

Assignment3

Jack

2024-10-01

1. Visualisation

1.1 Q1

```
hawksSmall <- drop_na(select(Hawks, Age, Day, Month, Year, CaptureTime, Species, Wing, Weight, Tail))
print(head(hawksSmall, 5))
```

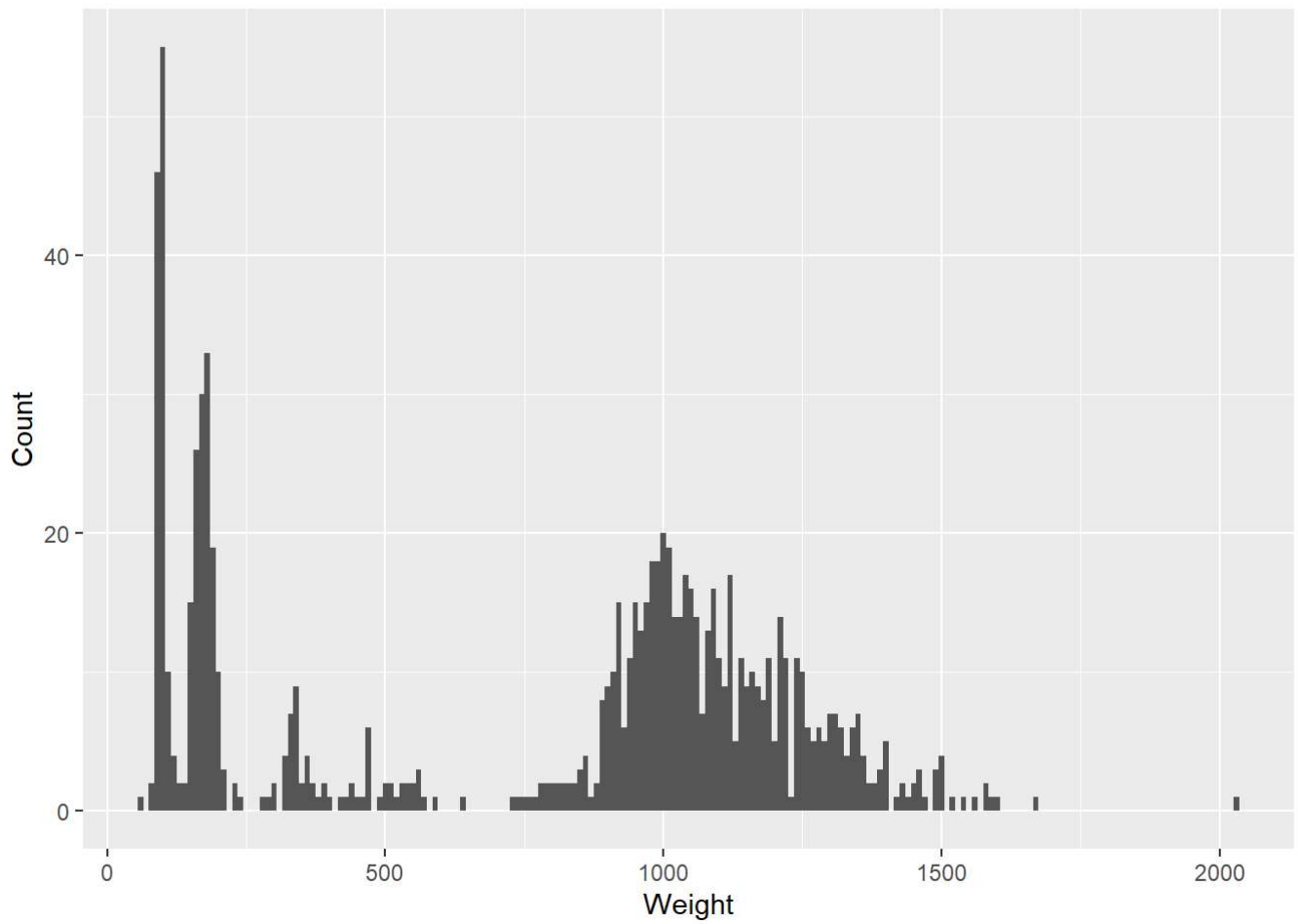
```
##   Age Day Month Year CaptureTime Species Wing Weight Tail
## 1   I  19     9 1992      13:30      RT  385    920  219
## 2   I  22     9 1992      10:30      RT  376    930  221
## 3   I  23     9 1992      12:45      RT  381    990  235
## 4   I  23     9 1992      10:50      CH  265    470  220
## 5   I  27     9 1992      11:15      SS  205    170  157
```

```
print(dim(hawksSmall))
```

```
## [1] 897    9
```

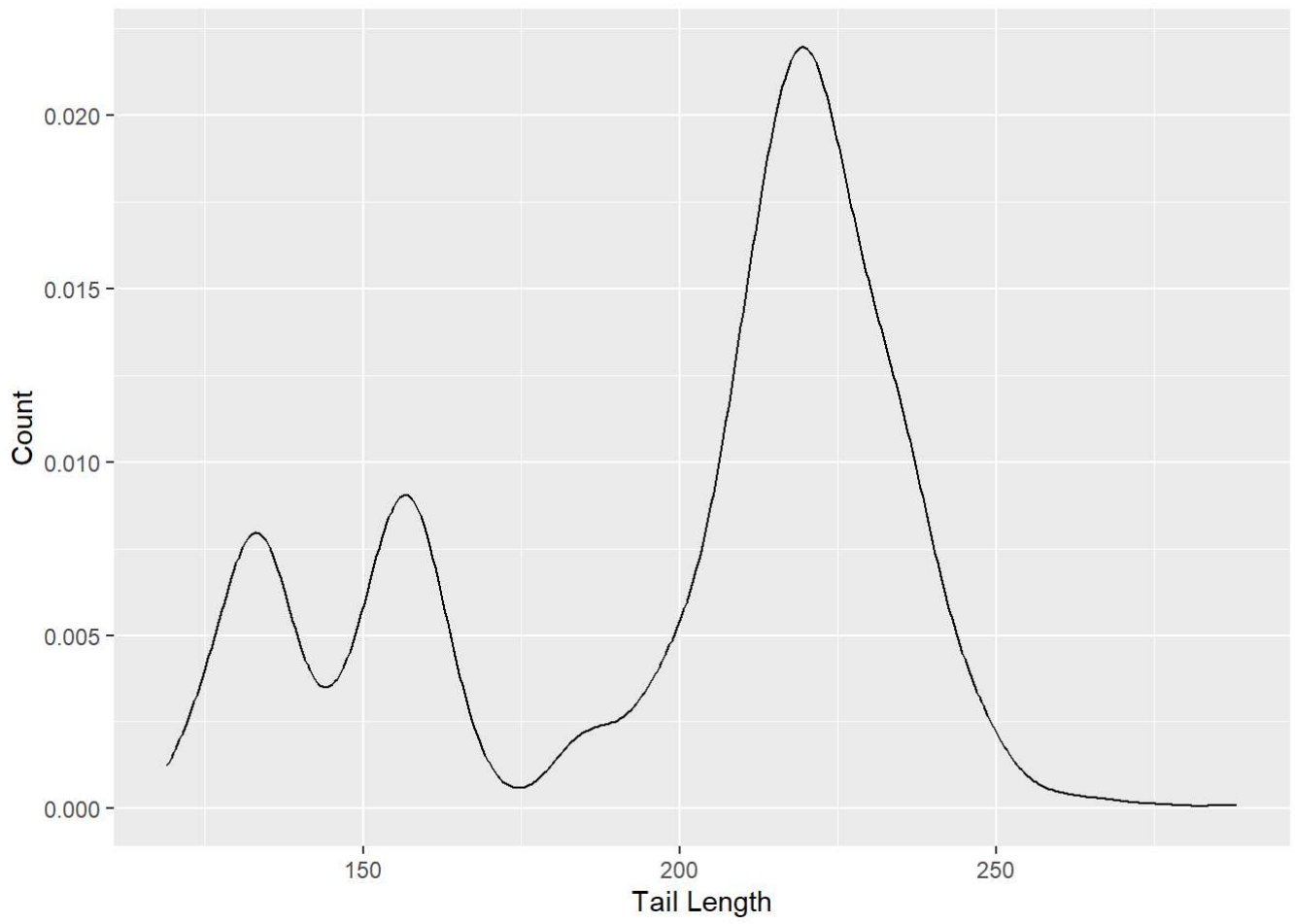
1.1 Q2

```
hawksSmall_weight <- ggplot(data=hawksSmall, aes(x=Weight))
hawksSmall_weight + geom_histogram(binwidth=10) + ylab("Count")
```

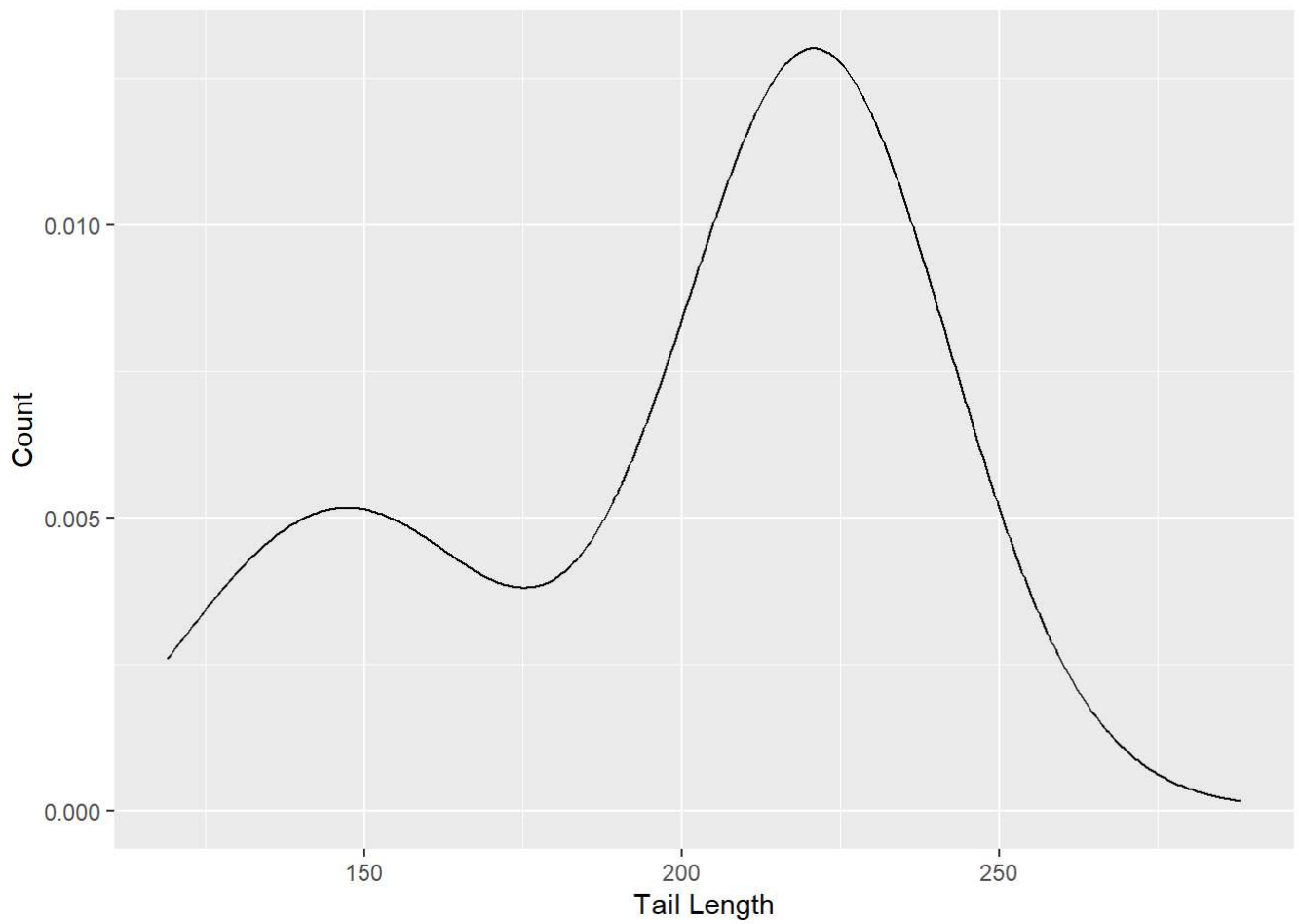


1.1 Q3

```
hawksSmall_density <- ggplot(data=hawksSmall, aes(x=Tail)) + xlab("Tail Length")  
hawksSmall_density + geom_density(adjust=0.5) + ylab("Count")
```



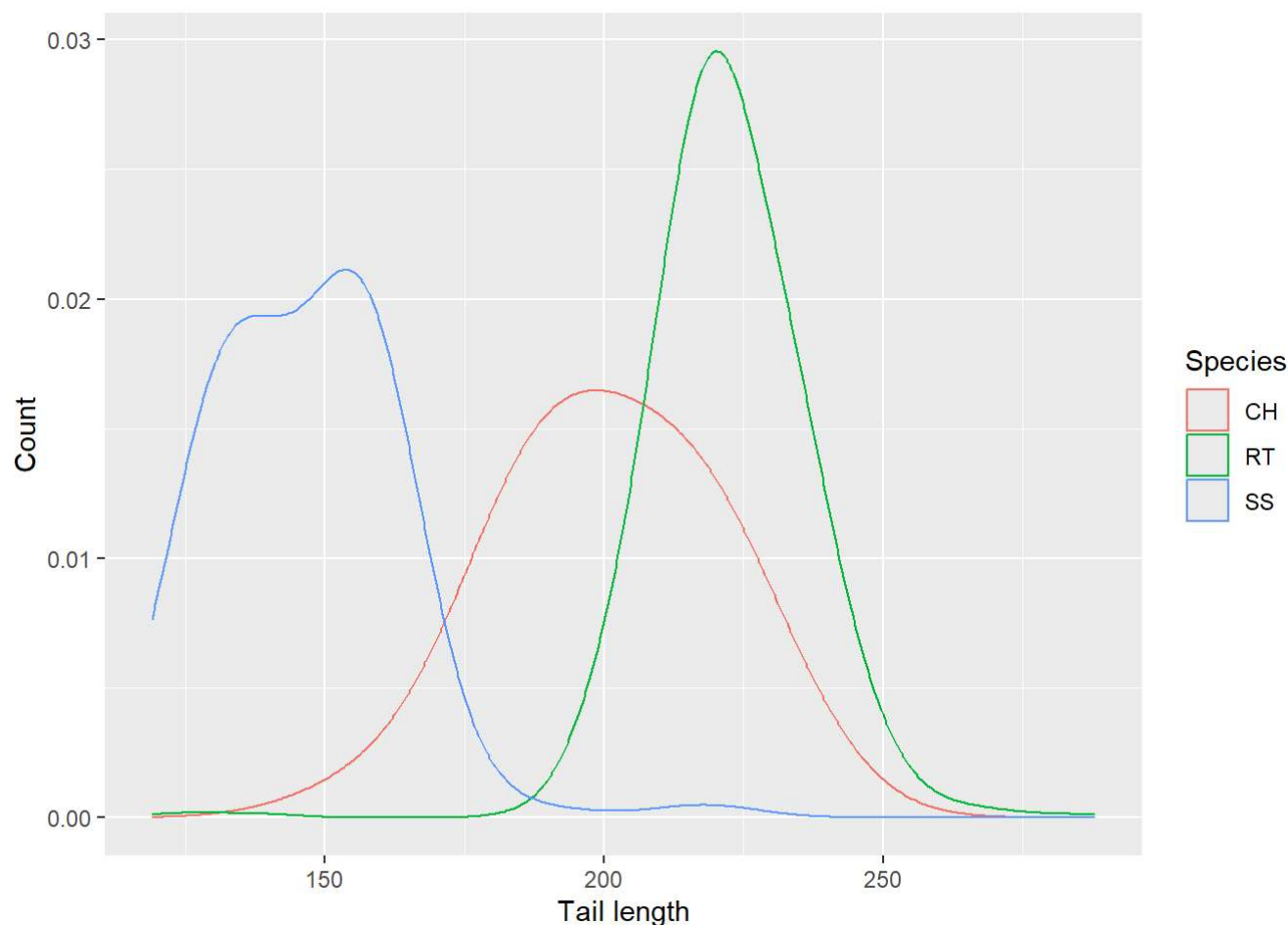
```
hawksSmall_density + geom_density(adjust=2) + ylab("Count")
```



These plots have different modes. There are 3 modes in plot with adjust 0.5 and there are 2 modes in plot with adjust 2

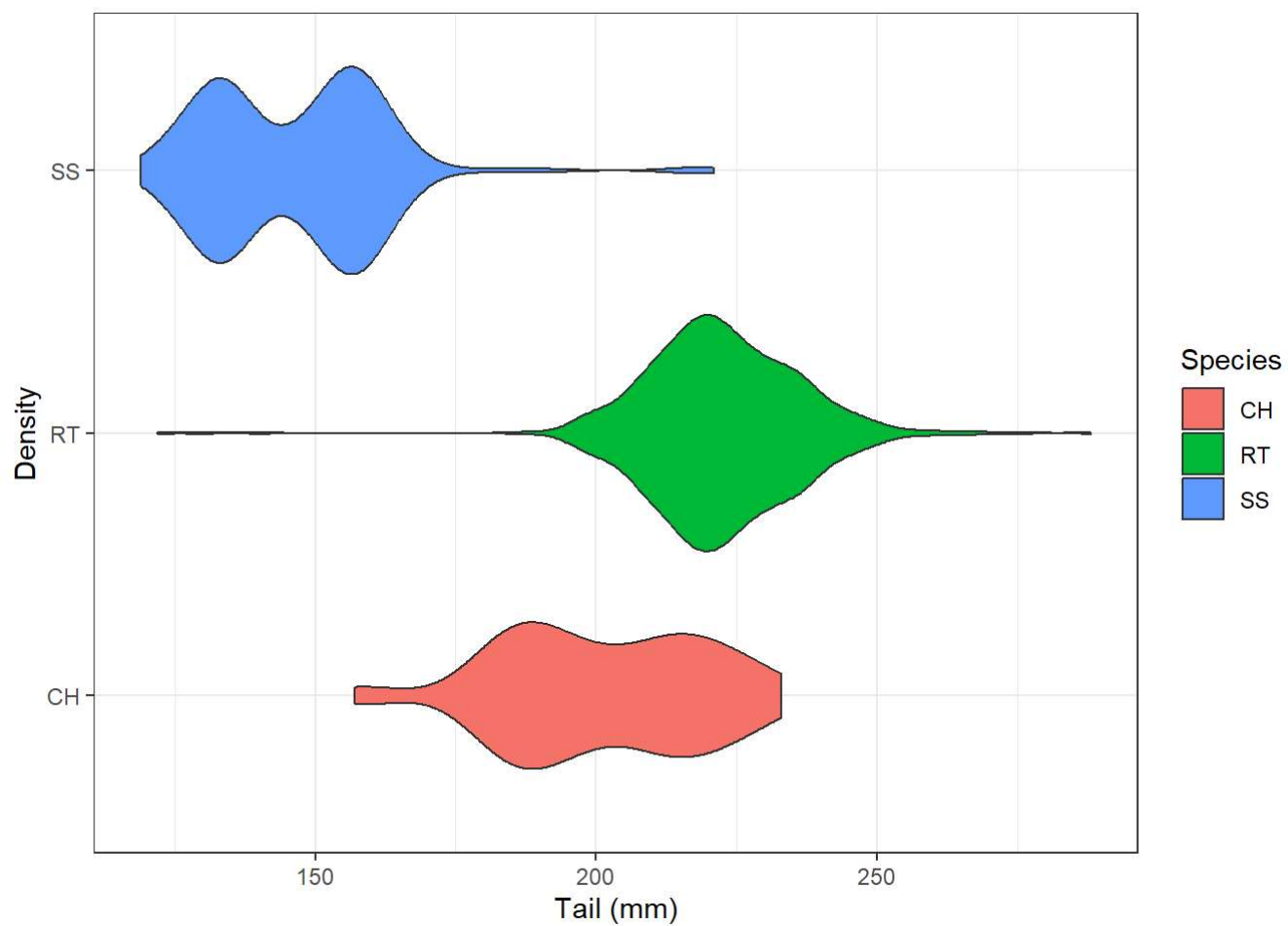
1.1 Q4

```
ggplot(data=hawksSmall, aes(x=Tail, color=Species)) + geom_density(adjust=2) + xlab("Tail length") + ylab("Count")
```



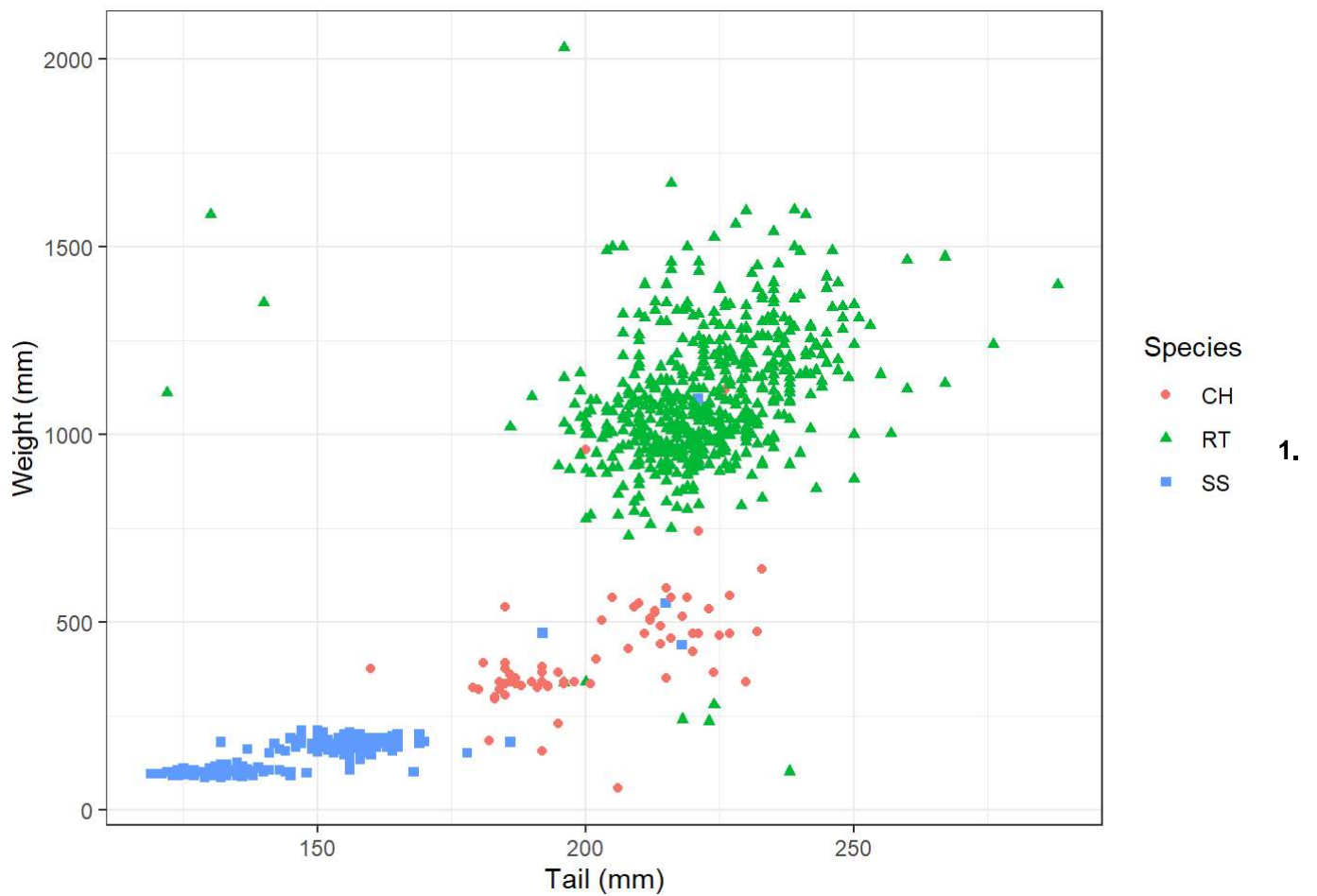
1.1 Q5

```
ggplot(data=hawksSmall, aes(x=Tail, y=Species, fill=Species)) + geom_violin() + theme_bw() + xlab("Tail (mm)") + ylab("Density")
```



1.1 Q6

```
hawksMass <- ggplot(data=hawksSmall, aes(y=Weight, x=Tail)) + xlab("Tail (mm)") + ylab("Weight (mm)")  
hawksMass + geom_point(aes(color=Species, shape=Species)) + theme_bw()
```



aesthetics: Species -> color, Species -> shape, Tail -> x, Weight -> y,

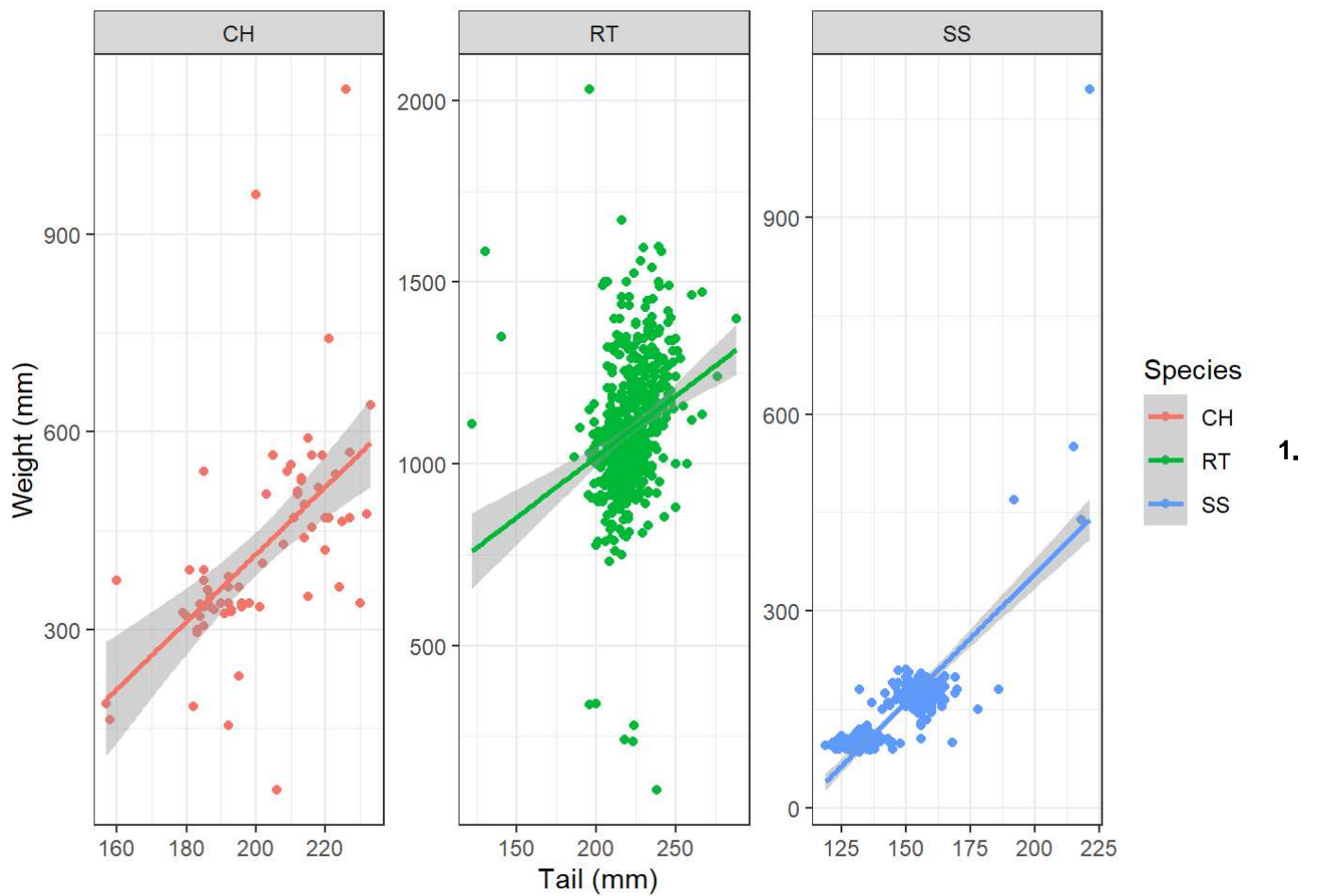
2. glyphs: points

3. Shape Color

1.1 Q7

```
hawksMass + geom_point(aes(color=Species)) + theme_bw() + facet_wrap(~Species, scales="free") +  
geom_smooth(method="lm", aes(color=Species))
```

```
## `geom_smooth()` using formula = 'y ~ x'
```



Area color Direction

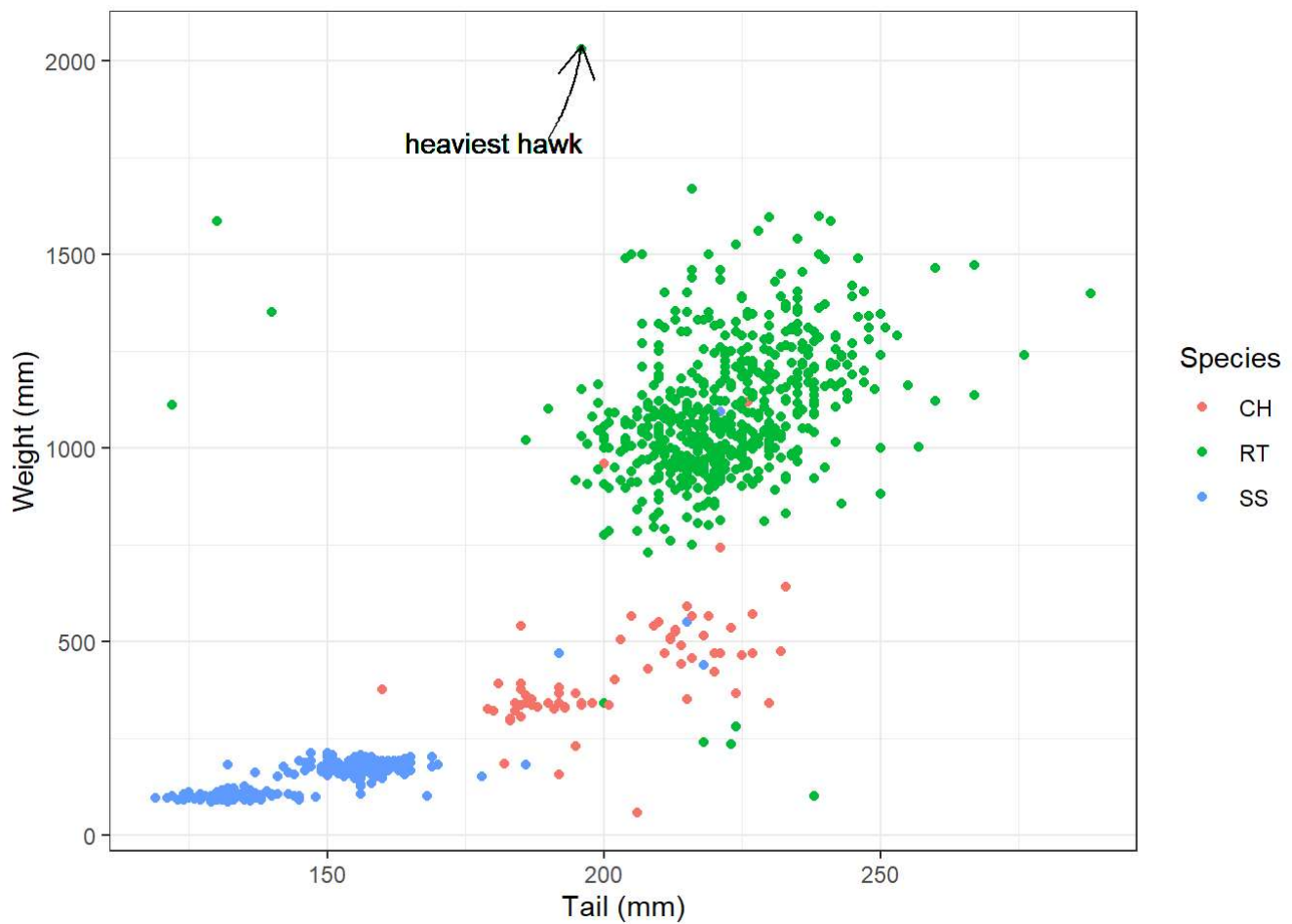
2. Proportional

1.1 Q8

```
heaviest <- max(filter(hawksSmall)$Weight, na.rm=TRUE)
print(heaviest)
```

```
## [1] 2030
```

```
hawksMass + geom_point(aes(color=Species)) + theme_bw() + geom_curve(x=190, xend=196, y=1800, yend=
=2040, arrow=arrow(length=unit(0.5, 'cm')), curvature=0.1) + geom_text(x=180, y=1790, label="heavies
t hawk")
```



2. Finite probability spaces

2.1 Q1

$$\binom{22}{z} \cdot 3^z \cdot 7^{22-z}$$

2.1 Q2

```
pro_red_spheres <- function(z) {
  stopifnot(is.numeric(z), z>=0)
  A <- choose(22, z) * 3^z * 7^(22-z)
  result <- A/(10^22)
  return (result)
}
print(pro_red_spheres(10))
```

```
## [1] 0.05285129
```

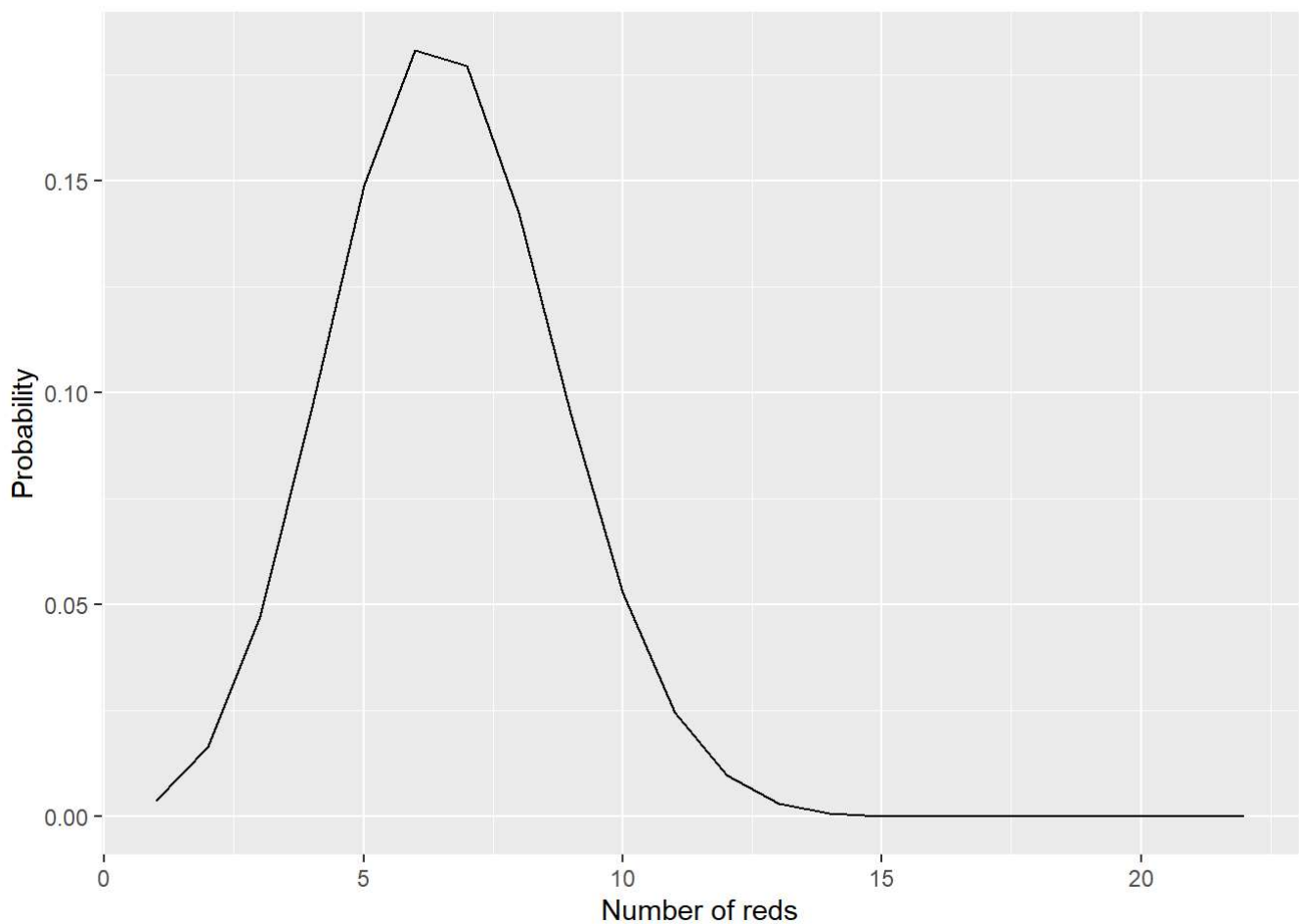

2.1 Q3

```
num_reds <- c()
prob <- c()
for (x in 1:22){
  num_reds <- append(num_reds, x)
  prob <- append(prob, pro_red_spheres(x))
}
prob_by_num_reds <- data.frame(num_reds, prob)
prob_by_num_reds %>% head(3)
```

```
##   num_reds      prob
## 1         1 0.003686403
## 2         2 0.016588812
## 3         3 0.047396606
```

2.1 Q4

```
ggplot(prob_by_num_reds, aes(x=num_reds,y=prob)) + geom_line() + xlab("Number of reds") + ylab("Probability")
```



2.1 Q5

```
# case 1: Setting the random seed just once
set.seed(0)
for(i in 1:5){
  print(sample(100, 5, replace=FALSE))
}
```

```
## [1] 14 68 39  1 34
## [1] 87 43 14 82 59
## [1] 51 97 85 21 54
## [1] 74  7 73 79 85
## [1] 37 89 100 34 99
```

```
# case 2: Resetting the random seed every time
set.seed(1)
print(sample(100, 5, replace=FALSE))
```

```
## [1] 68 39  1 34 87
```

```
set.seed(1)
print(sample(100, 5, replace=FALSE))
```

```
## [1] 68 39  1 34 87
```

```
set.seed(1)
print(sample(100, 5, replace=FALSE))
```

```
## [1] 68 39  1 34 87
```

```
# case 3: reproducing case 1 if we set a random seed at the beginning
set.seed(0)
for(i in 1:5){
  print(sample(100, 5, replace=FALSE))
}
```

```
## [1] 14 68 39  1 34
## [1] 87 43 14 82 59
## [1] 51 97 85 21 54
## [1] 74  7 73 79 85
## [1] 37 89 100 34 99
```

```
# The result will be 5 samples same as in case 1, because they have the same seed
```

```
itermap <- function(.x,.f){
  result <- list()
  for (item in .x) {
    result <- c(result, list(.f(item)))
  }
  return (result)
}
itermap(c(1,2,3),function(x){return (c(x,x^2))})
```

```
## [[1]]
## [1] 1 1
##
## [[2]]
## [1] 2 4
##
## [[3]]
## [1] 3 9
```

```
itermap_dbl <- function(.x,.f) {
  result <- numeric(length(.x))
  for(i in 1:length(.x)){
    result[i] <- .f(.x[[i]])
  }
  return (result)
}
itermap_dbl(c(1,2,3), function(x){return (x^3)})
```

```
## [1] 1 8 27
```

```
trial <- sequence(1000)
set.seed(0)
sampling_with_replacement_simulation <- data.frame(trial) %>% mutate(sample_balls = itermap(.x=
trial,function(x){sample(10,22,replace=TRUE)}))

sampling_with_replacement_simulation <- sampling_with_replacement_simulation %>% mutate(num_red
s = itermap_dbl(.x=trial,function(x){
  n <- 0
  for (i in sampling_with_replacement_simulation[[x,"sample_balls"]]){
    if (i <= 3){n <- n+1}
  }
  return (n)
})
))
head(sampling_with_replacement_simulation,5)
```

```
##      trial                                     sample_balls
## 1      1  9, 4, 7, 1, 2, 7, 2, 3, 1, 5, 5, 10, 6, 10, 7, 9, 5, 5, 9, 9, 5, 5
## 2      2  2, 10, 9, 1, 4, 3, 6, 10, 10, 6, 4, 4, 10, 9, 7, 6, 9, 8, 9, 7, 8, 6
## 3      3  10, 7, 3, 10, 6, 8, 2, 2, 6, 6, 1, 3, 3, 8, 6, 7, 6, 8, 7, 1, 4, 8
## 4      4  9, 9, 7, 4, 7, 6, 1, 5, 6, 1, 9, 7, 7, 3, 6, 2, 10, 10, 7, 3, 2, 10
## 5      5  1, 10, 10, 8, 10, 5, 7, 8, 5, 6, 8, 1, 3, 10, 3, 1, 6, 6, 4, 9, 5, 1
##      num_reds
## 1          5
## 2          3
## 3          7
## 4          6
## 5          6
```

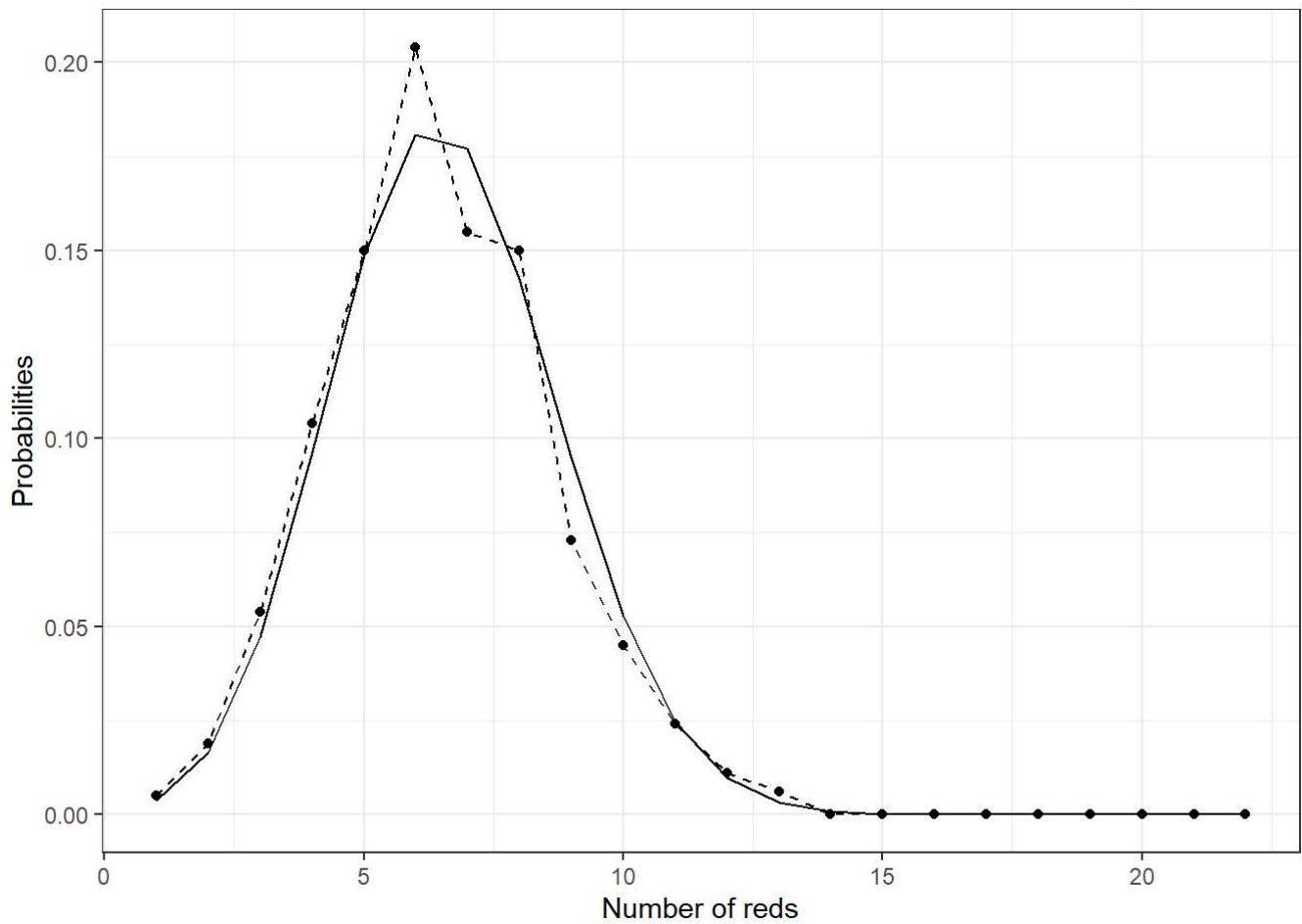
2.1 Q6

```
# we extract a vector corresponding to the number of reds in each trial
num_reds_in_simulation <- sampling_with_replacement_simulation %>% pull(num_reds)
# add a column which gives the number of trials with a given number of reds
prob_by_num_reds <- prob_by_num_reds %>% mutate(predicted_prob = itermap_dbl(.x = num_reds, function
(.x) sum(num_reds_in_simulation == .x)) / 1000)
print(prob_by_num_reds)
```

```
##      num_reds      prob predicted_prob
## 1          1 3.686403e-03          0.005
## 2          2 1.658881e-02          0.019
## 3          3 4.739661e-02          0.054
## 4          4 9.648595e-02          0.104
## 5          5 1.488640e-01          0.150
## 6          6 1.807635e-01          0.204
## 7          7 1.770744e-01          0.155
## 8          8 1.422919e-01          0.150
## 9          9 9.486130e-02          0.073
## 10         10 5.285129e-02          0.045
## 11         11 2.470970e-02          0.024
## 12         12 9.707380e-03          0.011
## 13         13 3.200235e-03          0.006
## 14         14 8.816975e-04          0.000
## 15         15 2.015309e-04          0.000
## 16         16 3.778704e-05          0.000
## 17         17 5.715686e-06          0.000
## 18         18 6.804388e-07          0.000
## 19         19 6.139298e-08          0.000
## 20         20 3.946691e-09          0.000
## 21         21 1.610894e-10          0.000
## 22         22 3.138106e-12          0.000
```

2.1 Q7

```
prob_by_num_reds %>% rename(TheoreticalProbability=prob, EstimatedProbability=predicted_prob) %>%  
  ggplot(aes(x=num_reds)) + theme_bw() + geom_line(aes(y=TheoreticalProbability)) + geom_line(  
    aes(y=EstimatedProbability), linetype="dashed") + geom_point(aes(y=EstimatedProbability)) + xlab(  
    "Number of reds") + ylab("Probabilities")
```



2.2 Q1

```

# 1. Random seed
set.seed(0)

# 2. set a number of trials
trials <- sequence(100)

# 3. generate samples
spheres <- c(rep("Red", 50), rep("Blue", 30), rep("Green", 20))
sampling_without_replacement_simulation <- data.frame(trials) %>% mutate(sample_balls = itermap
(.x=trials,function(x){sample(spheres,10,replace=TRUE)}))

# 4. Compute the number of red, green, and blue
# Red
sampling_without_replacement_simulation <- sampling_without_replacement_simulation %>% mutate(n
um_reds = itermap_dbl(.x=trials,function(x){
  n <- 0
  for (i in sampling_without_replacement_simulation[[x,"sample_balls"]]){
    if (i == "Red"){n <- n+1}
  }
  return (n)
}
))

# Green
sampling_without_replacement_simulation <- sampling_without_replacement_simulation %>% mutate(n
um_greens = itermap_dbl(.x=trials,function(x){
  n <- 0
  for (i in sampling_without_replacement_simulation[[x,"sample_balls"]]){
    if (i == "Green"){n <- n+1}
  }
  return (n)
}
))

#Blue
sampling_without_replacement_simulation <- sampling_without_replacement_simulation %>% mutate(n
um_blues = itermap_dbl(.x=trials,function(x){
  n <- 0
  for (i in sampling_without_replacement_simulation[[x,"sample_balls"]]){
    if (i == "Blue"){n <- n+1}
  }
  return (n)
}
))
print(sampling_without_replacement_simulation)

```

| ## | trials | sample_balls |
|-------|--------|---|
| ## 1 | 1 | Red, Blue, Red, Red, Red, Green, Red, Red, Green, Blue |
| ## 2 | 2 | Blue, Green, Green, Red, Blue, Blue, Red, Blue, Blue, Green |
| ## 3 | 3 | Red, Green, Red, Red, Green, Red, Blue, Red, Green, Red |
| ## 4 | 4 | Blue, Blue, Red, Red, Red, Red, Red, Red, Green, Blue |
| ## 5 | 5 | Red, Red, Red, Blue, Red, Blue, Red, Red, Red, Red |
| ## 6 | 6 | Red, Red, Red, Red, Red, Blue, Blue, Blue, Green, Blue |
| ## 7 | 7 | Blue, Green, Green, Red, Red, Green, Red, Green, Red, Green |
| ## 8 | 8 | Red, Red, Red, Green, Red, Red, Red, Red, Red, Red |
| ## 9 | 9 | Red, Blue, Green, Green, Green, Red, Blue, Green, Green, Blue |
| ## 10 | 10 | Blue, Green, Red, Red, Red, Red, Blue, Red, Red, Blue |
| ## 11 | 11 | Red, Red, Red, Red, Blue, Red, Red, Blue, Green, Green |
| ## 12 | 12 | Red, Green, Green, Green, Red, Red, Red, Red, Blue, Red |
| ## 13 | 13 | Blue, Red, Red, Blue, Red, Blue, Green, Green, Blue, Green |
| ## 14 | 14 | Blue, Red, Red, Blue, Blue, Red, Red, Green, Red, Red |
| ## 15 | 15 | Red, Blue, Blue, Red, Green, Green, Red, Red, Blue, Red |
| ## 16 | 16 | Blue, Green, Green, Blue, Red, Red, Red, Blue, Red, Red |
| ## 17 | 17 | Green, Red, Red, Blue, Red, Red, Red, Red, Green, Red |
| ## 18 | 18 | Red, Blue, Blue, Red, Red, Green, Red, Green, Red, Red |
| ## 19 | 19 | Red, Red, Red, Blue, Blue, Blue, Red, Green, Red, Red |
| ## 20 | 20 | Green, Red, Blue, Red, Red, Red, Red, Green, Blue, Red |
| ## 21 | 21 | Blue, Green, Blue, Green, Blue, Green, Red, Blue, Red, Green |
| ## 22 | 22 | Red, Blue, Blue, Red, Green, Red, Red, Blue, Blue, Blue |
| ## 23 | 23 | Green, Green, Blue, Red, Red, Red, Blue, Red, Blue, Red |
| ## 24 | 24 | Red, Red, Green, Blue, Blue, Blue, Red, Red, Red, Green |
| ## 25 | 25 | Red, Blue, Red, Red, Red, Blue, Blue, Red, Blue, Red |
| ## 26 | 26 | Red, Green, Blue, Red, Blue, Green, Red, Red, Red, Blue |
| ## 27 | 27 | Green, Green, Red, Red, Red, Red, Green, Green, Green, Green |
| ## 28 | 28 | Blue, Blue, Blue, Blue, Green, Red, Blue, Red, Green, Green |
| ## 29 | 29 | Blue, Red, Red, Red, Red, Red, Blue, Blue, Green, Blue |
| ## 30 | 30 | Red, Red, Green, Red, Green, Red, Green, Green, Green, Red |
| ## 31 | 31 | Blue, Blue, Green, Blue, Blue, Blue, Red, Green, Red, Blue |
| ## 32 | 32 | Blue, Blue, Red, Red, Green, Green, Blue, Red, Red, Red |
| ## 33 | 33 | Red, Red, Blue, Green, Green, Red, Red, Blue, Blue, Red |
| ## 34 | 34 | Red, Green, Green, Blue, Red, Green, Blue, Blue, Red, Green |
| ## 35 | 35 | Red, Blue, Red, Red, Blue, Red, Blue, Blue, Blue, Blue |
| ## 36 | 36 | Blue, Red, Red, Red, Red, Red, Red, Red, Blue, Green |
| ## 37 | 37 | Red, Red, Red, Red, Green, Green, Red, Red, Red, Red |
| ## 38 | 38 | Green, Red, Red, Green, Blue, Green, Blue, Blue, Green, Blue |
| ## 39 | 39 | Red, Red, Blue, Blue, Green, Blue, Blue, Blue, Green, Green |
| ## 40 | 40 | Red, Blue, Red, Blue, Red, Red, Red, Red, Red, Blue |
| ## 41 | 41 | Blue, Red, Green, Red, Blue, Red, Red, Red, Green, Red |
| ## 42 | 42 | Green, Red, Blue, Red, Blue, Blue, Blue, Red, Red, Red |
| ## 43 | 43 | Blue, Blue, Blue, Green, Green, Red, Red, Red, Red, Red |
| ## 44 | 44 | Red, Blue, Blue, Red, Red, Red, Red, Green, Red, Red |
| ## 45 | 45 | Green, Red, Blue, Red, Red, Red, Green, Blue, Blue, Green |
| ## 46 | 46 | Blue, Blue, Blue, Red, Blue, Green, Blue, Blue, Red, Green |
| ## 47 | 47 | Red, Red, Red, Red, Red, Red, Red, Red, Green, Red |
| ## 48 | 48 | Red, Red, Red, Green, Red, Red, Red, Red, Red, Blue |
| ## 49 | 49 | Blue, Blue, Green, Red, Green, Red, Green, Red, Red, Green |
| ## 50 | 50 | Red, Green, Green, Red, Red, Green, Red, Red, Red, Blue |
| ## 51 | 51 | Red, Red, Green, Red, Blue, Red, Blue, Red, Green, Green |
| ## 52 | 52 | Red, Blue, Blue, Green, Red, Red, Green, Green, Green, Red |
| ## 53 | 53 | Red, Red, Red, Blue, Red, Red, Blue, Blue, Blue, Green |
| ## 54 | 54 | Green, Green, Blue, Red, Red, Red, Red, Green, Blue, Red |

| | | |
|--------|-----|---|
| ## 55 | 55 | Green, Blue, Green, Red, Blue, Blue, Green, Green, Green, Red |
| ## 56 | 56 | Blue, Red, Red, Blue, Green, Red, Red, Blue, Red, Red |
| ## 57 | 57 | Red, Red, Red, Blue, Green, Red, Red, Blue, Red, Blue |
| ## 58 | 58 | Red, Blue, Blue, Red, Blue, Green, Blue, Blue, Red, Red |
| ## 59 | 59 | Red, Blue, Green, Green, Blue, Red, Blue, Green, Green, Green |
| ## 60 | 60 | Blue, Red, Green, Blue, Blue, Red, Red, Blue, Red, Red |
| ## 61 | 61 | Green, Green, Red, Blue, Green, Red, Red, Red, Red, Red |
| ## 62 | 62 | Green, Blue, Blue, Green, Green, Blue, Red, Red, Green, Blue |
| ## 63 | 63 | Green, Red, Red, Green, Blue, Green, Red, Blue, Red, Red |
| ## 64 | 64 | Red, Blue, Red, Red, Green, Green, Red, Red, Red, Green |
| ## 65 | 65 | Blue, Red, Blue, Blue, Blue, Red, Green, Blue, Red, Green |
| ## 66 | 66 | Red, Green, Red, Red, Red, Green, Blue, Green, Blue, Green |
| ## 67 | 67 | Blue, Green, Red, Red, Red, Green, Blue, Blue, Red, Red |
| ## 68 | 68 | Red, Blue, Red, Green, Green, Red, Green, Red, Red, Green |
| ## 69 | 69 | Blue, Green, Red, Blue, Green, Green, Green, Red, Blue, Red |
| ## 70 | 70 | Green, Red, Red, Red, Green, Red, Blue, Red, Blue, Red |
| ## 71 | 71 | Red, Red, Blue, Red, Red, Red, Red, Red, Red, Blue |
| ## 72 | 72 | Green, Green, Blue, Blue, Red, Blue, Green, Red, Blue, Green |
| ## 73 | 73 | Red, Green, Red, Red, Blue, Red, Red, Red, Blue, Red |
| ## 74 | 74 | Red, Green, Green, Blue, Red, Red, Red, Red, Red, Red |
| ## 75 | 75 | Blue, Blue, Green, Red, Blue, Red, Red, Red, Blue, Green |
| ## 76 | 76 | Red, Red, Blue, Blue, Green, Green, Blue, Blue, Green, Green |
| ## 77 | 77 | Red, Blue, Red, Red, Red, Green, Red, Blue, Blue, Red |
| ## 78 | 78 | Blue, Blue, Red, Green, Blue, Green, Red, Red, Green, Blue |
| ## 79 | 79 | Green, Red, Red, Blue, Green, Blue, Green, Red, Red, Blue |
| ## 80 | 80 | Blue, Red, Green, Red, Red, Red, Blue, Green, Red, Red |
| ## 81 | 81 | Blue, Red, Red, Red, Red, Green, Green, Blue, Red, Red |
| ## 82 | 82 | Green, Red, Red, Green, Red, Red, Red, Red, Red, Blue |
| ## 83 | 83 | Red, Red, Red, Blue, Red, Blue, Blue, Green, Red, Green |
| ## 84 | 84 | Blue, Blue, Blue, Green, Red, Red, Red, Red, Red, Blue |
| ## 85 | 85 | Blue, Blue, Green, Red, Blue, Blue, Red, Blue, Red, Red |
| ## 86 | 86 | Red, Green, Blue, Red, Green, Blue, Blue, Blue, Blue, Red |
| ## 87 | 87 | Blue, Red, Red, Red, Red, Red, Blue, Red, Green, Green |
| ## 88 | 88 | Green, Green, Red, Red, Blue, Blue, Red, Red, Blue, Blue |
| ## 89 | 89 | Green, Red, Blue, Green, Red, Blue, Red, Red, Green, Red |
| ## 90 | 90 | Red, Red, Red, Red, Red, Blue, Blue, Blue, Red, Red |
| ## 91 | 91 | Blue, Red, Red, Blue, Red, Red, Red, Green, Green, Red |
| ## 92 | 92 | Red, Blue, Green, Red, Blue, Red, Red, Blue, Green, Blue |
| ## 93 | 93 | Green, Red, Blue, Green, Red, Blue, Blue, Blue, Green, Green |
| ## 94 | 94 | Red, Blue, Red, Blue, Red, Green, Red, Red, Blue, Blue |
| ## 95 | 95 | Blue, Red, Blue, Green, Red, Blue, Red, Red, Blue, Red |
| ## 96 | 96 | Blue, Green, Red, Red, Blue, Blue, Red, Blue, Green, Red |
| ## 97 | 97 | Green, Red, Red, Red, Blue, Blue, Red, Red, Green, Red |
| ## 98 | 98 | Red, Red, Green, Green, Green, Blue, Blue, Red, Blue, Blue |
| ## 99 | 99 | Red, Green, Red, Blue, Blue, Blue, Blue, Blue, Red, Blue |
| ## 100 | 100 | Red, Blue, Red, Red, Red, Blue, Green, Blue, Red, Red |
| ## | | num_reds num_greens num_blues |
| ## 1 | 6 | 2 2 |
| ## 2 | 2 | 3 5 |
| ## 3 | 6 | 3 1 |
| ## 4 | 6 | 1 3 |
| ## 5 | 8 | 0 2 |
| ## 6 | 5 | 1 4 |
| ## 7 | 4 | 5 1 |
| ## 8 | 9 | 1 0 |
| ## 9 | 2 | 5 3 |

| | | | |
|-------|---|---|---|
| ## 10 | 6 | 1 | 3 |
| ## 11 | 6 | 2 | 2 |
| ## 12 | 6 | 3 | 1 |
| ## 13 | 3 | 3 | 4 |
| ## 14 | 6 | 1 | 3 |
| ## 15 | 5 | 2 | 3 |
| ## 16 | 5 | 2 | 3 |
| ## 17 | 7 | 2 | 1 |
| ## 18 | 6 | 2 | 2 |
| ## 19 | 6 | 1 | 3 |
| ## 20 | 6 | 2 | 2 |
| ## 21 | 2 | 4 | 4 |
| ## 22 | 4 | 1 | 5 |
| ## 23 | 5 | 2 | 3 |
| ## 24 | 5 | 2 | 3 |
| ## 25 | 6 | 0 | 4 |
| ## 26 | 5 | 2 | 3 |
| ## 27 | 4 | 6 | 0 |
| ## 28 | 2 | 3 | 5 |
| ## 29 | 5 | 1 | 4 |
| ## 30 | 5 | 5 | 0 |
| ## 31 | 2 | 2 | 6 |
| ## 32 | 5 | 2 | 3 |
| ## 33 | 5 | 2 | 3 |
| ## 34 | 3 | 4 | 3 |
| ## 35 | 4 | 0 | 6 |
| ## 36 | 7 | 1 | 2 |
| ## 37 | 8 | 2 | 0 |
| ## 38 | 2 | 4 | 4 |
| ## 39 | 2 | 3 | 5 |
| ## 40 | 7 | 0 | 3 |
| ## 41 | 6 | 2 | 2 |
| ## 42 | 5 | 1 | 4 |
| ## 43 | 5 | 2 | 3 |
| ## 44 | 7 | 1 | 2 |
| ## 45 | 4 | 3 | 3 |
| ## 46 | 2 | 2 | 6 |
| ## 47 | 9 | 1 | 0 |
| ## 48 | 8 | 1 | 1 |
| ## 49 | 4 | 4 | 2 |
| ## 50 | 6 | 3 | 1 |
| ## 51 | 5 | 3 | 2 |
| ## 52 | 4 | 4 | 2 |
| ## 53 | 5 | 1 | 4 |
| ## 54 | 5 | 3 | 2 |
| ## 55 | 2 | 5 | 3 |
| ## 56 | 6 | 1 | 3 |
| ## 57 | 6 | 1 | 3 |
| ## 58 | 4 | 1 | 5 |
| ## 59 | 2 | 5 | 3 |
| ## 60 | 5 | 1 | 4 |
| ## 61 | 6 | 3 | 1 |
| ## 62 | 2 | 4 | 4 |
| ## 63 | 5 | 3 | 2 |
| ## 64 | 6 | 3 | 1 |
| ## 65 | 3 | 2 | 5 |

| | | | |
|--------|---|---|---|
| ## 66 | 4 | 4 | 2 |
| ## 67 | 5 | 2 | 3 |
| ## 68 | 5 | 4 | 1 |
| ## 69 | 3 | 4 | 3 |
| ## 70 | 6 | 2 | 2 |
| ## 71 | 8 | 0 | 2 |
| ## 72 | 2 | 4 | 4 |
| ## 73 | 7 | 1 | 2 |
| ## 74 | 7 | 2 | 1 |
| ## 75 | 4 | 2 | 4 |
| ## 76 | 2 | 4 | 4 |
| ## 77 | 6 | 1 | 3 |
| ## 78 | 3 | 3 | 4 |
| ## 79 | 4 | 3 | 3 |
| ## 80 | 6 | 2 | 2 |
| ## 81 | 6 | 2 | 2 |
| ## 82 | 7 | 2 | 1 |
| ## 83 | 5 | 2 | 3 |
| ## 84 | 5 | 1 | 4 |
| ## 85 | 4 | 1 | 5 |
| ## 86 | 3 | 2 | 5 |
| ## 87 | 6 | 2 | 2 |
| ## 88 | 4 | 2 | 4 |
| ## 89 | 5 | 3 | 2 |
| ## 90 | 7 | 0 | 3 |
| ## 91 | 6 | 2 | 2 |
| ## 92 | 4 | 2 | 4 |
| ## 93 | 2 | 4 | 4 |
| ## 94 | 5 | 1 | 4 |
| ## 95 | 5 | 1 | 4 |
| ## 96 | 4 | 2 | 4 |
| ## 97 | 6 | 2 | 2 |
| ## 98 | 3 | 3 | 4 |
| ## 99 | 3 | 1 | 6 |
| ## 100 | 6 | 1 | 3 |

```
# 5. Compute the minimum of the three counts
sampling_without_replacement_simulation["min_colors"] <- pmin(sampling_without_replacement_simulation["num_reds"], sampling_without_replacement_simulation["num_greens"], sampling_without_replacement_simulation["num_blues"])
print(sampling_without_replacement_simulation)
```

| ## | trials | sample_balls |
|-------|--------|---|
| ## 1 | 1 | Red, Blue, Red, Red, Red, Green, Red, Red, Green, Blue |
| ## 2 | 2 | Blue, Green, Green, Red, Blue, Blue, Red, Blue, Blue, Green |
| ## 3 | 3 | Red, Green, Red, Red, Green, Red, Blue, Red, Green, Red |
| ## 4 | 4 | Blue, Blue, Red, Red, Red, Red, Red, Red, Green, Blue |
| ## 5 | 5 | Red, Red, Red, Blue, Red, Blue, Red, Red, Red, Red |
| ## 6 | 6 | Red, Red, Red, Red, Red, Blue, Blue, Blue, Green, Blue |
| ## 7 | 7 | Blue, Green, Green, Red, Red, Green, Red, Green, Red, Green |
| ## 8 | 8 | Red, Red, Red, Green, Red, Red, Red, Red, Red, Red |
| ## 9 | 9 | Red, Blue, Green, Green, Green, Red, Blue, Green, Green, Blue |
| ## 10 | 10 | Blue, Green, Red, Red, Red, Red, Blue, Red, Red, Blue |
| ## 11 | 11 | Red, Red, Red, Red, Blue, Red, Red, Blue, Green, Green |
| ## 12 | 12 | Red, Green, Green, Green, Red, Red, Red, Red, Blue, Red |
| ## 13 | 13 | Blue, Red, Red, Blue, Red, Blue, Green, Green, Blue, Green |
| ## 14 | 14 | Blue, Red, Red, Blue, Blue, Red, Red, Green, Red, Red |
| ## 15 | 15 | Red, Blue, Blue, Red, Green, Green, Red, Red, Blue, Red |
| ## 16 | 16 | Blue, Green, Green, Blue, Red, Red, Red, Blue, Red, Red |
| ## 17 | 17 | Green, Red, Red, Blue, Red, Red, Red, Red, Green, Red |
| ## 18 | 18 | Red, Blue, Blue, Red, Red, Green, Red, Green, Red, Red |
| ## 19 | 19 | Red, Red, Red, Blue, Blue, Blue, Red, Green, Red, Red |
| ## 20 | 20 | Green, Red, Blue, Red, Red, Red, Red, Green, Blue, Red |
| ## 21 | 21 | Blue, Green, Blue, Green, Blue, Green, Red, Blue, Red, Green |
| ## 22 | 22 | Red, Blue, Blue, Red, Green, Red, Red, Blue, Blue, Blue |
| ## 23 | 23 | Green, Green, Blue, Red, Red, Red, Blue, Red, Blue, Red |
| ## 24 | 24 | Red, Red, Green, Blue, Blue, Blue, Red, Red, Red, Green |
| ## 25 | 25 | Red, Blue, Red, Red, Red, Blue, Blue, Red, Blue, Red |
| ## 26 | 26 | Red, Green, Blue, Red, Blue, Green, Red, Red, Red, Blue |
| ## 27 | 27 | Green, Green, Red, Red, Red, Red, Green, Green, Green, Green |
| ## 28 | 28 | Blue, Blue, Blue, Blue, Green, Red, Blue, Red, Green, Green |
| ## 29 | 29 | Blue, Red, Red, Red, Red, Red, Blue, Blue, Green, Blue |
| ## 30 | 30 | Red, Red, Green, Red, Green, Red, Green, Green, Green, Red |
| ## 31 | 31 | Blue, Blue, Green, Blue, Blue, Blue, Red, Green, Red, Blue |
| ## 32 | 32 | Blue, Blue, Red, Red, Green, Green, Blue, Red, Red, Red |
| ## 33 | 33 | Red, Red, Blue, Green, Green, Red, Red, Blue, Blue, Red |
| ## 34 | 34 | Red, Green, Green, Blue, Red, Green, Blue, Blue, Red, Green |
| ## 35 | 35 | Red, Blue, Red, Red, Blue, Red, Blue, Blue, Blue, Blue |
| ## 36 | 36 | Blue, Red, Red, Red, Red, Red, Red, Red, Blue, Green |
| ## 37 | 37 | Red, Red, Red, Red, Green, Green, Red, Red, Red, Red |
| ## 38 | 38 | Green, Red, Red, Green, Blue, Green, Blue, Blue, Green, Blue |
| ## 39 | 39 | Red, Red, Blue, Blue, Green, Blue, Blue, Blue, Green, Green |
| ## 40 | 40 | Red, Blue, Red, Blue, Red, Red, Red, Red, Red, Blue |
| ## 41 | 41 | Blue, Red, Green, Red, Blue, Red, Red, Red, Green, Red |
| ## 42 | 42 | Green, Red, Blue, Red, Blue, Blue, Blue, Red, Red, Red |
| ## 43 | 43 | Blue, Blue, Blue, Green, Green, Red, Red, Red, Red, Red |
| ## 44 | 44 | Red, Blue, Blue, Red, Red, Red, Red, Green, Red, Red |
| ## 45 | 45 | Green, Red, Blue, Red, Red, Red, Green, Blue, Blue, Green |
| ## 46 | 46 | Blue, Blue, Blue, Red, Blue, Green, Blue, Blue, Red, Green |
| ## 47 | 47 | Red, Red, Red, Red, Red, Red, Red, Red, Green, Red |
| ## 48 | 48 | Red, Red, Red, Green, Red, Red, Red, Red, Red, Blue |
| ## 49 | 49 | Blue, Blue, Green, Red, Green, Red, Green, Red, Red, Green |
| ## 50 | 50 | Red, Green, Green, Red, Red, Green, Red, Red, Red, Blue |
| ## 51 | 51 | Red, Red, Green, Red, Blue, Red, Blue, Red, Green, Green |
| ## 52 | 52 | Red, Blue, Blue, Green, Red, Red, Green, Green, Green, Red |
| ## 53 | 53 | Red, Red, Red, Blue, Red, Red, Blue, Blue, Blue, Green |
| ## 54 | 54 | Green, Green, Blue, Red, Red, Red, Red, Green, Blue, Red |

| | | |
|--------|-----|---|
| ## 55 | 55 | Green, Blue, Green, Red, Blue, Blue, Green, Green, Green, Red |
| ## 56 | 56 | Blue, Red, Red, Blue, Green, Red, Red, Blue, Red, Red |
| ## 57 | 57 | Red, Red, Red, Blue, Green, Red, Red, Blue, Red, Blue |
| ## 58 | 58 | Red, Blue, Blue, Red, Blue, Green, Blue, Blue, Red, Red |
| ## 59 | 59 | Red, Blue, Green, Green, Blue, Red, Blue, Green, Green, Green |
| ## 60 | 60 | Blue, Red, Green, Blue, Blue, Red, Red, Blue, Red, Red |
| ## 61 | 61 | Green, Green, Red, Blue, Green, Red, Red, Red, Red, Red |
| ## 62 | 62 | Green, Blue, Blue, Green, Green, Blue, Red, Red, Green, Blue |
| ## 63 | 63 | Green, Red, Red, Green, Blue, Green, Red, Blue, Red, Red |
| ## 64 | 64 | Red, Blue, Red, Red, Green, Green, Red, Red, Red, Green |
| ## 65 | 65 | Blue, Red, Blue, Blue, Blue, Red, Green, Blue, Red, Green |
| ## 66 | 66 | Red, Green, Red, Red, Red, Green, Blue, Green, Blue, Green |
| ## 67 | 67 | Blue, Green, Red, Red, Red, Green, Blue, Blue, Red, Red |
| ## 68 | 68 | Red, Blue, Red, Green, Green, Red, Green, Red, Red, Green |
| ## 69 | 69 | Blue, Green, Red, Blue, Green, Green, Green, Red, Blue, Red |
| ## 70 | 70 | Green, Red, Red, Red, Green, Red, Blue, Red, Blue, Red |
| ## 71 | 71 | Red, Red, Blue, Red, Red, Red, Red, Red, Red, Blue |
| ## 72 | 72 | Green, Green, Blue, Blue, Red, Blue, Green, Red, Blue, Green |
| ## 73 | 73 | Red, Green, Red, Red, Blue, Red, Red, Red, Blue, Red |
| ## 74 | 74 | Red, Green, Green, Blue, Red, Red, Red, Red, Red, Red |
| ## 75 | 75 | Blue, Blue, Green, Red, Blue, Red, Red, Red, Blue, Green |
| ## 76 | 76 | Red, Red, Blue, Blue, Green, Green, Blue, Blue, Green, Green |
| ## 77 | 77 | Red, Blue, Red, Red, Red, Green, Red, Blue, Blue, Red |
| ## 78 | 78 | Blue, Blue, Red, Green, Blue, Green, Red, Red, Green, Blue |
| ## 79 | 79 | Green, Red, Red, Blue, Green, Blue, Green, Red, Red, Blue |
| ## 80 | 80 | Blue, Red, Green, Red, Red, Red, Blue, Green, Red, Red |
| ## 81 | 81 | Blue, Red, Red, Red, Red, Green, Green, Blue, Red, Red |
| ## 82 | 82 | Green, Red, Red, Green, Red, Red, Red, Red, Red, Blue |
| ## 83 | 83 | Red, Red, Red, Blue, Red, Blue, Blue, Green, Red, Green |
| ## 84 | 84 | Blue, Blue, Blue, Green, Red, Red, Red, Red, Red, Blue |
| ## 85 | 85 | Blue, Blue, Green, Red, Blue, Blue, Red, Blue, Red, Red |
| ## 86 | 86 | Red, Green, Blue, Red, Green, Blue, Blue, Blue, Blue, Red |
| ## 87 | 87 | Blue, Red, Red, Red, Red, Red, Blue, Red, Green, Green |
| ## 88 | 88 | Green, Green, Red, Red, Blue, Blue, Red, Red, Blue, Blue |
| ## 89 | 89 | Green, Red, Blue, Green, Red, Blue, Red, Red, Green, Red |
| ## 90 | 90 | Red, Red, Red, Red, Red, Blue, Blue, Blue, Red, Red |
| ## 91 | 91 | Blue, Red, Red, Blue, Red, Red, Red, Green, Green, Red |
| ## 92 | 92 | Red, Blue, Green, Red, Blue, Red, Red, Blue, Green, Blue |
| ## 93 | 93 | Green, Red, Blue, Green, Red, Blue, Blue, Blue, Green, Green |
| ## 94 | 94 | Red, Blue, Red, Blue, Red, Green, Red, Red, Blue, Blue |
| ## 95 | 95 | Blue, Red, Blue, Green, Red, Blue, Red, Red, Blue, Red |
| ## 96 | 96 | Blue, Green, Red, Red, Blue, Blue, Red, Blue, Green, Red |
| ## 97 | 97 | Green, Red, Red, Red, Blue, Blue, Red, Red, Green, Red |
| ## 98 | 98 | Red, Red, Green, Green, Green, Blue, Blue, Red, Blue, Blue |
| ## 99 | 99 | Red, Green, Red, Blue, Blue, Blue, Blue, Blue, Red, Blue |
| ## 100 | 100 | Red, Blue, Red, Red, Red, Blue, Green, Blue, Red, Red |
| ## | | num_reds num_greens num_blues min_colors |
| ## 1 | | 6 2 2 2 |
| ## 2 | | 2 3 5 2 |
| ## 3 | | 6 3 1 1 |
| ## 4 | | 6 1 3 1 |
| ## 5 | | 8 0 2 0 |
| ## 6 | | 5 1 4 1 |
| ## 7 | | 4 5 1 1 |
| ## 8 | | 9 1 0 0 |
| ## 9 | | 2 5 3 2 |

| | | | | |
|-------|---|---|---|---|
| ## 10 | 6 | 1 | 3 | 1 |
| ## 11 | 6 | 2 | 2 | 2 |
| ## 12 | 6 | 3 | 1 | 1 |
| ## 13 | 3 | 3 | 4 | 3 |
| ## 14 | 6 | 1 | 3 | 1 |
| ## 15 | 5 | 2 | 3 | 2 |
| ## 16 | 5 | 2 | 3 | 2 |
| ## 17 | 7 | 2 | 1 | 1 |
| ## 18 | 6 | 2 | 2 | 2 |
| ## 19 | 6 | 1 | 3 | 1 |
| ## 20 | 6 | 2 | 2 | 2 |
| ## 21 | 2 | 4 | 4 | 2 |
| ## 22 | 4 | 1 | 5 | 1 |
| ## 23 | 5 | 2 | 3 | 2 |
| ## 24 | 5 | 2 | 3 | 2 |
| ## 25 | 6 | 0 | 4 | 0 |
| ## 26 | 5 | 2 | 3 | 2 |
| ## 27 | 4 | 6 | 0 | 0 |
| ## 28 | 2 | 3 | 5 | 2 |
| ## 29 | 5 | 1 | 4 | 1 |
| ## 30 | 5 | 5 | 0 | 0 |
| ## 31 | 2 | 2 | 6 | 2 |
| ## 32 | 5 | 2 | 3 | 2 |
| ## 33 | 5 | 2 | 3 | 2 |
| ## 34 | 3 | 4 | 3 | 3 |
| ## 35 | 4 | 0 | 6 | 0 |
| ## 36 | 7 | 1 | 2 | 1 |
| ## 37 | 8 | 2 | 0 | 0 |
| ## 38 | 2 | 4 | 4 | 2 |
| ## 39 | 2 | 3 | 5 | 2 |
| ## 40 | 7 | 0 | 3 | 0 |
| ## 41 | 6 | 2 | 2 | 2 |
| ## 42 | 5 | 1 | 4 | 1 |
| ## 43 | 5 | 2 | 3 | 2 |
| ## 44 | 7 | 1 | 2 | 1 |
| ## 45 | 4 | 3 | 3 | 3 |
| ## 46 | 2 | 2 | 6 | 2 |
| ## 47 | 9 | 1 | 0 | 0 |
| ## 48 | 8 | 1 | 1 | 1 |
| ## 49 | 4 | 4 | 2 | 2 |
| ## 50 | 6 | 3 | 1 | 1 |
| ## 51 | 5 | 3 | 2 | 2 |
| ## 52 | 4 | 4 | 2 | 2 |
| ## 53 | 5 | 1 | 4 | 1 |
| ## 54 | 5 | 3 | 2 | 2 |
| ## 55 | 2 | 5 | 3 | 2 |
| ## 56 | 6 | 1 | 3 | 1 |
| ## 57 | 6 | 1 | 3 | 1 |
| ## 58 | 4 | 1 | 5 | 1 |
| ## 59 | 2 | 5 | 3 | 2 |
| ## 60 | 5 | 1 | 4 | 1 |
| ## 61 | 6 | 3 | 1 | 1 |
| ## 62 | 2 | 4 | 4 | 2 |
| ## 63 | 5 | 3 | 2 | 2 |
| ## 64 | 6 | 3 | 1 | 1 |
| ## 65 | 3 | 2 | 5 | 2 |

| | | | | |
|--------|---|---|---|---|
| ## 66 | 4 | 4 | 2 | 2 |
| ## 67 | 5 | 2 | 3 | 2 |
| ## 68 | 5 | 4 | 1 | 1 |
| ## 69 | 3 | 4 | 3 | 3 |
| ## 70 | 6 | 2 | 2 | 2 |
| ## 71 | 8 | 0 | 2 | 0 |
| ## 72 | 2 | 4 | 4 | 2 |
| ## 73 | 7 | 1 | 2 | 1 |
| ## 74 | 7 | 2 | 1 | 1 |
| ## 75 | 4 | 2 | 4 | 2 |
| ## 76 | 2 | 4 | 4 | 2 |
| ## 77 | 6 | 1 | 3 | 1 |
| ## 78 | 3 | 3 | 4 | 3 |
| ## 79 | 4 | 3 | 3 | 3 |
| ## 80 | 6 | 2 | 2 | 2 |
| ## 81 | 6 | 2 | 2 | 2 |
| ## 82 | 7 | 2 | 1 | 1 |
| ## 83 | 5 | 2 | 3 | 2 |
| ## 84 | 5 | 1 | 4 | 1 |
| ## 85 | 4 | 1 | 5 | 1 |
| ## 86 | 3 | 2 | 5 | 2 |
| ## 87 | 6 | 2 | 2 | 2 |
| ## 88 | 4 | 2 | 4 | 2 |
| ## 89 | 5 | 3 | 2 | 2 |
| ## 90 | 7 | 0 | 3 | 0 |
| ## 91 | 6 | 2 | 2 | 2 |
| ## 92 | 4 | 2 | 4 | 2 |
| ## 93 | 2 | 4 | 4 | 2 |
| ## 94 | 5 | 1 | 4 | 1 |
| ## 95 | 5 | 1 | 4 | 1 |
| ## 96 | 4 | 2 | 4 | 2 |
| ## 97 | 6 | 2 | 2 | 2 |
| ## 98 | 3 | 3 | 4 | 3 |
| ## 99 | 3 | 1 | 6 | 1 |
| ## 100 | 6 | 1 | 3 | 1 |

```
# 6. Compute the proportion of rows for which the minimum number of the three counts is zero
zero <- 0
for (i in sampling_without_replacement_simulation[["min_colors"]]){
  if (i == 0){
    zero <- zero + 1
  }
}
proportion <- zero/length(trials)
print(proportion)
```

```
## [1] 0.11
```

2.2 Q2

```
subset <- choose(100,10)

miss_red <- choose(50,10)
miss_blue <- choose(70,10)
miss_green <- choose(80,10)
miss_red_blue <- choose(20,10)
miss_red_green <- choose(30,10)
miss_blue_green <- choose(50,10)

# avoid double count
miss_one_or_two_colors <- miss_red + miss_green + miss_blue - miss_blue_green - miss_red_green
- miss_red_blue

pro <- miss_one_or_two_colors / subset
print(pro)
```

```
## [1] 0.1180318
```