What every game developer should know about

Mathematical Optimization (Part 3)

Intelligent Computational Media
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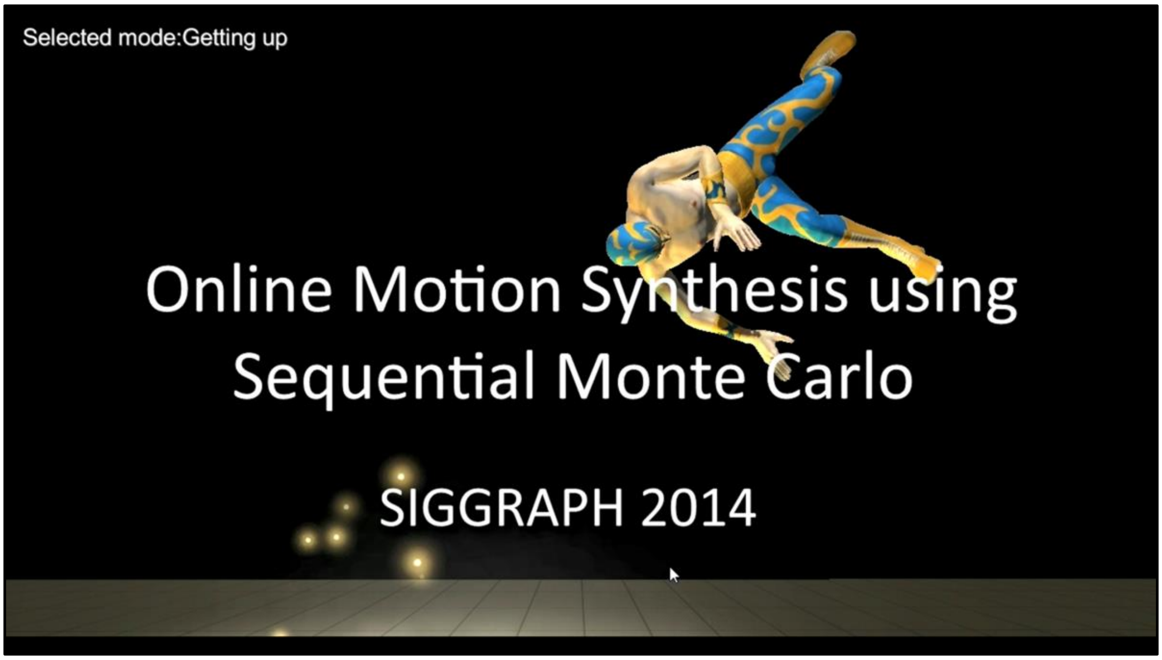
Contents: Alternatives to Deep RL in optimizing action sequences

- Forward search methods
- Neuroevolution

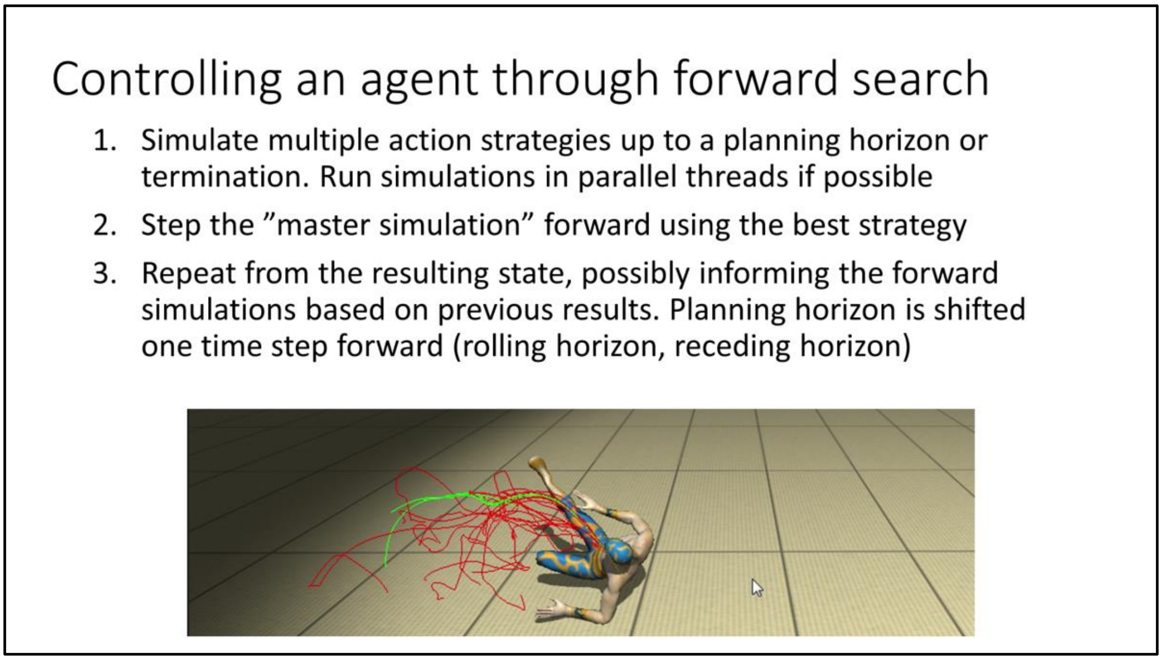

Selected mode:Getting up

Online Motion Synthesis using Sequential Monte Carlo

SIGGRAPH 2014

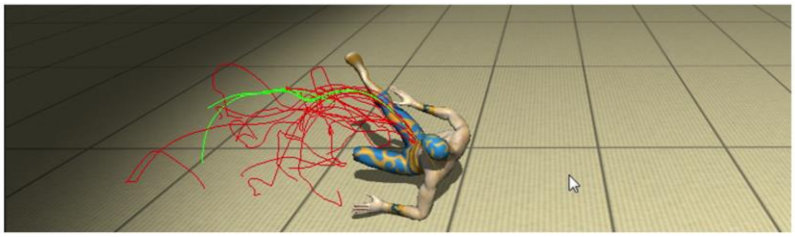
Let’s start with a video.

Live demo



Controlling an agent through forward search

1. Simulate multiple action strategies up to a planning horizon or termination. Run simulations in parallel threads if possible
2. Step the ”master simulation” forward using the best strategy
3. Repeat from the resulting state, possibly informing the forward simulations based on previous results. Planning horizon is shifted one time step forward (rolling horizon, receding horizon)



In this demo, there’s no learning or training of neural networks. One starts the software and it just works.

This is because it’s using forward search instead.

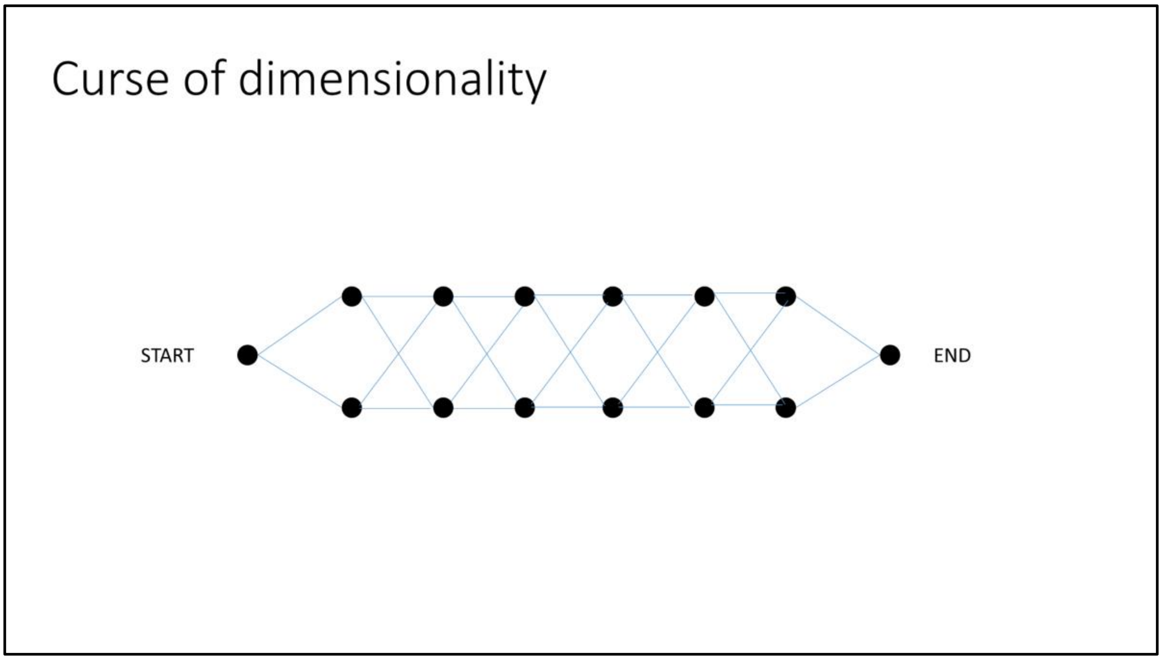
Deep RL vs. Forward Search

Deep RL:
+ Once trained, very computationally efficient
- Training can be very slow and may not converge

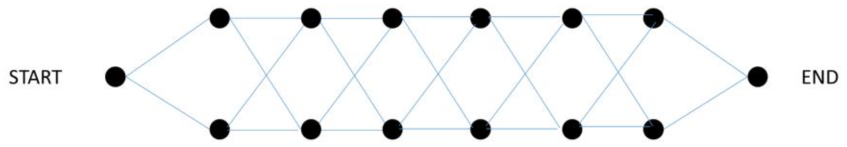
Search methods:
+ Get good results fast, without training neural networks
- Requires forward simulation (many times faster than real-time simulation)
- Requires capability to save and load simulation state

Combining search & neural networks can allow adjusting the tradeoffs.

Why is searching for action sequences hard?



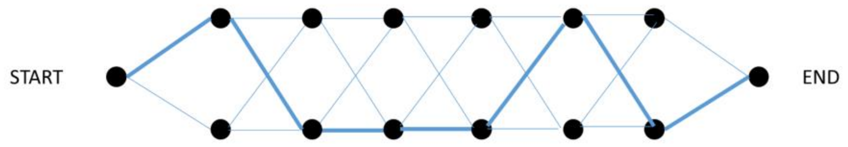
Curse of dimensionality



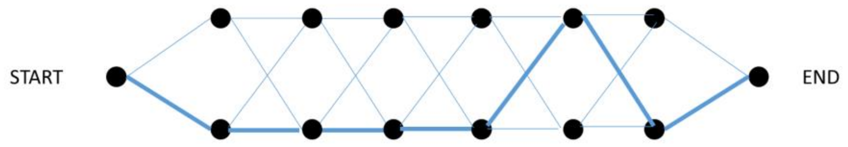
Consider finding the sequence of actions (moves) that take us from START to END along the shortest possible path.

Even in this simple graph, there are surprisingly many paths from start to end

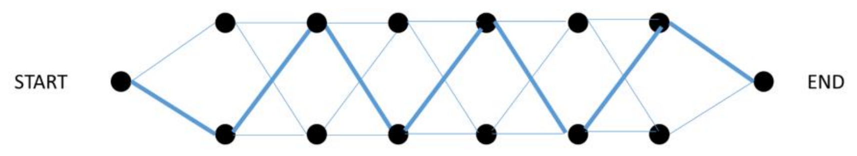
Curse of dimensionality

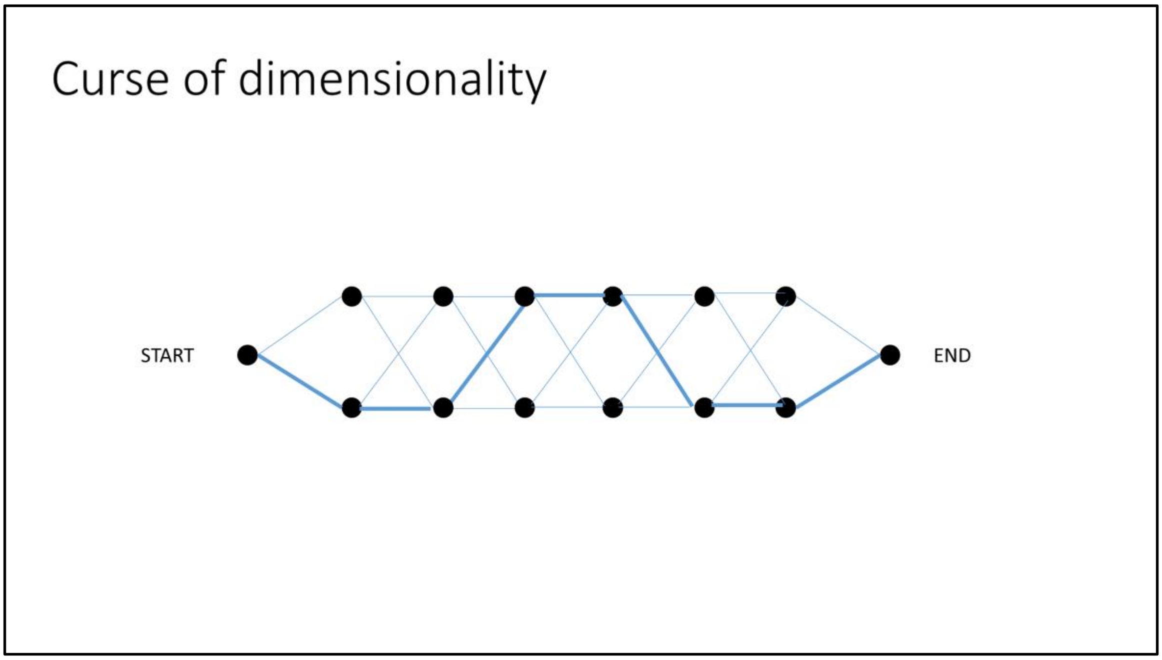


Curse of dimensionality



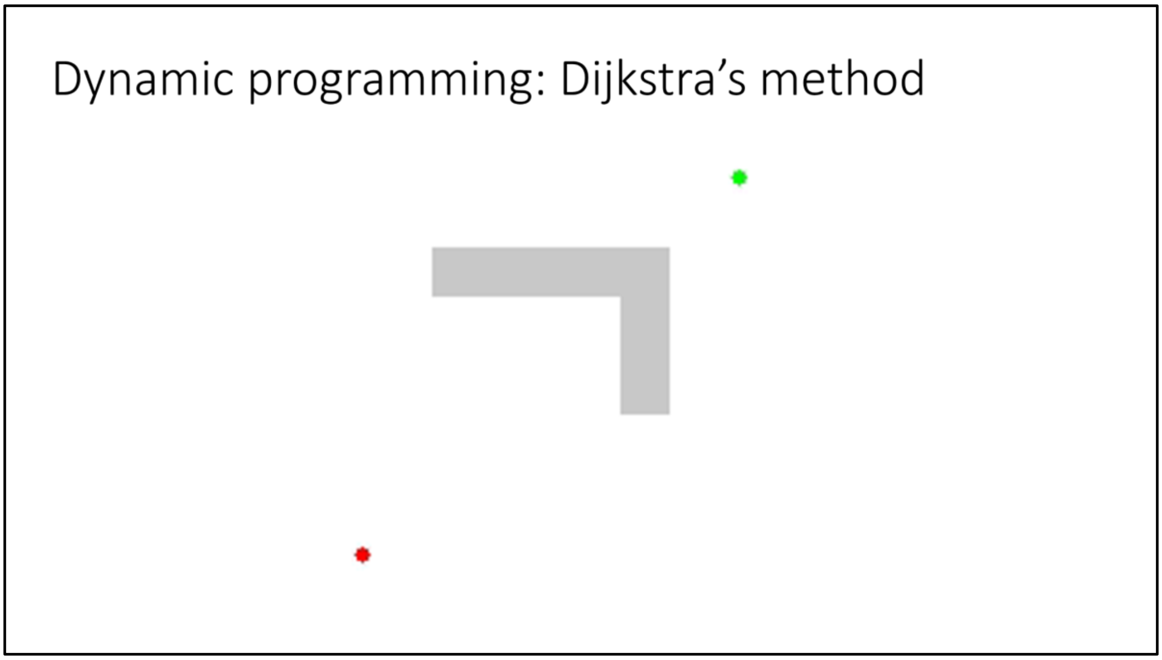
Curse of dimensionality





Curse of dimensionality

In fact, the number of possible paths grows exponentially with the number of vertices



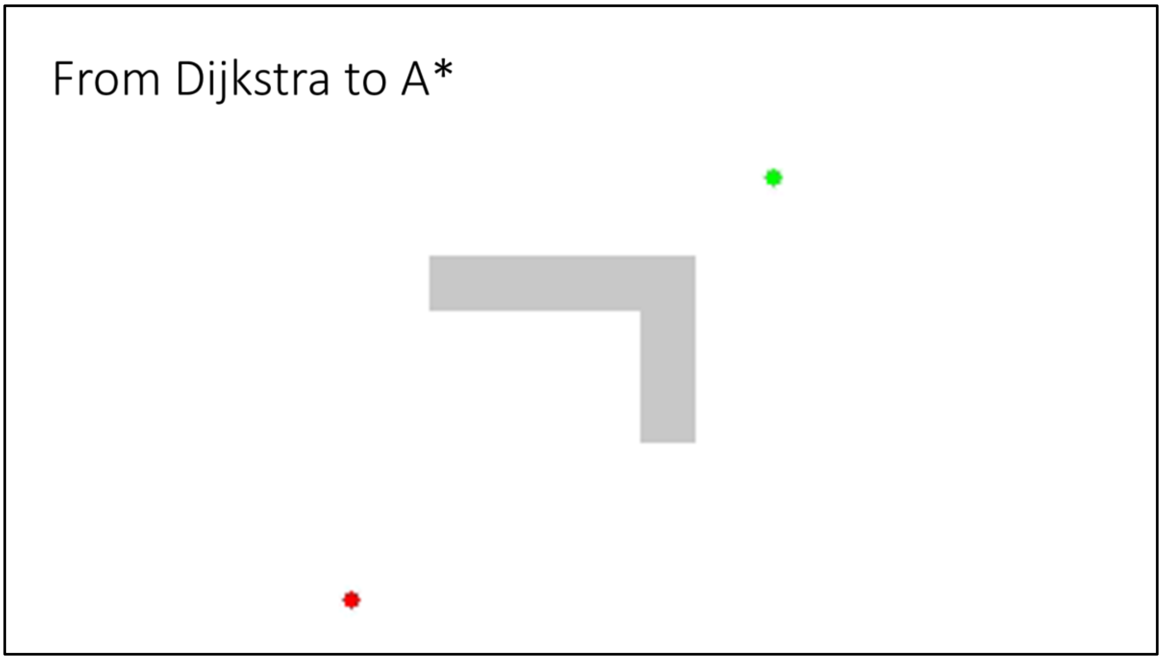
Dynamic programming: Dijkstra's method

Clearly, brute-force checking all possible paths is not possible. Perhaps the most well-known algorithm, and one taught in CS 101 is Dijkstra's algorithm.

Green is the target, red is the starting point. At each iteration, the neighbors (blue circles) of the already visited nodes (colored) are considered, and the closest one to the starting point is visited. When visiting, the total distance from the starting point is stored in the node, here visulalized as the color gradient. It is computed recursively as the sum of the visited node's distance from the closest node C in the already visited set, and the distance stored in C.

Although Dijkstra is slow, it's immensely faster than iterating over all possible paths. The number of paths grows exponentially with path length, whereas Dijkstra's worst case complexity is only O(N^2), where N is the number of nodes. This is due to the use of the dynamic programming principle, i.e., breaking the problem down into a collection of simpler subproblems, solving each of those subproblems just once, and storing their solutions. In Dijkstra's case, the solutions are the shortest path lengths from the nodes in the visited set – a new added node's shortest path is then updated simply by adding its distance to the stored value.

Animation by Subh83 - Own work, CC BY 3.0,
https://commons.wikimedia.org/w/index.php?curid=14916903

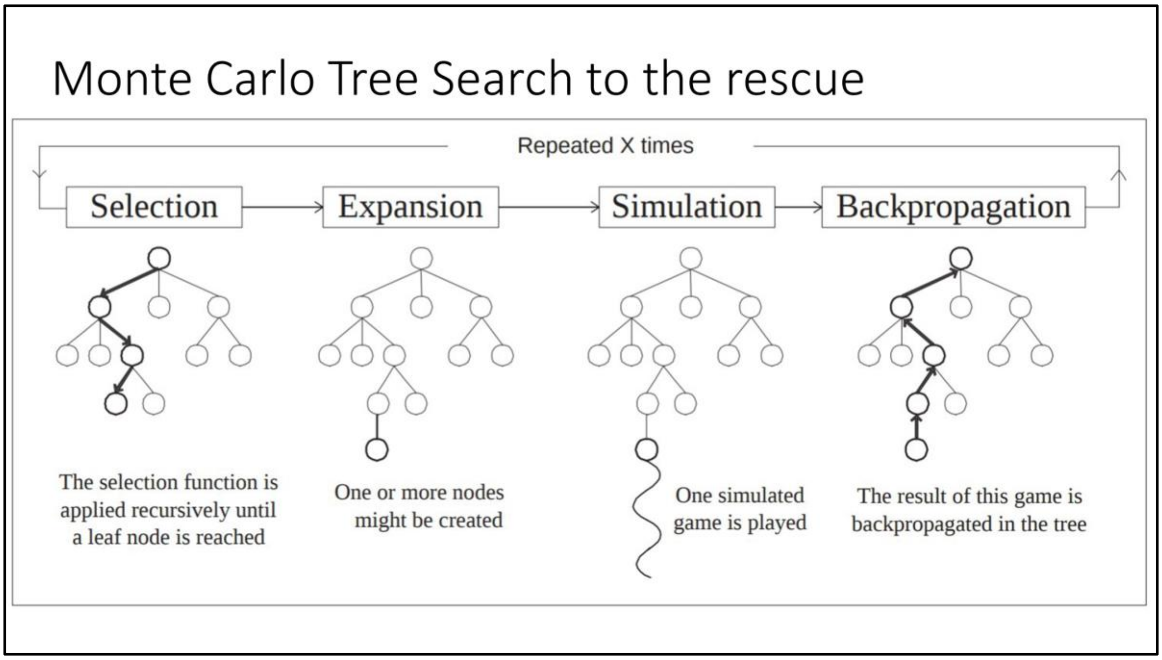


From Dijkstra to A*

Instead of visiting the neighbor with shortest distance to the start node, one selects neighbor K with the shortest estimated whole path length between start and end. This is a sum of the realized distance from start to K and a heuristic distance from K to end, for example, Euclidian distance.

Why is it still hard?

- In 2d or 3d, this is trivial
- Unity’s NavMesh implements pathfinding, no need to code yourself
- However, the search process becomes very slow with dozens or hundreds of possible actions
- A* heuristics hard to design for problems beyond pathfinding



Monte Carlo Tree Search to the rescue

Repeated X times
Selection
Expansion
Simulation
Backpropagation
The selection function is applied recursively until a leaf node is reached
One or more nodes might be created
One simulated game is played
The result of this game is backpropagated in the tree

MCTS is a common technique for adaptively exploring the tree of possible actions, without visiting every possible node. It also handles both single and multiplayer games.

Backpropagation: node utility = mean of child node utilities.

Diagram from: Chaslot, Guillaume, et al. "Monte-Carlo Tree Search: A New Framework for Game AI." *AIIDE*. 2008.

https://www.aaai.org/Papers/AIIDE/2008/AIIDE08-036.pdf

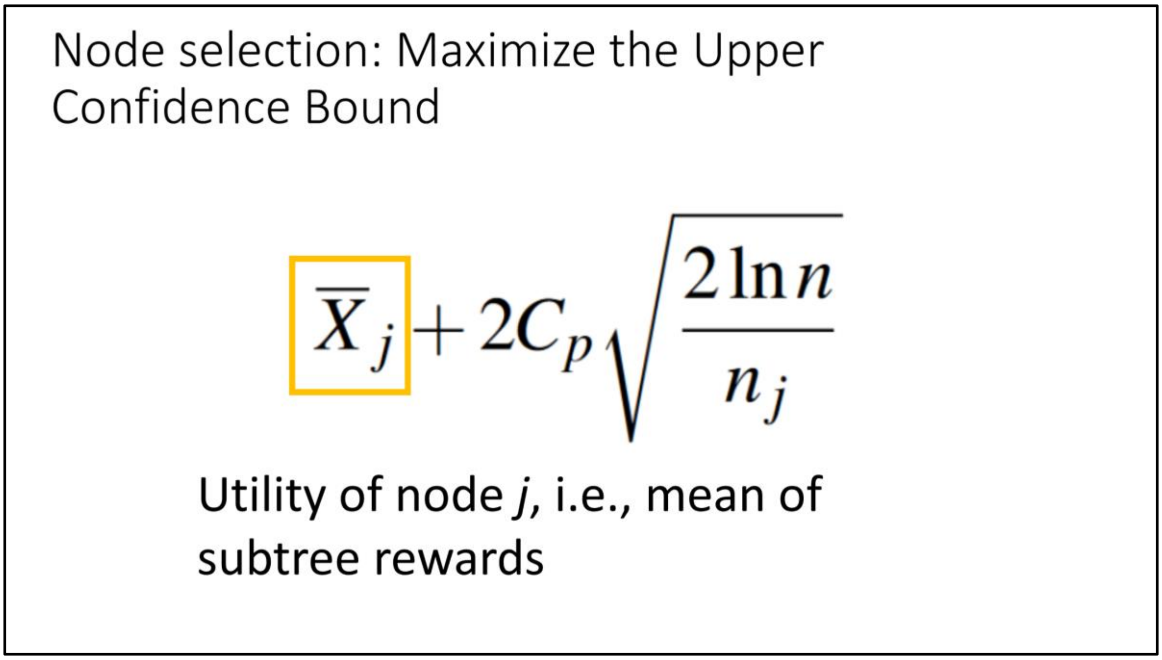
Node selection: Maximize the Upper Confidence Bound

$$\overline{X}_j + 2C_p\sqrt{\frac{2\ln n}{n_j}}$$

X_j is the average reward of all nodes beneath node j, C_p is an exploration constant (theoretical optimum 1/√2), n is the number of times the parent node has been visited, and n_j the number of times the node j has been visited.

It is important to note that while UCB1 is the most popular for-mula used for action selection it is certainly not the only one available. Other options include epsilon-greedy, Thompson sampling, and Bayesian bandits.

Source: http://gameaibook.org/book.pdf, page 46



Node selection: Maximize the Upper Confidence Bound

$$\boxed{\overline{X}_j} + 2C_p\sqrt{\frac{2\ln n}{n_j}}$$

Utility of node *j*, i.e., mean of subtree rewards

X_j is the average reward of all nodes beneath node j, C_p is an exploration constant (theoretical optimum 1/√2), n is the number of times the parent node has been visited, and n_j the number of times the node j has been visited.

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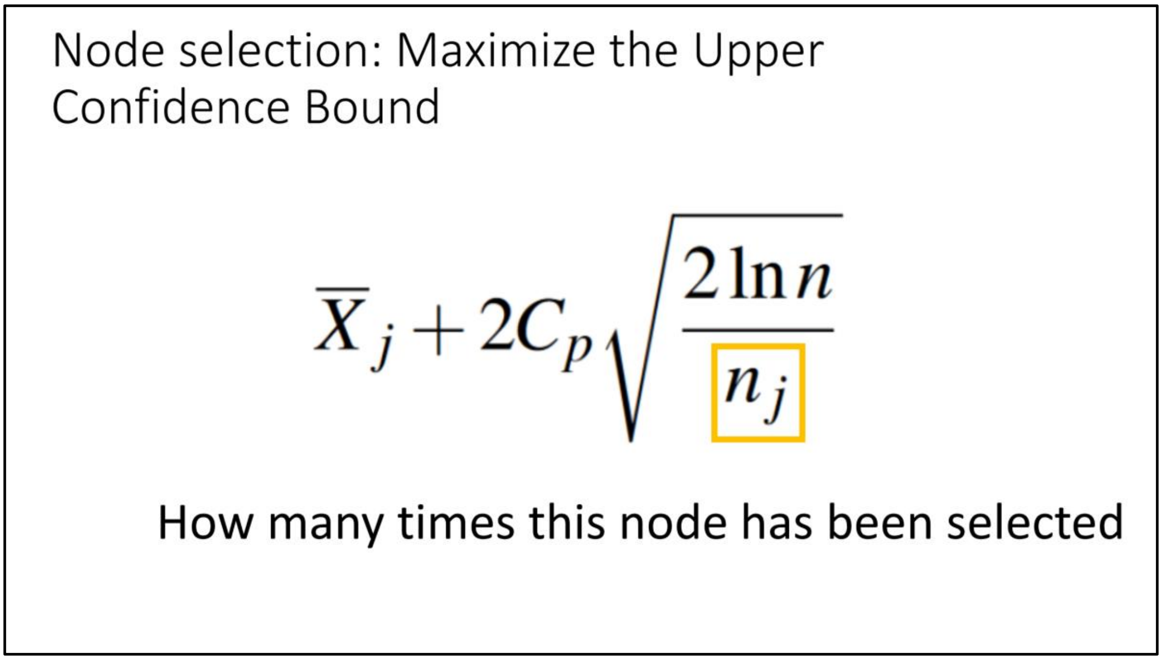

Node selection: Maximize the Upper Confidence Bound

$$\overline{X}_j + 2C_p\sqrt{\frac{2\ln n}{\boxed{n_j}}}$$

How many times this node has been selected

X_j is the average reward of all nodes beneath node j, C_p is an exploration constant (theoretical optimum 1/√2), n is the number of times the parent node has been visited, and n_j the number of times the node j has been visited.

It is important to note that while UCB1 is the most popular for-mula used for action selection it is certainly not the only one available. Other options include epsilon-greedy, Thompson sampling, and Bayesian bandits.

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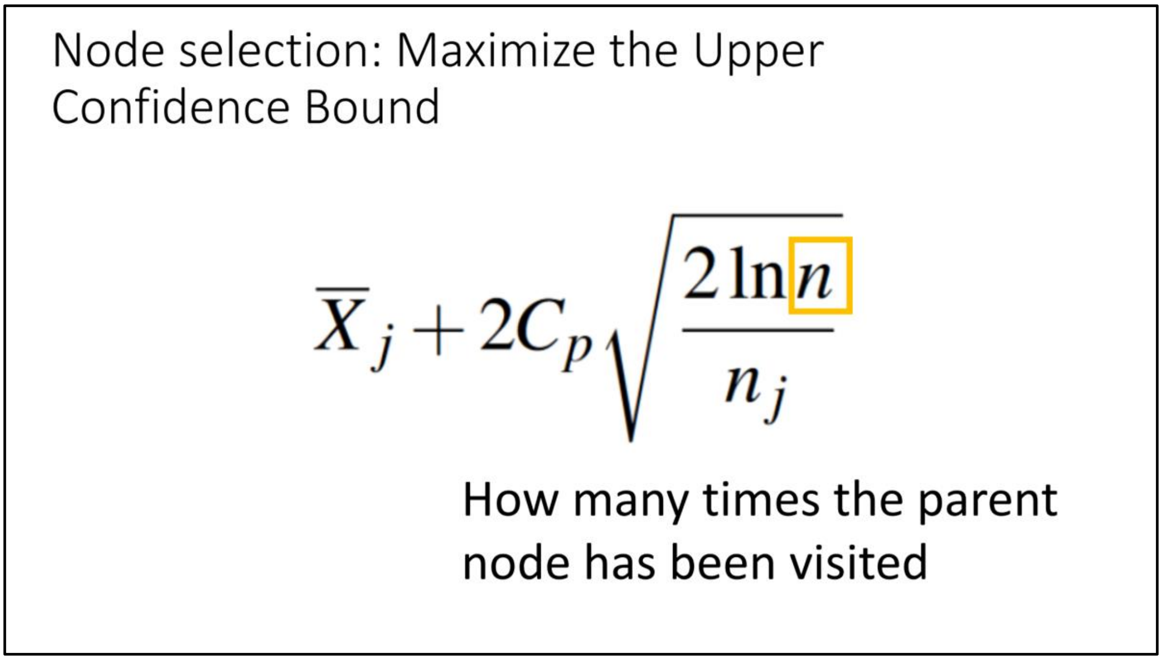

Node selection: Maximize the Upper Confidence Bound

$$\overline{X}_j + 2C_p\sqrt{\frac{2\ln \boxed{n}}{n_j}}$$

How many times the parent node has been visited

X_j is the average reward of all nodes beneath node j, C_p is an exploration constant (theoretical optimum 1/√2), n is the number of times the parent node has been visited, and n_j the number of times the node j has been visited.

It is important to note that while UCB1 is the most popular for-mula used for action selection it is certainly not the only one available. Other options include epsilon-greedy, Thompson sampling, and Bayesian bandits.

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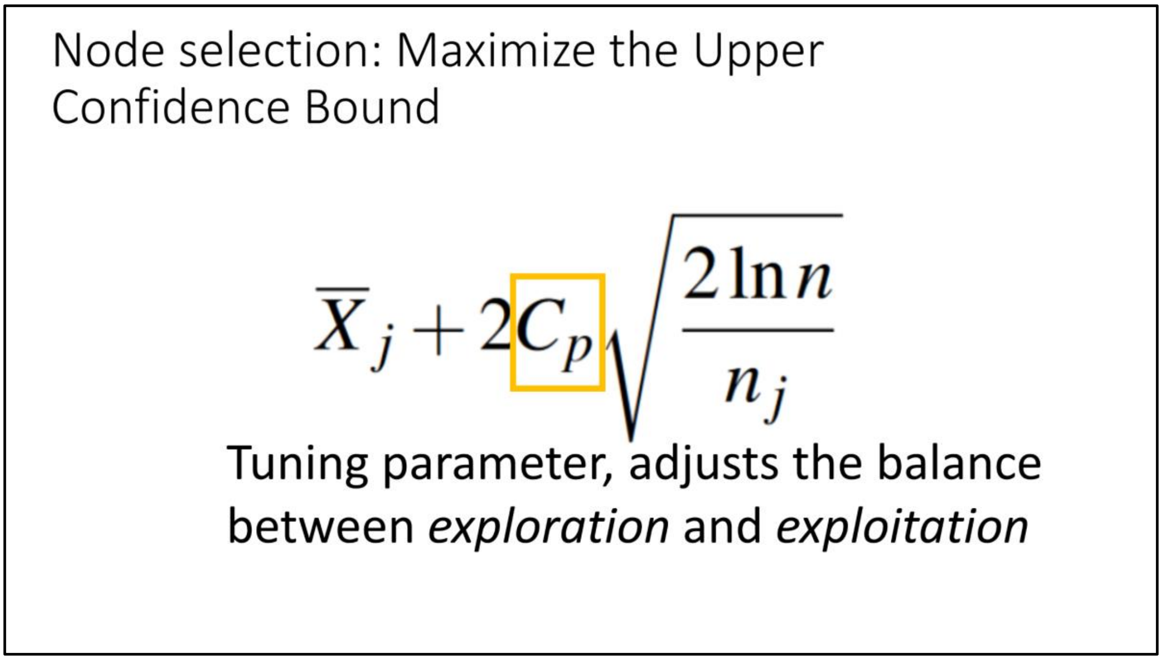


Node selection: Maximize the Upper Confidence Bound

$$\overline{X}_j + 2\boxed{C_p}\sqrt{\frac{2\ln n}{n_j}}$$

Tuning parameter, adjusts the balance between *exploration* and *exploitation*

Exploitation=repeating actions that yielded high utility in the past

Node selection: Maximize the Upper Confidence Bound

$$\boxed{\overline{X}_j} + 2C_p\sqrt{\frac{2\ln n}{n_j}}$$

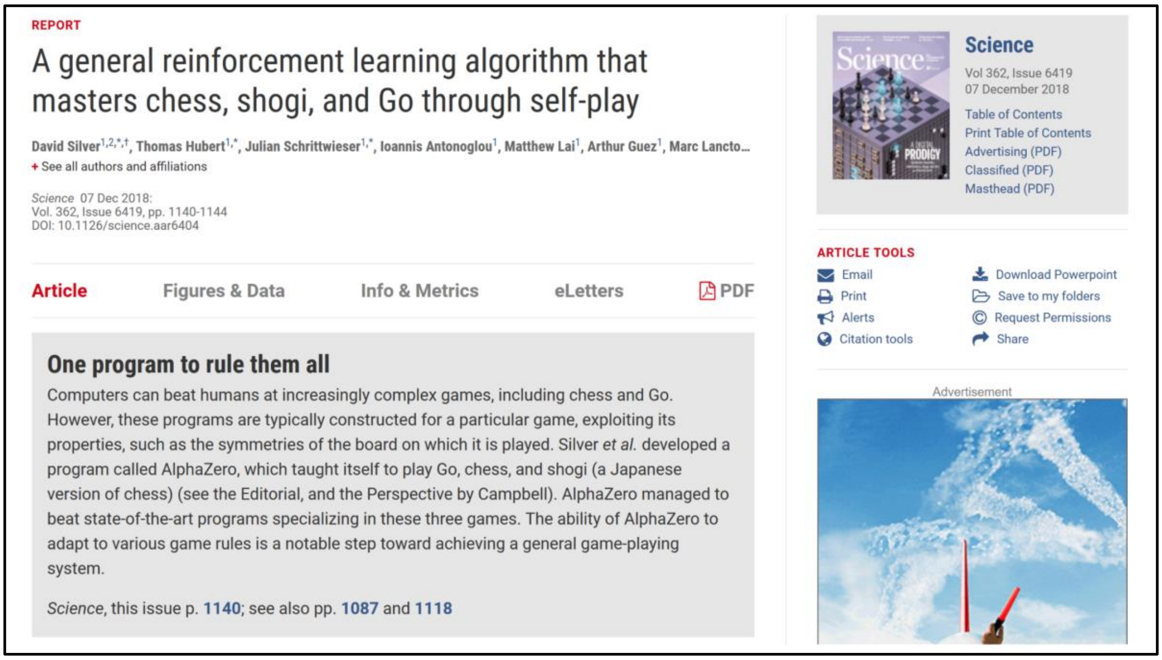
Exploitation:
maximize utility
(e.g., average number of wins)

Node selection: Maximize the Upper Confidence Bound

$$\overline{X}_j + \boxed{2C_p \sqrt{\frac{2 \ln n}{n_j}}}$$

Exploration:
High for nodes with a small number of visits n_j

https://www.youtube.com/watch?v=HRiEUUC9TUA



REPORT

A general reinforcement learning algorithm that masters chess, shogi, and Go through self-play

David Silver[1,2,*,†], Thomas Hubert[1,*], Julian Schrittwieser[1,*], Ioannis Antonoglou[1], Matthew Lai[1], Arthur Guez[1], Marc Lancto...
+ See all authors and affiliations

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DOI: 10.1126/science.aar6404

Article | Figures & Data | Info & Metrics | eLetters | PDF

One program to rule them all

Computers can beat humans at increasingly complex games, including chess and Go. However, these programs are typically constructed for a particular game, exploiting its properties, such as the symmetries of the board on which it is played. Silver *et al.* developed a program called AlphaZero, which taught itself to play Go, chess, and shogi (a Japanese version of chess) (see the Editorial, and the Perspective by Campbell). AlphaZero managed to beat state-of-the-art programs specializing in these three games. The ability of AlphaZero to adapt to various game rules is a notable step toward achieving a general game-playing system.

Science, this issue p. 1140; see also pp. 1087 and 1118

Science
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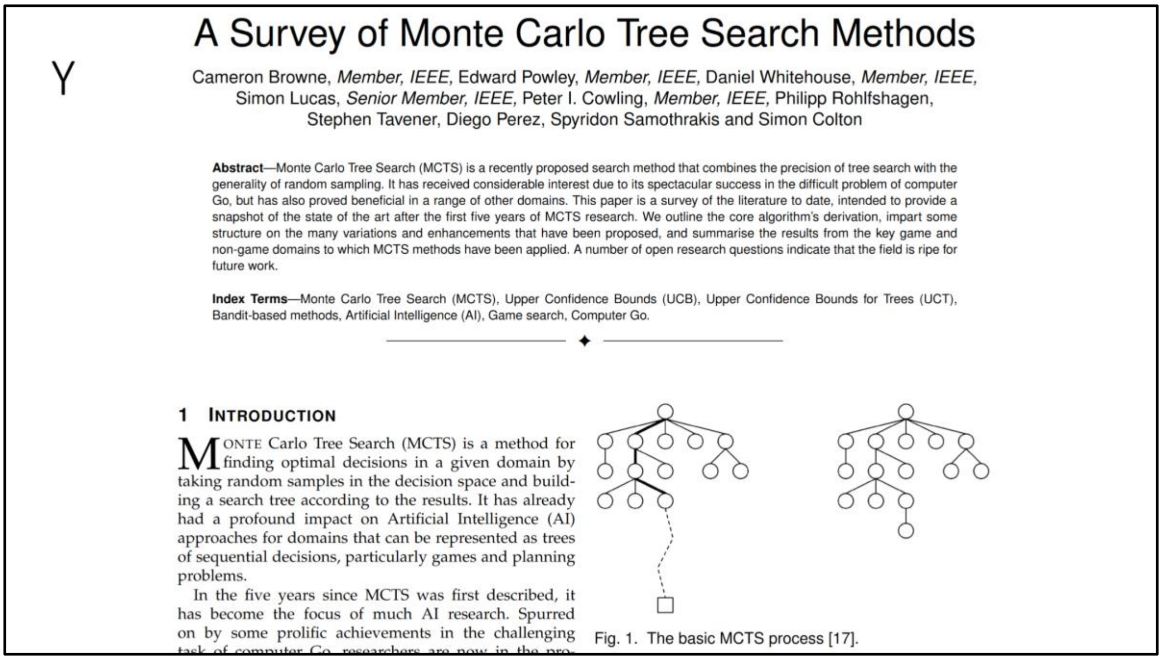
A combination of MCTS and RL is also what powers AlphaZero, Google Deepmind's general game-playing AI that is the current master of Go and Chess, to name a few.



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A Survey of Monte Carlo Tree Search Methods

Cameron Browne, *Member, IEEE,* Edward Powley, *Member, IEEE,* Daniel Whitehouse, *Member, IEEE,* Simon Lucas, *Senior Member, IEEE,* Peter I. Cowling, *Member, IEEE,* Philipp Rohlfshagen, Stephen Tavener, Diego Perez, Spyridon Samothrakis and Simon Colton

Abstract—Monte Carlo Tree Search (MCTS) is a recently proposed search method that combines the precision of tree search with the generality of random sampling. It has received considerable interest due to its spectacular success in the difficult problem of computer Go, but has also proved beneficial in a range of other domains. This paper is a survey of the literature to date, intended to provide a snapshot of the state of the art after the first five years of MCTS research. We outline the core algorithm's derivation, impart some structure on the many variations and enhancements that have been proposed, and summarise the results from the key game and non-game domains to which MCTS methods have been applied. A number of open research questions indicate that the field is ripe for future work.

Index Terms—Monte Carlo Tree Search (MCTS), Upper Confidence Bounds (UCB), Upper Confidence Bounds for Trees (UCT), Bandit-based methods, Artificial Intelligence (AI), Game search, Computer Go.

✦

1 Introduction

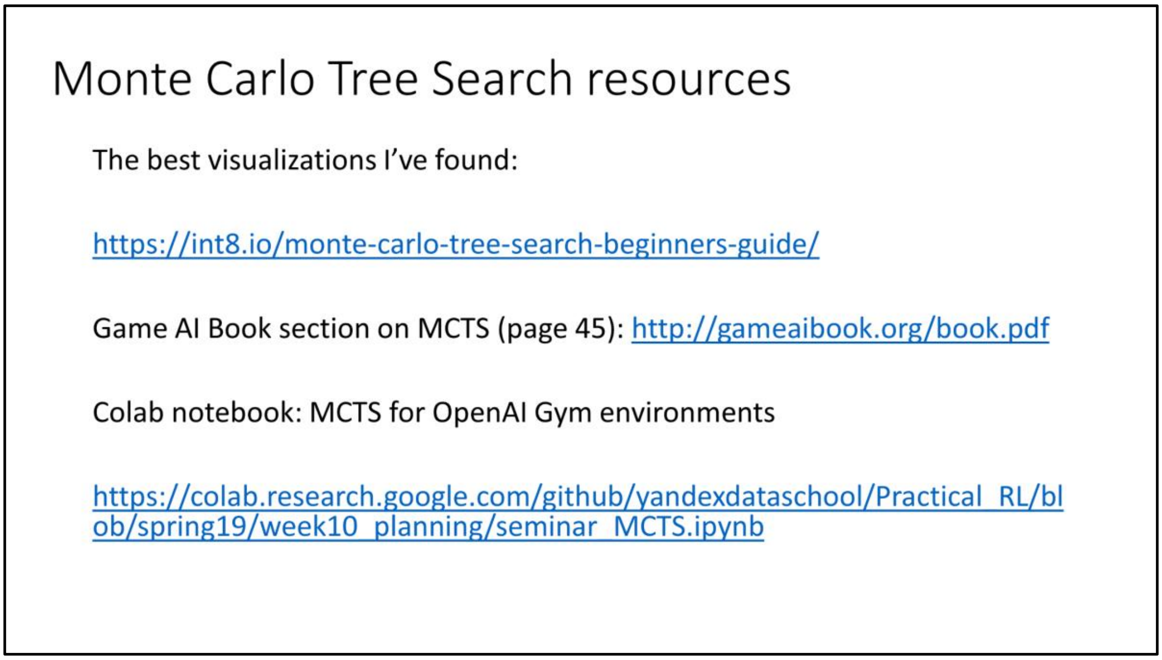
MONTE Carlo Tree Search (MCTS) is a method for finding optimal decisions in a given domain by taking random samples in the decision space and building a search tree according to the results. It has already had a profound impact on Artificial Intelligence (AI) approaches for domains that can be represented as trees of sequential decisions, particularly games and planning problems.

In the five years since MCTS was first described, it has become the focus of much AI research. Spurred on by some prolific achievements in the challenging task of computer Go, researchers are now in the pro-

Fig. 1. The basic MCTS process [17].

http://mcts.ai/pubs/mcts-survey-master.pdf

A useful article explaining MCTS and its variants



Monte Carlo Tree Search resources

The best visualizations I've found:

https://int8.io/monte-carlo-tree-search-beginners-guide/

Game AI Book section on MCTS (page 45): http://gameaibook.org/book.pdf

Colab notebook: MCTS for OpenAI Gym environments

https://colab.research.google.com/github/yandexdataschool/Practical_RL/blob/spring19/week10_planning/seminar_MCTS.ipynb

The notebook extends the environments with state saving and loading. This is what you will always need if you try MCTS instead of RL.

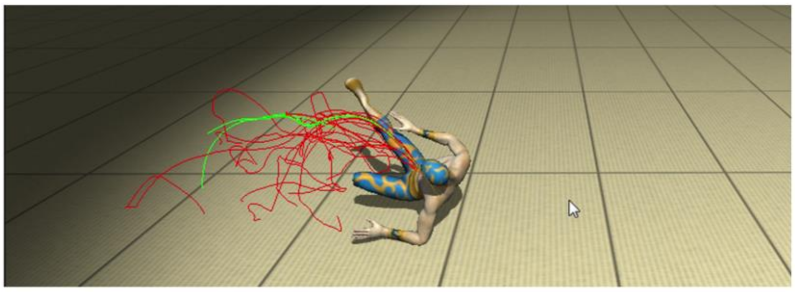
MCTS as a Deep Reinforcement Learning alternative

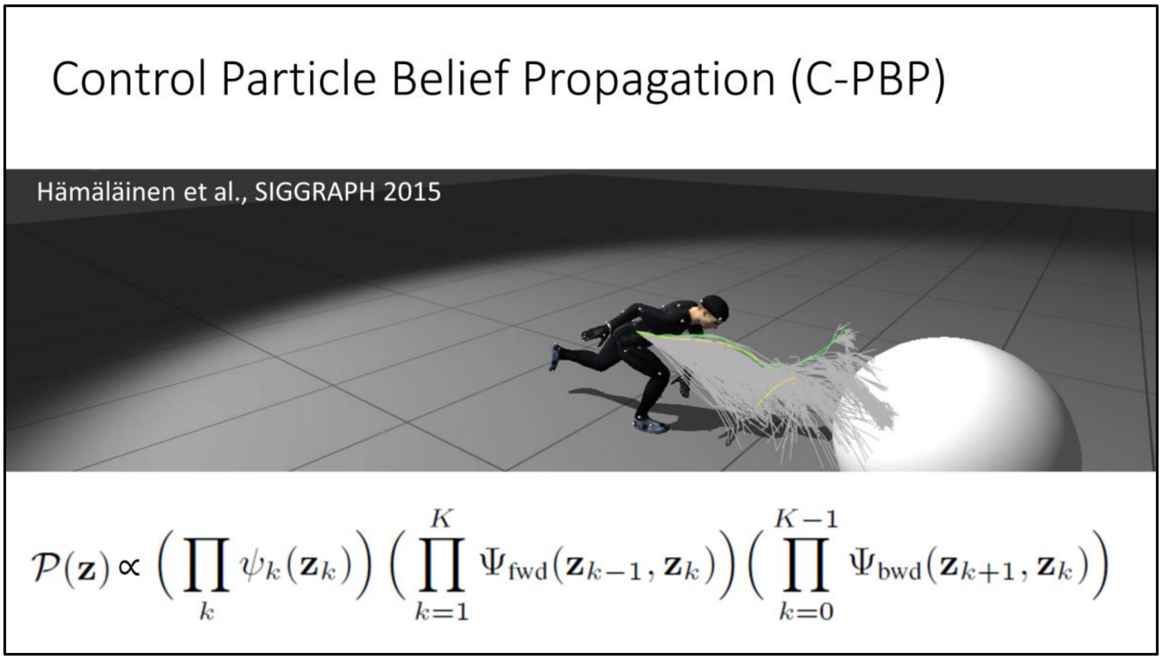
- Both find solutions to a Markov Decision Process defined by states, actions, and rewards
- MCTS benefit: usually gets results faster, but requires the ability to save and restore simulation state
  - IMPORTANT: You can quickly iterate on the MDP design, e.g., what kinds of rewards to use, without always waiting for a day for RL training
- MCTS drawback: requires more CPU
- Recommended use: If you can save & restore state, start with MCTS. Switch to RL when you're sure that your reward function produces good results.

What about continuous actions?

Our SIGGRAPH 2014 work: no tree search

- Simply run a number of rollouts from the current state
- Search space reduced by encoding action sequences as pose splines
- Take action following the best rollout
- The distribution of rollout actions is adapted





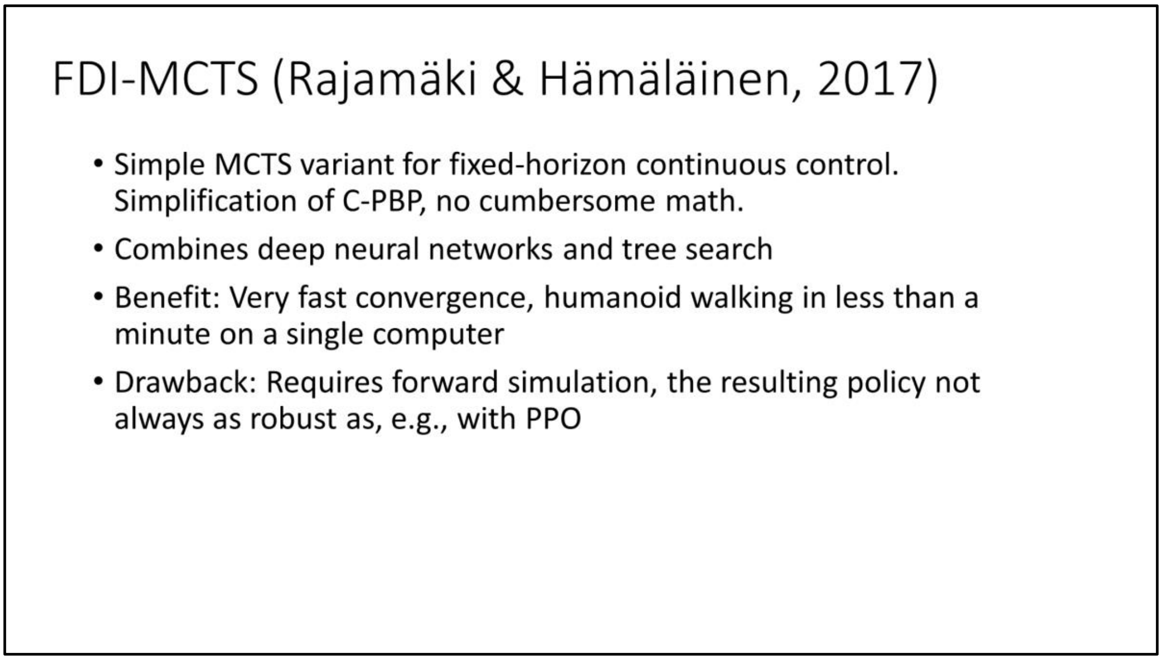
Control Particle Belief Propagation (C-PBP)

Hämäläinen et al., SIGGRAPH 2015

$$\mathcal{P}(\mathbf{z}) \propto \left(\prod_{k} \psi_k(\mathbf{z}_k) \right) \left(\prod_{k=1}^{K} \Psi_{\text{fwd}}(\mathbf{z}_{k-1}, \mathbf{z}_k) \right) \left(\prod_{k=0}^{K-1} \Psi_{\text{bwd}}(\mathbf{z}_{k+1}, \mathbf{z}_k) \right)$$

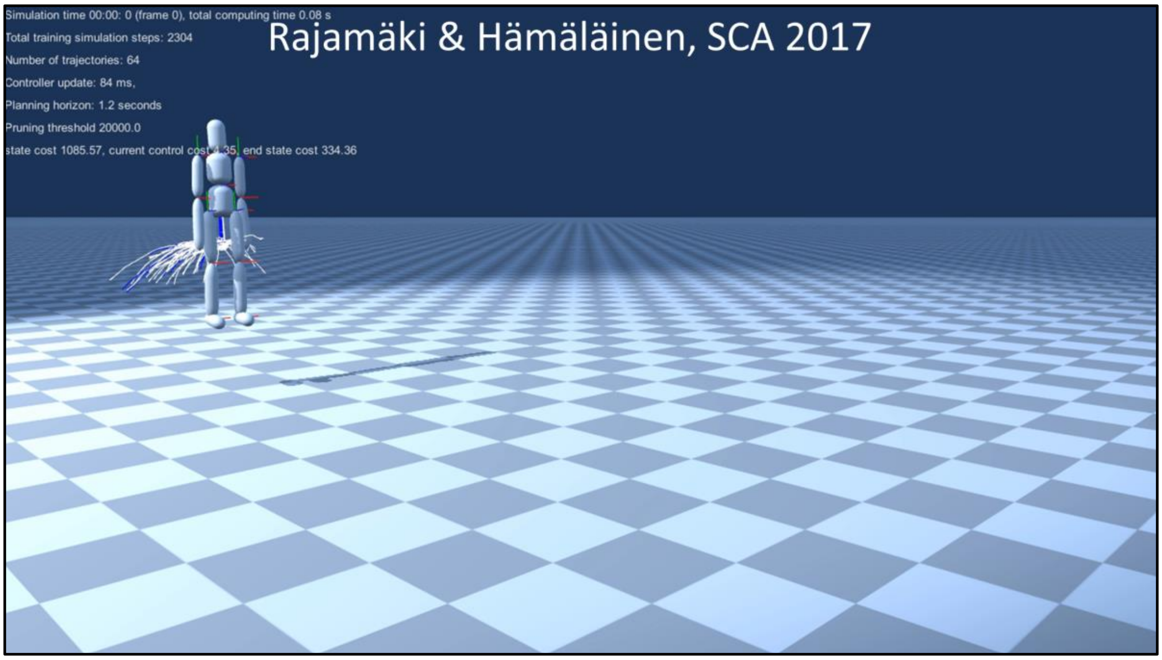
In this paper, we for the first time implemented a form of sampling-based or Monte Carlo Tree Search. However, the mathematical framework and reasoning was based on Belief Propagation. In hindsight, this was overly cumbersome, although the results were certainly better

Hämäläinen, Perttu, Joose Rajamäki, and C. Karen Liu. "Online control of simulated humanoids using particle belief propagation." *ACM Transactions on Graphics (TOG)* 34.4 (2015): 81.



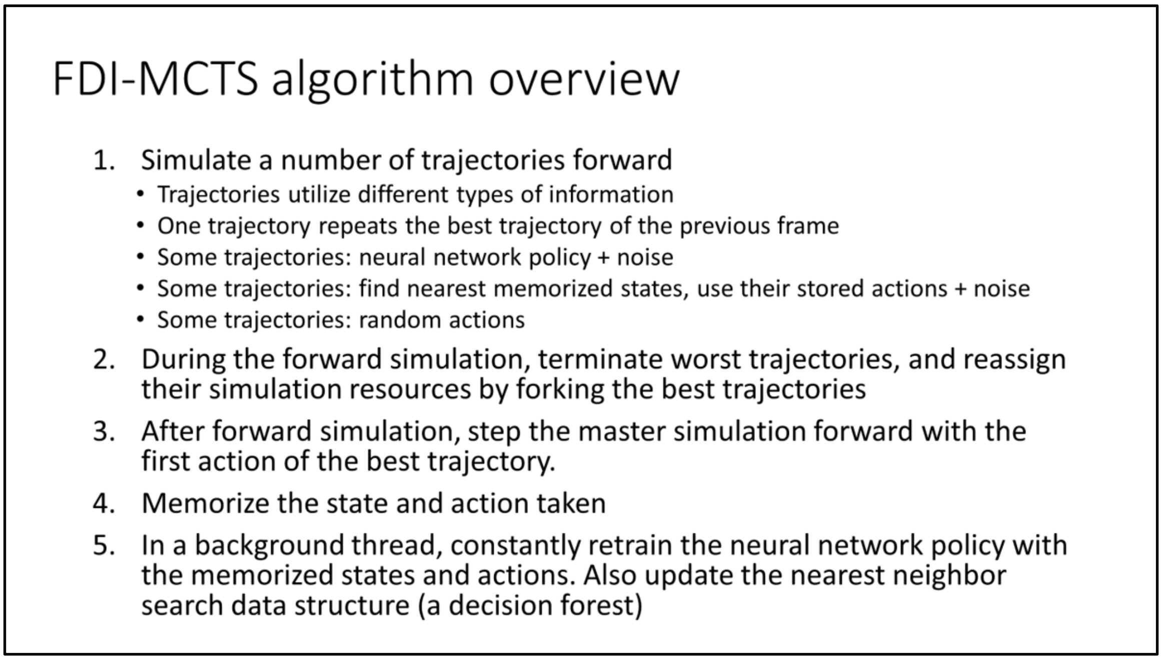
FDI-MCTS (Rajamäki & Hämäläinen, 2017)

- Simple MCTS variant for fixed-horizon continuous control. Simplification of C-PBP, no cumbersome math.
- Combines deep neural networks and tree search
- Benefit: Very fast convergence, humanoid walking in less than a minute on a single computer
- Drawback: Requires forward simulation, the resulting policy not always as robust as, e.g., with PPO



```
Simulation time 00:00: 0 (frame 0), total computing time 0.08 s
Total training simulation steps: 2304
Number of trajectories: 64
Controller update: 84 ms,
Planning horizon: 1.2 seconds
Pruning threshold 20000.0
state cost 1085.57, current control cost 4.35, end state cost 334.36
```

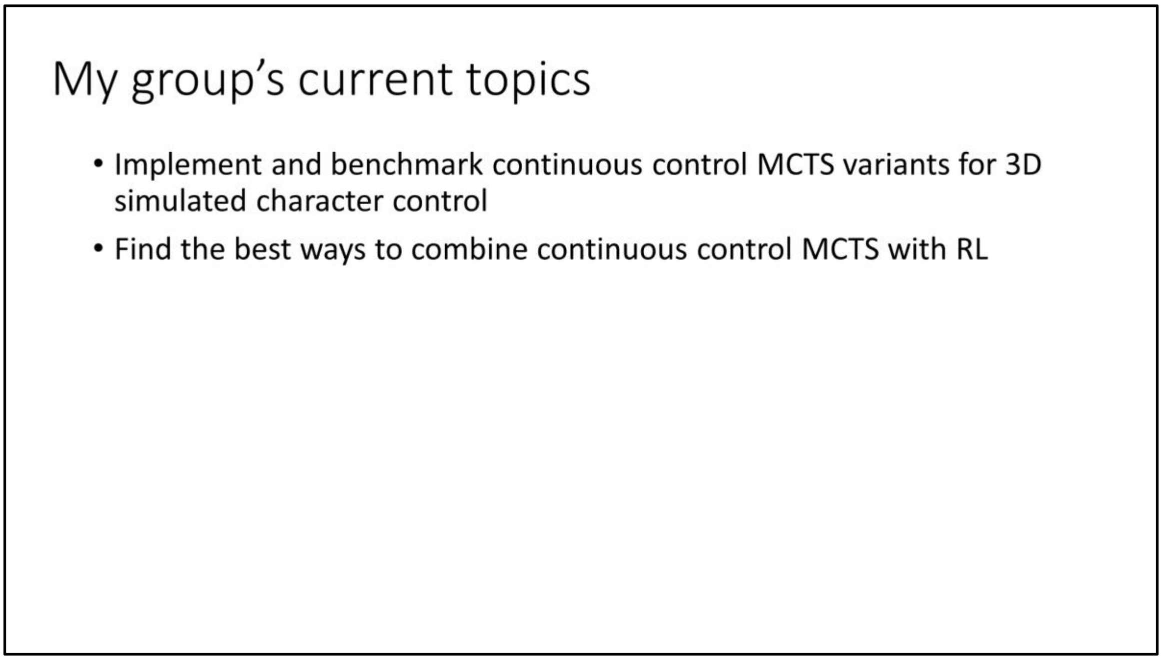
This video shows a humanoid agent learning to walk from scratch. Stable locomotion emerges in less than a minute of CPU time.



FDI-MCTS algorithm overview

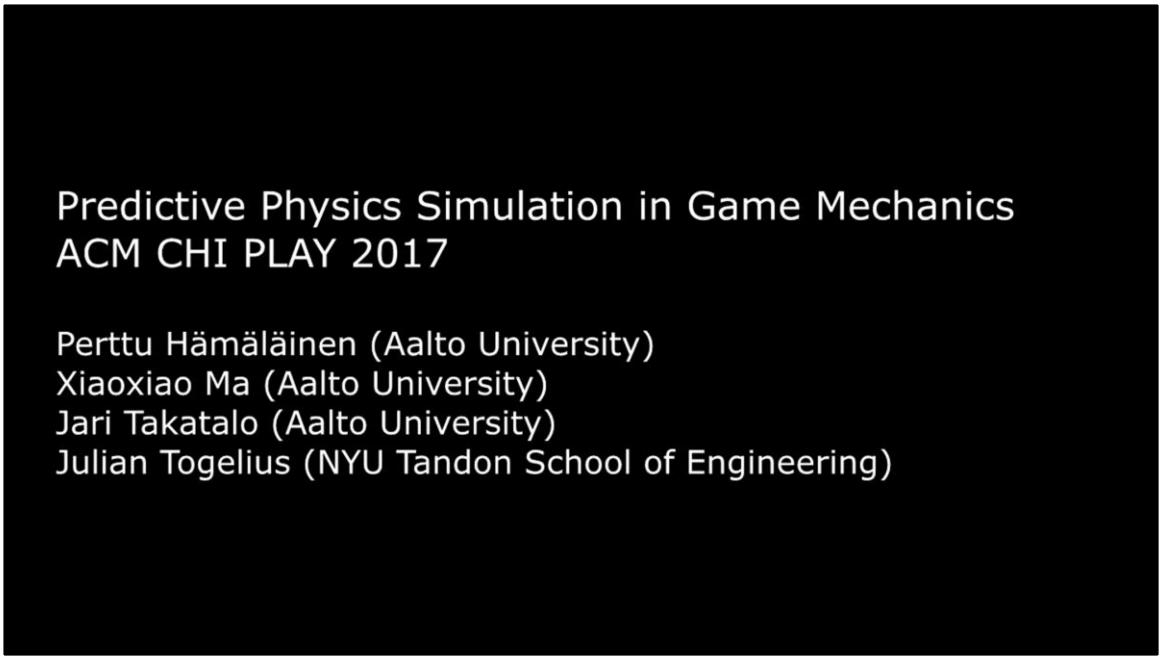
1. Simulate a number of trajectories forward
   - Trajectories utilize different types of information
   - One trajectory repeats the best trajectory of the previous frame
   - Some trajectories: neural network policy + noise
   - Some trajectories: find nearest memorized states, use their stored actions + noise
   - Some trajectories: random actions
2. During the forward simulation, terminate worst trajectories, and reassign their simulation resources by forking the best trajectories
3. After forward simulation, step the master simulation forward with the first action of the best trajectory.
4. Memorize the state and action taken
5. In a background thread, constantly retrain the neural network policy with the memorized states and actions. Also update the nearest neighbor search data structure (a decision forest)

Memory buffer limited to make the algorithm forget initial states and actions. Size of the memory buffer determines speed of convergence. Small buffer: fast convergence, but higher chance of getting stuck in less optimal behavior patterns.



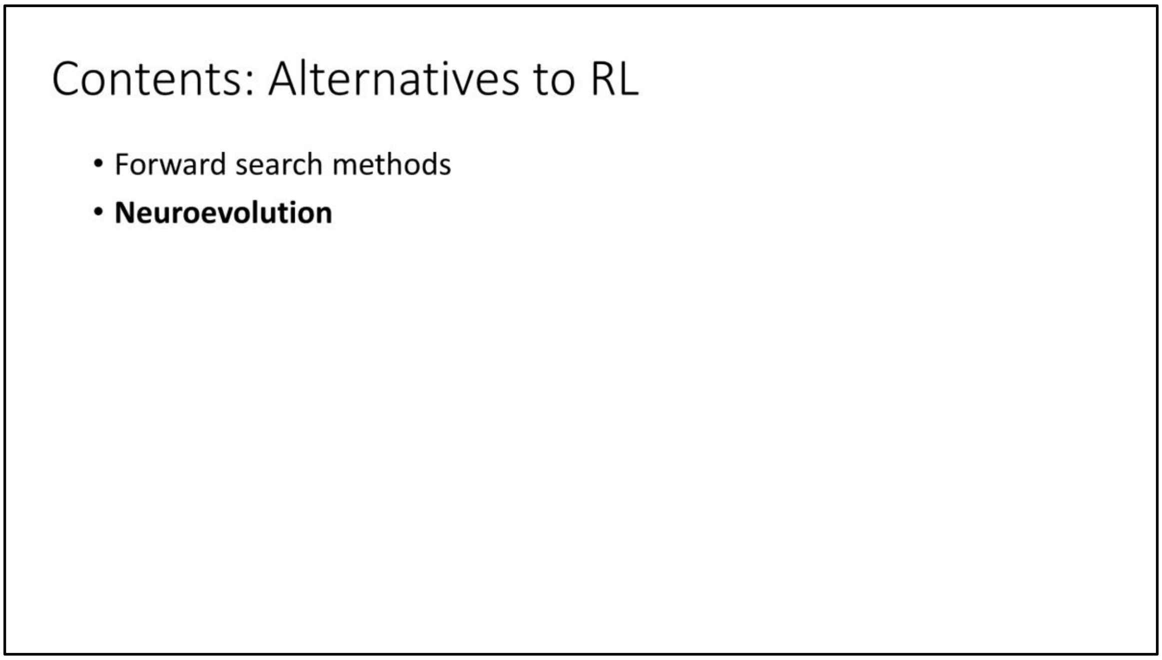
My group’s current topics

- Implement and benchmark continuous control MCTS variants for 3D simulated character control
- Find the best ways to combine continuous control MCTS with RL


Predictive Physics Simulation in Game Mechanics
ACM CHI PLAY 2017

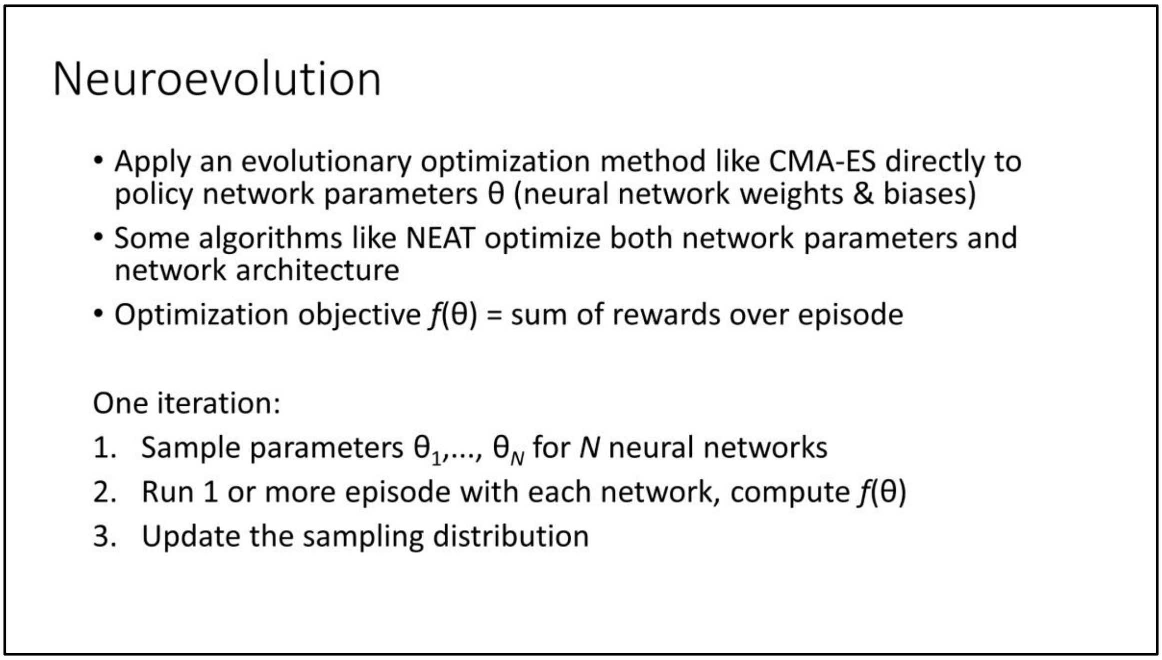
Perttu Hämäläinen (Aalto University)
Xiaoxiao Ma (Aalto University)
Jari Takatalo (Aalto University)
Julian Togelius (NYU Tandon School of Engineering)

Finally, I'd like to point out that forward search can lead to interesting game mechanics, if designed for the player instead of AI.



Contents: Alternatives to RL

- Forward search methods
- **Neuroevolution**



Neuroevolution

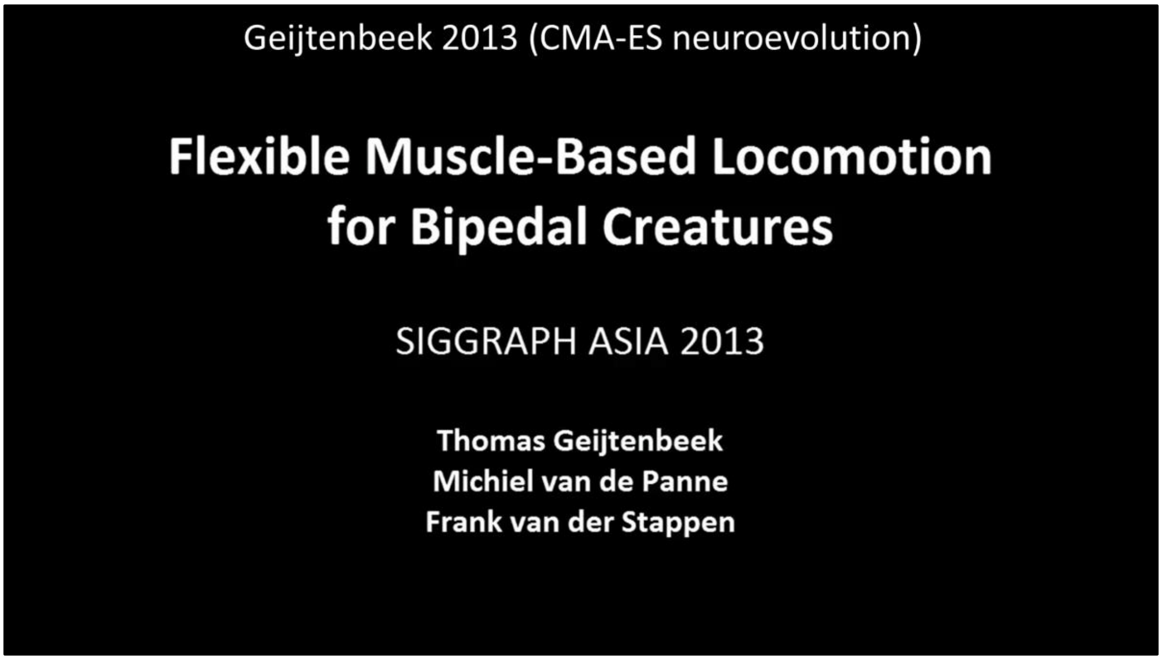
- Apply an evolutionary optimization method like CMA-ES directly to policy network parameters θ (neural network weights & biases)
- Some algorithms like NEAT optimize both network parameters and network architecture
- Optimization objective $f(\theta)$ = sum of rewards over episode

One iteration:

1. Sample parameters $\theta_1,\ldots,\theta_N$ for N neural networks
2. Run 1 or more episode with each network, compute $f(\theta)$
3. Update the sampling distribution

Very similar to RL, but more simple, as one doesn’t need to train any networks, compute advantages etc.! Instead, one just samples random network parameters.

Evolutionary optimization = optimization methods that mutate and recombine ”populations” of individuals, inspired by biological evolution. CMA-ES is inspired by this tradition, although later on, it has been analyzed and understood from different perspectives, e.g., sampling-based gradient descend with adaptive step size.


Geijtenbeek 2013 (CMA-ES neuroevolution)

Flexible Muscle-Based Locomotion for Bipedal Creatures

SIGGRAPH ASIA 2013

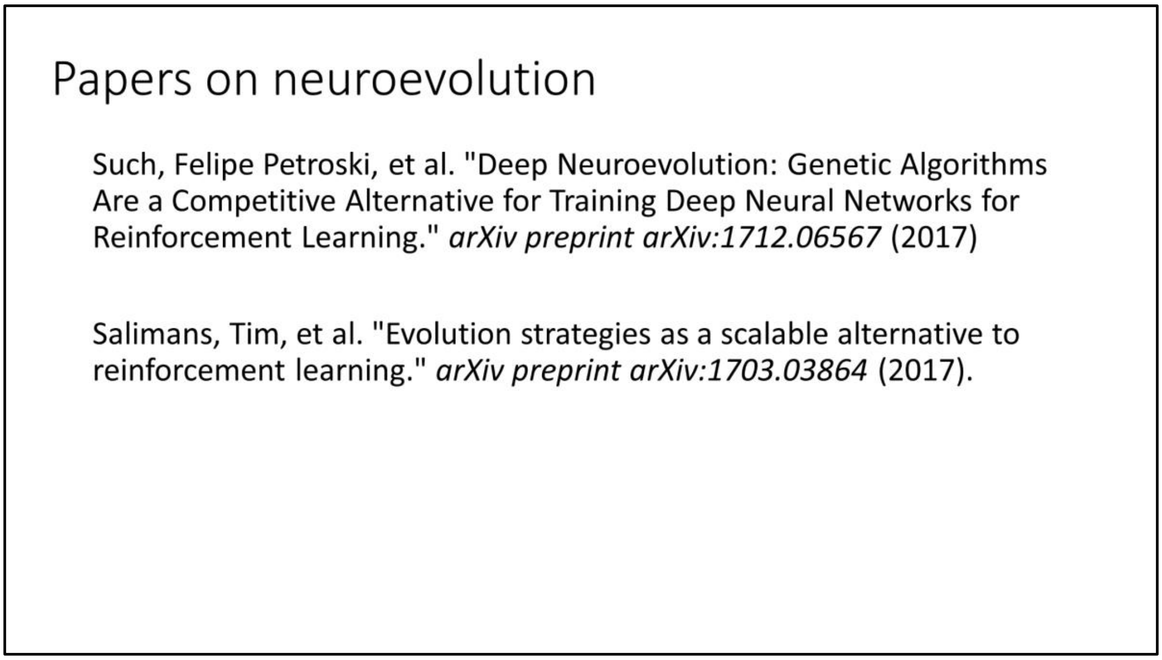
Thomas Geijtenbeek
Michiel van de Panne
Frank van der Stappen

This paper from way before deep reinforcement learning still has the best looking humanoid locomotion results.

It combines CMA-ES neuroevolution with some problem-specific engineering like using a state machine that switches between different policy networks for different parts of the gait.

CMA-ES works well in this case, as the dedicated networks can be small, with only hundreds of parameters for CMA-ES to optimize.

LM-MA-ES can easily optimize networks with a few thousand parameters. Beyond that, PPO or PPO-CMA provide faster convergence in my experience.

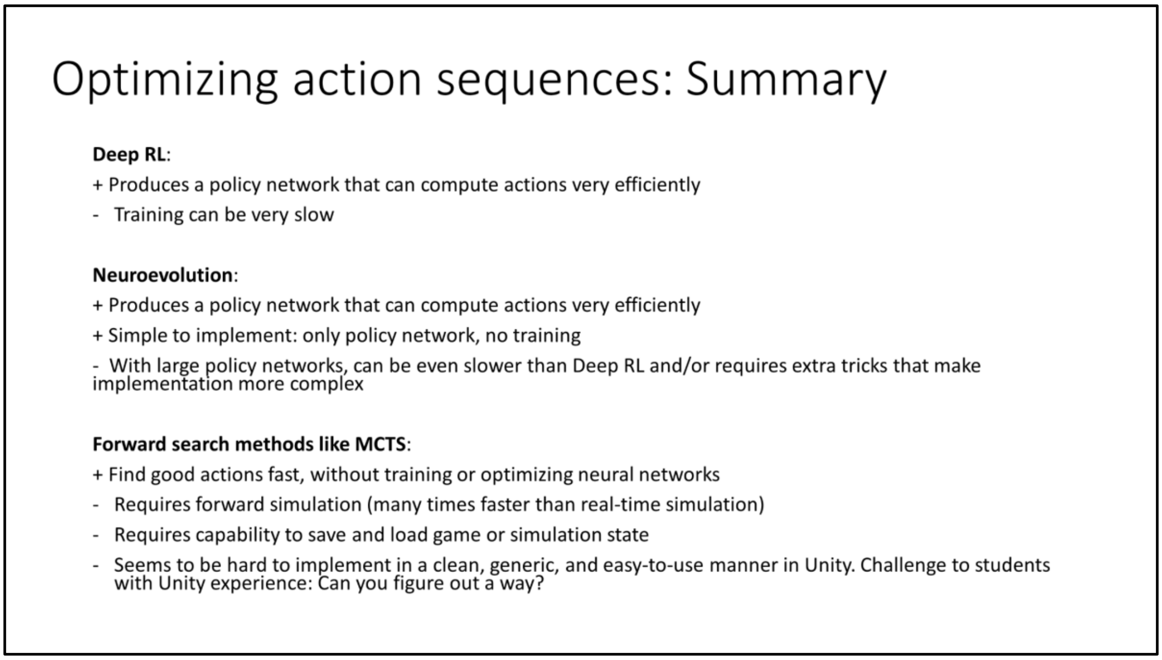


Papers on neuroevolution

Such, Felipe Petroski, et al. "Deep Neuroevolution: Genetic Algorithms Are a Competitive Alternative for Training Deep Neural Networks for Reinforcement Learning." *arXiv preprint arXiv:1712.06567* (2017)

Salimans, Tim, et al. "Evolution strategies as a scalable alternative to reinforcement learning." *arXiv preprint arXiv:1703.03864* (2017).

These papers propose methods for making neuroevolution efficient with large neural networks.



Optimizing action sequences: Summary

Deep RL:

+ Produces a policy network that can compute actions very efficiently
- Training can be very slow

Neuroevolution:

+ Produces a policy network that can compute actions very efficiently
+ Simple to implement: only policy network, no training
- With large policy networks, can be even slower than Deep RL and/or requires extra tricks that make implementation more complex

Forward search methods like MCTS:

+ Find good actions fast, without training or optimizing neural networks
- Requires forward simulation (many times faster than real-time simulation)
- Requires capability to save and load game or simulation state
- Seems to be hard to implement in a clean, generic, and easy-to-use manner in Unity. Challenge to students with Unity experience: Can you figure out a way?