EXERCISES

Solutions to exercises marked with an asterisk (*) involve extensive computations. Formulate these problems as dynamic programs and provide representative computations to indicate the nature of the dynamic programming recursions; solve to completion only if a computer system is available.

1. In solving the minimum-delay routing problem in Section 11.1, we assumed the same delay along each street (arc) in the network. Suppose, instead, that the delay when moving along any arc upward in the network is 2 units greater than the delay when moving along any arc downward. The delay at the intersections is still given by the data in Fig. 11.1. Solve for the minimum-delay route by both forward and backward induction.

2. Decatron Mills has contracted to deliver 20 tons of a special coarsely ground wheat flour at the end of the current month, and 140 tons at the end of the next month. The production cost, based on which the Sales Department has bargained with prospective customers, is $c_1(x_1) = 7500 + (x_1 - 50)^2$ per ton for the first month, and $c_2(x_2) = 7500 + (x_2 - 40)^2$ per ton for the second month; x_1 and x_2 are the number of tons of the flour produced in the first and second months, respectively. If the company chooses to produce more than 20 tons in the first month, any excess production can be carried to the second month at a storage cost of $3 per ton.

   Assuming that there is no initial inventory and that the contracted demands must be satisfied in each month (that is, no back-ordering is allowed), derive the production plan that minimizes total cost. Solve by both backward and forward induction. Consider x_1 and x_2 as continuous variables, since any fraction of a ton may be produced in either month.

3. A construction company has four projects in progress. According to the current allocation of manpower, equipment, and materials, the four projects can be completed in 15, 20, 18, and 25 weeks. Management wants to reduce the completion times and has decided to allocate an additional $35,000 to all four projects. The new completion times as functions of the additional funds allocated to each projects are given in Table E11.1.

   How should the $35,000 be allocated among the projects to achieve the largest total reduction in completion times? Assume that the additional funds can be allocated only in blocks of $5000.

Table E11.1 Completion times (in weeks)

Additional funds ($\times$1000 dollars)	*Project 1*	*Project 2*	*Project 3*	*Project 4*
0	15	20	18	25
5	12	16	15	21
10	10	13	12	18
15	8	11	10	16
20	7	9	9	14
25	6	8	8	12
30	5	7	7	11
35	4	7	6	10

4. The following table specifies the unit weights and values of five products held in storage. The quantity of each item is unlimited.

Product	*Weight* (W_i)	*Value* (V_i)
1	7	9
2	5	4
3	4	3
4	3	2
5	1	$\frac{1}{2}$

A plane with a capacity of 13 weight units is to be used to transport the products. How should the plane be loaded to maximize the value of goods shipped? (Formulate the problem as an integer program and solve by dynamic programming.)

5. Any linear-programming problem with n decision variables and m constraints can be converted into an n-stage dynamic-programming problem with m state parameters.

   Set up a dynamic-programming formulation for the following linear program:

$$\text{Minimize} \sum_{j=1}^{n} c_j x_j,$$

subject to:

$$\begin{aligned} \sum_{j=1}^{n} a_{ij} x_j &\leq b_i \qquad (i = 1, 2, \ldots, m), \\ x_j &\geq 0 \qquad (j = 1, 2, \ldots, n). \end{aligned}$$

   Why is it generally true that the simplex method rather than dynamic programming is recommended for solving linear programs?

6. Rambling Roger, a veteran of the hitchhiking corps, has decided to leave the cold of a Boston winter and head for the sunshine of Miami. His vast experience has given him an indication of the expected time in hours it takes to hitchhike over certain segments of the highways. Knowing he will be breaking the law in several states and wishing to reach the warm weather quickly, Roger wants to know the least-time route to take. He summarized his expected travel times on the map in Fig. E11.1. Find his shortest time route.

7. J. J. Jefferson has decided to move from the West Coast, where he lives, to a mid-western town, where he intends to buy a small farm and lead a quiet life. Since J. J. is single and has accumulated little furniture, he decides to rent a small truck for \$200 a week or fraction of a week (one-way, no mileage charge) and move his belongings by himself. Studying the map, he figures that his trip will require four stages, regardless of the particular routing. Each node shown in Fig. E11.2 corresponds to a town where J. J. has either friends or relatives and where he plans to spend one day resting and visiting if he travels through the town. The numbers in brackets in Fig. E11.2 specify the travel time in days between nodes. (The position of each node in the network is not necessarily related to its geographical position on the map.) As he will travel through different states, motel rates, tolls, and gas prices vary significantly; Fig. E11.2 also shows the cost in dollars for traveling (excluding truck rental charges) between every two nodes. Find J. J.'s cheapest route between towns 1 and 10, including the truck rental charges.

8. At THE CASINO in Las Vegas, a customer can bet only in dollar increments. Betting a certain amount is called ''playing a round.'' Associated with each dollar bet on a round, the customer has a 40% chance to win another dollar and a 60% chance to lose his, or her, dollar. If the customer starts with \$4 and wants to maximize the chances of finishing with at least \$7 after two rounds, how much should be bet on each round? [*Hint.* Consider the number of dollars available at the beginning of each round as the state variable.]

*9. In a youth contest, Joe will shoot a total of ten shots at four different targets. The contest has been designed so that Joe will not know whether or not he hits any target until after he has made all ten shots. He obtains 6 points if any shot hits target 1, 4 points for hitting target 2, 10 points for hitting target 3, and 7 points for hitting target 4. At each shot there is an 80% chance that he will miss target 1, a 60% chance of missing target 2, a 90% chance of missing target 3, and a 50% chance of missing target 4, given that he aims at the appropriate target.

   If Joe wants to maximize his expected number of points, how many shots should he aim at each target?

10. A monitoring device is assembled from five different components. Proper functioning of the device depends upon its total weight q so that, among other tests, the device is weighted; it is accepted only if $r_1 \leq q \leq r_2$, where the two limits r_1 and r_2 have been prespecified.

    The weight $q_j (j = 1, 2, \ldots, 5)$ of each component varies somewhat from unit to unit in accordance with a normal distribution with mean μ_j and variance σ_j^2. As $q_1, q_2, \ldots, q_5$ are independent, the total weight q will also be a normal variable with mean $\mu = \sum_{j=1}^{5} \mu_j$ and variance $\sigma^2 = \sum_{j=1}^{5} \sigma_j^2$.

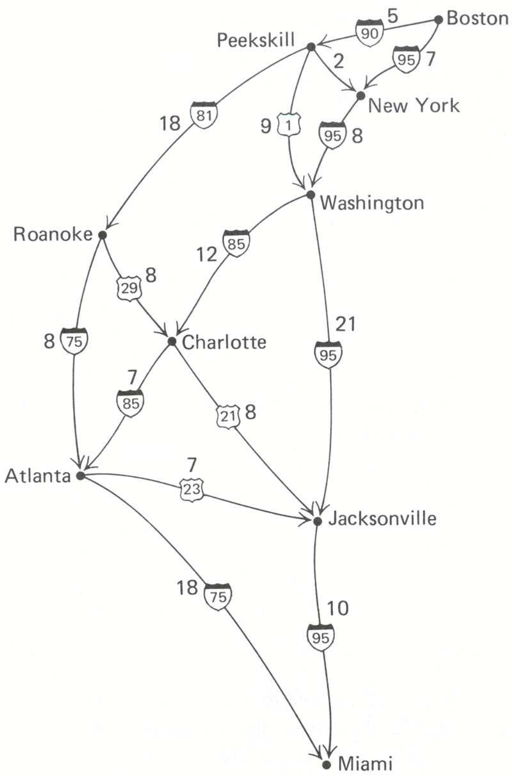

Figure E11.1 Travel times to highways.

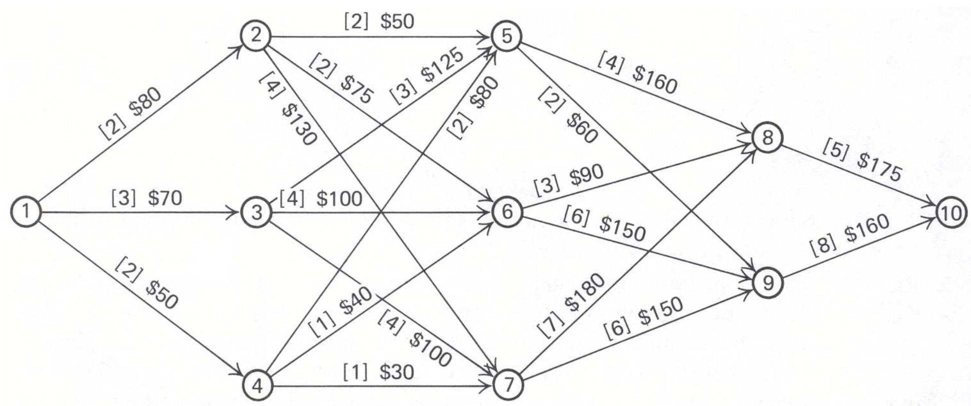

Figure E11.2 Routing times and costs.

Clearly, even if μ can be adjusted to fall within the interval $[r_1, r_2]$, the rejection rate will depend upon σ^2; in this case, the rejection rate can be made as small as desired by making the variance σ^2 sufficiently small. The design department has decided that $\sigma^2 = 5$ is the largest variance that would make the rejection rate of the monitoring device acceptable. The cost of manufacturing component j is $c_j = 1/\sigma_j^2$.

Determine values for the design parameters σ_j^2 for $j = 1, 2, \ldots, 5$ that would minimize the manufacturing cost of the components while ensuring an acceptable rejection rate. [*Hint.* Each component is a stage; the state variable is that portion of the total variance σ^2 not yet distributed. Consider σ_j^2's as continuous variables.]

*11. A scientific expedition to Death Valley is being organized. In addition to the scientific equipment, the expedition also has to carry a stock of spare parts, which are likely to fail under the extreme heat conditions prevailing in that area. The estimated number of times that the six critical parts, those sensitive to the heat conditions, will fail during the expedition are shown below in the form of probability distributions.

Part 1

# of Failures	*Probability*
0	0.5
1	0.3
2	0.2

Part 2

# of Failures	*Probability*
0	0.4
1	0.3
2	0.2
3	0.1

Part 3

# of Failures	*Probability*
0	0.7
1	0.2
2	0.1

Part 4

# of Failures	*Probability*
0	0.9
1	0.1

Part 5

# of Failures	*Probability*
0	0.8
1	0.1
2	0.1

Part 6

# of Failures	*Probability*
0	0.8
1	0.2

The spare-park kit should not weight more than 30 pounds. If one part is needed and it is not available in the spare-park kit, it may be ordered by radio and shipped by helicopter at unit costs as specified in Table E11.2, which also gives the weight of each part.

Table E11.2 Spare-Part Data

Part	*Weight* (pounds/unit)	*Shipping cost* (\$/unit)
1	4	100
2	3	70
3	2	90
4	5	80
5	3	60
6	2	50

Determine the composition of the spare-park kit to minimize total expected ordering costs.

*12. After a hard day at work I frequently wish to return home as quickly as possible. I must choose from several alternate routes (see Fig. E11.3); the travel time on any road is uncertain and depends upon the congestion at the nearest major

Table E11.3 Travel time on the road

		Travel-time distribution	
Road i–j	*Congestion at initial intersection* (i)	*Travel time* (minutes)	*Probability*
5–4	Heavy	4	$\frac{1}{4}$
		6	$\frac{1}{2}$
		10	$\frac{1}{4}$
	Light	2	$\frac{1}{3}$
		3	$\frac{1}{3}$
		5	$\frac{1}{3}$
5–3	Heavy	5	$\frac{1}{2}$
		12	$\frac{1}{2}$
	Light	3	$\frac{1}{2}$
		6	$\frac{1}{2}$
4–2	Heavy	7	$\frac{1}{3}$
		14	$\frac{2}{3}$
	Light	4	$\frac{1}{2}$
		6	$\frac{1}{2}$
3–2	Heavy	5	$\frac{1}{4}$
		11	$\frac{3}{4}$
	Light	3	$\frac{1}{3}$
		5	$\frac{1}{3}$
		7	$\frac{1}{3}$
3–1	Heavy	3	$\frac{1}{2}$
		5	$\frac{1}{2}$
	Light	2	$\frac{1}{2}$
		3	$\frac{1}{2}$
2–1	Heavy	2	$\frac{1}{2}$
		4	$\frac{1}{2}$
	Light	1	$\frac{1}{2}$
		2	$\frac{1}{2}$

intersection preceding that route. Using the data in Table E11.3, determine my best route, given that the congestion at my starting point is heavy.

Assume that if I am at intersection i with heavy congestion and I take road i–j, then

$$\text{Prob (intersection } j \text{ is heavy)} = 0.8.$$

If the congestion is light at intersection i and I take road i–j, then

$$\text{Prob (intersection } j \text{ is heavy)} = 0.3.$$

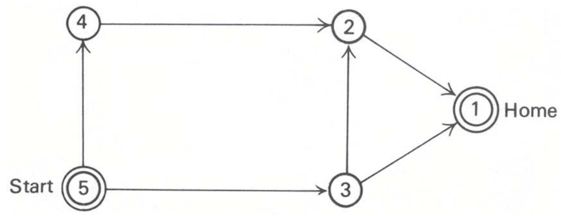

Figure E11.3

13. Find the shortest path from node 1 to every other node in the network given in Fig. E11.4, using the shortest-route algorithm for acyclic networks. The number next to each arc is the ''length'' of that arc.

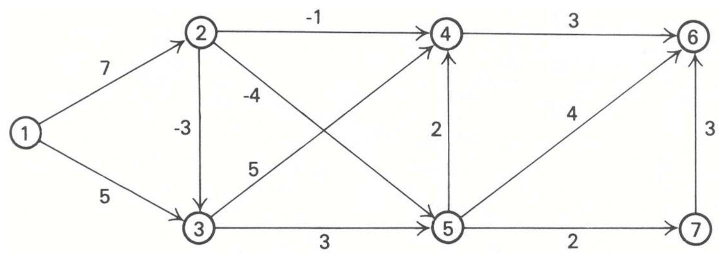

Figure E11.4

14. Find the shortest path from node 1 to every other node in the network given in Fig. E11.5.

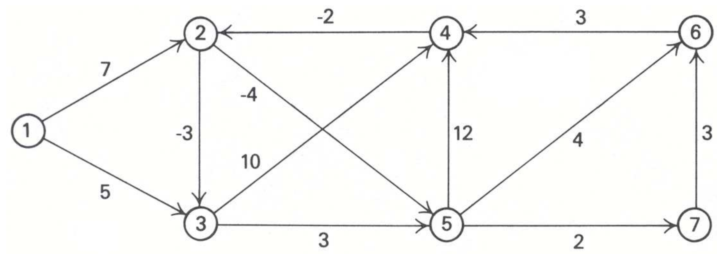

Figure E11.5

15. a) Give a dynamic-programming recursion for finding the shortest path from every node to a particular node, node k, in a network without negative cycles.
    b) Apply the recursion from part (a) to find the shortest path from every node to node 6 in the network specified in the previous exercise.

16. A state's legislature has R representatives. The state is sectioned into s districts, where District j has a population p_j and $s < R$. Under strictly proportional representation. District j would receive $Rp_j/(\sum_{j=1}^{s} p_j) = r_j$ representatives; this allocation is not feasible, however, because r_j may not be integer-valued. The objective is to allocate y_j representatives to District j for $j = 1, 2, \ldots, s$, so as to minimize, over all the districts, the maximum difference between y_j and r_j; that is, minimize [maximum $(|y_1 - r_1|, |y_2 - r_2|, \ldots, |y_s - r_s|)$].

    a) Formulate the model in terms of a dynamic-programming recursion.
    b) Apply your method to the data $R = 4$, $s = 3$, and $r_1 = 0.4$, $r_2 = 2.4$, and $r_3 = 1.2$.
    c) Discuss whether the solution seems reasonable, given the context of the problem.

17. In a textile plant, cloth is manufactured in rolls of length L. Defects sometimes occur along the length of the cloth. Consider a specific roll with $(N - 1)$ defects appearing at distances $y_1, y_2, \ldots, y_{N-1}$ from the start of the roll $(y_{i+1} > y_i$ for all $i)$. Denote the start of the roll by y_0, the end by y_N.

The roll is cut into pieces for sale. The value of a piece depends on its length and the number of defects. Let

$$v(x, m) = \text{Value of a piece of length } x \text{ having } m \text{ defects.}$$

Assume that all cuts are made *through* defects and that such cutting removes the defect.

Specify how to determine where to cut the cloth to maximize total value.

18. A manufacturing company, receiving an order for a special product, has worked out a production plan for the next 5 months. All components will be manufactured internally except for one electronic part that must be purchased. The purchasing manager in charge of buying the electronic part must meet the requirements schedule established by the production department. After negotiating with several suppliers, the purchasing manager has determined the best possible price for the electronic part for each of the five months in the planning horizon. Table E11.4 summarizes the requirement schedule and purchase price information.

Table E11.4 Requirements schedule and purchasing prices

Month	*Requirements* (thousands)	*Purchasing price* ($/thousand pieces)
1	5	10
2	10	11
3	6	13
4	9	10
5	4	12

The storage capacity for this item is limited to 12,000 units; there is no initial stock, and after the five-month period the item will no longer be needed. Assume that the orders for the electronic part are placed once every month (at the beginning of each month) and that the delivery lead time is very short (delivery is made practically instantaneously). No back-ordering is permitted.

a) Derive the monthly purchasing schedule if total purchasing cost is to be minimized.
b) Assume that a storage charge of $250 is incurred for each 1000 units found in inventory at the end of a month. What purchasing schedule would minimize the purchasing and storage costs?

Table E11.5 Profits in response to advertising

Additional investment in advertising (in $100,000)	*Profits* (in $100,000)				
	Product A	*Product* B	*Product* C	*Product* D	*Product* E
0	0	0	0	0	0
1	0.20	0.18	0.23	0.15	0.25
2	0.35	0.30	0.43	0.30	0.45
3	0.50	0.42	0.60	0.45	0.65
4	0.63	0.54	0.75	0.58	0.80
5	0.75	0.64	0.80	0.70	0.90
6	0.83	0.74	0.92	0.81	0.95
7	0.90	0.84	0.98	0.91	0.98
8	0.95	0.92	1.02	1.00	1.01
9	0.98	1.00	1.05	1.04	1.02
10	0.98	1.05	1.06	1.07	1.03

19. Rebron, Inc., a cosmetics manufacturer, markets five different skin lotions and creams: A, B, C, D, E. The company has decided to increase the advertising budget allocated to this group of products by 1 million dollars for next year. The marketing department has conducted a research program to establish how advertising affects the sales levels

of these products. Table E11.5 shows the increase in each product's contribution to net profits as a function of the additional advertisement expenditures.

Given that maximization of net profits is sought, what is the optimal allocation of the additional advertising budget among the five products? Assume, for simplicity, that advertising funds must be allocated in blocks of $100,000.

*20. A machine tool manufacturer is planning an expansion program. Up to 10 workers can be hired and assigned to the five divisions of the company. Since the manufacturer is currently operating with idle machine capacity, no new equipment has to be purchased.

Hiring new workers adds $250/day to the indirect costs of the company. On the other hand, new workers add value to the company's output (i.e., sales revenues in excess of direct costs) as indicated in Table E11.6. Note that the value added depends upon both the number of workers hired and the division to which they are assigned.

Table E11.6 Value added by new workers

New workers (x_n)	*Increase in contribution to overhead* ($/day)				
	Division 1	*Division 2*	*Division 3*	*Division 4*	*Division 5*
0	0	0	0	0	0
1	30	25	35	32	28
2	55	50	65	60	53
3	78	72	90	88	73
4	97	90	110	113	91
5	115	108	120	133	109
6	131	124	128	146	127
7	144	138	135	153	145
8	154	140	140	158	160
9	160	150	144	161	170
10	163	154	145	162	172

The company wishes to hire workers so that the value that they add exceeds the $250/day in indirect costs. What is the minimum number of workers the company should hire and how should they be allocated among the five divisions?

21. A retailer wishes to plan the purchase of a certain item for the next five months. Suppose that the demand in these months is known and given by:

Month	*Demand* (units)
1	10
2	20
3	30
4	30
5	20

The retailer orders at the beginning of each month. Initially he has no units of the item. Any units left at the end of a month will be transferred to the next month, but at a cost of 10 ¢ per unit. It costs $20 to place an order. Assume that the retailer can order only in lots of 10, 20, ... units and that the maximum amount he can order each month is 60 units. Further assume that he receives the order immediately (no lead time) and that the demand occurs just after he receives the order. He attempts to stock whatever remains but cannot stock more than 40 units—units in excess of 40 are discarded at no additional cost and with no salvage value. How many units should the retailer order each month?

*22. Suppose that the retailer of the previous exercise does not know demand with certainty. All assumptions are as in Exercise 21 except as noted below. The demand for the item is the same for each month and is given by the following distribution:

Demand	*Probability*
10	0.2
30	0.5
30	0.3

Each unit costs \$1. Each unit demanded in excess of the units on hand is lost, with a penalty of \$2 per unit. How many units should be ordered each month to minimize total expected costs over the planning horizon? Outline a dynamic-programming formulation and complete the calculations for the last two stages only.

*23. The owner of a hardware store is surprised to find that he is completely out of stock of ''Safe-t-lock,'' an extremely popular hardened-steel safety lock for bicycles. Fortunately, he became aware of this situation before anybody asked for one of the locks; otherwise he would have lost \$2 in profits for each unit demanded but not available. He decides to use his pickup truck and immediately obtain some of the locks from a nearby warehouse.

Although the demand for locks is uncertain, the probability distribution for demand is known; it is the same in each month and is given by:

Demand	*Probability*
0	0.1
100	0.3
200	0.4
300	0.2

The storage capacity is 400 units, and the carrying cost is \$1 per unit per month, charged to the month's *average inventory* [i.e., (initial + ending)/2]. Assume that the withdrawal rate is uniform over the month. The lock is replenished monthly, at the beginning of the month, in lots of one hundred.

What is the replenishment strategy that minimizes the expected costs (storage and shortage costs) over a planning horizon of four months? No specific inventory level is required for the end of the planning horizon.

*24. In anticipation of the Olympic games, Julius, a famous Danish pastry cook, has opened a coffee-and-pastry shop not far from the Olympic Village. He has signed a contract to sell the shop for \$50,000 after operating it for 5 months.

Julius has several secret recipes that have proved very popular with consumers during the last Olympic season, but now that the games are to be held on another continent, variations in tastes and habits cast a note of uncertainty over his chances of renewed success.

The pastry cook plans to sell all types of common pastries and to use his specialties to attract large crowds to his shop. He realizes that the popularity of his shop will depend upon how well his specialties are received; consequently, he may alter the offerings of these pastries from month to month when he feels that he can improve business. When his shop is not popular, he may determine what type of specialties to offer by running two-day market surveys. Additionally, Julius can advertise in the *Olympic Herald Daily* and other local newspapers to attract new customers.

The shop's popularity may change from month to month. These transitions are uncertain and depend upon advertising and market-survey strategies. Table E11.7 summarizes the various possibilities. The profit figures in this table include advertising expenditures and market-survey costs.

Note that Julius has decided either to advertise or to run the market survey whenever the shop is not popular.

Assume that, during his first month of operation, Julius passively waits to see how popular his shop will be. What is the optimal strategy for him to follow in succeeding months to maximize his expected profits?

25. A particular city contains six significant points of interest. Figure E11.6 depicts the network of major two-way avenues connecting the points; the figure also shows travel time (in both directions) along each avenue. Other streets, having travel times exceeding those along the major avenues, link the points but have been dropped from this analysis.

In an attempt to reduce congestion of traffic in the city, the city council is considering converting some two-way avenues to one-way avenues.

The city council is considering two alternative planning objectives:

Table E11.7 Profit possibilities (p = probability; E = expected profit)

	Popular next month		*Not popular next month*	
Popular this month, no advertising	$p = \frac{6}{10}$,	$E = \$6000$	$p = \frac{4}{10}$,	$E = \$2000$
Popular this month, advertising	$p = \frac{3}{4}$,	$E = \$4000$	$p = \frac{1}{10}$,	$E = \$3000$
Not popular this month, market survey	$p = \frac{1}{3}$,	$E = \$3000$	$p = \frac{2}{3}$,	$E = -\$2000$
Not popular this month, advertising	$p = \frac{6}{10}$,	$E = \$1000$	$p = \frac{4}{10}$,	$E = -\$5000$

a) Given that point (1) is the tourist information center for the city, from which most visitors depart, which avenues should be made one-way so as to minimize the travel times from point (1) to every other point?
b) If the travel times from each point to every other point were to be minimized, which avenues would be converted to one-way?

In both cases, assume that the travel times shown in Fig E11.6 would not be affected by the conversion. If the total conversion cost is proportional to the number of avenues converted to one-way, which of the above solutions has the lowest cost?

*26. Consider the following one-period problem: a certain item is produced centrally in a factory and distributed to four warehouses. The factory can produce up to 12 thousand pieces of the item. The transportation cost from the factory to warehouse n is t_n dollars per thousand pieces.

From historical data, it is known that the demand per period from warehouse n for the item is governed by a Poisson distribution† with mean λ_n (in thousands of pieces). If demand exceeds available stock a penalty of π_n dollars per thousand units out of stock is charged at warehouse n.

The current inventory on hand at warehouse n is q_n thousand units.

a) Formulate a dynamic program for determining the amount to be produced and the optimal allocation to each warehouse, in order to minimize transportation and expected stockout costs.
b) Solve the problem for a four-warehouse system with the data given in Table E11.8.

† The Poisson distribution is given by Prob. $(\tilde{k} = k) = \frac{\lambda^k e^{-\lambda}}{k!}$.

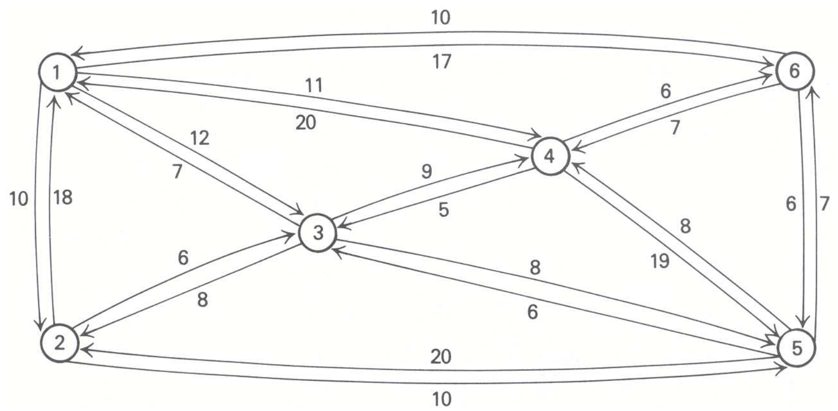

Figure E11.6

Table E11.8

Warehouse (n)	*Demand* λ_n (thousand units)	*Inventory* q_n (thousand units)	*Transportation cost* t_n (\$ per 1000 units)	*Stockout penalty* π_n (\$ per 1000 units)
1	3	1	100	1500
2	4	2	300	2000
3	5	1	250	1700
4	2	0	200	2200

*27. Precision Equipment, Inc., has won a government contract to supply 4 pieces of a high precision part that is used in the fuel throttle-valve of an orbital module. The factory has three different machines capable of producing the item. They differ in terms of setup cost, variable production cost, and the chance that every single item will meet the high-quality standards (see Table E11.9).

Table E11.9

Machine	*Setup cost* (\$)	*Variable cost* (\$/unit)	*Probability of meeting standards*
A	100	20	0.50
B	300	40	0.80
C	500	60	0.90

After the parts are produced, they are sent to the engine assembly plant where they are tested. There is no way to recondition a rejected item. Any parts in excess of four, even if good, must be scrapped. If less than 4 parts are good, the manufacturer has to pay a penalty of \$200 for each undelivered item.

How many items should be produced on each machine in order to minimize total expected cost? [*Hint*. Consider each machine as a stage and define the state variable as the number of acceptable parts still to be produced.]

*28. One of the systems of a communications satellite consists of five electronic devices connected in series; the system as a whole would fail if any one of these devices were to fail. A common engineering design to increase the reliability of the system is to connect several devices of the same type in parallel, as shown in Fig E11.7. The parallel devices in each group are controlled by a monitoring system, so that, if one device fails, another one immediately becomes operative.

The total weight of the system may not exceed 20 pounds. Table E11.10 shows the weight in pounds and the probability of failure for each device in group j of the system design. How many devices should be connected in

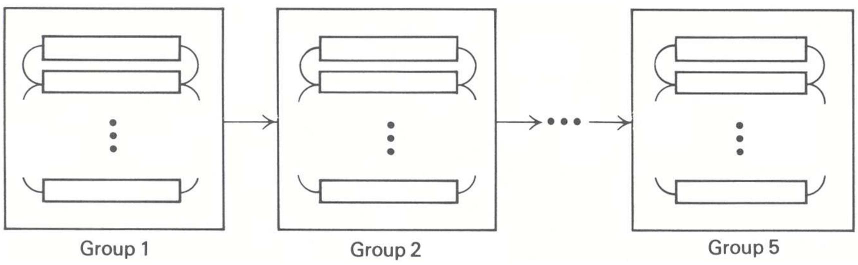

Figure E11.7

parallel in each group so as to maximize the reliability of the overall system?

*29. The production manager of a manufacturing company has to devise a production plan for item AK102 for the

Table E11.10

Group	*Weight* (lbs./device)	*Probability of failure for each device*
1	1	0.20
2	2	0.10
3	1	0.30
4	2	0.15
5	3	0.05

next four months. The item is to be produced at most once monthly; because of capacity limitations the monthly production may not exceed 10 units. The cost of one setup for any positive level of production in any month is $10.

The demand for this item is uncertain and varies from month to month; from past experience, however, the manager concludes that the demand in each month can be approximated by a Poisson distribution with parameter λ_n (n shows the month to which the distribution refers).

Inventory is counted at the end of each month and a holding cost of $10 is charged for each unit; if there are stockouts, a penalty of $20 is charged for every unit out of stock. There is no initial inventory and no outstanding back-orders; no inventory is required at the end of the planning period. Assume that the production lead time is short so that the amount released for production in one month can be used to satisfy demand within the same month.

What is the optimal production plan, assuming that the optimality criterion is the minimum expected cost? Assume that $\lambda_1 = 3$, $\lambda_2 = 5$, $\lambda_3 = 2$, $\lambda_4 = 4$ units.

*30. Just before the Christmas season, Bribham of New England, Inc., has signed a large contract to buy four varieties of Swiss chocolate from a local importer. As it was already late, the distributor could arrange for only a limited transportation of 20 tons of Swiss chocolate to be delivered in time for Christmas.

Chocolate is transported in containers; the weight and the transportation cost per container are given in Table E11.11.

Table E11.11

Variety	*Weight* (tons/container)	*Transportation* ($/container)	*Shortage cost* ($/container)	λ_n (tons)
1	2	50	500	3
2	3	100	300	4
3	4	150	800	2
4	4	200	1000	1

A marketing consulting firm has conducted a study and has estimated the demand for the upcoming holiday season as a Poisson distribution with parameter $\lambda_n (n = 1, 2, 3, 4$ indicates the variety of the chocolate). Bribham loses contribution (i.e., shortage cost) for each container that can be sold (i.e., is demanded) but is not available.

How many containers of each variety should the company make available for Christmas in order to minimize total expected cost (transportation and shortage costs)?

ACKNOWLEDGMENTS

The example in Section 11.6 is taken from the State Department of Public Health case by Richard F. Meyer.
Exercise 10 is based on Section 10.6 of *Nonlinear and Dynamic Programming*, Addison-Wesley, 1962, by George Hadley.
Exercise 24 is inspired by *Dynamic Programming and Markov Processes*, John Wiley & Sons, 1960, by Ronald A. Howard.