MESH ADAPTATION:
A SHORT INTRODUCTION

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PLAN

- What is mesh adaptation?
- A priori error estimator
- How to use it in practice

WHAT IS MESH ADAPTATION

- The numerical models leads to spatial discretization errors
- We want to adapt the mesh size to minimize/control this error
- => Need of a posteriori estimators (that use the approximated solution)
- 2 strategies:
  - uniform refinement of the mesh until the solution convergency (mesh independency): very expensive
  - non-uniform refinement: lot of nodes where the solution has large variations, few where the solution doesn't vary.
- Possibility to take into account the directions of the solution: anisotropic mesh

A PRIORI ERROR ESTIMATOR: UPPER BOUND OF THE ERROR IN THE LINEAR INTERPOLATION

- We suppose that we know the exact solution of our problem, *u*
- We want to found an upper bound of the distance between u and its linear interpolation over the mesh : $u - P_h u$
- The starting point is a Taylor development of $u - P_h u$
- At the end, we can show that :

$$\|u - P_h u\|_{\infty,T} \leq c_d \max_{x \in T} \max_{\vec{e} \in E_T} \langle \vec{e}, Hu(x)\vec{e} \rangle$$

With T a mesh element, c_d a constant that depends on the dimension, E_T the list of the edges of T and Hu the Hessian of u;

(detailed proof: https://www.ljll.math.upmc.fr/frey/publications/RR-4759.pdf, p18 -> 23)

- More particularly, in 2D, Cd = 2/9

How to use this majoration in practice

- If H is linear over T and if we note Vi the vertices of the element T:

$$\text{For } \bar{M} = \max_{v \in V_i} H_u(v),$$
$$\max_{x \in T} \max_{\vec{e} \in E_T} < \vec{e}, H_u(x) | \vec{e} > \leq \max_{\vec{e} \in E_T} < \vec{e}, \bar{M}\vec{e} >$$

- Thus, the final upper bound of the error of interpolation is:

$$\|u - P_h u\|_{\infty,T} \leq c_d \max_{\vec{e} \in E_T} \langle \vec{e}, \bar{M}\vec{e} \rangle$$

- => The error of interpolation depends on the squared edge length and we can control this error by the edge length control.

How to use this majoration in practice

- We want a maximal error of ϵ :

$$\epsilon = c_d < \vec{e}, \bar{M}\vec{e} >$$

and $$1 =< \vec{e}, M\vec{e} > \text{ with } M = \frac{c_d}{\epsilon}\bar{M}$$

- Then, to compute an anisotropic size map : just compute M at each mesh node;

- to compute an isotropic size map, compute the smallest size (s) prescribed by M: if we note L the largest eigenvalue of M, $s^2 = 1/L$.