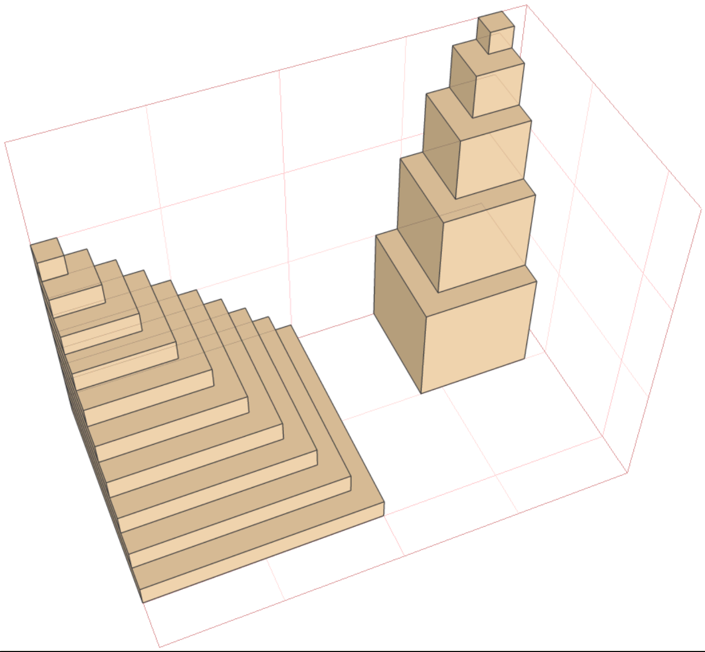
Skaičiuojama, kiek yra kubelių 1×1×1 dydžio (viršutiniai):



$$S_n = \sum_{i=1}^{n} i^2$$

$$S_n = 1\times(n-0)+3\times(n-1)+5\times(n-2)+7\times(n-3)+\ldots+(2n-1)\times 1 =$$

$$= \sum_{i=1}^{n}(n+1-i)\times(2i-1) = \sum_{i=1}^{n}\left(-2i^2+2(n+1)i+i-n-1\right) = -2S_n+(2n+3)\sum_{i=1}^{n} i - n(n+1) =$$

$$= -2S_n + \frac{n(n+1)(2n+3)}{2} - n(n+1) = -2S_n + \frac{n(n+1)(2n+3-2)}{2}$$

$$S_n = \frac{n(n+1)(2n+1)}{6}$$

$$S_n = \sum_{i=1}^{n} i^3$$

$$S_n = 1\times\sum_{j=1}^{n} j + 3\times\sum_{j=2}^{n} j + 5\times\sum_{j=3}^{n} j + \ldots + (2n-1)\times n =$$

$$= \sum_{i=1}^{n}\left((2i-1)\times\sum_{j=i}^{n} j\right) = \sum_{i=1}^{n}\left((2i-1)\times\frac{(i+n)(n-i+1)}{2}\right) =$$

$$= \frac{1}{2}\sum_{i=1}^{n}(2i-1)\times\left(n^2+n-i^2+i\right) = \frac{1}{2}\sum_{i=1}^{n}(2i-1)\times n(n+1) + \frac{1}{2}\sum_{i=1}^{n}(2i-1)\times\left(i-i^2\right) =$$

$$= \frac{1}{2}(n+1)\times n^3 + \frac{1}{2}\sum_{i=1}^{n}\left(3i^2-2i^3-i\right) = \frac{1}{2}\left(n^2+n\right)\times n^2 + \frac{n(n+1)(2n+1)}{4} - S_n - \frac{n(n+1)}{4}$$

$$2S_n = \frac{1}{2}(n+1)\times n^3 + \frac{n(n+1)(2n+1)}{4} - \frac{n(n+1)}{4} = \frac{n(n+1)}{4}\left(2n^2+2n+1-1\right) = \frac{n^2(n+1)^2}{2}$$

$$S_n = \frac{n^2(n+1)^2}{4}$$

4.3-1

$T(n) = 4T\left(\frac{n}{2}\right) + n$

$T(n) = 4T\left(\frac{n}{2}\right) + n^2$

$T(n) = 4T\left(\frac{n}{2}\right) + n^3$

Įrodyti, kad $T(n) = \Theta(\log_2 n)$ yra rekuriantinės lygties $T(n) = T\left(\frac{n}{2}\right) + \Theta(1)$ sprendinys.

4.3-4

Ar galima taikyti pagrindinę teoremą rekuriantinei lygčiai $T(n) = 4T\left(\frac{n}{2}\right) + n^2 \log_2 n$.