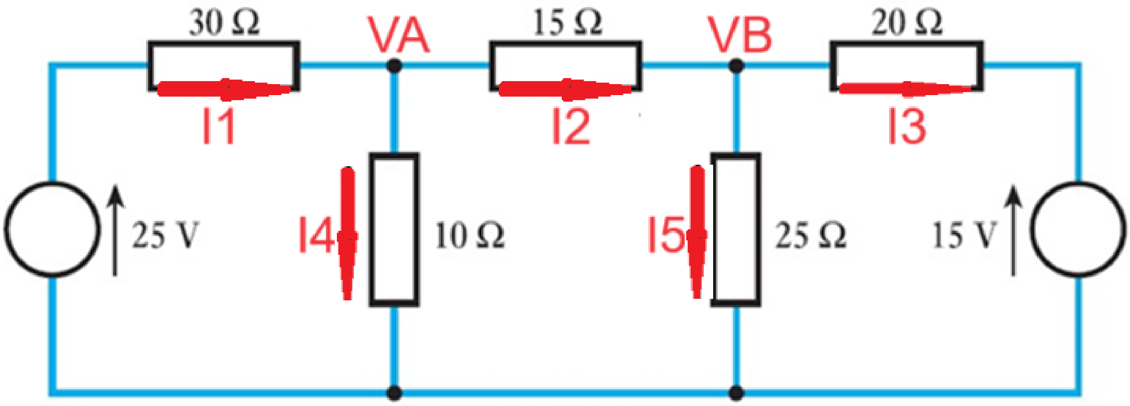
Coursework 1

The coursework is finished by Jiale Fu (2543035) and Zijun Liu (2543336) in Transportation D2201.

Q1



Q1-1

Let us suppose that the current pass through the five resistors is I_1, I_2, I_3, I_4, I_5 , and the position of the upper joints are V_A, V_B.

We can get:

$I_1 = I_4 + I_2$

$I_2 = I_5 + I_3$

$I_1 = \frac{25-V_A}{30}$

$I_4 = \frac{V_A}{10}$

$I_2 = \frac{V_A-V_B}{15}$

$I_5 = \frac{V_B}{25}$

$I_3 = \frac{V_B-15}{20}$

In this case, we can get:

$30I_4 + 30I_5 + 30I_3 + V_A = 25$

$10I_4 - V_A = 0$

$15I_5 + 15I_3 - V_A + V_B = 0$

$25I_5 - V_B = 0$

$20I_3 - V_B = -15$

Using the following python code to solve the equations:

```python
import numpy as np

A = np.array([[30, 30, 30,  1,  0],
              [10,  0,  0, -1,  0],
              [ 0, 15, 15, -1,  1],
              [ 0, 25,  0,  0, -1],
              [ 0,  0, 20,  0, -1]])
b = np.array([25, 0, 0, 0, -15])

x = np.linalg.solve(A, b)

print("solution is:")
print(x)
```

The solution is:

[0.6714876   0.30578512 -0.3677686   6.71487603 7.6446281]

Which means that:

$I_1 = 0.6095A$

$I_2 = -0.0620A$

$I_3 = -0.3678A$

$I_4 = 0.6715A$

$I_5 = 0.3058A$

$V_A = 6.7149V$

$V_B = 7.6446V$

So, the voltage drops are:

$V_{D1} = 25V - V_A = 18.2851V$

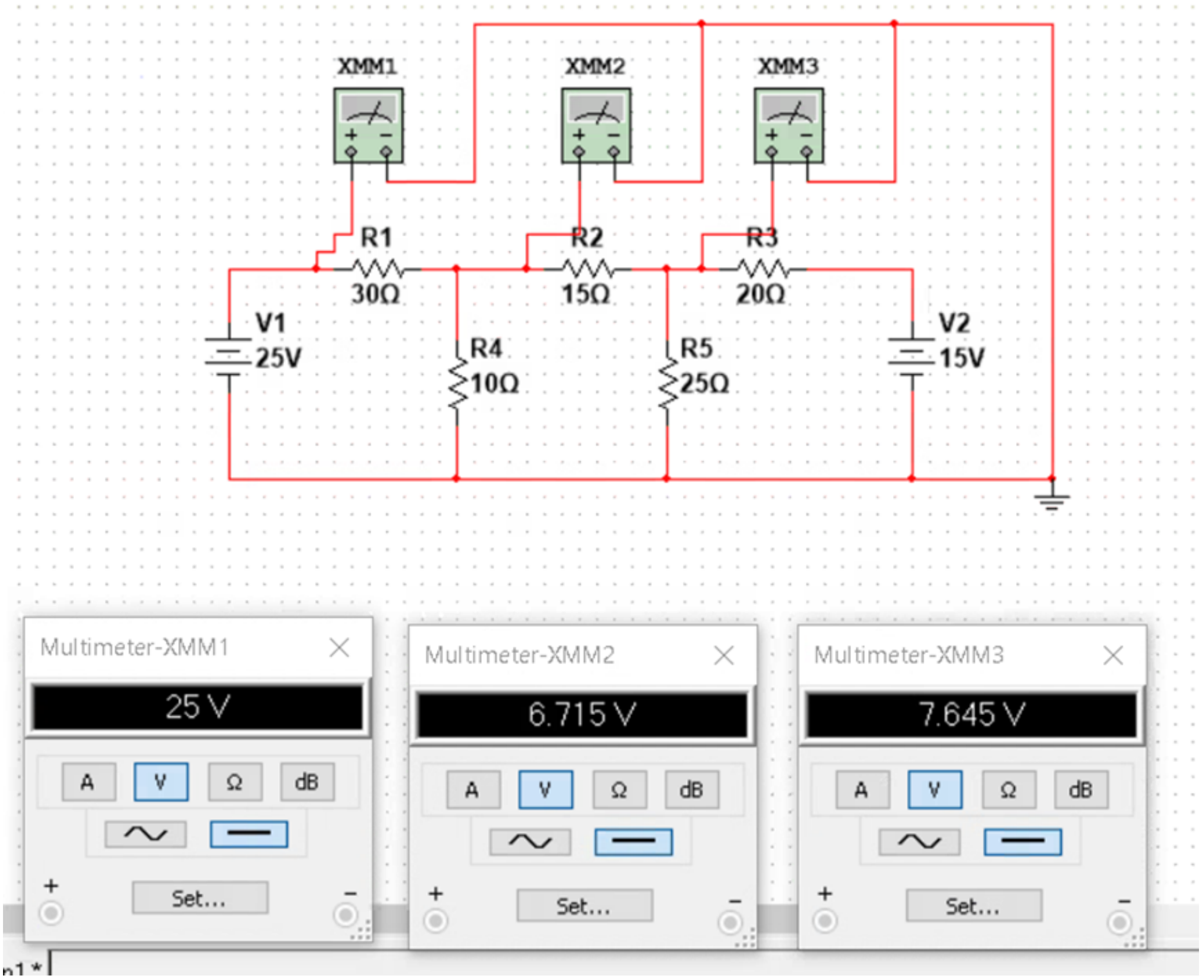
$V_{D2} = V_A - V_B = -0.9297V$

$V_{D3} = V_B - 15 = -7.3554V$

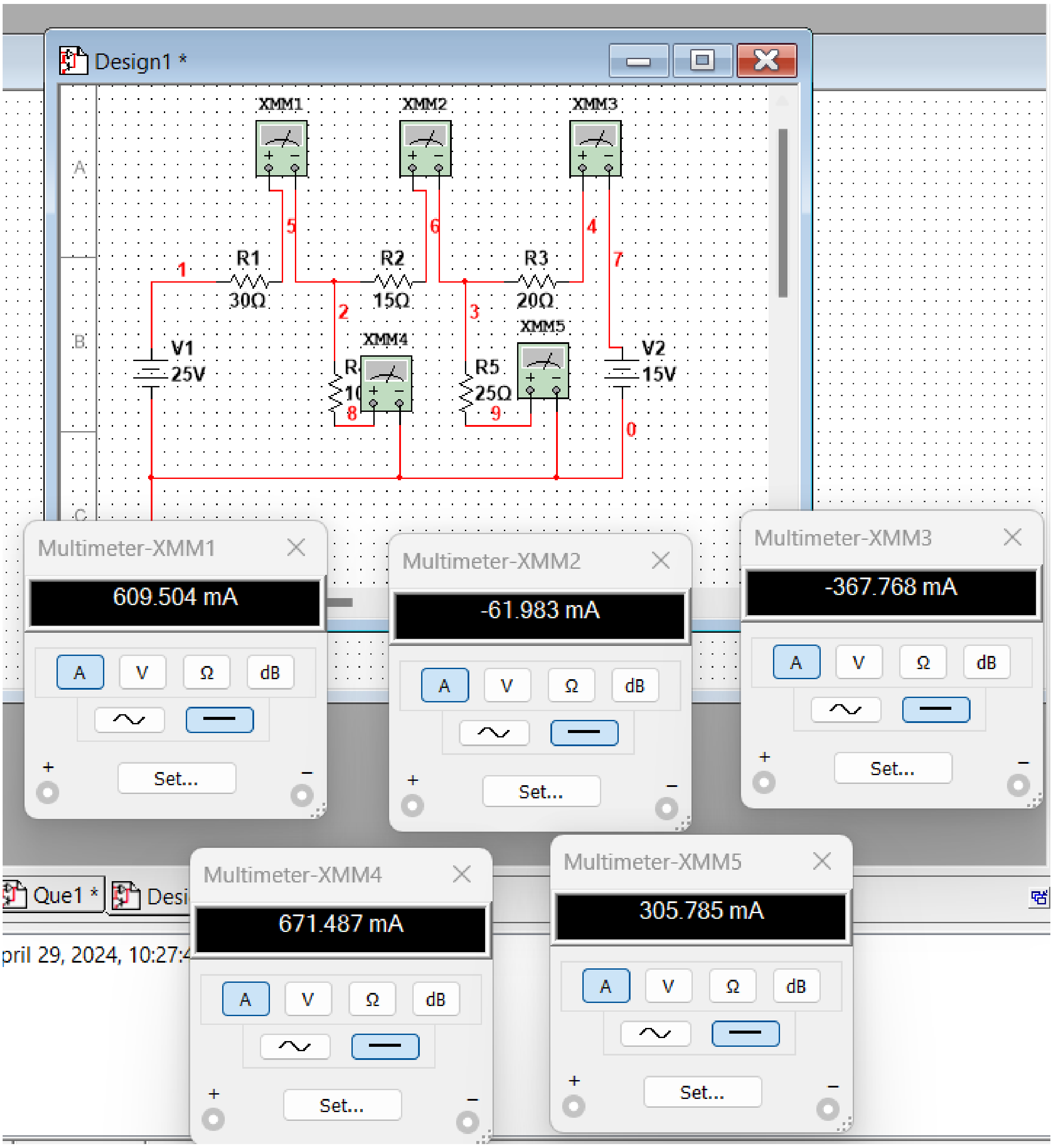
$V_{D4} = V_A = -6.7149V$

$V_{D5} = V_B = 7.6446V$

Q1-2



The results of simulation are as follows:


Design1 *
XMM1
XMM2
XMM3
R1
30Ω
R2
15Ω
R3
20Ω
V1
25V
V2
15V
XMM4
XMM5
R5
25Ω
Multimeter-XMM1
609.504 mA
Multimeter-XMM2
-61.983 mA
Multimeter-XMM3
-367.768 mA
Multimeter-XMM4
671.487 mA
Multimeter-XMM5
305.785 mA
A
V
Ω
dB
Set...
Que1 *
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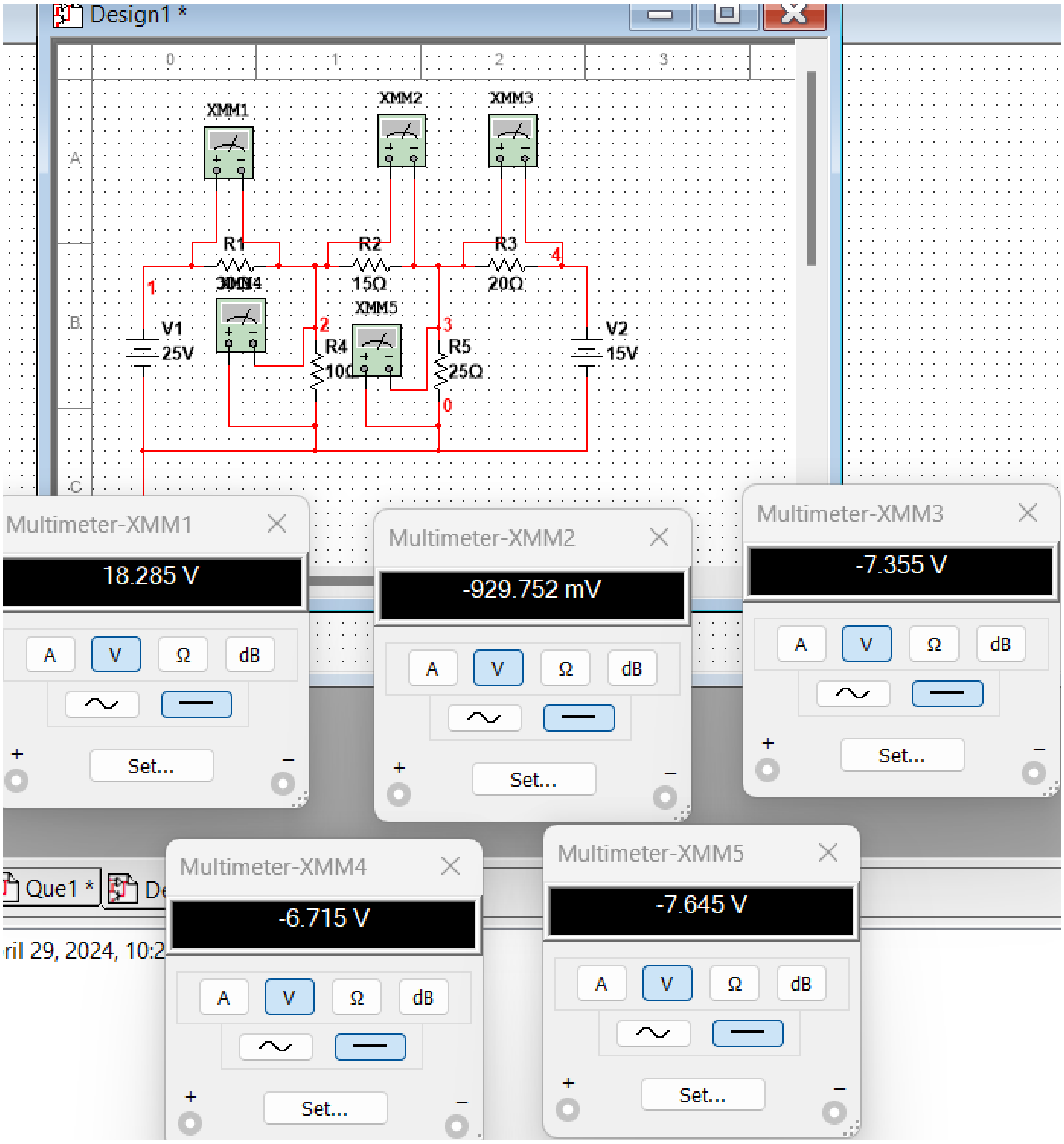


Summery

Resistor	R1	R2	R3	R4	R5
Calculated Voltage Drop	18.2851 V	-0.9297 V	-7.3554 V	-6.7149 V	7.6446 V
Calculated Current	609.5 mA	-62.0 mA	-367.8 mA	671.5 mA	305.8 A
Simulated Voltage Drop	18.285 V	-0.930 V	-7.355 V	-6.715 V	-7.645 V
Simulated Current	609.504 mA	-61.983 mA	-367.768 mA	671.487 mA	305.785 mA

The simulated result fits the calculation well.

Q2

Consider the following circuit. A capacitor with C = 0.25 μF is in series with two resistors R1 = 1500 kΩ and R2 = 3500 kΩ. The RMS value of the AC voltage supplied to the circuit is 100 VAC and the frequency is 50 Hz.

Q2-1

The amplitude is $100V \times \sqrt{2} = 100\sqrt{2} = 141.4V$

$Z = \sqrt{(R_1 + R_2)^2 + X_C^2}$ where $X_C = \frac{1}{2\pi fC} = 12.7k\Omega$

Hence, $Z = 5000.02k\Omega$

$I_{max} = \frac{V_{max}}{Z} = 28.2\mu A$

$V_{max,R_1} = R_1 I_{max} = 42.3V$

$V_{max,R_2} = R_2 I_{max} = 98.7V$

$V_{max,X_C} = X_C I_{max} = 0.36V$

For resistors, the phase angle between the current and the voltage is 0°, so $\Phi_{R_1} = \Phi_{R_2} = 0$; for capacitor, $I = \frac{dQ(t)}{dt} = \frac{CdV(t)}{dt} = C\frac{dV_{max}sin(\omega t)}{dt} = CV_{max}sin(\omega t + \frac{\pi}{2})$

So the phase angle of the capasitor is $\Phi_{C_2} = 0$

Q2-2

$Z = 5000.02k\omega$

$\phi = tan^{-1}\frac{V_{max,X_C}}{V_{max,R_1}+V_{max,R_2}} = 0.146^\circ$

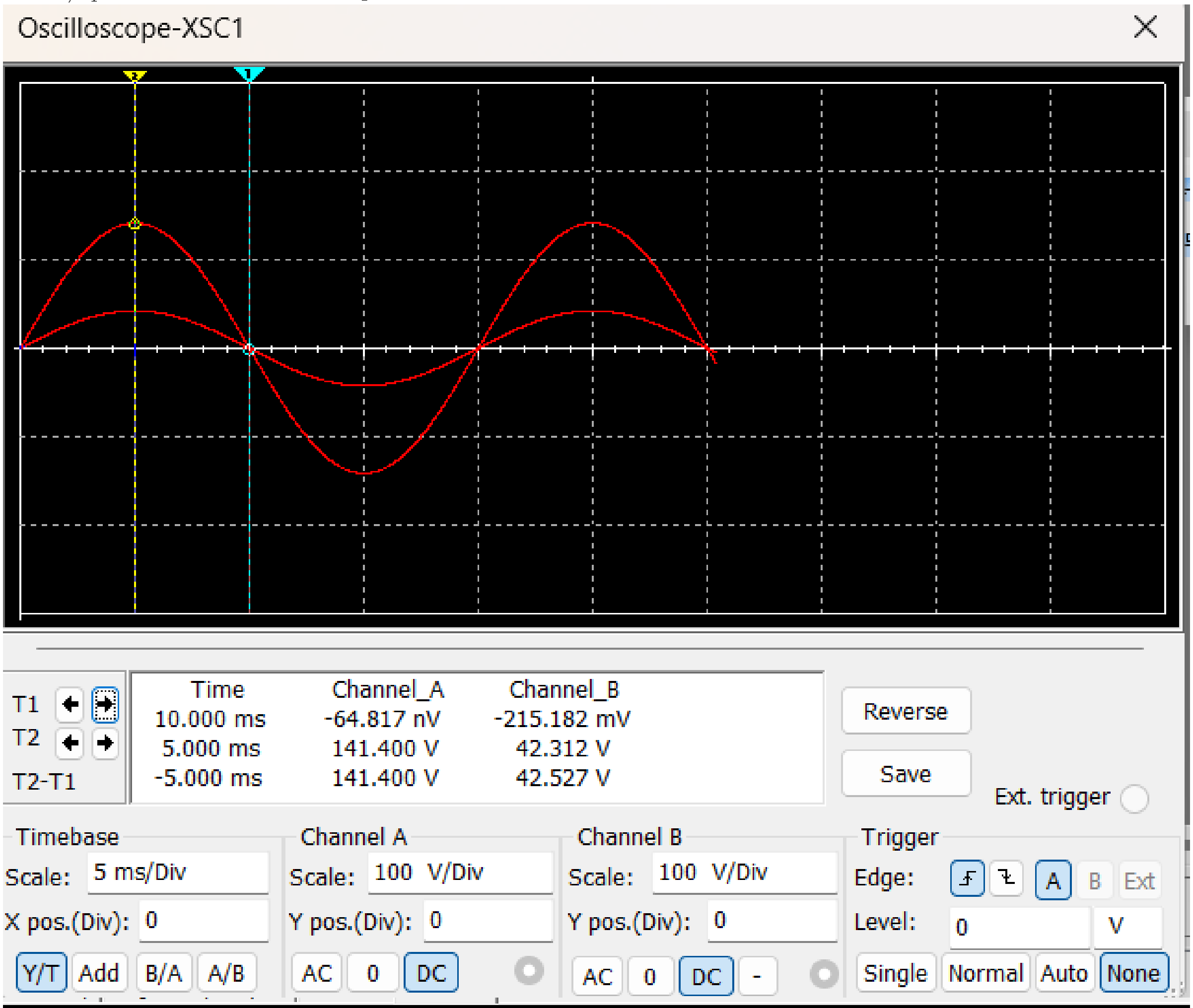
Q2-3

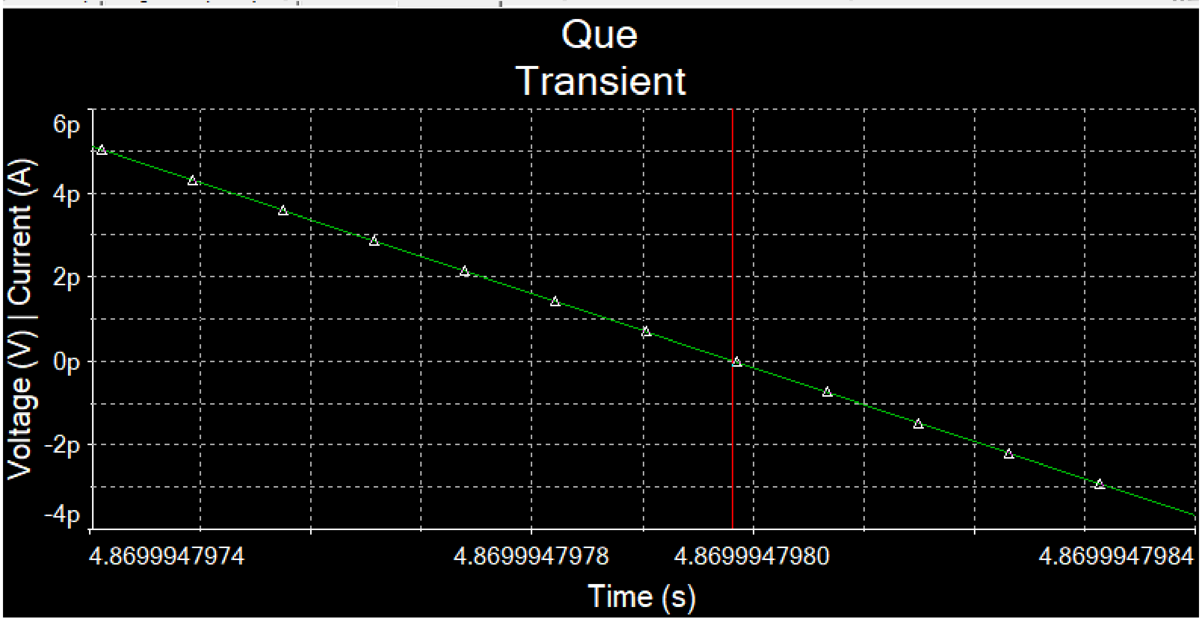
The circuit is run in the multisim.

Simulation of Q2-1

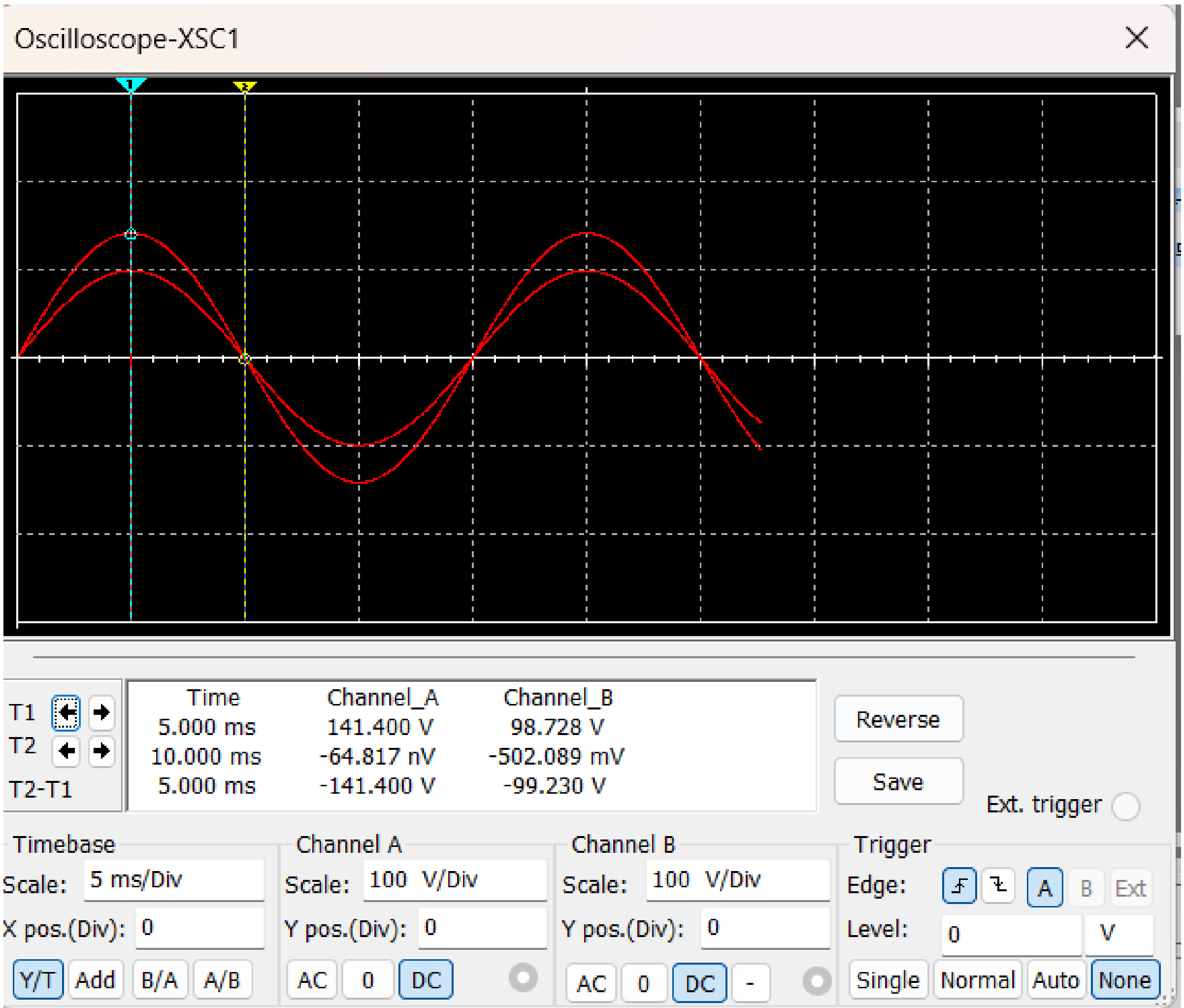
The voltage of the AC power source is set as $141.4V$.

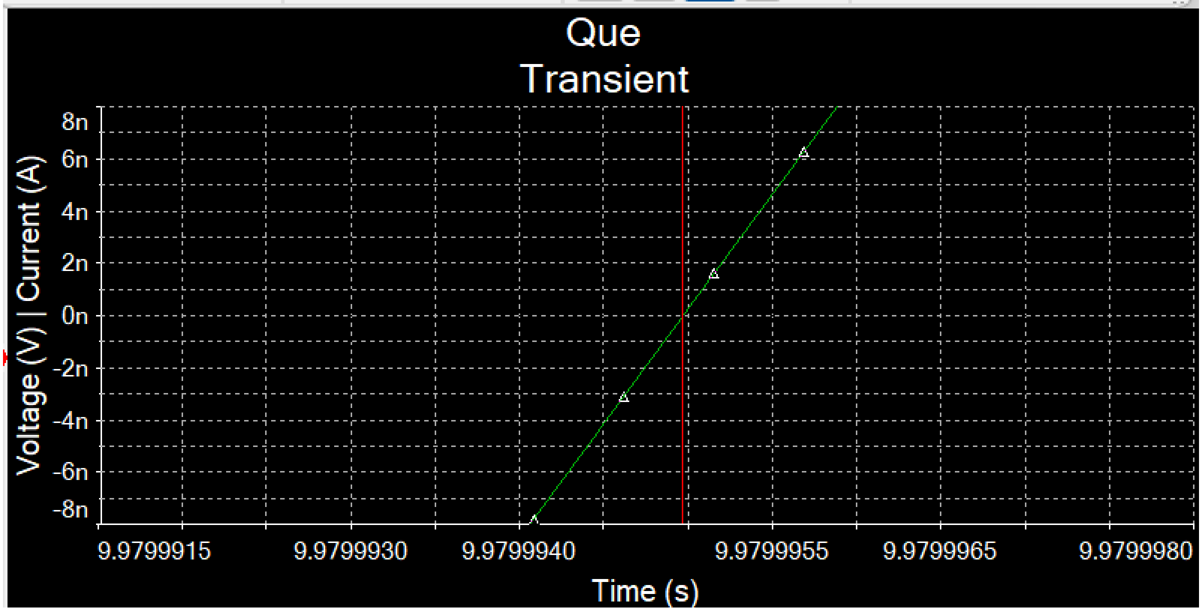
$V_{max,R_1} = 42.312V$, and $\Phi_{R_1} = 0$





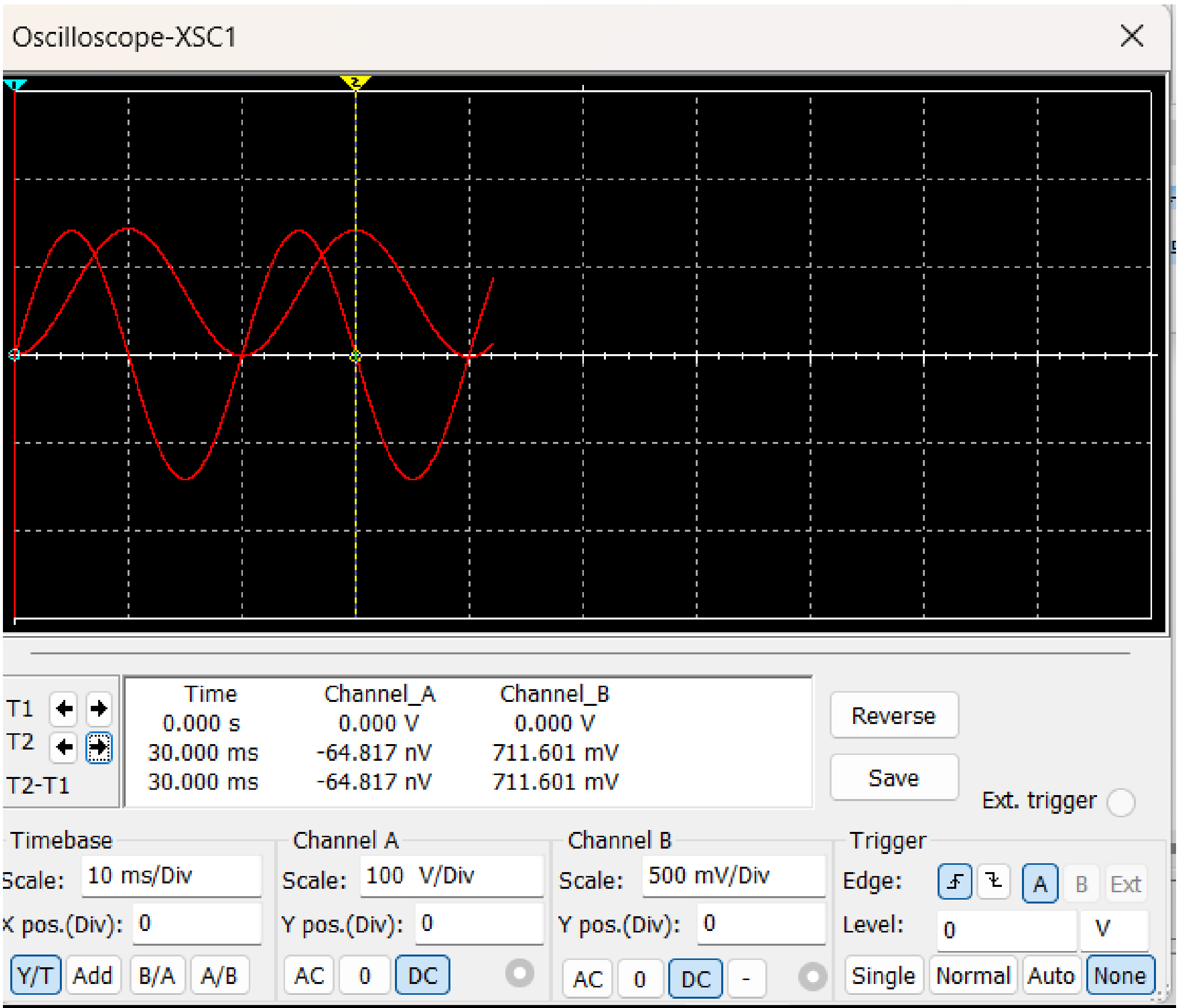
$V_{max,R_2} = 98.728V$, and $\Phi_{R_2} = 0$

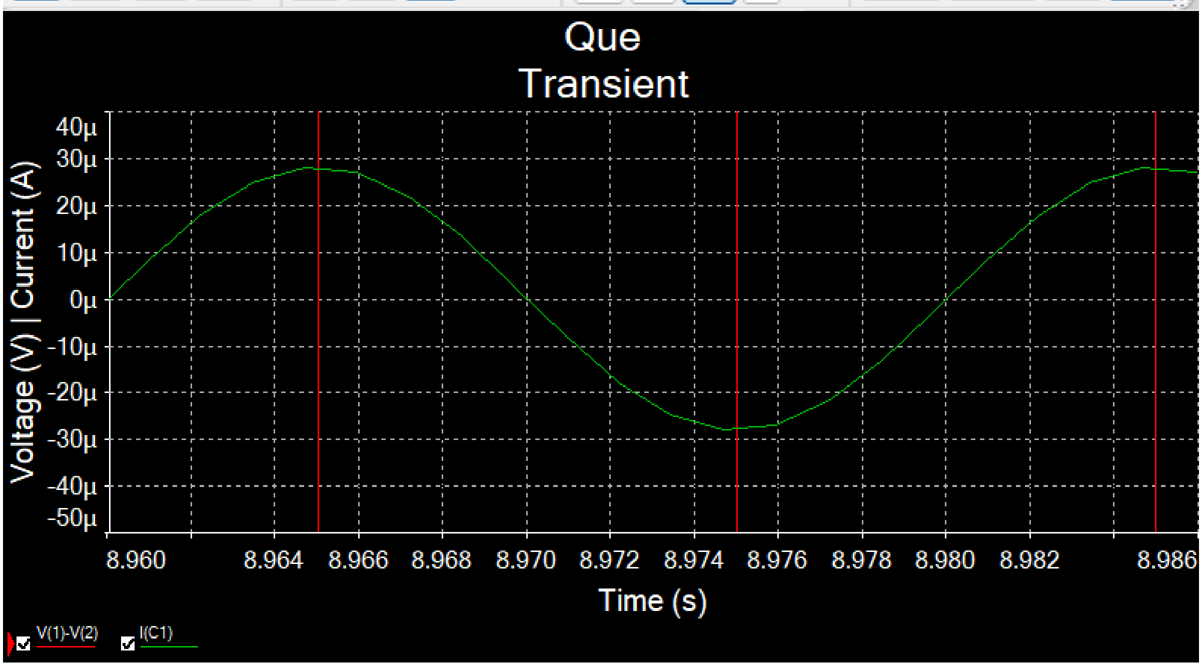




$V_{max,X_C} = 0.356V$

(Notive that the actural voltage should be divided by 2), and $\Phi_{X_C} = 90^\circ$.





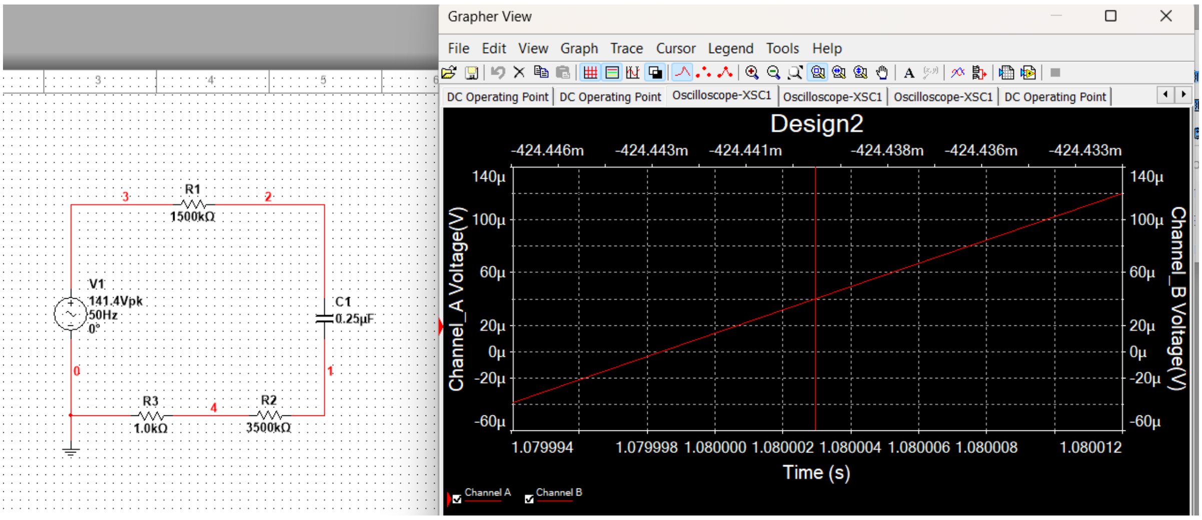
$I_{max} = 28\mu A$

Resistor	R1	R2	C1
Calculated Phase Drop	0°	0°	90°
Calculated V_{max}	42.3 V	98.7 V	0.36 V
Calculated I_{max}	42.3 V	98.7 V	0.36 V

Resistor	R1	R2	C1
Simulated Phase Drop	0°	0°	90°
Simulated V_{max}	42.312 V	98.728 V	0.356 V

Calculated Current	Simulated Current
$28.2\mu A$	$28\mu A$

Simulation of Q2-2



By adding a testing resistor, the time lag is $4.5 \times 10^{-6} s$, and the phase difference is 0.081°.

Q2-4

All phase shift are ignorable. The theorical phase shift is as followed.

Capacitance	Calculated phase shift	Simulated phase shift
$0.1\mu F$	0.3647°	0.365°
$0.2\mu F$	0.1823°	0.182°
$0.3\mu F$	0.1215°	0.122°
$0.4\mu F$	0.0911°	0.091°
$0.5\mu F$	0.1729°	0.173°
$0.6\mu F$	0.0607°	0.061°
$0.7\mu F$	0.0521°	0.052°
$0.8\mu F$	0.0455°	0.045°
$0.9\mu F$	0.0405°	0.040°
$1.0\mu F$	0.0364°	0.036°

Q2-5

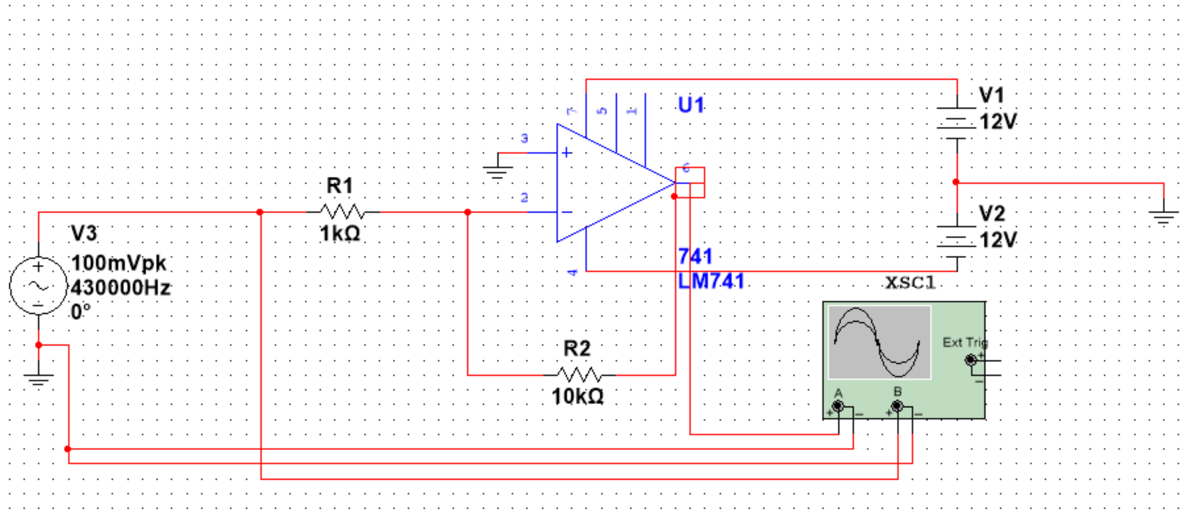
All phase shift are ignorable. The theorical phase shift is as followed.

freq	Calculated phase shift	Simulated phase shift
$50Hz$	0.1459°	0.146°
$100Hz$	0.0729°	0.073°
$150Hz$	0.0486°	0.049°
$200Hz$	0.0364°	0.036°
$250Hz$	0.0291°	0.029°
$300Hz$	0.0243°	0.024°
$350Hz$	0.0208°	0.021°
$400Hz$	0.0182°	0.018°
$450Hz$	0.0162°	0.016°
$500Hz$	0.0145°	0.014°

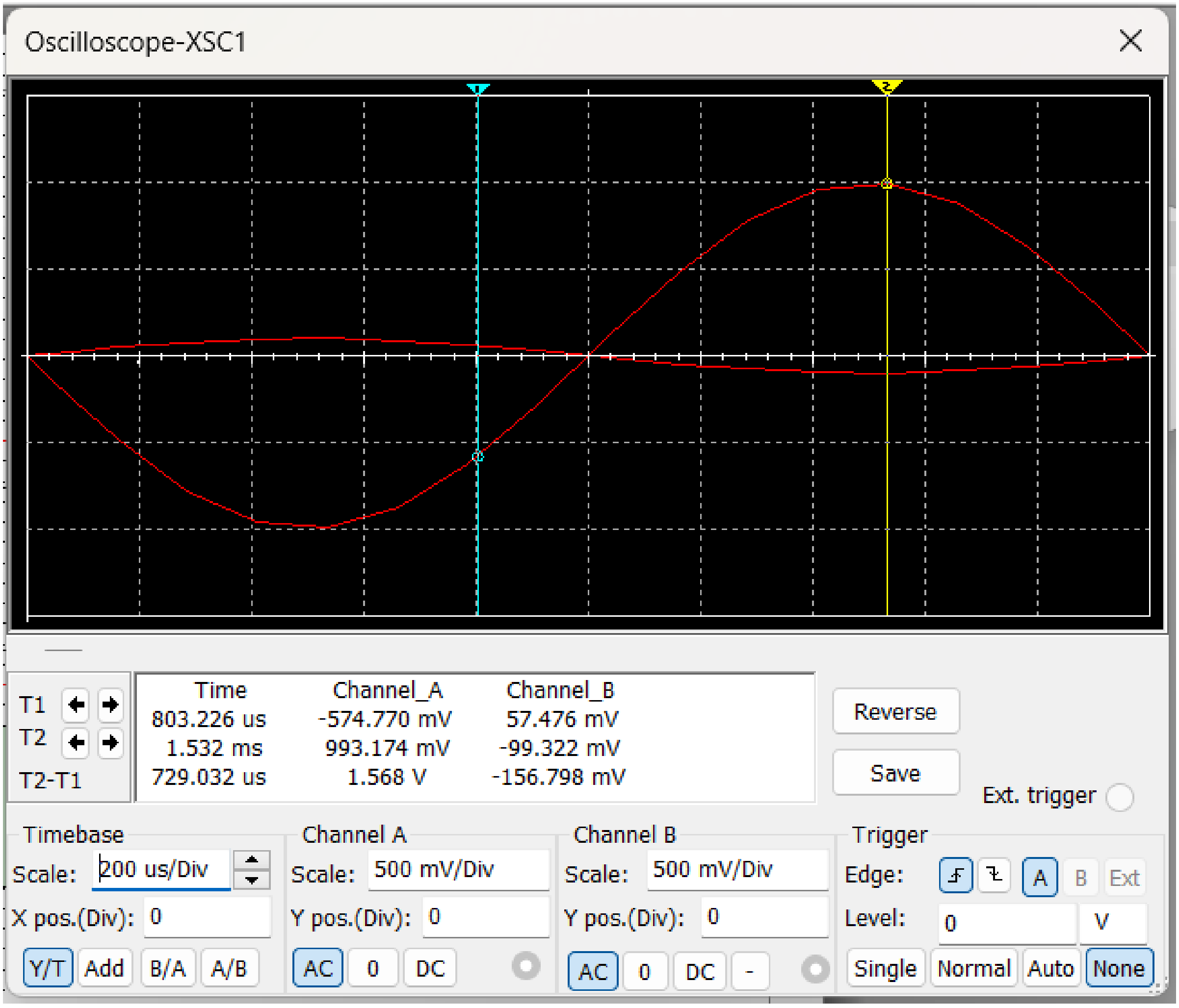
As shown above, when the frequency raise, the phase difference decreased.

Q3

The circuit in Multisim is as followed:

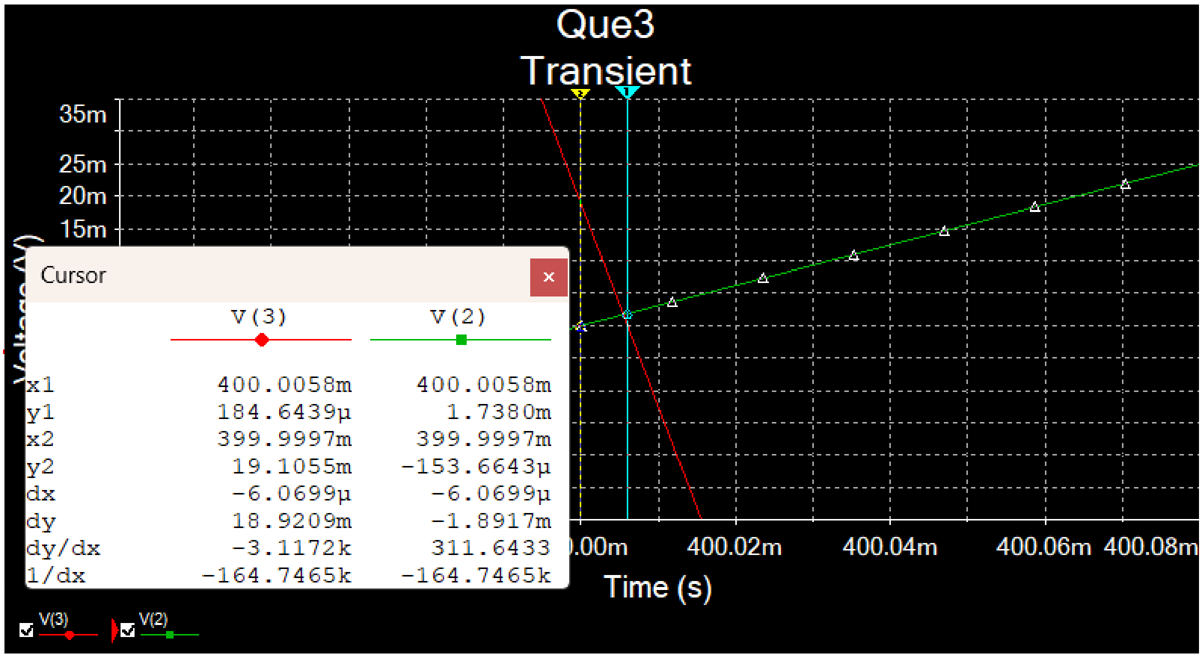


Q3-1



The output voltage is close to 1000 mV, and the voltage gain is $10log_{10}\left(\frac{1000mV}{100mV}\right) = 10$

Q3-2



The phase difference is $2\pi \times \frac{(400.0058ms - 399.9997ms)}{2ms} = 0.01916rad$

Q3-3

freq	Voltage gain	GBPW
$10^2 Hz$	$10.051 dB$	N/A
$10^3 Hz$	$10.051 dB$	N/A
$10^4 Hz$	$10.025 dB$	N/A
$5 \times 10^4 Hz$	$9.44 dB$	472000
$10^5 Hz$	$8.073 dB$	807300
$5 \times 10^5 Hz$	$2.0951 dB$	1047550
$8.6 \times 10^5 Hz$	$1 dB$	860000

From the results above, when the frequency increase, the voltage gain first remains unchanged until the freq reaches about $5 \times 10^4 Hz$, and the frequency decrease, and the gain–bandwidth product (GBWP) is around 9×10^5.

Q3-4

the bandwidth at $A = 10$ is $4.3 \times 10^5 Hz$

Q4

Q4-1

Since the ideal voltage gain is $A_{1/2/3} = -\frac{R_F}{R_{1/2/3}}$, the output should be $V_{out} = -\frac{R_F}{R_1} V_1 - \frac{R_F}{R_2} V_2 - \frac{R_F}{R_3} V_3$. The requirements is $V_{out} = -5V_1 - 2V_2 - 0.1V_3$.

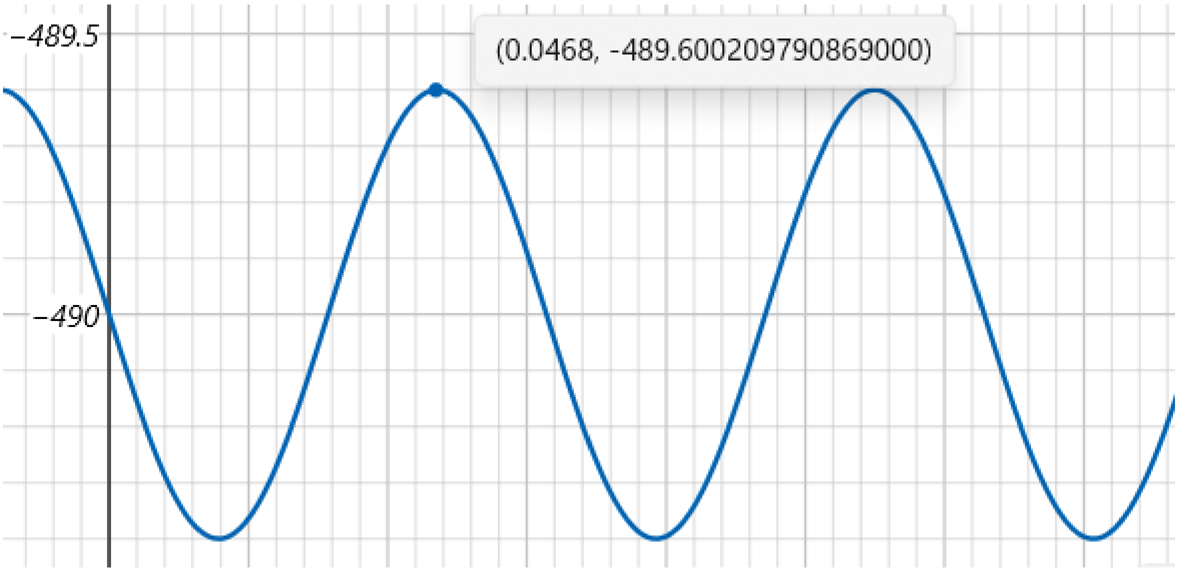
Hence, the ratio is: $R_f : R_1 : R_2 : R_3 = 0.2 : 0.5 : 10 : 1$

If $R_f = 100k\Omega$, $R_1 = 20k\Omega, R_2 = 50k\Omega, R_3 = 1M\Omega$

Q4-2

Since $V_1 = 100V_{DC}, V_2 = -5V_{DC}, V_3 = 4sin(2\pi ft)V = 4sin(100\pi t)V$, $V_{out} = -500 + 10 - 0.4sin(100\pi t) = -490 - 0.4sin(100\pi t)$.

The theoretical minimum and maximum value are $-490.4V$ and $-489.6V$. The $V - t$ diagram by Microsoft Calculator is as followed:



The detected minimum and maximum value are $-490.374V$ and $489.590V$. The result is as followed: