## TP2: Mesh adaptation with the Mmg platform

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#### 1 Goals:

- Adapt a mesh to a given size map;
- Compute an isotropic/anisotropic size map based on the interpolation error.

## 2 Mmg platform in short

The Mmg platform gathers software dedicated to simplicial mesh modifications:

- mmg2d : for 2D meshes;
- mmgs: for 3D surface meshes;
- mmg3d : for 3D volume meshes.

All this software allow quality improvement, mesh adaptation to a size map (isotropic/anisotropic) and level-set discretization.

Additional documentation can be founded here:

```
http://www.mmgtools.org/mmg-remesher-try-mmg/mmg-remesher-tutorials
and here:
http://www.mmgtools.org/mmg-remesher-try-mmg/mmg-remesher-options
```

#### 2.1 Installation

- 1. Download and untar the last stable binaries :
  - For Linux users:

```
$ wget https://www.mmgtools.org/files/2017/11/mmg-5.3.6-Linux-4.9.0-2-
amd64-appli.tar.gz
$ tar -xvzf mmg-5.3.6-Linux-4.9.0-2-amd64-appli.tar.gz
```

• for Mac users:

```
$ wget https://www.mmgtools.org/files/2017/11/mmg-5.3.6-Darwin-16.7.0-
appli.tar.gz
$ tar -xvzf mmg-5.3.6-Darwin-16.7.0-appli.tar.gz
```

2. Add the path of the bin folder to you PATH variable to be able to run the applications from your terminal without adding the full binary path:

```
$ echo "PATH=$PATH_TO_BIN:$PATH" >> ~/.bashrc
$ source ~/.bashrc
```

### 3 A first run of the remesher

To run the remesher, you must give the application name followed by the path and mesh name (the naca mesh of the previous TP is provided in the TP/Data directory of this repository):

```
$ cd TP
$ mmg2d_03 Data/naca_embedded.mesh
```

```
- PHASE 1 : DATA ANALYSIS
                                    Input histogram
-- MESH QUALITY
              62506
                                      0.498423 (8)
                       0.953331 WRST.
  BEST
       1.000000 AVRG.
  HISTOGRAMM: 100.00 % > 0.12
  PHASE 1 COMPLETED.
                     0.038s
                            Step and time passed in it
 PHASE 2 : ISOTROPIC MESHING
        0 splitted,
                      482 collapsed,
                                       183 swapped, 3 iter.
-- GRADATION : 1.300000
        3 splitted,
                     29503 collapsed,
                                       1186 swapped, 4 iter.
                     1324 collapsed,
                                                      2312 moved, 4 iter.
      374 splitted,
                                       217 swapped,
  PHASE 2 COMPLETED.
                      0.585s
END OF MODULE MMG2D: IMB-LJLL
Output histog
-- MESH QUALITY 905
  BEST 0.999972 AVRG.
                       0.940556 WRST.
                                      0.770953 (376)
  HISTOGRAMM: 100.00 % > 0.12
  MESH PACKED UP
  NUMBER OF VERTICES
                        486
                             CORNERS
                                          3
  NUMBER OF TRIANGLES
                        905
  NUMBER OF EDGES
                         67
```

FIGURE 1 – Default Mmg output.

```
MESH QUALITY
              62506
                                                          Qualities and index of
      1.000000 AVRG.
                        0.953331 WRST.
                                          0.498423 (8)
BEST
HISTOGRAMM:
             100.00 % > 0.12
                                                            the worst triangle
             100.00 % > 0.5
              1.0
                      61967
                               99.14 %
  0.6 < 0 <
              0.8
                        526
                                0.84 %
                                0.02 %
```

FIGURE 2 – Detailed quality histogram.

By default, Mmg creates a mesh in the same path and of the same extension than the input mesh (.mesh here) with the .o prefix to the extension so here: Data/naca\_embedded.o.mesh.

#### 3.1 The Mmg output

You can see the Mmg output in the konsole. By default, Mmg prints (see figure 1):

- The different phases of the algorithm (analysis step, remeshing step...) and the time spent in each of this step;
- some info about the input/output qualities histogram;
- the final mesh statistics (number of nodes, of elements and of edges).

You can change the default verbosity of Mmg with the -v option. By default, the verbosity value is setted to 1. If you set the verbosity to 5,  $mmg2d_03 \, Data/naca_embedded.mesh -v 5$ , you will obtain:

- detailed quality histograms (see figure 2);
- detailed remeshing steps;
- edge length histogram (see figure 3).

#### 3.2 Mesh improvement with edge length preservation: -optim option

Open your output mesh in Gmsh: by default, Mmg tries to create a mesh that respect the asked boundary approximation (-hausd option, 0.1 by default), the maximal ratio between two adjacent edges

RESULTING EDGE LENGT	HS 132	24 Numb	er of edges
AVERAGE LENGTH		1.1402	
SMALLEST EDGE LENGTH		0.6384	18443 274
LARGEST EDGE LENGTH		1.7260	31195 27246
0.60 < L < 1.30	1066	80.51 %	<b>\</b>
			largest edge
HISTOGRAMM:			extremities
0.60 < L < 0.71	6	0.45 %	
0.71 < L < 0.90	139	10.50 %	
0.90 < L < 1.30	921	69.56 %	
1.30 < L < 1.41	173	13.07 %	
1.41 < L < 2.00	85	6.42 %	

FIGURE 3 – Edge length histogram.

(-hgrad option, 1.3 by default) and that contains the smallest possible number of points.

If you want to preserve the edge length of the input mesh, you can run Mmg with the -optim option:

```
$ mmg2d_03 Data/naca_embedded.mesh -optim -v 5
```

Open your mesh in Gmsh and check that the edge length ar preserved. You can compare the input and output quality histograms.

If you want, you can play with the some other options: try for example to disable the gradation (-hgrad -1) or to ask for a better boundary approximation (-hmax 0.5 -hausd 0.0005).

## 4 Mesh adaptation to a size map

You can provide to Mmg a size map in a .sol file. This file lists, for each node, the prescribed edge length. In isotropic case, the file contains 1 scalar data per node (s). In anisotropic case, it contains a metric tensor :

$$M = R \Lambda R^T$$

#### With:

- $\Lambda = (\lambda_i)_i$  a diagonal matrix such as  $\lambda_i = 1/s^2$  with s the wanted edge length.
- $R = (r_{ij})_{ij}$ , an orthonormal matrix such as the  $r_j$  vectors gives the direction in which we want the  $1/s^2$  length.

See figure 4 to see an explanation of the .sol file format for an isotropic and an anisotropic size map.

You will find in the Data directory 2 size maps, naca\_iso.sol and naca\_aniso.sol. Try to adapt your mesh to each map. For example, for the isotropic map:

```
$ mmg2d_03 Data/naca_embedded.mesh -sol naca_iso.sol -hausd 0.001 -v 5
```

Again, you can play with the gradation parameter.

## 5 Size map computation to control the error of interpolation of an analytic function over the mesh

#### 5.1 Computation of the nodal values of a 2D analytic function

- 1. Choose a function, for example, a Gaussian function : f(x,y) = 10 \* exp(x \* x/30 + y \* y/30) or f(x,y) = sin(x);
- 2. Compute its nodal values on the mesh nodes (you can load the Gmsh mesh in Octave);

```
MeshVersionFormatted 2

MeshVersionFormatted 22=Double precision

Dimension 2

SolAtVertices

31418

1 3 3=Tensorial size map

400 0 1 Metric tensor at node 1
1 0 400 Metric tensor at node 1
1 0 400 Metric tensor at node 2
400 0 1
1 0 1
1 0 1
1 0 1
1 0 1
1 0 0 5
Scalar sizes
1 0 1
400 0 400
400 0 400
400 0 400
400 0 400
400 0 400
400 0 400
400 0 400
400 0 400
400 0 400
400 0 400
400 0 400
400 0 400
400 0 400
400 0 400
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400 0 400
400 0 400
400 0 400
400 0 400
400 0 400
400 0 400
400 0 400
400 0 400
```

FIGURE 4 - Isotropic (left) and anisotropic (right) size maps at Medit file format.

- 3. Save this values in a .sol file;
- 4. Convert your .mesh/.sol file at Gmsh format with the following command:

5. Now you can open your mesh inside Gmsh to check the solution data.

## 5.2 Computation of an anisotropic size map to control the interpolation error over the mesh

- 1. compute  $M_V = \frac{2}{9\epsilon} H_u(V)$  with V a mesh node;
- 2. save  $M_V$  in a .sol file.

Adapt your mesh and check the results.

# 5.3 Computation of an isotropic size map to control the interpolation error over the mesh

- 1. Compute  $M_V$ ;
- 2. compute the eigenvalues of  $M_V$ ;
- 3. compute the maximal eigenvalue  $\lambda$ ;
- 4. Compute  $s_v = \frac{1}{\sqrt{\lambda}}$ ;
- 5. save  $s_v$  in a .sol file.

Adapt your mesh and check the results.