Example Ising State

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The Ising state is a graph homomorphism

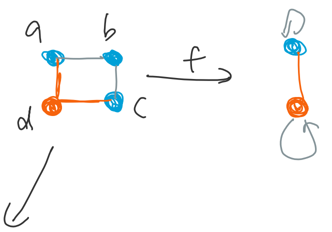
$$f : G \to H$$

where H 'encodes' the interaction pattern.

e.g.

let's label blue nodes w/ '1'
orange nodes w/ '2'

Then



$$V(G) = \{(a,1), (b,1), (c,1), (d,2)\}$$

$$E(G) = \{((a,1),(b,1)), ((a,1),(d,2)),$$
$$\cap \quad ((b,1),(c,1)), ((c,1),(d,2)),$$
$$V(G) \times V(G) \quad ((a,1),(a,1)), \dots, ((d,2),(d,2))\}$$

Note that $\pi_1 V(G) = \{\pi_1 d \mid d \in V(G)\}$ gives only the position a label and we lose information about the state

and $\pi_2 V(G)$ tells us about state but not position.

Also, the 'color' (since more than half are N. American) of the edge gives interaction type.

Hence if $e \in E(G)$ and

$$\pi_2 \circ \pi_1 (e) = \pi_2 \circ \pi_2 (e)$$

we have grey edges and if

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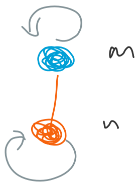
$\pi_2 \circ \pi_1(e) \neq \pi_2 \circ \pi_2(e)$

we have orange edge.

Now we need to find a homomorphism f into

$H :=$ 

which has

$V(H) = \{(m,1), (n,2)\}$

$E(H) = \{((m,1),(n,2)), ((m,1),(m,1)), ((n,n),(n,2))\}$

Action of f on vertices

Let $\underset{V(G)}{\underset{\cap}{f(v)}} = \underset{V(H)}{\underset{\cap}{v'}} \iff \pi_2 v = \pi_2 v'$

Thus, f sends blue nodes of G to the blue node of H, and same for orange.

Action of f on edges

$$f(e) = e' \iff \left(\begin{array}{c} \pi_2 \circ \pi_1(e) = \pi_2 \circ \pi_2(e) \\ \text{and} \\ \pi_2 \circ \pi_1(e') = \pi_2 \circ \pi_2(e') \end{array} \right)$$

so, for example, $f(((a,1),(b,1))) = ((n,1),(n,1))$

C-sets over the schema below.

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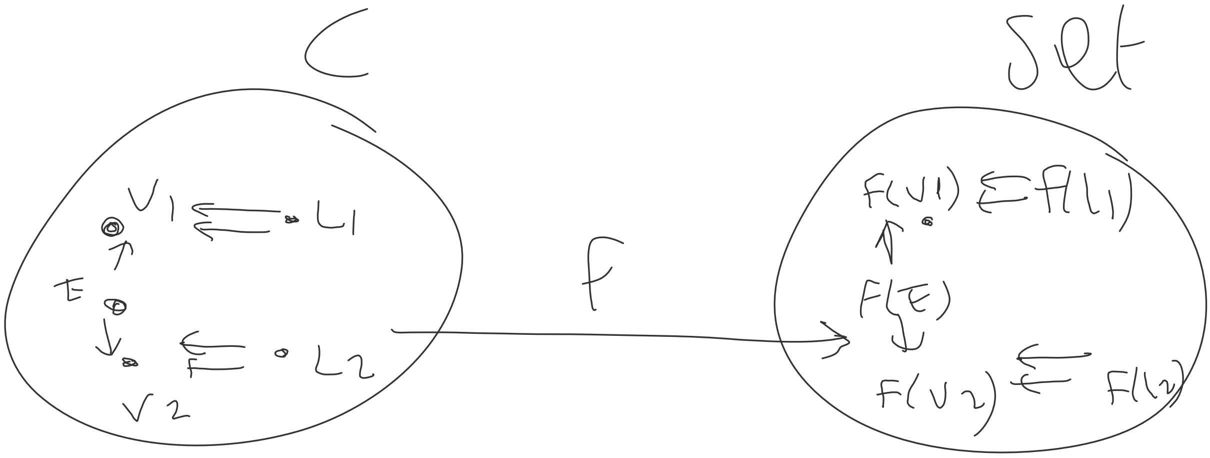
$$\begin{tikzcd} V_1 & L_1 \arrow[l, shift left] \arrow[l, shift right] \\ E \arrow[u] \arrow[d] & \\ V_2 & L_2 \arrow[l, shift left] \arrow[l, shift right] \end{tikzcd}$$

are copresheaves

$$\boxed{C \longrightarrow set}$$



In order to define the C-set F I need to

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F I need to define its action
on objects and morphisms.

$F(V_1)$ will be the set of
blue vertices

i.e in the example before, it
will be

$$F(V_1) = \{ (a,1), (b,1), (c,1) \}.$$

and V_2 will be the set of
orange vertices

$$F(V_2) = \{ (d,2) \}.$$

$F(E)$ will be the set of orange edges

$$\{ ((a,1), (d,2)), ((c,1), (d,2)) \}.$$

and so $E \to V_1$ will just

and so $E \to V_1$ will just be the projection of $e \in E$ onto the vertex labelled '1', and $E \to V_2$ will be the projection of e onto the vertex labelled '2'

$F(L_1)$ consists of edges between blue nodes, so here will be:

$$F(L_1) = \{((a, '1'), (b, '2')), ((b, '1'), (c, '1'))\}$$

and $L_1 \rightrightarrows V_1$

will project both vertices in $e \in L_1$ into V_1.

Same for L_2,
(except $L_2 = \{((d, 2), (d, 2))\}$
So the grey edges here are kind of trivial.)