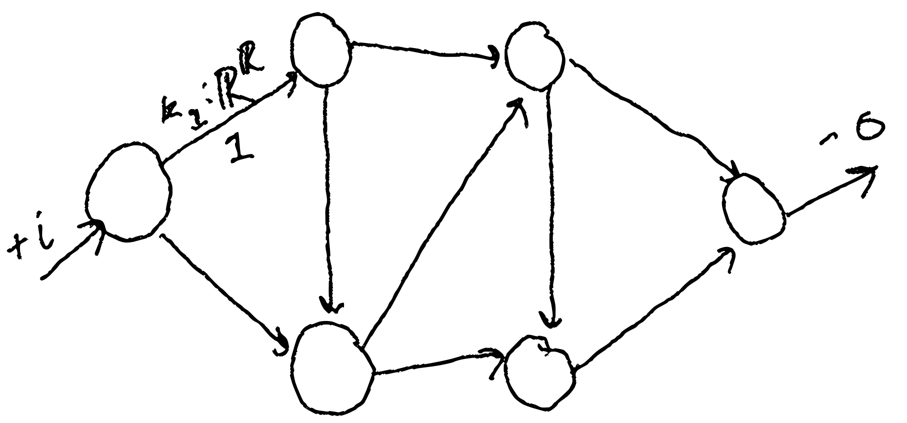

Flow-problem formulation

$$A_{ij} := \begin{cases} 1 & \text{if } s(j) = i \\ -1 & \text{if } t(j) = i \\ 0 & \text{o/w} \end{cases}$$

x_i represents flow on edge i.

Flow conservation equation: $Ax = b$

So want to min $\sum_i k_i(x_i)$

s.t. $Ax = b$

How to do this compositionally?

flow graph on V:

$$K \xleftarrow{\ell} E \underset{t}{\overset{s}{\rightrightarrows}} V \xrightarrow{b} \mathbb{R} \mid \mathbb{1}^T b = 0$$

Can combine flow graphs

$$\varphi_{N,M}((E_1, s_1, t_1, \ell_1, b_1), (E_2, s_2, t_2, \ell_2, b_2))$$

$$:= (E_1 + E_2, s_1 + s_2, t_1 + t_2, [\ell_1, \ell_2], [b_1, b_2]).$$

Merge nodes:

$\phi : N \to M$

$$\mathrm{FlowGraph}(\phi)(E, s, t, \ell, b) = (E, \phi s, \phi t, \ell, \phi_* b)$$

Translation from FG to CP is compositional:

$$\mathrm{netflow}_N : \mathrm{FlowGraph}(N) \to \mathrm{Conc}(N)$$

$$(E, s, t, \ell, b) \mapsto$$

$$(\lambda \mapsto \inf_{x \in \mathbb{R}^E} \sum_{e \in E} \ell(e)(x(e)) + \lambda^T(Ax - b))$$

$$\begin{tikzcd} \mathrm{FlowGraph}(N) \arrow[r, "FG(\phi)"] \arrow[d, "nf_N"'] & \mathrm{FlowGraph}(M) \arrow[d, "nf_M"] \\ \mathrm{Conc}(N) \arrow[r, "\mathrm{Conc}(\phi)"'] & \mathrm{Conc}(M) \end{tikzcd}$$