

UF

Discrete Exterior Calculus Tips & Tricks:

All you need is $\star d \wedge$

GATAS Lab Seminar Series, Fall 2023

Luke Morris

\$HOME=/UF/HWCOE/CISE



Motivation

72 A. M. TURING ON THE CHEMICAL BASIS OF MORPHOGENESIS

from one pattern into another, rather than from homogeneity into a pattern. One would like to be able to follow this more general process mathematically also. The difficulties are, however, such that one cannot hope to have any very embracing *theory* of such processes, beyond the statement of the equations. It might be possible, however, to treat a few particular cases in detail with the aid of a digital computer. This method has the advantage that it is not so necessary to make simplifying assumptions as it is when doing a more theoretical type of analysis. It might even be possible to take the mechanical aspects of the problem into account as well as the chemical, when applying this type of method. The essential disadvantage of the method is that one only gets results for particular cases. But this disadvantage is probably of comparatively little importance. Even with the ring problem, considered in this paper, for which a reasonably complete mathematical analysis was possible, the computational treatment of a particular case was most illuminating. The

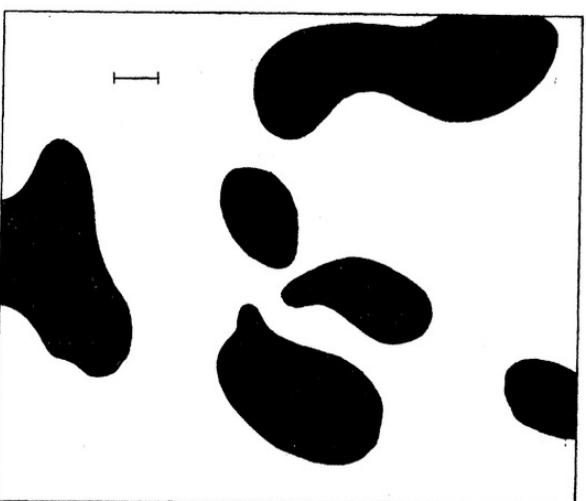
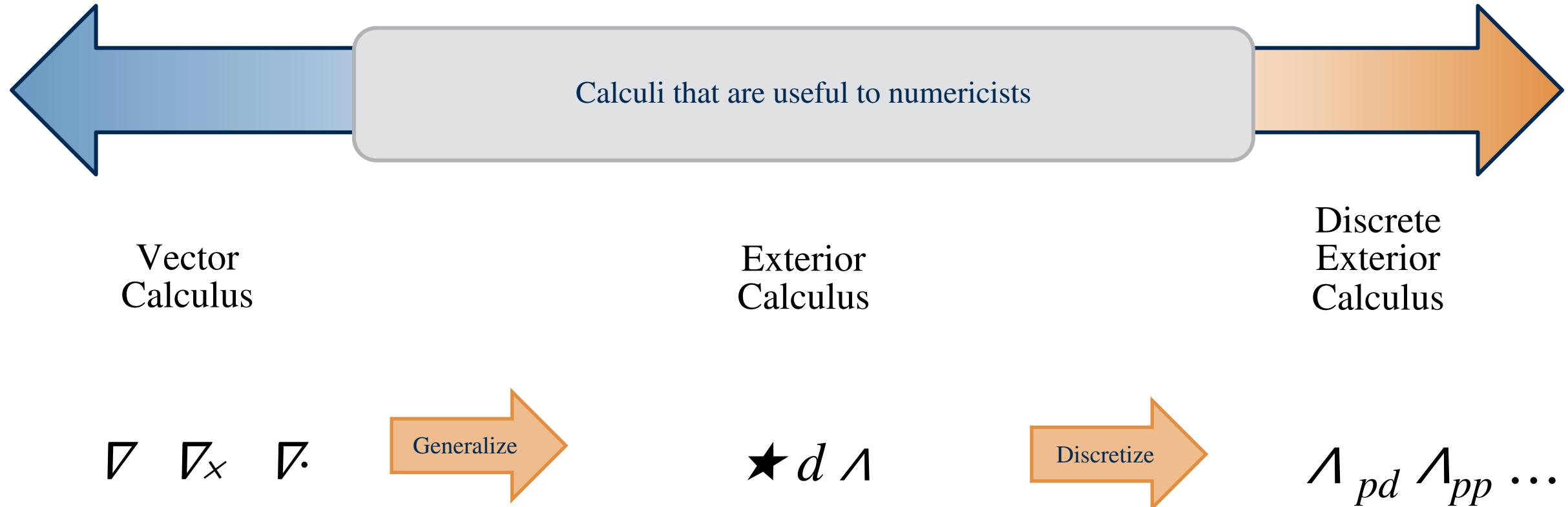


FIGURE 2. An example of a 'dappled' pattern as resulting from a type (a) morphogen system.
A marker of unit length is shown. See text, §9, 11.

Spectrum of Calculus “Utility”



de Rahm Complex

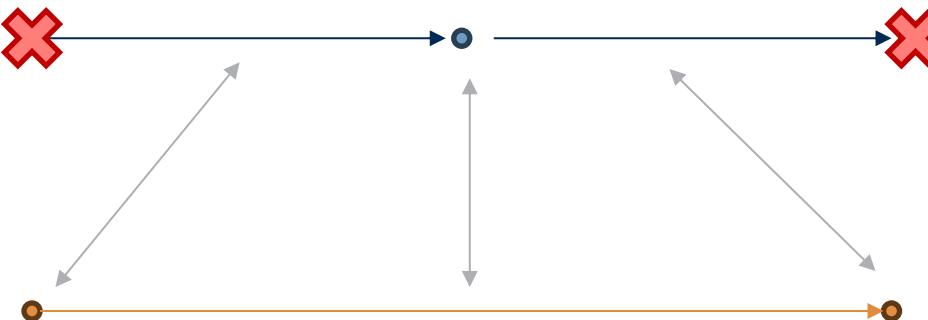
$$\begin{array}{ccc}
 \Omega_0 & \xrightarrow{d} & \Omega_1 \\
 \star \downarrow & \uparrow \star^{-1} & \downarrow \star \\
 \tilde{\Omega}_n & \xleftarrow{\tilde{d}} & \tilde{\Omega}_{n-1}
 \end{array}$$

n=1 DEC de Rahm Complex

$$\begin{array}{ccccccc}
 \Omega_0 & \xrightarrow{d} & \Omega_1 & \xrightarrow{d} & \Omega_2 & \xrightarrow{d} & \Omega_3 \\
 \star \downarrow & \uparrow \star^{-1} & \star \downarrow & \uparrow \star^{-1} & \star \downarrow & \uparrow \star^{-1} & \star \downarrow \\
 \tilde{\Omega}_n & \xleftarrow{\tilde{d}} & \tilde{\Omega}_{n-1} & \xleftarrow{\tilde{d}} & \tilde{\Omega}_{n-2} & \xleftarrow{\tilde{d}} & \tilde{\Omega}_{n-3}
 \end{array}$$

n=2 DEC de Rahm Complex

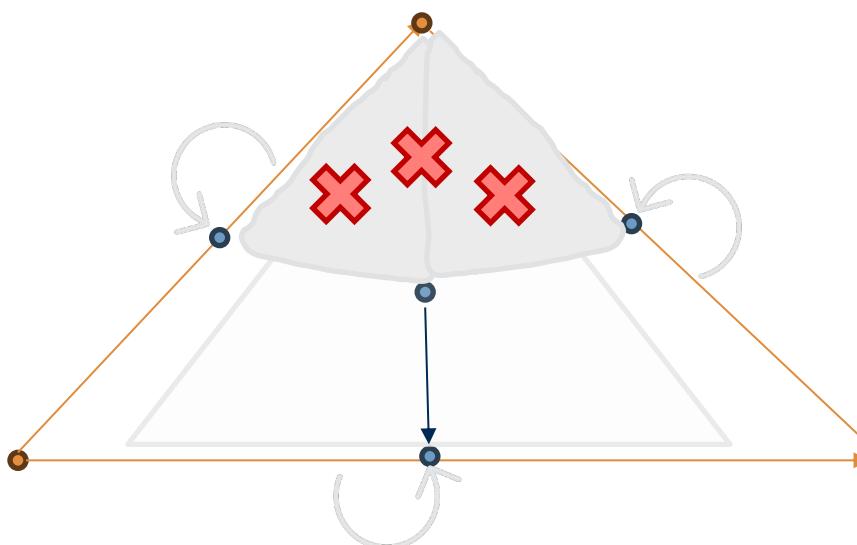
de Rahm Complex Meshes



Legend

- Primal Mesh (Orange circle)
- Dual Mesh (Blue circle)
- Hodge Dual (Grey circle)
- Support simplex (Red circle with 'X')

$n=1$ DEC Representative



$n=2$ DEC Representative

d: Exterior Derivative



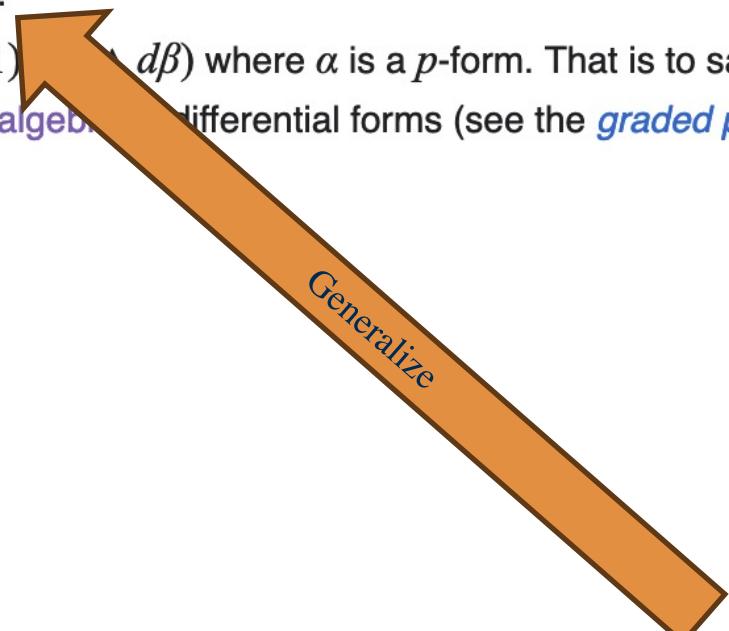
Exterior derivative

Contents [hide]

(Top)

Definition

1. df is the differential of f for a 0-form f .
2. $d(df) = 0$ for a 0-form f .
3. $d(\alpha \wedge \beta) = d\alpha \wedge \beta + (-1)^{|\alpha|} \alpha \wedge d\beta$ where α is a p -form. That is to say, d is an antiderivation of degree 1 on the exterior algebra of differential forms (see the *graded product rule*).



Contents [hide]

(Top)

Curl of gradient is zero [edit]

The curl of the gradient of any continuously twice-differentiable scalar field φ (i.e., differentiability class C^2) is always the zero vector:

$$\nabla \times (\nabla \varphi) = \mathbf{0}$$

Vector calculus identities

Article Talk

From Wikipedia, the free encyclopedia

Exterior Derivative



Exterior derivative

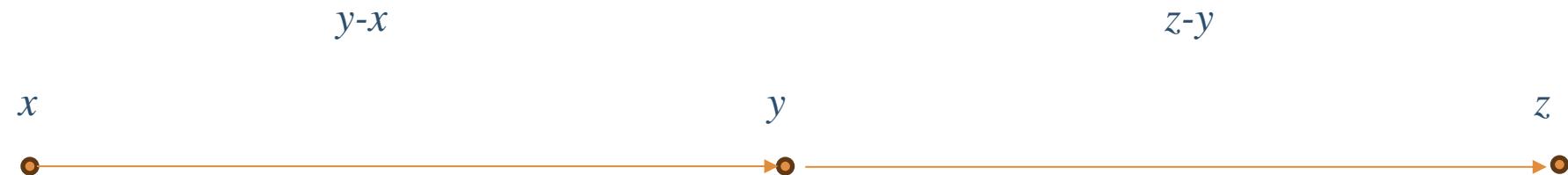
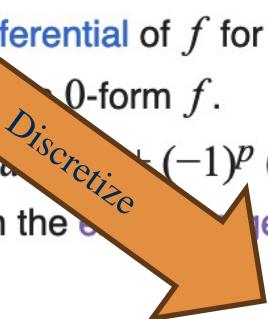
Contents [hide]

(Top)

Definition

1. df is the differential of f for a 0-form f .
2. $d(df) = 0$ for any 0-form f .
3. $d(\alpha \wedge \beta) = d\alpha \wedge \beta + (-1)^p (\alpha \wedge d\beta)$ where α is a p -form. That is to say, d is an antiderivation of degree 1 on the exterior algebra of differential forms (see the *graded product rule*).

Discretize



Exterior Derivative



Exterior derivative

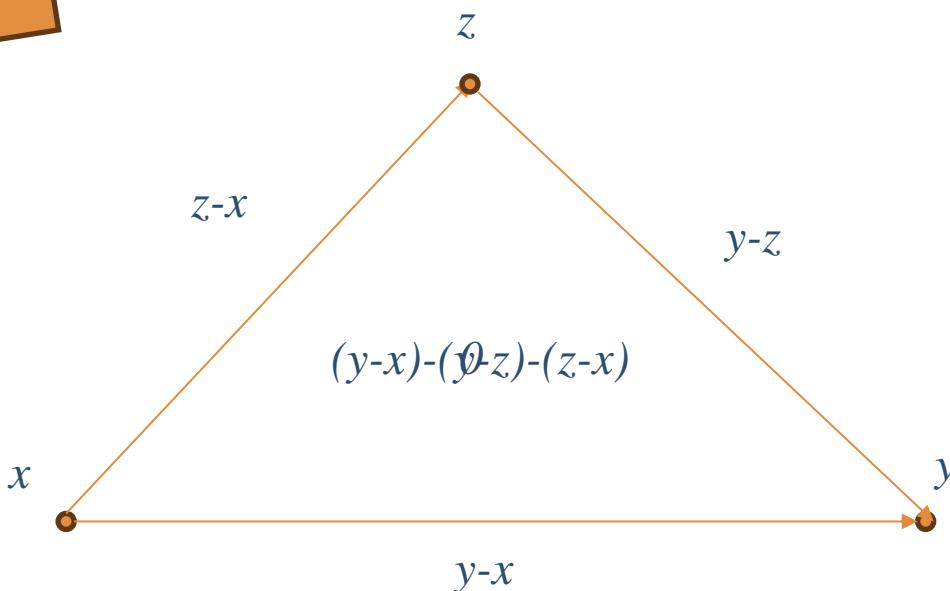
Contents [hide]

(Top)

Definition

1. df is the differential of f for a 0-form f .
2. $d(df) = 0$ for a 0-form f .
3. $d(\alpha \wedge \beta) = d\alpha \wedge \beta + (-1)^{|\alpha|} \alpha \wedge d\beta$ where α is a p -form. That is to say, d is an antiderivation of degree 1 on the exterior algebra of differential forms (see the *graded product rule*).

Discretize

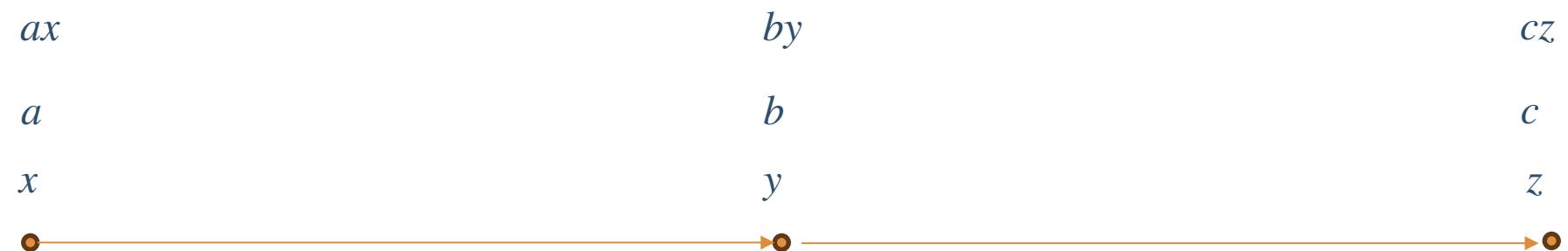


Curl of gradient is zero [edit]

The curl of the gradient of any continuously twice-differentiable scalar field φ (i.e., differentiability

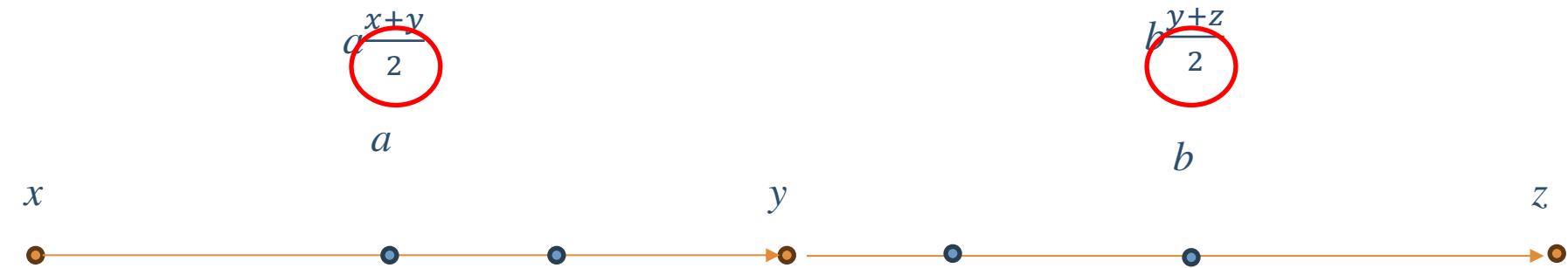
$$\nabla \times (\nabla \varphi) = \mathbf{0}$$

\wedge : Wedge Product *0-Form and 0-Form*



~~A: Wedge Product~~ *0-Form and 1-Form*

★: Hodge Star



Compute





Burgers' equation

Contents [hide]

(Top)

Article Talk

From Wikipedia, the free encyclopedia

$$\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} = \nu \frac{\partial^2 u}{\partial x^2}$$

Variable

Compute what?



Burgers' equation

Contents [hide]

(Top)

Article Talk

From Wikipedia, the free encyclopedia

$$\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} = \nu \frac{\partial^2 u}{\partial x^2}$$

Variable

Compute what?



Burgers' equation

Contents [hide]

(Top)

Article Talk

From Wikipedia, the free encyclopedia

$$\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} = \nu \frac{\partial^2 u}{\partial x^2}$$

The diagram shows the Burgers' equation with arrows indicating its components. A blue arrow points from the term $\frac{\partial u}{\partial t}$ to the label "Variable". A red arrow points from the term $u \frac{\partial u}{\partial x}$ to the label "Unary Operator". A green arrow points from the term $\nu \frac{\partial^2 u}{\partial x^2}$ to the label "Variable".

Variable

Unary Operator

Compute what?



Burgers' equation

Contents [hide]

(Top)

Article Talk

From Wikipedia, the free encyclopedia

$$\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} = \nu \frac{\partial^2 u}{\partial x^2}$$

The diagram shows the Burgers' equation with arrows pointing to its components:

- Variable:** Points to the variable u .
- Unary Operator:** Points to the term $\frac{\partial u}{\partial t}$.
- Binary Operator:** Points to the term $u \frac{\partial u}{\partial x}$.
- Unary Operator:** Points to the term $\nu \frac{\partial^2 u}{\partial x^2}$.

Compute what?



Burgers' equation

Contents [hide]

(Top)

Article Talk

From Wikipedia, the free encyclopedia

$$\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} = \nu \frac{\partial^2 u}{\partial x^2}$$

The diagram illustrates the components of the Burgers' equation. It features a central mathematical expression: $\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} = \nu \frac{\partial^2 u}{\partial x^2}$. Four arrows point from labels to specific parts of the equation: a blue arrow labeled "Summation" points to the first term $\frac{\partial u}{\partial t}$; a green arrow labeled "Variable" points to the variable u ; a blue arrow labeled "Binary Operator" points to the term $u \frac{\partial u}{\partial x}$; and an orange arrow labeled "Unary Operator" points to the term $\nu \frac{\partial^2 u}{\partial x^2}$.

Compute what?

 Search

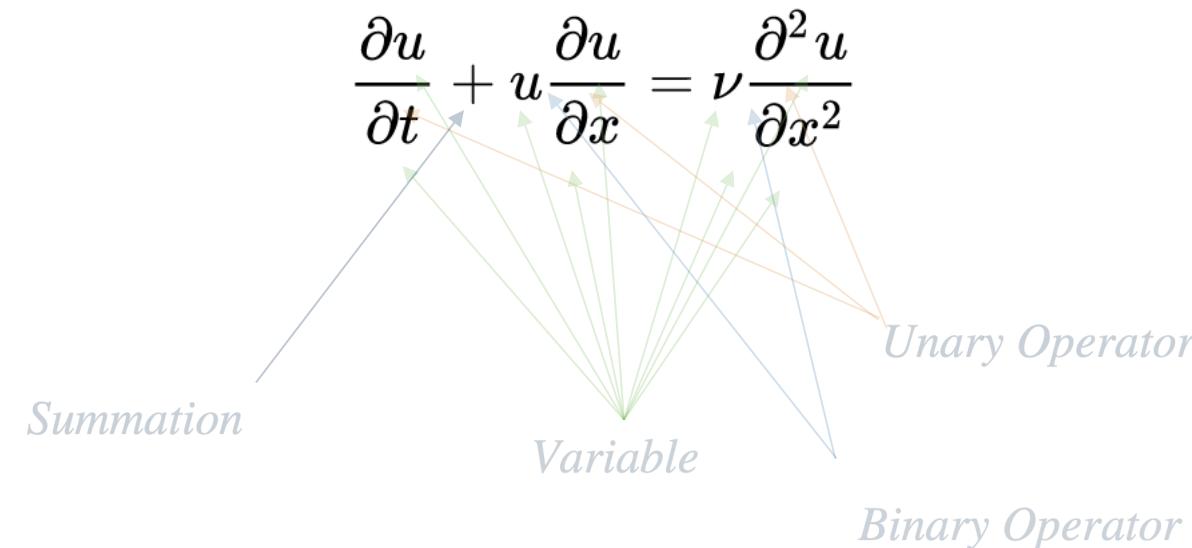
Burgers' equation

Contents [hide]

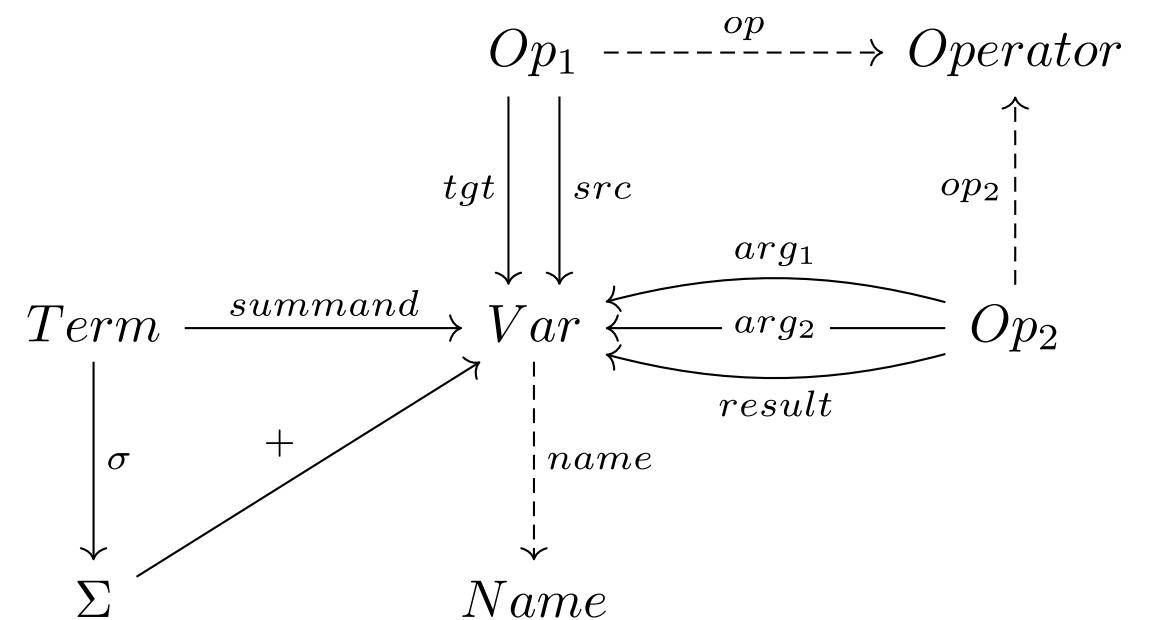
(Top)

Article Talk

From Wikipedia, the free encyclopedia

$$\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} = \nu \frac{\partial^2 u}{\partial x^2}$$


The diagram illustrates the components of the Burgers' equation. It shows the equation $\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} = \nu \frac{\partial^2 u}{\partial x^2}$ with arrows pointing from labels to specific terms: "Summation" points to the first term $\frac{\partial u}{\partial t}$; "Variable" points to the variable u in the second term; "Unary Operator" points to the derivative $\frac{\partial u}{\partial x}$ in the second term; and "Binary Operator" points to the derivative $\frac{\partial^2 u}{\partial x^2}$ in the third term.



Compute what?



Search Wikipedia Search

Burgers' equation

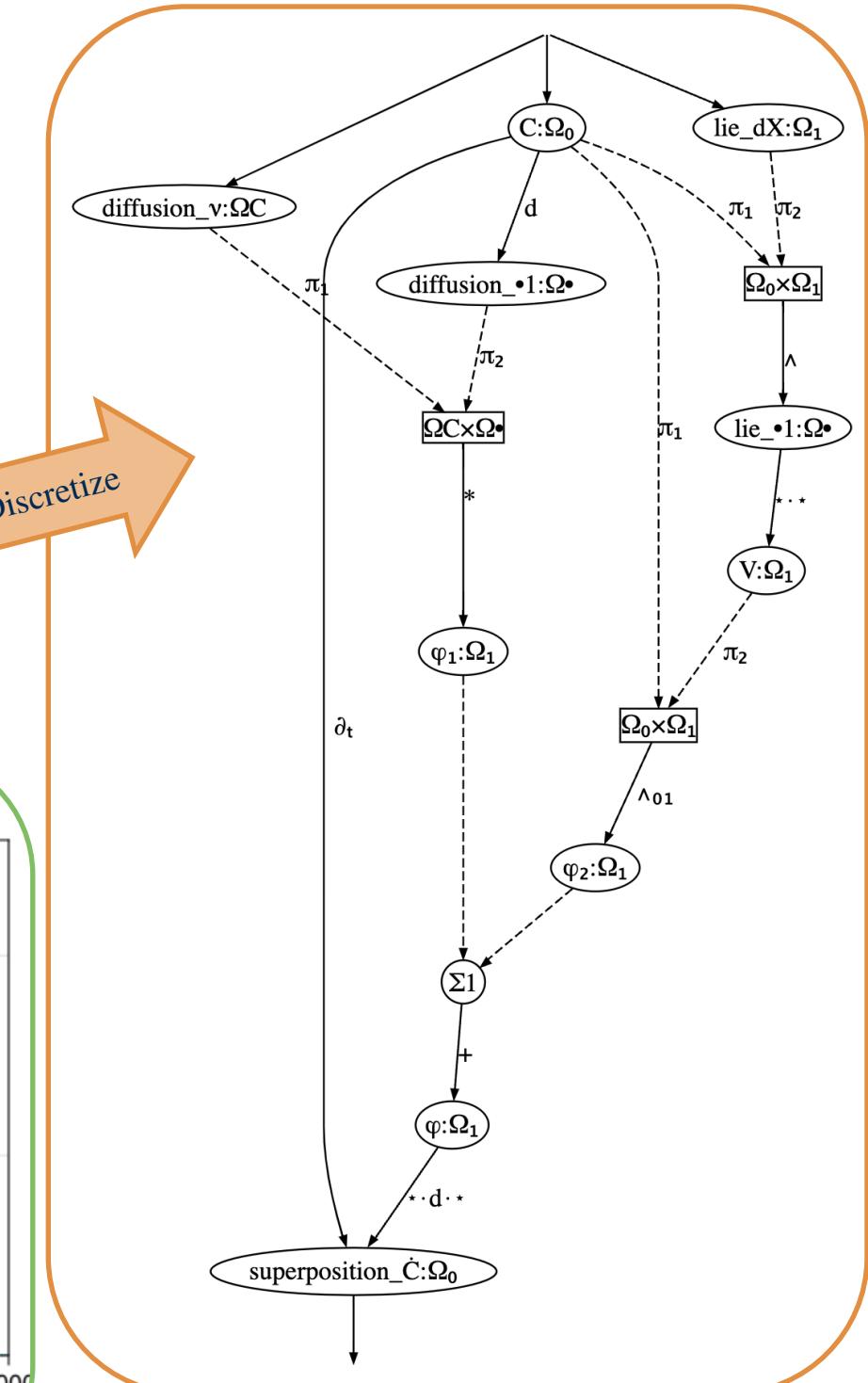
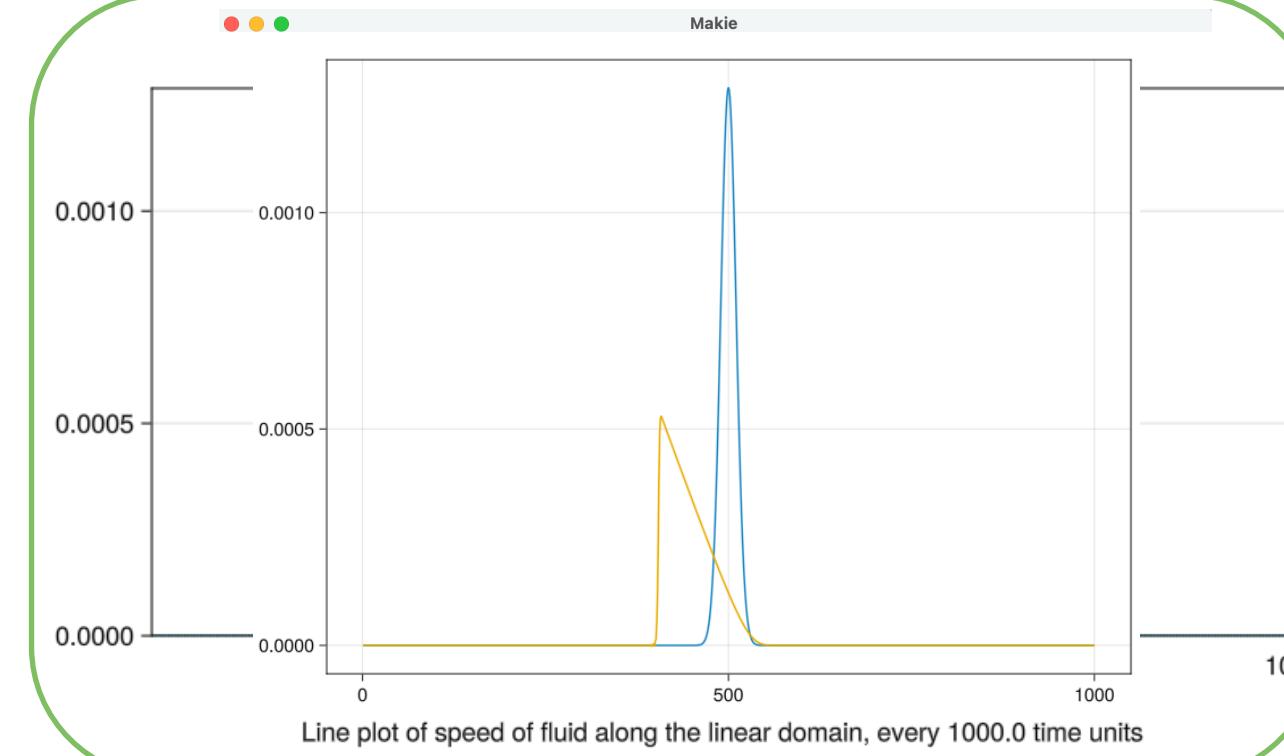
Contents [hide]

(Top)

Article Talk

From Wikipedia, the free encyclopedia

$$\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} = \nu \frac{\partial^2 u}{\partial x^2}$$



“The Computational Treatment of a Particular Case”

72 A. M. TURING ON THE CHEMICAL BASIS OF MORPHOGENESIS

from one pattern into another, rather than from homogeneity into a pattern. One would like to be able to follow this more general process mathematically also. The difficulties are, however, such that one cannot hope to have any very embracing *theory* of such processes, beyond the statement of the equations. It might be possible, however, to treat a few particular cases in detail with the aid of a digital computer. This method has the advantage that it is not so necessary to make simplifying assumptions as it is when doing a more theoretical type of analysis. It might even be possible to take the mechanical aspects of the problem into account as well as the chemical, when applying this type of method. The essential disadvantage of the method is that one only gets results for particular cases. But this disadvantage is probably of comparatively little importance. Even with the ring problem, considered in this paper, for which a reasonably complete mathematical analysis was possible, the computational treatment of a particular case was most illuminating. The

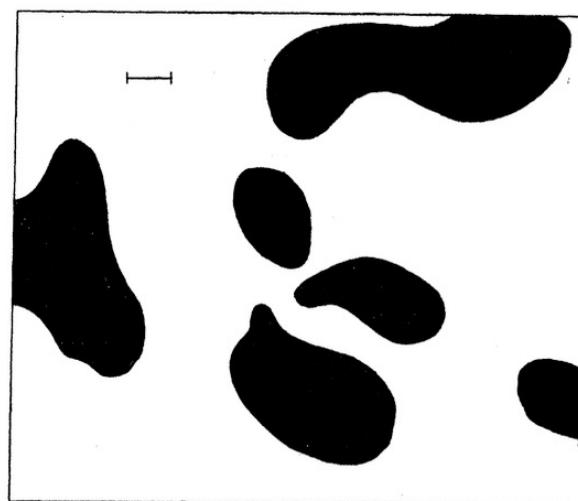
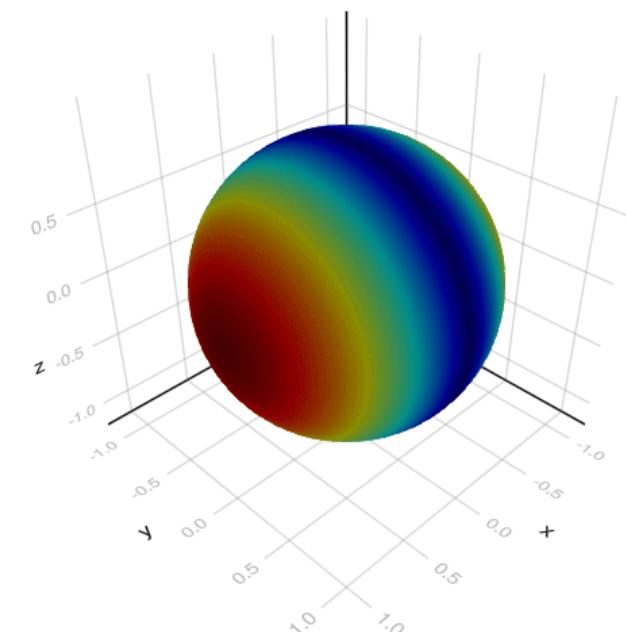


FIGURE 2. An example of a ‘dappled’ pattern as resulting from a type (a) morphogen system. A marker of unit length is shown. See text, §9, 11.

```
GrayScott = @decapode begin
  (U, V)::Form0{X}
  (f, k, ru, rv)::Constant{X}

  UV2 == (U .* (V .* V))
  ∂t(U) == ru * Δ(U) - UV2 + f * (1 .- U)
  ∂t(V) == rv * Δ(U) + UV2 - (f + k) .* V
end
```

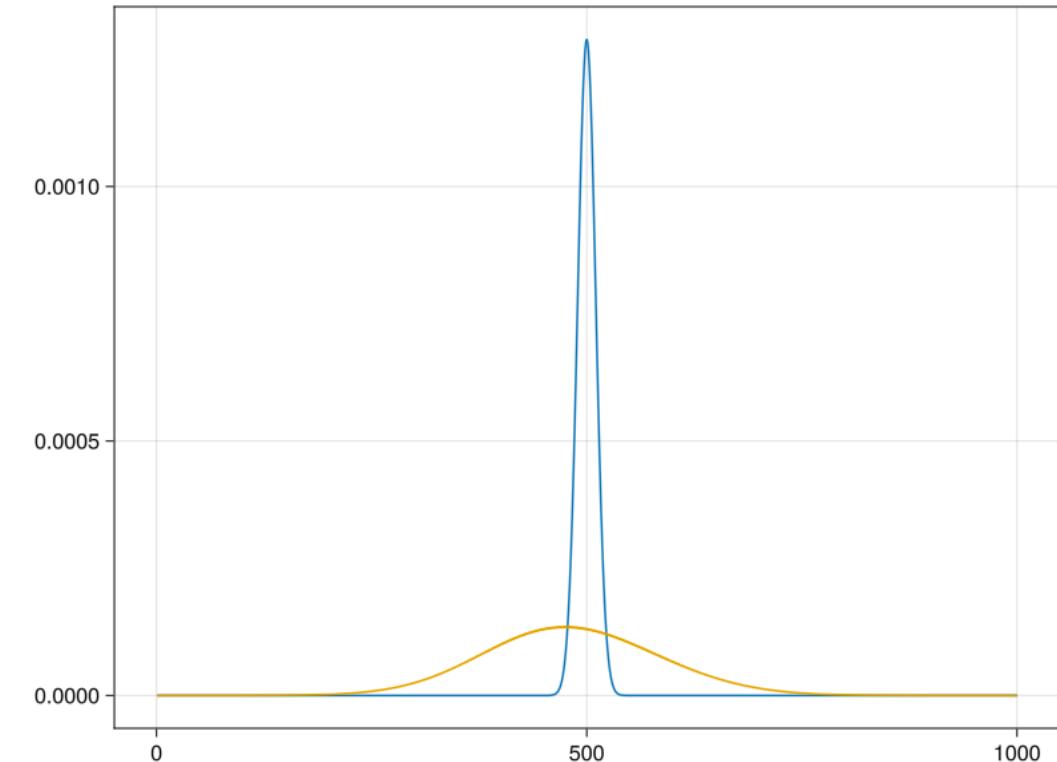


Trick 1: Divergence

- Given a 1-form, tell me how much it diverges *at each point*
- In the DEC, this is $\star d \star$

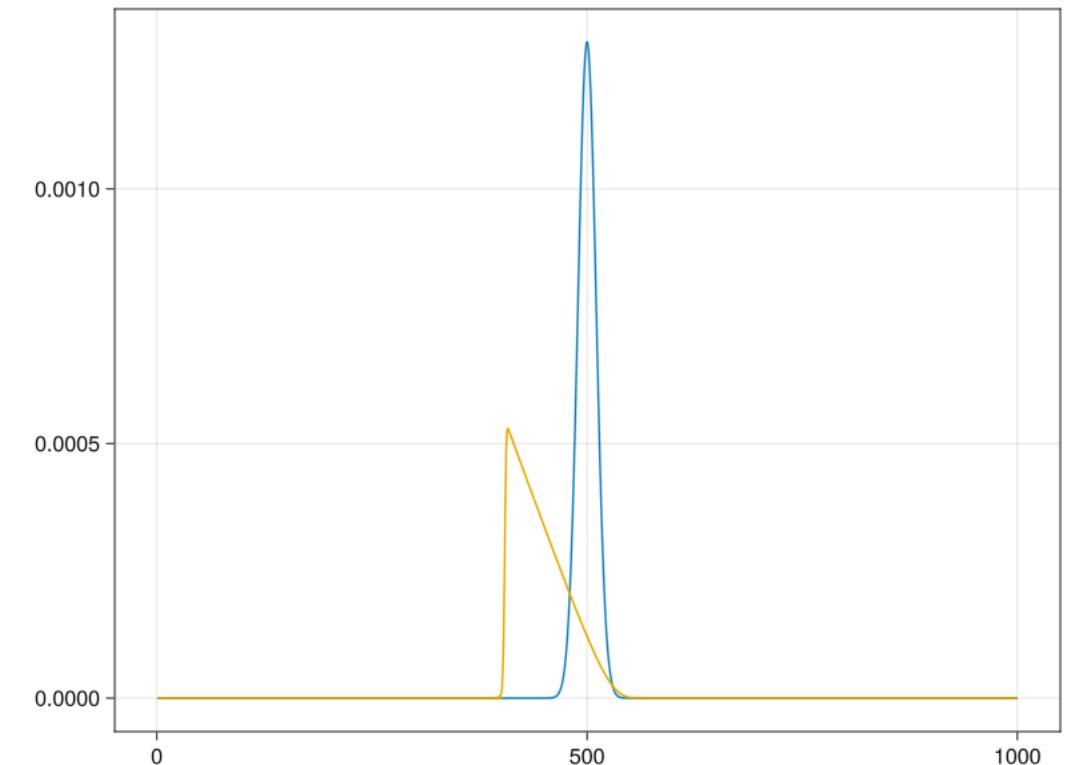
Trick 2: Diffusion

- When you hear “diffusion” think Laplacian, Δ
- In the DEC, this is $\star d \star d$
- *a.k.a.* divergence of gradient



Trick 3: Advection

- When you hear “advection” think movement along a 1-form
- In the DEC, this is $\star d \star (X \wedge Y)$
- *a.k.a.* divergence of wedge



Trick 4: Diffusion is sometimes hidden

- The $\star d \star d$ pattern may not be contiguous in ~~memory~~ syntax

JOURNAL OF GEOPHYSICAL RESEARCH, VOL. 86, NO. C11, PAGES 11,065–11,072, NOVEMBER 20, 1981

On the Dynamics of the Ice Sheets

P. HALFAR

Max-Planck-Institut für Meteorologie, Hamburg, Federal Republic of Germany

A similarity solution of the equation that describes the time evolution of an ice sheet is obtained by separation of variables. It describes the motion of an initial δ function ice sheet distribution and is asymptotically stable with respect to all perturbations that leave the total volume invariant. Homogeneity in one horizontal direction is assumed.

1. INTRODUCTION

On time scales larger than 10^3 years, the motion of the ice sheets plays an important role in the climatic system of the earth. To explain these variations, two mechanisms have mainly been discussed. One is the external forcing via the orbital parameters of the earth [Hays *et al.*, 1976]. There is good evidence that the obliquity of the earth's axis and the precession of the equinoxes influence the ice sheet variations. However, a physical connection between the large 10^3 years peak in the climatic spectrum and the period of eccentricity of the earth's orbit remains doubtful [Rooth *et al.*, 1978]. The other mechanism is a feedback between the variables of the climatic system [Imbre and Imbre, 1980; Källén *et al.*, 1979; Oerlemans, 1979]. The aim of such feedback models is to find an eigenmode with a characteristic time scale of 10^3 years.

It is important for any model of ice sheet dynamics to master the simplest case when external forcing and feedback mechanisms are absent. Therefore, I shall consider only that part of the motion of an ice sheet that is due to the flow law of the ice. A two-dimensional case is treated (length and height). The ice is assumed to rest on a flat ground where the velocities vanish. It has not yet been realized that the nonlinear equation, which describes in the small slope approximation the time evolution of the surface [Mahaffy, 1976], has a similarity solution of finite extent that grows in length and shrinks in height. By linearizing the equation of motion around the similarity solution, any deviation from this solution which leaves the volume invariant can be shown to fade away in the course of time (i.e., this solution is asymptotically stable). The eigenvalues and eigenfunctions of the equation of motion that are linearized in the deviation are represented through standard analytical expressions. The result suggests that any solution of the equation of motion approaches during its time evolution of a similarity solution.

The linearization around a given solution fits into the general scheme developed by Nye [1969] and Weertman [1958] in which the authors consider expansions around solutions, which are in equilibrium with sources and sinks, but the sawtooth pattern of isotopic records [Rooth *et al.*, 1978] indicates that the ice age ice sheets were far from equilibrium.

I hope that the similarity solution will also be useful to describe some of the more realistic cases when accumulation and ablation change the volume, sliding occurs on the bed, temperature varies in space and time, or ice-ocean interaction takes place along the marine margins of the ice sheet. If the internal friction of the ice that is responsible for the asymptotic stability of the similarity solution dominates some of

Copyright © 1981 by the American Geophysical Union.

$$\partial_t(h) = \circ(\star, d, \star)(\Gamma \quad d(h) \quad \text{avg}_{01}|d(h)^\sharp|^{n-1} \quad \text{avg}_{01}(h^{n+2}))$$

these effects, then an expansion around similarity solutions might be valuable. As this task goes beyond the present paper, let me sketch how it could be done.

The above-mentioned more realistic properties add as perturbations to the equation of motion. At every time a solution of this perturbed equation can be approximated through a similarity solution by adjustment of volume, center of mass and length. The remaining difference is expanded in terms of the eigenfunctions of the unperturbed and in the deviations from this similarity solution linearized equation of motion. The three lowest modes do not occur, for these degrees of freedom are already fixed by adjustment of volume, center of mass, and length (cf. section 7). The perturbed equation is then linearized in the amplitudes of the higher eigenfunctions and yields a first-order differential equation for the time evolution of volume, center of mass, length, and these amplitudes. If the damping of the amplitudes, which is already there in the unperturbed case, is large compared with the perturbations, then they will remain small, the first-order expansion remains valid, and, therefore, the procedure is self-consistent. To repeat the essential assumption, the damping of deviations from the shapes of the similarity solutions that is caused by internal friction limits their growth, which is due to the perturbations.

In this way the model could consider all degrees of freedom of an ice sheet, and only a few of the lowest ones must be taken into account owing to the stronger damping of the higher modes that would save a lot of computer time. On the other hand, there would be more than 1 degree of freedom, which is necessary to build feedback models with cyclic motions. This would be a difference to the Weertman [1964] model, which has only the length as degree of freedom.

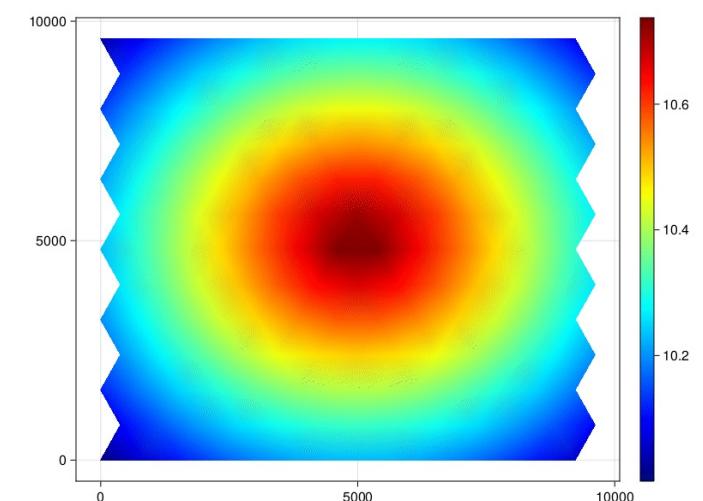
2. THE MODEL

Consider a two-dimensional ice sheet resting on a flat ground (Figure 1). Its motion is determined by Glen's flow law of the ice that connects the second invariants of the deviatoric stress tensor and the strain-rate tensor [Nye, 1957]:

$$\dot{\varepsilon} = \left[\frac{\sigma'}{A} \right]^n \quad (1)$$

and by the boundary condition of vanishing velocity on the bed. Neglecting longitudinal stress gradients, the vertically integrated equation of motion becomes in the small slope approximation [Mahaffy, 1976]

$$\frac{\partial h}{\partial t} = \frac{2}{(n+2)} \left[\frac{\rho g}{A} \right]^\frac{n}{n+2} \frac{\partial}{\partial x} \left[\frac{\partial h}{\partial x} \right]^\frac{n-1}{n+2} h^{n+2} \quad (2)$$



Trick 5: Domain

- Scientists do not usually say “these dynamics occur on a 1-manifold!”
- They may describe an indexing scheme
- They may collapse or average along a dimension

Trick 5: Domain (Cont.)

- They may say...

THE CHEMICAL BASIS OF MORPHOGENESIS

By A. M. TURING, F.R.S. *University of Manchester*

(Received 9 November 1951—Revised 15 March 1952)

It is suggested that a system of chemical substances, called morphogens, reacting together and diffusing through a tissue, is adequate to account for the main phenomena of morphogenesis. Such a system, although it may originally be quite homogeneous, may later develop a pattern or structure due to an instability of the homogeneous equilibrium, which is triggered off by random disturbances. Such reaction-diffusion systems are considered in some detail in the case of an isolated ring of cells, a mathematically convenient, though biologically unusual system. The investigation is chiefly concerned with the onset of instability. It is found that there are six essentially different forms which this may take. In the most interesting form stationary waves appear on the ring. It is suggested that this might account, for instance, for the tentacle patterns on *Hydra* and for whorled leaves. A system of reactions and diffusion on a sphere is also considered. Such a system appears to account for gastrulation. Another reaction system in two dimensions gives rise to patterns reminiscent of dappling. It is also suggested that stationary waves in two dimensions could account for the phenomena of phyllotaxis.

The purpose of this paper is to discuss a possible mechanism by which the genes of a zygote may determine the anatomical structure of the resulting organism. The theory does not make any new hypotheses; it merely suggests that certain well-known physical laws are sufficient to account for many of the facts. The full understanding of the paper requires a good knowledge of mathematics, some biology, and some elementary chemistry. Since readers cannot be expected to be experts in all of these subjects, a number of elementary facts are explained, which can be found in text-books, but whose omission would make the paper difficult reading.

1. A MODEL OF THE EMBRYO. MORPHOGENS

In this section a mathematical model of the growing embryo will be described. This model will be a simplification and an idealization, and consequently a falsification. It is to be hoped that the features retained for discussion are those of greatest importance in the present state of knowledge.

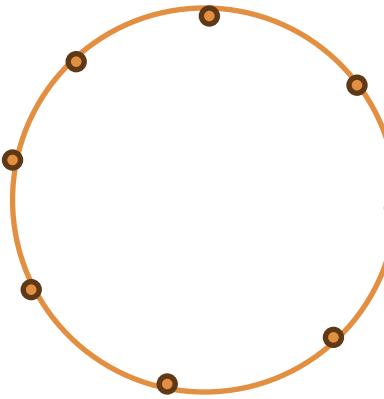
The model takes two slightly different forms. In one of them the cell theory is recognized but the cells are idealized into geometrical points. In the other the matter of the organism is imagined as continuously distributed. The cells are not, however, completely ignored, for various physical and physico-chemical characteristics of the matter as a whole are assumed to have values appropriate to the cellular matter.

With either of the models one proceeds as with a physical theory and defines an entity called 'the state of the system'. One then describes how that state is to be determined from the state at a moment very shortly before. With either model the description of the state consists of two parts, the mechanical and the chemical. The mechanical part of the state describes the positions, masses, velocities and elastic properties of the cells, and the forces between them. In the continuous form of the theory essentially the same information is given in the form of the stress, velocity, density and elasticity of the matter. The chemical part of the state is given (in the cell form of theory) as the chemical composition of each separate cell; the diffusibility of each substance between each two adjacent cells must also

CHEMICAL BASIS OF MORPHOGENESIS

47

cell r may be written X_r , and Y_r has a similar meaning. It is convenient to regard 'cell N ' and 'cell O ' as synonymous, and likewise 'cell 1' and cell ' $N+1$ '. One can then say that for each r satisfying $1 \leq r \leq N$ cell r exchanges material by diffusion with cells $r-1$ and $r+1$.



Trick 5: Domain (Cont.)

- Or, they may say...

On the Dynamics of the Ice Sheets

P. HALFAR

Max-Planck-Institut für Meteorologie, Hamburg, Federal Republic of Germany

A similarity solution of the equation that describes the time evolution of an ice sheet is obtained by separation of variables. It describes the motion of an initial δ function ice sheet distribution and is asymptotically stable with respect to all perturbations that leave the total volume invariant. Homogeneity in one horizontal direction is assumed.

1. INTRODUCTION

On time scales larger than 10^3 years, the motion of the ice sheets plays an important role in the climatic system of the earth. To explain these variations, two mechanisms have mainly been discussed. One is the external forcing via the orbital parameters of the earth [Hays *et al.*, 1976]. There is good evidence that the obliquity of the earth's axis and the precession of the equinoxes influence the ice sheet variations. However, a physical connection between the large 10^3 years peak in the climatic spectrum and the period of eccentricity of the earth's orbit remains doubtful [Rooth *et al.*, 1978]. The other mechanism is a feedback between the variables of the climatic system [Imbrie and Imbrie, 1980; Källen *et al.*, 1979; Oerlemans, 1979]. The aim of such feedback models is to find an eigenmode with a characteristic time scale of 10^3 years.

It is important for any model of ice sheet dynamics to master the simplest case when external forcing and feedback mechanisms are absent. Therefore, I shall consider only that part of the motion of an ice sheet that is due to the flow law of the ice. A two-dimensional case is treated (length and height). The ice is assumed to rest on a flat ground where the velocities vanish. It has not yet been realized that the nonlinear equation, which describes in the small slope approximation the time evolution of the surface [Mahaffy, 1976], has a similarity solution of finite extent that grows in length and shrinks in height. By linearizing the equation of motion around the similarity solution, any deviation from this solution which leaves the volume invariant can be shown to fade away in the course of time (i.e., this solution is asymptotically stable). The eigenvalues and eigenfunctions of the equation of motion that are linearized in the deviation are represented through standard analytical expressions. The result suggests that any solution of the equation of motion approaches during its time evolution a similarity solution.

The linearization around a given solution fits into the general scheme developed by Nye [Paterson, 1969] and Weertman [1958] in which the authors consider expansions around solutions, which are in equilibrium with sources and sinks, but the sawtooth pattern of isotopic records [Rooth *et al.*, 1978] indicates that the ice age ice sheets were far from equilibrium.

I hope that the similarity solution will also be useful to describe some of the more realistic cases when accumulation and ablation change the volume, sliding occurs on the bed, temperature varies in space and time, or ice-ocean interaction takes place along the marine margins of the ice sheet. If the internal friction of the ice that is responsible for the asymptotic stability of the similarity solution dominates some of

these effects, then an expansion around similarity solutions might be valuable. As this task goes beyond the present paper, let me sketch how it could be done.

The above-mentioned more realistic properties add as perturbations to the equation of motion. At every time a solution of this perturbed equation can be approximated through a similarity solution by adjustment of volume, center of mass, and length. The remaining difference is expanded in terms of the eigenfunctions of the unperturbed and in the deviations from this similarity solution linearized equation of motion. The three lowest modes do not occur, for these degrees of freedom are already fixed by adjustment of volume, center of mass, and length (cf. section 7). The perturbed equation is then linearized in the amplitudes of the higher eigenfunctions and yields a first-order differential equation for the time evolution of volume, center of mass, length, and these amplitudes. If the damping of these amplitudes, which is already there in the unperturbed case, is large compared with the perturbations, then they will remain small, the first-order expansion remains valid, and, therefore, the procedure is self-consistent. To repeat the essential assumption, the damping of deviations from the shapes of the similarity solutions that is caused by internal friction limits their growth, which is due to the perturbations.

In this way the model could consider all degrees of freedom of an ice sheet, and only a few of the lowest ones must be taken into account owing to the stronger damping of the higher modes that would save a lot of computer time. On the other hand, there would be more than 1 degree of freedom, which is necessary to build feedback models with cyclic motions. This would be a difference to the Weertman [1964] model, which has only the length as degree of freedom.

2. THE MODEL

Consider a two-dimensional ice sheet resting on a flat ground (Figure 1). Its motion is determined by Glen's flow law of the ice that connects the second invariants of the deviatoric stress tensor and the strain-rate tensor [Nye, 1957]:

$$\dot{\epsilon} = \left(\frac{\sigma'}{A} \right)^n \quad (1)$$

and by the boundary condition of vanishing velocity on the bed. Neglecting longitudinal stress gradients, the vertically integrated equation of motion becomes in the small slope approximation [Mahaffy, 1976]



$$\frac{\partial h}{\partial t} = \frac{2}{(n+2)} \left(\frac{\rho g}{A} \right)^{1/n} \frac{\partial}{\partial x} \left(\frac{\partial h}{\partial x} \right)^{n-1} h^{n+2} \quad (2)$$

Trick 5: Domain (Cont.)

- Or, they may say...

50 MAGNETIC FIELD ANNIHILATION*

HARRY E. PETSCHER
*Avco-Everett Research Laboratory
Everett, Mass.*

Sweet's mechanism for the rate of annihilation of the magnetic field at the boundary between two regions of plasma containing oppositely directed field lines is re-examined. It is pointed out that previous analyses overlooked standing magneto-hydrodynamic waves as a possible mechanism for converting magnetic energy to plasma energy. An estimate of the annihilation rate including such waves is made. Using this rate, it is found that the energy required for a flare can be released in 10^8 sec. This time is short enough to account for the observed solar flare times if the source of the flare energy is stored magnetic energy.

One of the principal objections which has been raised to the suggestion that solar flares result from the rapid release of magnetically stored energy has been that the rate at which magnetic energy can be released is too slow. The model of the boundary between regions in which oppositely directed field lines exist, which was originally suggested by Sweet (References 1 and 2) and evaluated quantitatively by Parker (Reference 3), leads to times for the release of energy which are too large by a factor of 10 to 100. This is true even when it is assumed that the gas remains partially ionized and the resulting decrease in the effective conductivity due to ambipolar diffusion is included. It has been pointed out by Jaggi (Reference 4) that on the basis of the resistive instability analysis of Furth, Killeen, and Rosenbluth (Reference 5) the boundary in the Parker-Sweet model would be unstable. It is thus clear that the Parker-Sweet annihilation rates underestimate the actual rate of magnetic field annihilation. However, the linearized instability analysis does not lead immediately to an estimate of what the actual rate would be.

The existence of this instability leads to the suggestion that the flow becomes turbulent and

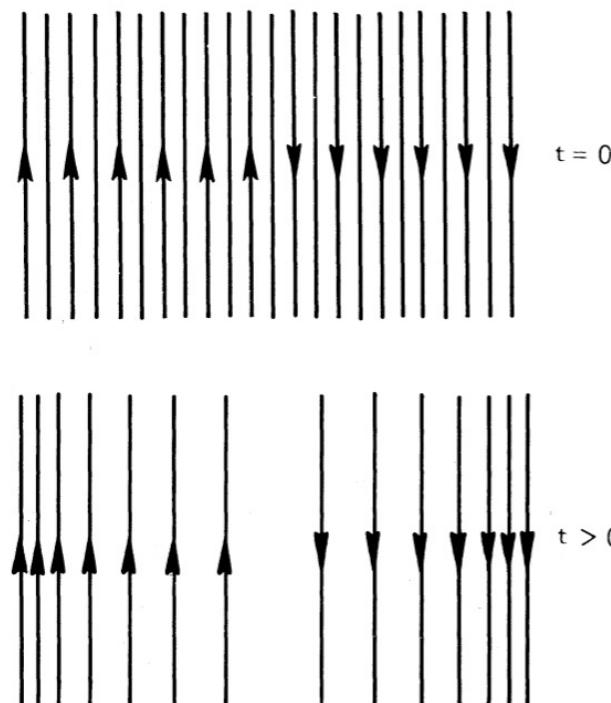
that the increased annihilation rate results from the increased dissipation due to turbulence (References 6 and 7). The purpose of the present paper is to point to another possibility. This alternate possibility is based on the fact that Parker's analysis has overlooked a significant mechanism for the dissipation of magnetic field energy. If this mechanism is also included, it is possible to construct a steady flow configuration with much faster annihilation rates. Parker's solution should be regarded as a special case in a series of possible solutions. The existence of the instability of this particular solution should probably be considered largely as an indication that the particular solution underestimates the actual rate at which annihilation will occur.

To illustrate the mechanisms by which magnetic energy can be converted into plasma energy, let us consider for the moment a completely one-dimensional time-dependent situation in an incompressible fluid in which two regions with oppositely directed magnetic field lines are placed in contact at zero time (the actual situation to be discussed in the body of this paper will not be one-dimensional or time-dependent). In the present case, two mechanisms exist for the annihilation of magnetic field energy: (1) dissipation due to the finite conductivity of the plasma which may be regarded as diffusion of the magnetic field, and (2) annihilation of the magnetic energy by the propagation of an Alfvén wave.

Figure 50-1(a) illustrates the diffusion case. The thickness of the region of reduced magnetic field is given by the ordinary skin depth formula. If, on the other hand, there is a component of magnetic field normal to the boundary, waves will travel outward in both directions and give

To illustrate the mechanisms by which magnetic energy can be converted into plasma energy, let us consider for the moment a completely one-dimensional time-dependent situation in an incompressible fluid in which two regions with oppositely directed magnetic field lines are placed in contact at zero time (the

DIFFUSION



*This work was sponsored by the Department of the Navy, Office of Naval Research, under contract No. Nonr-2824(00).

Trick 5: Domain (Cont.)

- Or, they may say...

50 MAGNETIC FIELD ANNIHILATION*

HARRY E. PETSCHER
Avco-Everett Research Laboratory
Everett, Mass.

Sweet's mechanism for the rate of annihilation of the magnetic field at the boundary between two regions of plasma containing oppositely directed field lines is re-examined. It is pointed out that previous analyses overlooked standing magneto-hydrodynamic waves as a possible mechanism for converting magnetic energy to plasma energy. An estimate of the annihilation rate including such waves is made. Using this rate, it is found that the energy required for a flare can be released in 10^8 sec. This time is short enough to account for the observed solar flare times if the source of the flare energy is stored magnetic energy.

One of the principal objections which has been raised to the suggestion that solar flares result from the rapid release of magnetically stored energy has been that the rate at which magnetic energy can be released is too slow. The model of the boundary between regions in which oppositely directed field lines exist, which was originally suggested by Sweet (References 1 and 2) and evaluated quantitatively by Parker (Reference 3), leads to times for the release of energy which are too large by a factor of 10 to 100. This is true even when it is assumed that the gas remains partially ionized and the resulting decrease in the effective conductivity due to ambipolar diffusion is included. It has been pointed out by Jaggi (Reference 4) that on the basis of the resistive instability analysis of Furth, Killeen, and Rosenbluth (Reference 5) the boundary in the Parker-Sweet model would be unstable. It is thus clear that the Parker-Sweet annihilation rates underestimate the actual rate of magnetic field annihilation. However, the linearized instability analysis does not lead immediately to an estimate of what the actual rate would be.

The existence of this instability leads to the suggestion that the flow becomes turbulent and

that the increased annihilation rate results from the increased dissipation due to turbulence (References 6 and 7). The purpose of the present paper is to point to another possibility. This alternate possibility is based on the fact that Parker's analysis has overlooked a significant mechanism for the dissipation of magnetic field energy. If this mechanism is also included, it is possible to construct a steady flow configuration with much faster annihilation rates. Parker's solution should be regarded as a special case in a series of possible solutions. The existence of the instability of this particular solution should probably be considered largely as an indication that the particular solution underestimates the actual rate at which annihilation will occur.

To illustrate the mechanisms by which magnetic energy can be converted into plasma energy, let us consider for the moment a completely one-dimensional time-dependent situation in an incompressible fluid in which two regions with oppositely directed magnetic field lines are placed in contact at zero time (the actual situation to be discussed in the body of this paper will not be one-dimensional or time-dependent). In the present case, two mechanisms exist for the annihilation of magnetic field energy: (1) dissipation due to the finite conductivity of the plasma which may be regarded as diffusion of the magnetic field, and (2) annihilation of the magnetic energy by the propagation of an Alfvén wave.

Figure 50-1(a) illustrates the diffusion case. The thickness of the region of reduced magnetic field is given by the ordinary skin depth formula. If, on the other hand, there is a component of magnetic field normal to the boundary, waves will travel outward in both directions and give

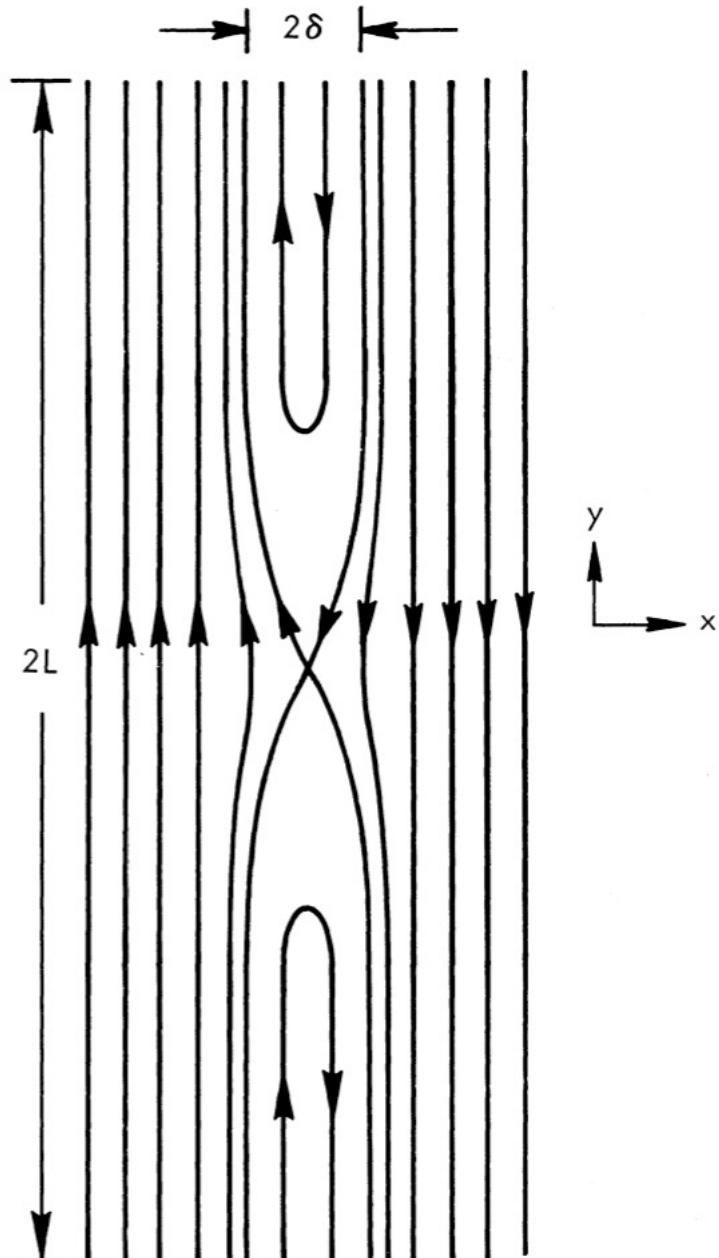
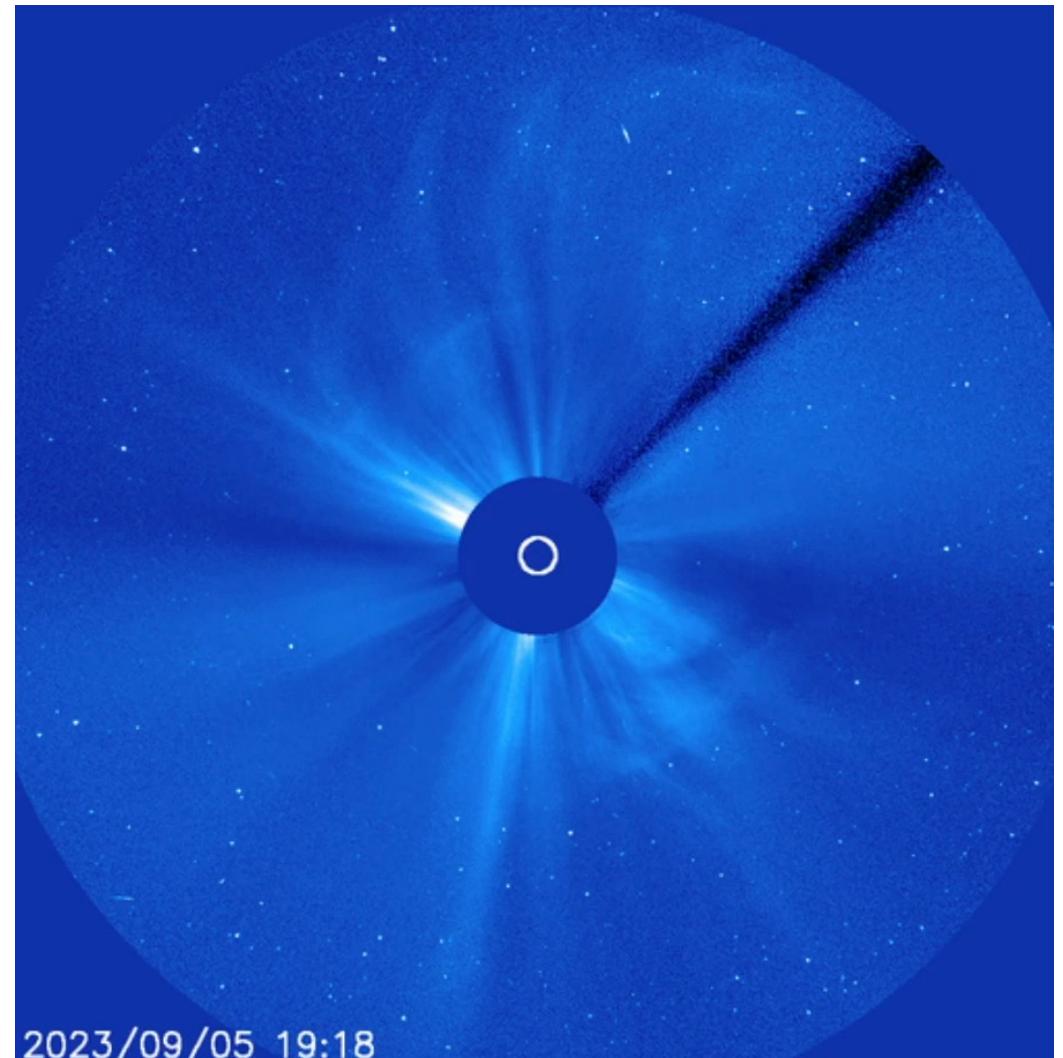


FIGURE 50-2. Magnetic field configuration for Parker's analysis of the Sweet mechanism. The fluid moves toward the boundary from both sides and is ejected along it.



2023/09/05 19:18

*This work was sponsored by the Department of the Navy, Office of Naval Research, under contract No. Nonr-2524(00).

Trick 5: Domain (Cont.)

- Or, they may say...

Dispersion of soluble matter in solvent flowing slowly through a tube

By SIR GEOFFREY TAYLOR, F.R.S.

(Received 31 March 1953)

When a soluble substance is introduced into a fluid flowing slowly through a small-bore tube it spreads out under the combined action of molecular diffusion and the variation of velocity over the cross-section. It is shown analytically that the distribution of concentration produced in this way is centred on a point which moves with the mean speed of flow and is symmetrical about it in spite of the asymmetry of the flow. The dispersion along the tube is governed by a virtual coefficient of diffusivity which can be calculated from observed distributions of concentration. Since the analysis relates the longitudinal diffusivity to the coefficient of molecular diffusion, observations of concentration along a tube provide a new method for measuring diffusion coefficients. The coefficient so obtained was found, with potassium permanganate, to agree with that measured in other ways.

The results may be useful to physiologists who may wish to know how a soluble salt is dispersed in blood streams.

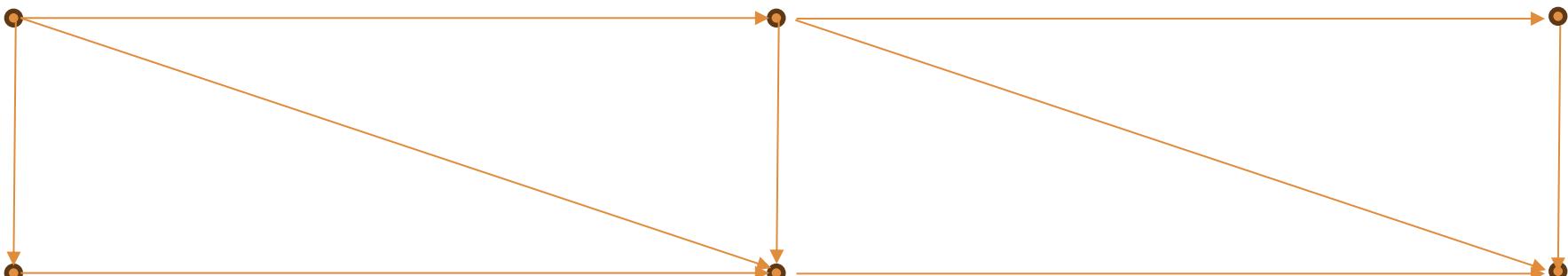
1. INTRODUCTION

If a conducting solution (e.g. brine) is injected into a tube through which water is flowing, the region in which it is concentrated moves downstream. At a fixed point the conductivity will rise as the solution reaches it, and if the conductivity is measured there the conductivity-time curve can be used as a means of measuring the stream velocity. If the injected material would remain concentrated in a small volume the method would be simple, but the stream velocity varies over the cross-section of a pipe. A part of the injected material which was initially near the centre of the tube would be carried to the measuring point faster than parts which were near the walls. To use the method as a means of measuring the mean speed of flow therefore it is necessary to know which point on the conductivity-time curve corresponds to this mean speed. This method has been used to measure the flow in large water mains where it is turbulent (Allen & Taylor 1923) and in small blood vessels where it may be non-turbulent (Stewart 1894; White 1947).

A similar method was used by Griffiths (1911) in experiments designed to measure viscosity of water at very low speeds of flow. A drop of fluorescent solution was inserted as a marker or index in a stream of water flowing slowly through a capillary tube. Griffiths found experimentally that the colouring matter spreads out in a symmetrical manner from a point which moves with the mean velocity of the water in the tube. Since the present paper contains an analysis of the situation observed by Griffiths it is worth quoting his theoretical reasoning on the subject. He wrote (1911, p. 190):

'In this paper the full mathematical treatment is not attempted; but an elementary consideration, although not complete, will be of advantage in dealing with the experiments. It can easily be shown that if the intensity of colour were constant over a cross-section of the tube the colour would diffuse along the tube exactly as if the water travelled in a solid column. The intensity of the colour cannot be absolutely

It will be assumed that the concentration is symmetrical about the central line of the pipe so that C is a function of r , x and t only. The equation for diffusion is



Trick 6: Recognizing Diffusion

- Diffusion terms often appear as a “sum of second derivatives”

Macrotransport theory for diffusiophoretic colloids and chemotactic microorganisms

Henry C.W. Chu^{1,†}, Stephen Garoff², Robert D. Tilton³ and Aditya S. Khair⁴

¹Department of Chemical Engineering, University of Florida, Gainesville, FL 32611, USA

²Department of Physics and Center for Complex Fluids Engineering, Carnegie Mellon University, Pittsburgh, PA 15213, USA

³Department of Chemical Engineering, Department of Biomedical Engineering, and Center for Complex Fluids Engineering, Carnegie Mellon University, Pittsburgh, PA 15213, USA

⁴Department of Chemical Engineering and Center for Complex Fluids Engineering, Carnegie Mellon University, Pittsburgh, PA 15213, USA

(Received 9 November 2020; revised 3 March 2021; accepted 2 April 2021)

We conduct an asymptotic analysis to derive a macrotransport equation for the long-time transport of a chemotactic/diffusiophoretic colloidal species in a uniform circular tube under a steady, laminar, pressure-driven flow and transient solute gradient. The solute gradient drives a ‘log-sensing’ advective flux of the colloidal species, which competes with Taylor dispersion due to the hydrodynamic flow. We demonstrate excellent agreement between the macrotransport equation and direct numerical solution of the full advection–diffusion equation for the colloidal species transport. In addition to its accuracy, the macrotransport equation requires $O(10^3)$ times less computational runtime than direct numerical solution of the advection–diffusion equation. Via scaling arguments, we identify three regimes of the colloidal species macrotransport, which span from chemotactic/diffusiophoretic-dominated macrotransport to the familiar Taylor dispersion regime, where macrotransport is dominated by the hydrodynamic flow. Finally, we discuss generalization of the macrotransport equation to channels of arbitrary (but constant) cross-section and to incorporate more sophisticated models of chemotactic fluxes. The macrotransport framework developed here will broaden the scope of designing chemotactic/diffusiophoretic transport systems by elucidating the interplay of macrotransport due to chemotaxis/diffusiophoresis and hydrodynamic flow.

Key words: mixing and dispersion, colloids, microfluidics

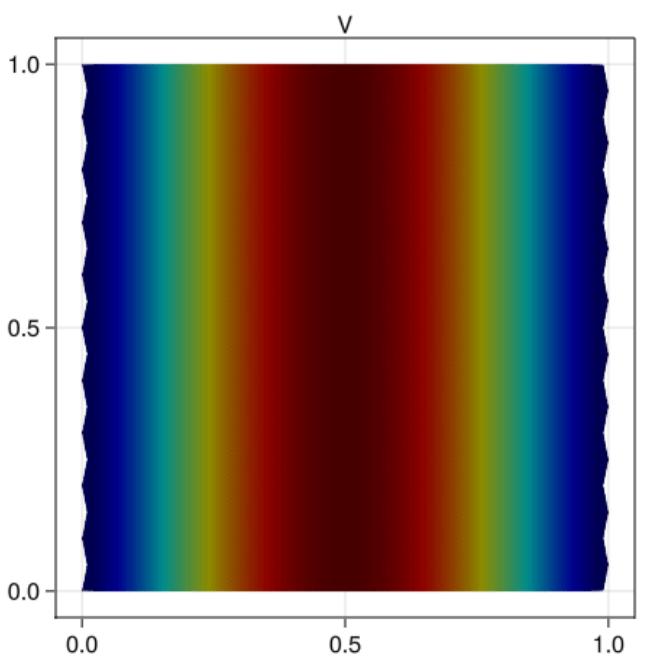
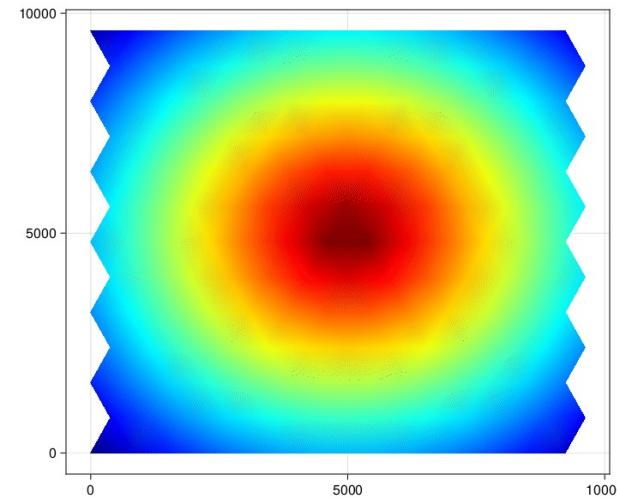
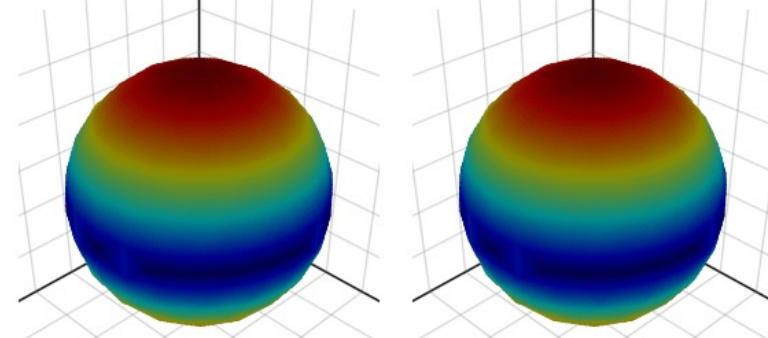
Under the chemical and hydrodynamic flow, the advection–diffusion–reaction equation for the diffusiophoretic/chemotactic colloids is

$$\frac{\partial C}{\partial t} + v \frac{\partial C}{\partial z} + \frac{1}{r} \frac{\partial(r u_r C)}{\partial r} + \frac{\partial(u_z C)}{\partial z} = \frac{D_c}{r} \frac{\partial}{\partial r} \left(r \frac{\partial C}{\partial r} \right) + D_c \frac{\partial^2 C}{\partial z^2} - \Gamma C, \quad (2.3)$$

$$\frac{D_c}{r} \frac{\partial}{\partial r} \left(r \frac{\partial C}{\partial r} \right)$$

† Email address for correspondence: h.chu@ufl.edu

Thanks!



Luke Morris CS PhD Student

University of Florida [CISE](#)

Contact:
Email: myfirstname[dot]mylastname@ufl.edu

[My LinkedIn](#)



About me:
I'm a 3rd-year PhD student in the Herbert Wertheim College of Engineering at the University of Florida.
Last summer, I was a research associate at [The Topos Institute](#).
I graduated with my Bachelor's in Computer Science from the University of Kentucky in 2021, *summa cum laude*.
My advisor here in Gainesville is [Dr. James Fairbanks](#) of the [GATAS Lab](#).

My Current Research Involves:

- Applied Category Theory
- Opinion Dynamics
- Multiphysics Simulations
- High Performance Computing
- Space Weather



