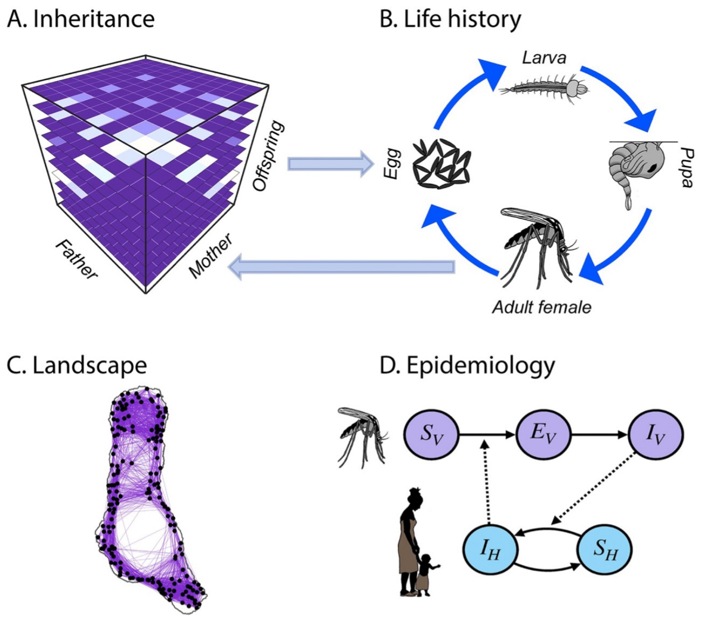
1. model checking
2. facilitate high-level critiques
3. features of type system imply features of model



4. guardrails for composition
5. Models typed by P_{type} are organized in a slice category

I. Motivation

Stratified Models



Wu et al., 2021

I. Motivation

Comparing metapopulation dynamics of infectious diseases under different models of human movement

Daniel T. Citron[a,1], Carlos A. Guerra[b], Andrew J. Dolgert[a], Sean L. Wu[c], John M. Henry[a], Héctor M. Sánchez C.[c], and David L. Smith[a]

disease models

- SIR

$$\frac{dS}{dt} = -\beta \frac{SI}{N}$$

$$\frac{dI}{dt} = \beta \frac{SI}{N} - \gamma I.$$

$$\frac{dR}{dt} = \gamma I$$

movement models

- flux

$$\frac{dN_i}{dt} = -\sum_{j=1}^{K} f_{i,j} N_i + \sum_{j=1}^{K} f_{j,i} N_j,$$

I. Motivation

Comparing metapopulation dynamics of infectious diseases under different models of human movement

Daniel T. Citron[a,1], Carlos A. Guerra[b], Andrew J. Dolgert[a], Sean L. Wu[c], John M. Henry[a], Héctor M. Sánchez C.[c], and David L. Smith[a]

disease models

- SIR

$$\frac{dS}{dt} = -\beta \frac{SI}{N}$$
$$\frac{dI}{dt} = \beta \frac{SI}{N} - \gamma I.$$
$$\frac{dR}{dt} = \gamma I$$

movement models

- flux
- simple trip

$$\frac{dN_{i,i}}{dt} = -\sum_{j=1}^{K} \phi_{i,j} N_{i,i} + \sum_{j=1}^{K} \tau_{i,j} N_{i,j}.$$
$$\frac{dN_{i,j}}{dt} = -\tau_{i,j} N_{i,j} + \phi_{i,j} N_{i,i}$$

I. Motivation

Comparing metapopulation dynamics of infectious diseases under different models of human movement

Daniel T. Citron[a,1], Carlos A. Guerra[b], Andrew J. Dolgert[a], Sean L. Wu[c], John M. Henry[a], Héctor M. Sánchez C.[c], and David L. Smith[a]

disease models

- SIR
- SIS
- Ross-Macdonald

⋮

movement models

- flux
- simple trip

⋮

Products

- A type of limit
- Relatively simple, but important
- An abstraction of
  Cartesian products of sets

Products

def A category $\mathcal{C}$ has products if we can choose

- for every pair of objects x and y, an object $x \times y$

- such that morphisms into $x \times y$ are in natural bijection with pairs of morphisms into x and y.

$$\mathrm{Hom}_{\mathcal{C}}(z, x \times y) \simeq \mathrm{Hom}_{\mathcal{C}}(z, x) \times \mathrm{Hom}_{\mathcal{C}}(z, y)$$

II. Limits

Products

$\mathcal{C} = \mathrm{Set}$

def A category $\mathcal{C}$ has products if we can choose

- for every pair of objects x and y, an object $x \times y$

  $\mathbb{N}$ $\mathbb{R}$ $\mathbb{N} \times \mathbb{R}$

- such that morphisms into $x \times y$ are in natural bijection with pairs of morphisms into x and y.

$$\mathrm{Hom}_{\mathcal{C}}(z, x \times y) \cong \mathrm{Hom}_{\mathcal{C}}(z, x) \times \mathrm{Hom}_{\mathcal{C}}(z, y)$$

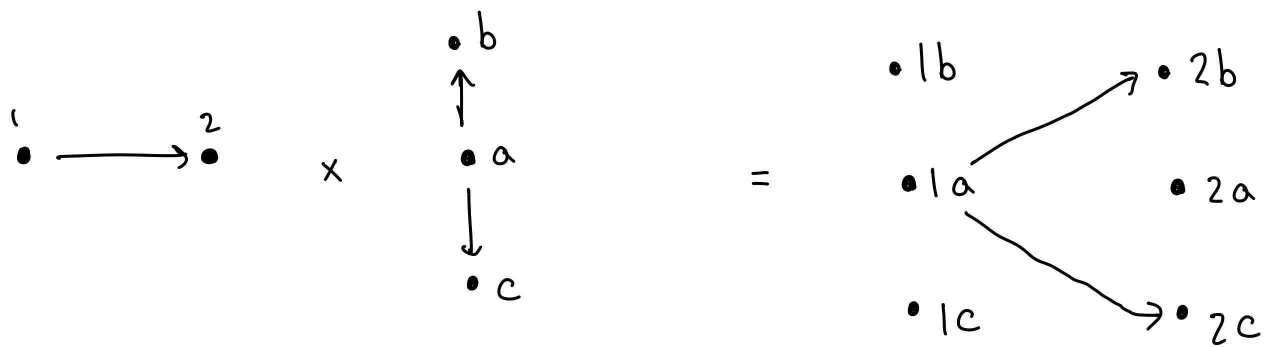
$$(\mathrm{age}, \mathrm{height}) : \mathrm{Ppl} \to \mathbb{N} \times \mathbb{R} \longleftrightarrow (\mathrm{age} : \mathrm{Ppl} \to \mathbb{N},\ \mathrm{height} : \mathrm{Ppl} \to \mathbb{R})$$

II. Limits

Products

In Graph:

Recall a graph is a $\mathcal{C}$-Set with schema $E \underset{tgt}{\overset{src}{\rightrightarrows}} V$

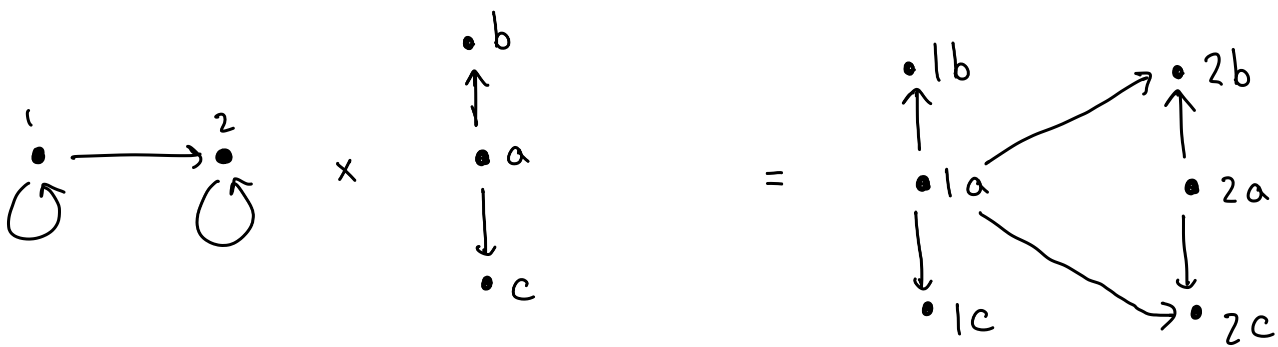


II. Limits

Products

In Graph:

Recall a graph is a $\mathcal{C}$-Set with schema $E \underset{tgt}{\overset{src}{\rightrightarrows}} V$

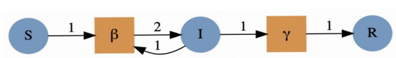


Products

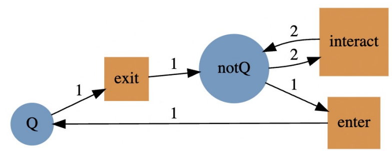
In Petri :

Recall a Petri net is a $\mathcal{C}$-Set with schema

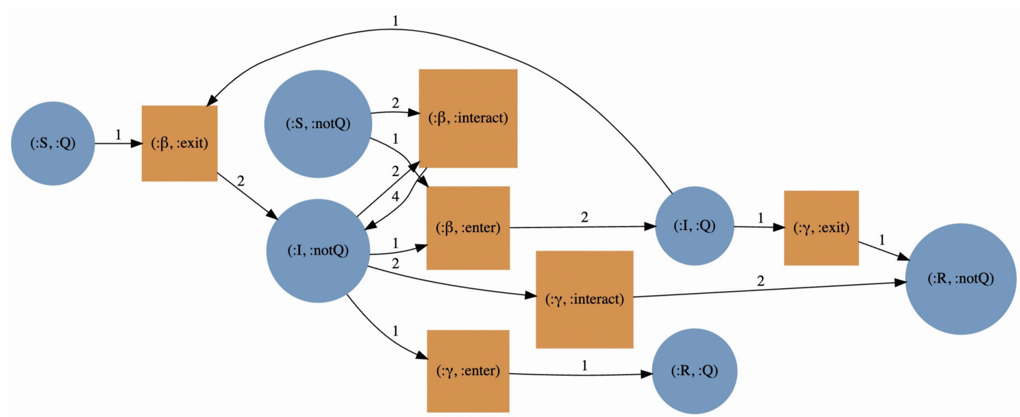
$$\begin{tikzcd} & S & \\ I \arrow[ur] \arrow[dr] & & O \arrow[ul] \arrow[dl] \\ & T & \end{tikzcd}$$



×

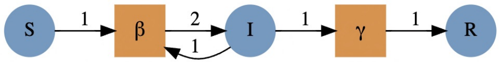


=



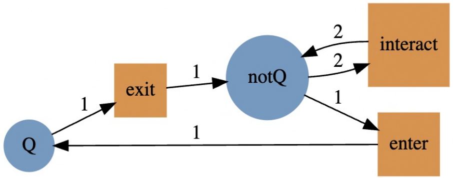
```
sir = LabelledPetriNet([:S, :I, :R],
    :β => ((:S, :I) => (:I, :I)),
    :γ => (:I => :R)
)

Graph(sir)
```



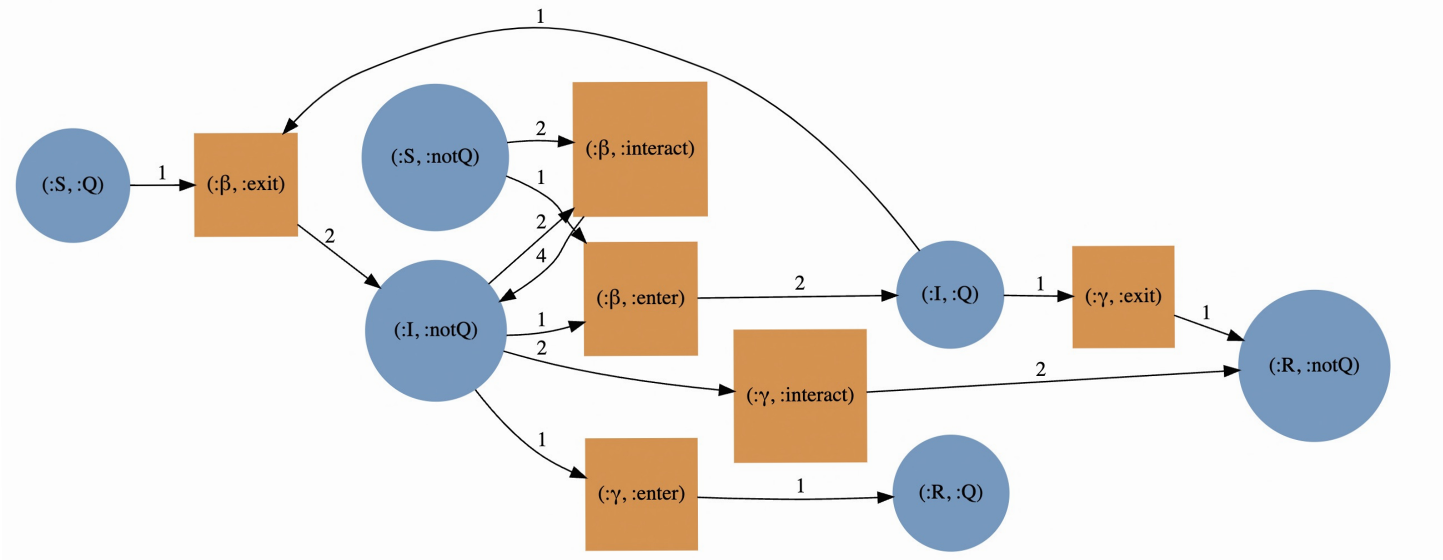

```
quarantine = LabelledPetriNet([:Q, :notQ],
    :enter => (:notQ => :Q),
    :exit => (:Q => :notQ),
    :interact => ((:notQ, :notQ) => (:notQ, :notQ))
)

Graph(quarantine)
```

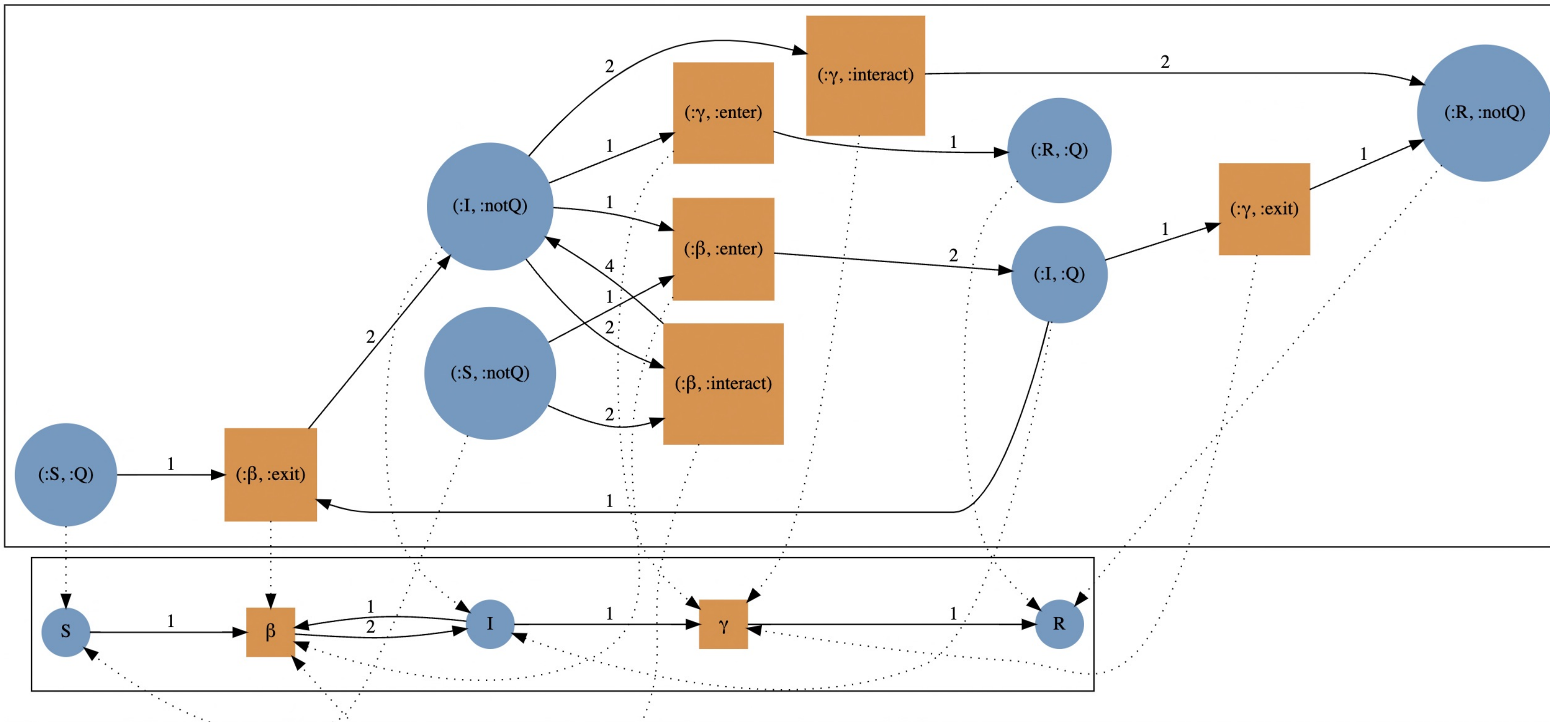


```
product_model = product(sir, quarantine)

Graph(apex(product_model))
```



```
Graph(legs(product_model)[1])
```



Pushouts

- A type of limit
- A generalization of product

Pushouts

- A type of limit
- A generalization of product

II. Limits

Pushouts

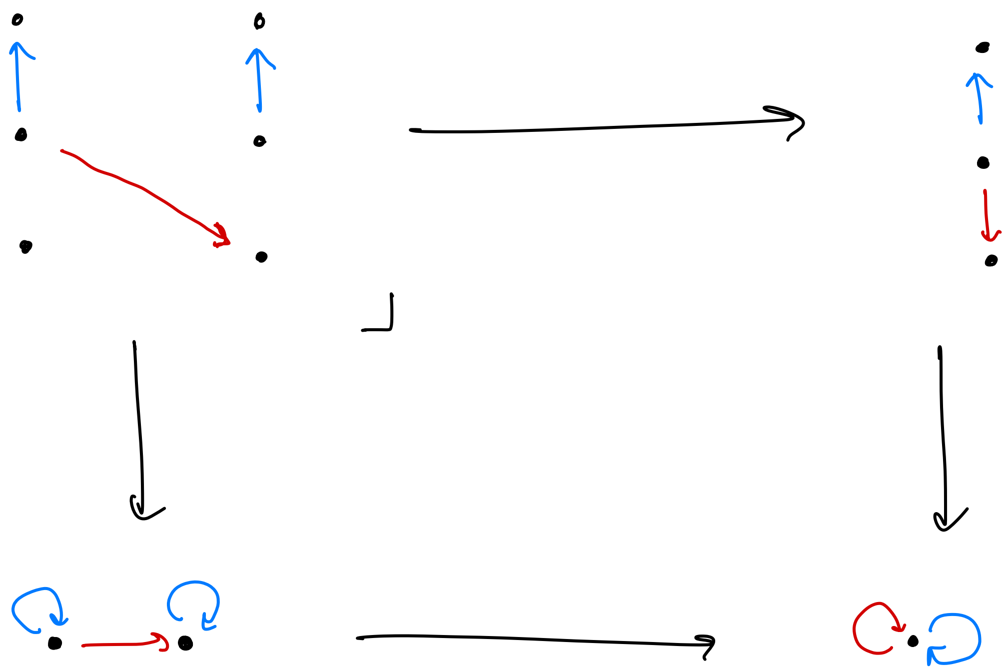
In Set

$$\begin{tikzcd}
 & \{(x,y) \mid f(x)=g(y)\} \arrow[dl] \arrow[dr] & \\
X \arrow[dr, "f"'] & & Y \arrow[dl, "g"] \\
 & Z &
\end{tikzcd}$$

$$\begin{tikzcd}
\text{Ppl} \arrow[dr, "\text{favorite animal}"'] & & \text{Pets} \arrow[dl, "\text{species}"] \\
 & \text{species} &
\end{tikzcd}$$

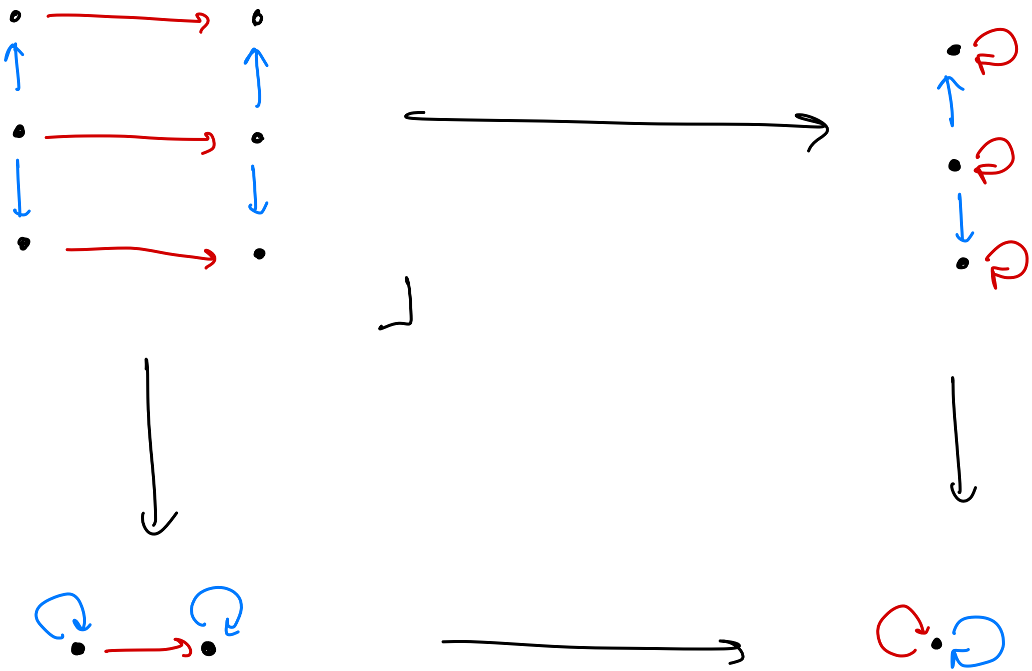
Pushouts

In Graph:



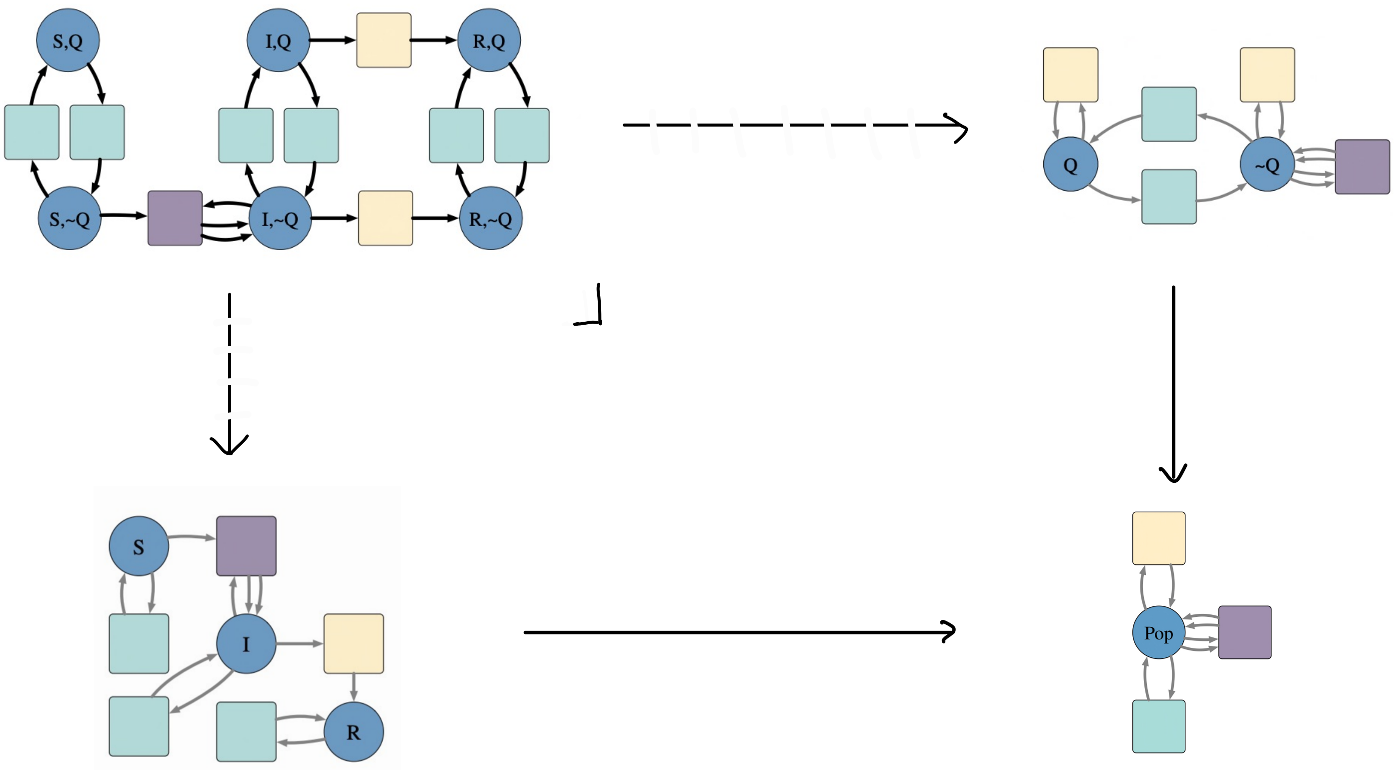
Pushouts

In Graph:



II. Limits

Pushouts in Petri:

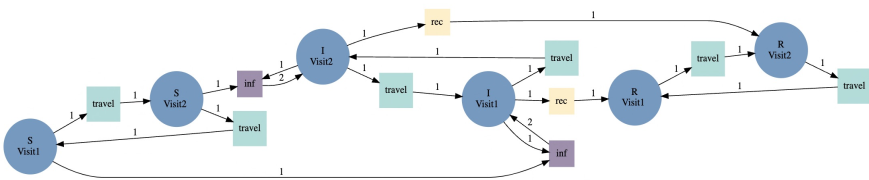


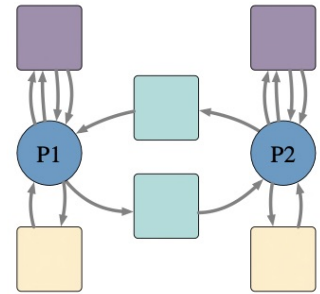
III. Stratified Models

A product in $\mathbf{Petri}/P_{\text{type}}$ is a stratified Petri net

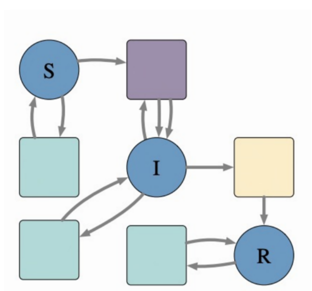
$$\begin{tikzcd}
P_{\text{stratified}} \arrow[r, dashed] \arrow[d, dashed] \arrow[dr, phantom, "\lrcorner", very near start] & P_{\text{strata}} \arrow[d] \\
P_{\text{disease}} \arrow[r] & P_{\text{infectious}}
\end{tikzcd}$$

III. Stratified Models

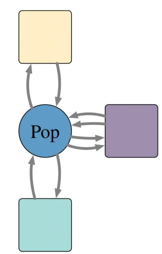




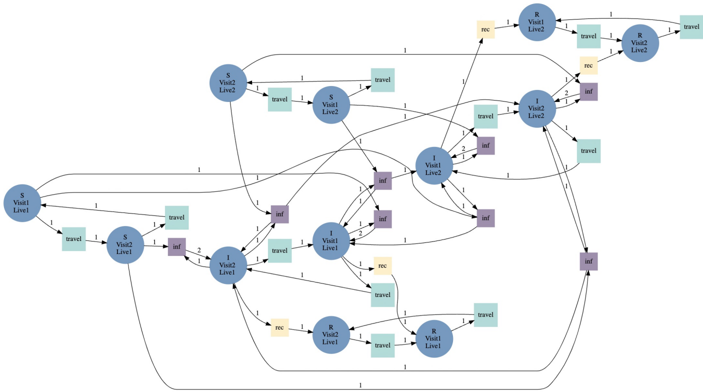
movement model

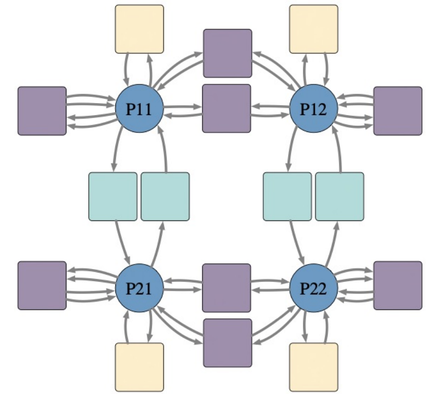


disease model

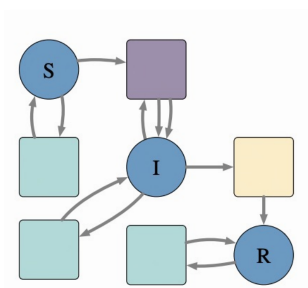


III. Stratified Models

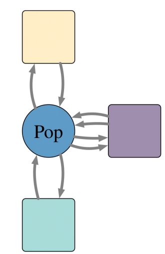




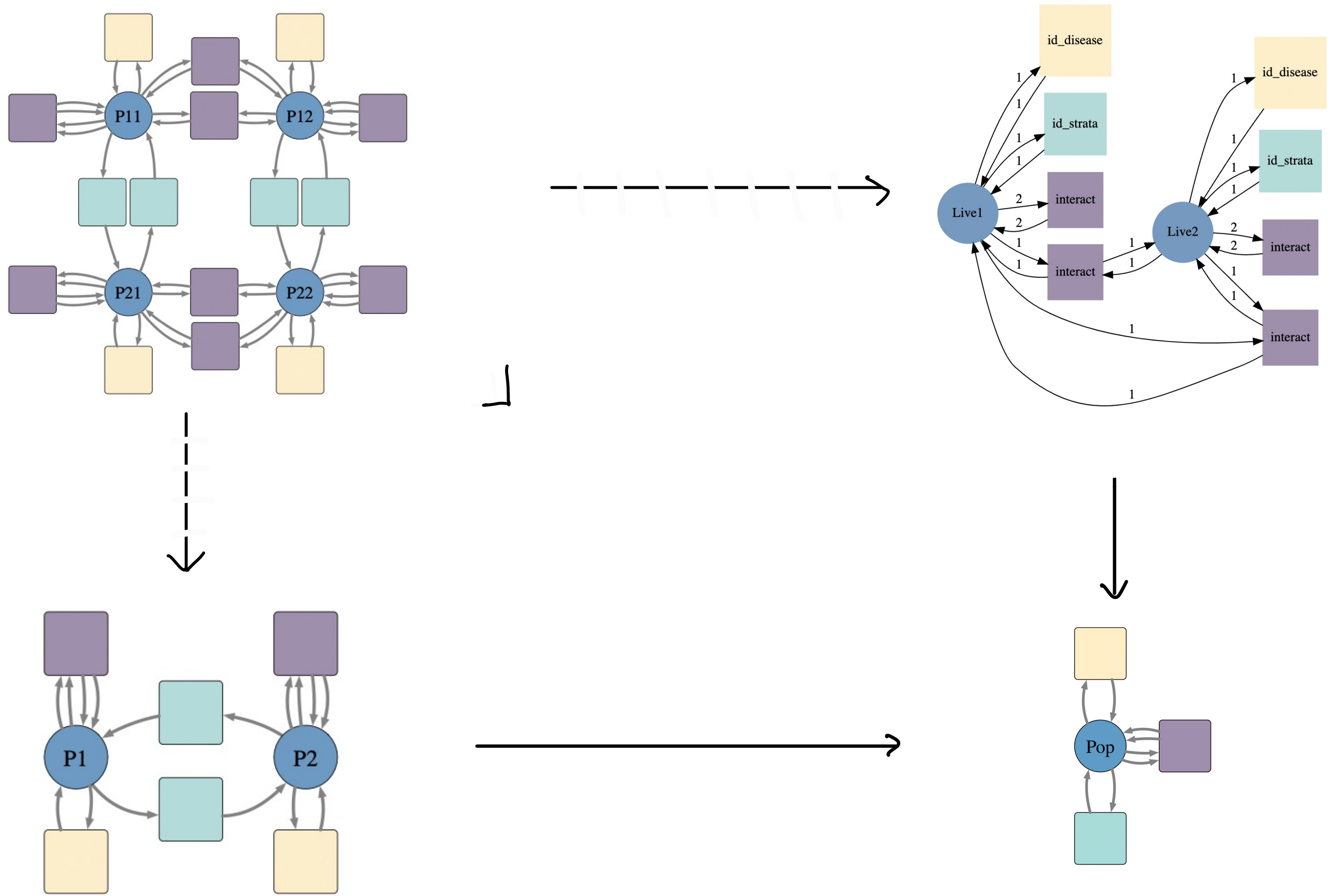
movement model



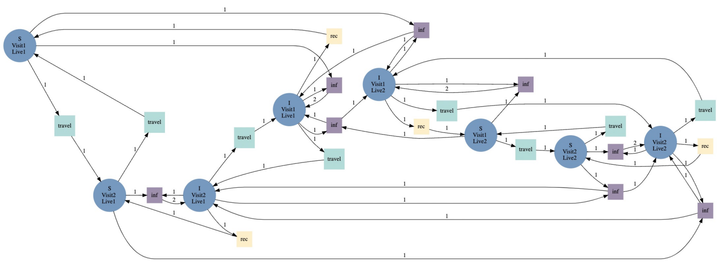
disease model

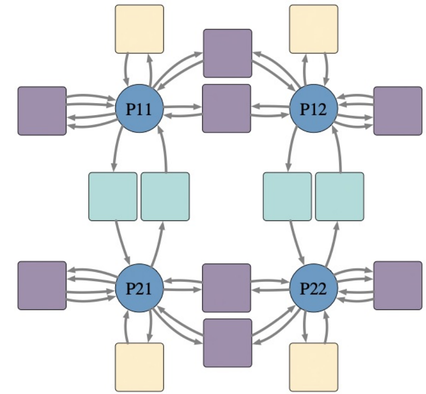


III. Stratified Models

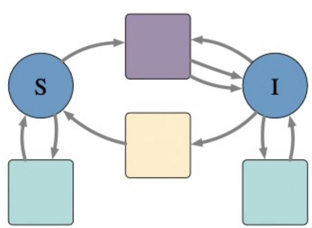


III. Stratified Models





movement model



disease model