# 目标规划

**1. 目标规划的常用解法**

**例1**某企业生产*A*、*B*、*C*三种产品，装备工作在同一生产线上完成，三种产品装备时的工时消耗分别为6小时、8小时和10小时，生产线每月正常工作时间为200小时，三种产品销售后每件可分别获利500元，650元和800元，每月预计销售量分别为12台、10台和6台，有关经营目标如下：

（1）利润指标不少于每月16000元；

（2）充分利用生产能力；

（3）加班时间不超过24小时；

（4）产量以预计销量为标准。

为确定生产计划，试建立该问题的目标规划模型。

解：设*A*、*B*、*C*三种产品的生产量分别为*x*1、*x*2、*x*3。

（1）设为未达到利润目标的差值，为超出利润目标的差值。

当利润小于16000时且，有成立。

当利润大于16000时且，有成立。

当利润恰好等于16000时且，有成立。

实际上利润只有上述三种情形之一发生，因而可以将三个等式写成一个等式



利润不少于16000理解为达到或超过16000，即使不能达到也要尽可能接近16000，可以表达成目标函数取最小值，则有



（2）设和分别为未超过生产能力和超过生产能力的偏差变量，充分利用生产能力理解为尽可能接近200小时，则有



（3）设和分别为加班时间未超过24小时和超过24小时的偏差变量，加班时间不超过24小时可以理解为可以不足24小时，但超过时间尽量少，则有



（4）设和为产品A未超过和超过预计销量的偏差变量，设和为产品B未超过和超过预计销量的偏差变量，设和为产品C未超过和超过预计销量的偏差变量，产量以预计销量为标准，等价于正负偏差同时取最小。







设分别为四个目标的优先因子，则问题的目标规划数学模型为：



例2



解：化成标准型



*k*=1

|  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | | | 0 | 0 | 0 |  | 0 |  |  |  |
|  |  | *b* |  |  |  |  |  |  |  |
| 0 |  | 100 | 1 | 1 | 1 | 0 | 0 | 0 | 0 | 100 |
|  |  | 45 | 1 | -1 | 0 | 1 | -1 | 0 | 0 | 45 |
|  |  | 60 | [2] | 3 | 0 | 0 | 0 | 1 | -1 | 30 |
|  | |  | 1  2 | -1  3 |  |  | -1 |  | -2 |  |

|  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | | | 0 | 0 | 0 |  | 0 |  |  |  |
|  |  | *b* |  |  |  |  |  |  |  |
| 0 |  | 70 | 0 | -1/2 | 1 | 0 | 0 | -1/2 | 1/2 | 140 |
|  |  | 15 | 0 | -5/2 | 0 | 1 | -1 | -1/2 | [1/2] | 30 |
| 0 |  | 30 | 1 | 3/2 | 0 | 0 | 0 | 1/2 | -1/2 | - |
|  | |  |  | -5/2 |  |  | -1 | -1/2  -1 | 1/2  -1 |  |

*k*=2

|  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | | | 0 | 0 | 0 |  | 0 |  |  |  |
|  |  | *b* |  |  |  |  |  |  |  |
| 0 |  | 55 | 0 | 2 | 1 | -1 | 1 | 0 | 0 |  |
|  |  | 30 | 0 | -5 | 0 | 2 | -2 | -1 | 1 |  |
| 0 |  | 45 | 1 | -1 | 0 | 1 | -1 | 0 | 0 |  |
|  | |  |  | -5 |  | -1  2 | -2 | -2 |  |  |

解：化成max标准型



*k*=1

|  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | | | 0 | 0 | 0 |  | 0 |  |  |  |
|  |  | *b* |  |  |  |  |  |  |  |
| 0 |  | 100 | 1 | 1 | 1 | 0 | 0 | 0 | 0 | 100 |
|  |  | 45 | 1 | -1 | 0 | 1 | -1 | 0 | 0 | 45 |
|  |  | 60 | [2] | 3 | 0 | 0 | 0 | 1 | -1 | 30 |
|  | |  | 1  2 | -1  3 |  |  | -1 |  | -2 |  |

|  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | | | 0 | 0 | 0 |  | 0 |  |  |  |
|  |  | *b* |  |  |  |  |  |  |  |
| 0 |  | 70 | 0 | -1/2 | 1 | 0 | 0 | -1/2 | 1/2 | 140 |
|  |  | 15 | 0 | -5/2 | 0 | 1 | -1 | -1/2 | [1/2] | 30 |
| 0 |  | 30 | 1 | 3/2 | 0 | 0 | 0 | 1/2 | -1/2 | - |
|  | |  |  | -5/2 |  |  | -1 | -1/2  -1 | 1/2  -1 |  |

*k*=2

|  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | | | 0 | 0 | 0 |  | 0 |  |  |  |
|  |  | *b* |  |  |  |  |  |  |  |
| 0 |  | 55 | 0 | 2 | 1 | -1 | 1 | 0 | 0 |  |
|  |  | 30 | 0 | -5 | 0 | 2 | -2 | -1 | 1 |  |
| 0 |  | 45 | 1 | -1 | 0 | 1 | -1 | 0 | 0 |  |
|  | |  |  | -5 |  | -1  2 | -2 | -2 |  |  |

例3



k=1

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | | | 0 | 0 |  |  |  | 0 |  |  | 0 |  |  |
|  |  | *b* |  |  |  |  |  |  |  |  |  |  |
|  |  | 30 | 1 | 0 | 1 | 0 | 0 | 0 | -1 | 0 | 0 | 0 | - |
|  |  | 15 | 0 | [1] | 0 | 1 | 0 | 0 | 0 | -1 | 0 | 0 | 15 |
|  |  | 1000 | 8 | 12 | 0 | 0 | 1 | 0 | 0 | 0 | -1 | 0 | 1000 |
| 0 |  | 40 | 1 | 2 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | -1 | 20 |
|  | |  | 0  8  0  1 | 0  12  0  1.5 |  |  |  |  | -1  0  0  -1 | -1  0  0  -1.5 | 0  -1  0  0 | 0  0  -1  0 |  |

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | | | 0 | 0 |  |  |  | 0 |  |  | 0 |  |  |
|  |  | *b* |  |  |  |  |  |  |  |  |  |  |
|  |  | 30 | 1 | 0 | 1 | 0 | 0 | 0 | -1 | 0 | 0 | 0 | 30 |
| 0 |  | 15 | 0 | 1 | 0 | 1 | 0 | 0 | 0 | -1 | 0 | 0 | - |
|  |  | 820 | 8 | 0 | 0 | -12 | 1 | 0 | 0 | 12 | -1 | 0 | 820 |
| 0 |  | 10 | [1] | 0 | 0 | -2 | 0 | 1 | 0 | 2 | 0 | -1 | 10 |
|  | |  | 0  8  0  1 |  |  | 0  -12  0  -1.5 |  |  | -1  0  0  -1 | -1  -12  0  0 | 0  -1  0  0 | 0  0  -1  0 |  |

k=2

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | | | 0 | 0 |  |  |  | 0 |  |  | 0 |  |  |
|  |  | *b* |  |  |  |  |  |  |  |  |  |  |
|  |  | 20 | 0 | 0 | 1 | 2 | 0 | -1 | -1 | -2 | 0 | [1] | 20 |
| 0 |  | 15 | 0 | 1 | 0 | 1 | 0 | 0 | 0 | -1 | 0 | 0 | - |
|  |  | 740 | 0 | 0 | 0 | 4 | 1 | -8 | 0 | -4 | -1 | 8 | 92.5 |
| 0 |  | 10 | 1 | 0 | 0 | -2 | 0 | 1 | 0 | 2 | 0 | -1 | - |
|  | |  |  |  |  | 0  4  0  0.5 |  | 0  -8  0  1 | -1  0  0  -1 | -1  -4  0  2 | 0  -1  0  0 | 0  -8  -1  1 |  |

k=3

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | | | 0 | 0 |  |  |  | 0 |  |  | 0 |  |  |
|  |  | *b* |  |  |  |  |  |  |  |  |  |  |
|  |  | 20 | 0 | 0 | 1 | 2 | 0 | -1 | -1 | -2 | 0 | 1 |  |
| 0 |  | 15 | 0 | 1 | 0 | 1 | 0 | 0 | 0 | -1 | 0 | 0 |  |
|  |  | 580 | 0 | 0 | -8 | -12 | 1 | 0 | 8 | 12 | -1 | 0 |  |
| 0 |  | 30 | 1 | 0 | 1 | 0 | 0 | 0 | -1 | 0 | 0 | 0 |  |
|  | |  |  |  | 0  -8  1  -1 | 0  -12  2  -1.5 |  | 0  0  -1  0 | -1  8  1  0 | -1  12  -2  0 | 0  -1  0  0 |  |  |

**2. 目标规划的灵敏度分析**

在目标规划的灵敏度分析中，各项系数的变化计算与线性规划相同，不再重复，这里主要分析优先因子的变换对目标规划的影响。

例4



|  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | | | 0 | 0 |  |  | 0 |  |  | 0 |  | 0 |  |
|  |  | *b* |  |  |  |  |  |  |  |  |  |  |
|  |  | 20 | 1 | 0 | 1 | -1 | 0 | 0 | 0 | 0 | 0 | 0 | - |
| 0 |  | 35 | 0 | [1] | 0 | 0 | 1 | -1 | 0 | 0 | 0 | 0 | 35 |
|  |  | 220 | -5 | 3 | 0 | 0 | 0 | 0 | 1 | -1 | 0 | 0 | 220/3 |
|  |  | 60 | 1 | -1 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | -1 | - |
|  | |  | 0  -3  1 | 0  1  0 |  | -1  0  -1 |  | -1  0  0 |  | 0  -1  0 |  | 0  -2  0 |  |
|  |  | 20 | 1 | 0 | 1 | -1 | 0 | 0 | 0 | 0 | 0 | 0 |  |
| 0 |  | 35 | 0 | 1 | 0 | 0 | 1 | -1 | 0 | 0 | 0 | 0 |  |
|  |  | 115 | -5 | 0 | 0 | 0 | -3 | 3 | 1 | -1 | 0 | 0 |  |
|  |  | 95 | 1 | 0 | 0 | 0 | 1 | -1 | 0 | 0 | 1 | -1 |  |
|  | |  | 0  -3  1 |  |  | -1  0  -1 | 0  -1  0 | -1  1  0 |  | 0  -1  0 |  | 0  -2  0 |  |

满意解为：。

由单纯形表知，初始基变量为、、、，则最终单纯形表中基矩阵对应的逆矩阵为



当的优先因子由改为0时



则的检验数变为



由此可知，优先因子的改变引起基的变化，以作为入基变量重新进行迭代

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | | | 0 | 0 |  |  | 0 | 0 |  | 0 |  | 0 |  |
|  |  | *b* |  |  |  |  |  |  |  |  |  |  |
|  |  | 20 | 1 | 0 | 1 | -1 | 0 | 0 | 0 | 0 | 0 | 0 | - |
| 0 |  | 35 | 0 | 1 | 0 | 0 | 1 | -1 | 0 | 0 | 0 | 0 | 35 |
|  |  | 115 | -5 | 0 | 0 | 0 | -3 | [3] | 1 | -1 | 0 | 0 | 115/3 |
|  |  | 95 | 1 | 0 | 0 | 0 | 1 | -1 | 0 | 0 | 1 | -1 | - |
|  | |  | 0  -3  1 |  |  | -1  0  -1 | 0  -1  0 | 0  1  0 |  | 0  -1  0 |  | 0  -2  0 |  |

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | | | 0 | 0 |  |  | 0 | 0 |  | 0 |  | 0 |  |
|  |  | *b* |  |  |  |  |  |  |  |  |  |  |
|  |  | 20 | 1 | 0 | 1 | -1 | 0 | 0 | 0 | 0 | 0 | 0 |  |
| 0 |  | 220/3 | -5/3 | 1 | 0 | 0 | 0 | 0 | 1/3 | -1/3 | 0 | 0 |  |
| 0 |  | 115/3 | -5/3 | 0 | 0 | 0 | -1 | 1 | 1/3 | -1/3 | 0 | 0 |  |
|  |  | 400/3 | -2/3 | 0 | 0 | 0 | 0 | 0 | 1/3 | -1/3 | 1 | -1 |  |
|  | |  | 0  -4/3  1 |  |  | -1  0  -1 |  |  | 0  -1/3  0 | 0  -2/3  0 |  | 0  -2  0 |  |

满意解为：。

**3. 目标规划的应用**

目标规划比线性规划更灵活，它已被广泛地应用于生产计划、人力资源分配等各个方面。

**例** 某电子厂生产录音机和电视机两种产品，分别经由甲、乙两个车间生产，已知除外购件以外，生产一台录音机需甲车间加工2小时，乙车间装备1小时；生产一台电视机需甲车间加工1小时，乙车间装备3小时。这两种产品生产出来后均需经检验、销售等环节。已知每台录音机的检验、销售费用需50元，每台电视机的检验、销售费用需30元。甲车间每月可用的生产工时为120小时，每小时费用为80元；乙车间每月可用的生产工时为150小时，每小时费用为20元。估计每台录音机的利润为100元，每台电视机的利润为75元，下一年度平均每月可销售录音机50台，电视机80台。

工厂确定月度计划的目标如下：

第一优先级：检验、销售费用每月不超过4600元；

第二优先级：每月售出录音机不少于50台；

第三优先级：甲、乙两车间的生产工时得到充分利用（重要性权系数按两个车间每小时费用的比例确定）；

第四优先级：甲车间加班不超过20小时；

第五优先级：每月销售电视机不少于80台；

第六优先级：两个车间加班的总时间要有控制（权系数分配与第三优先级相同）。

试确定该厂达到以上目标的最优月度计划生产数。

解：设为每月生产录音机的台数，为每月生产电视机的台数。根据题中给出的条件，各级目标约束情况如下：



(录音机)

(甲车间)

(乙车间)



(电视机)

甲、乙两车间总加班时间最小

因甲车间每小时生产费用为80元，乙车间每小时生产费用为20元，其权数比为4:1，故得目标规划模型为：

