AJAY KUMAR GARG ENGINEERING COLLEGE, GHAZIABAD

Department of Applied Science & Humanities

Pre-University Test

Course: B. Tech.
Session: 2024-25
Subject: Engineering Mathematics – I
Max Marks: 70

Semester: Ist
Section: (S1-S9&S11-S20)
Sub. Code: BAS103
Time: 3 hrs.

OBE Remarks:

Q.No.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17
CO No.	1	1	2	2	3	4	5	2	2	3	1	1	4	4	5	5	3
Bloom's Level	L1	L3	L1	L1	L4	L5	L4	L3	L5	L3	L1	L4	L2	L5	L2	L5	L5
Weightage CO4: 16																	Weightage CO5: 16

Note: Answer **all** the sections.

Section-A

A. Attempt **all** the parts. (7 X 2 =14)

1. If Vectors $(0,1,a)$, $(1,a,1)$ and $(a,1,0)$ are Linear Dependent then find the value of a.
2. If A is a skew-Hermitian matrix then show that iA is Hermitian.
3. Find the possible percentage error in computing the Parallel resistance r of two resistances r_1 and r_2, if it is given that error in r_1 and r_2 each is 2 %.
4. Find asymptote in the given curve $x^2y + y = x^2$.
5. If $u = \left(\frac{x^2y^2}{x+y}\right)$, then find $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y}$.
6. Find $\Gamma\left(-\frac{3}{2}\right)$.
7. State Stokes's theorem.

Section- B

B. Attempt **any three.** (3 X 7=21)

$e^{m(\cos^{-1}x)}$

8. If $y = e^{m(\cos^{-1}x)}$ then prove that $(1-x^2)y_{n+2} - (2n+1)xy_{n+1} - (n^2+m^2)y_n = 0$ and hence evaluate $y_n(0)$.
9. Trace the Curve: $x^2y^2 = (a+y)^2(a^2-y^2)$
10. Expand $f(x,y) = e^x \cos y$ in powers of x and y up to third degree terms.
11. Find the value of λ for which the system of linear equations $(3\lambda-8)x+3y+3z=0$, $3x+(3\lambda-8)y+3z=0$ and $3x+3y+(3\lambda-8)z=0$ has a non-trivial solution.
12. Find the characteristics equation of matrix $A=\begin{bmatrix}2 & 1 & 1\\ 0 & 1 & 0\\ 1 & 1 & 2\end{bmatrix}$ and hence compute A^{-1}. Also find the $A^8 - 5A^7 + 7A^6 - 3A^5 + A^4 - 5A^3 + 8A^2 - 2A + I$.

Section- C

C. Attempt **all** the parts. (5 X 7 = 35)

13. Attempt any one.

(a) To Show that $\Gamma(m)\cdot\Gamma\left(m+\frac{1}{2}\right)=\frac{\sqrt{\pi}}{2^{2m-1}}\Gamma(2m)$, $m>0$ and hence prove $\beta(m,m)=2^{(1-2m)}\beta\left(m,\frac{1}{2}\right)$.

(b) Change into polar coordinate and evaluate $\int_0^{\infty}\int_0^{\infty} e^{-(x^2+y^2)}dy\,dx$. Hence so that $\int_0^{\infty} e^{-x^2}dx=\frac{\sqrt{\pi}}{2}$.

14. Attempt any one.

(a) Evaluate $\int_0^1\int_{\sqrt{x}}^{2-x} y\,dx\,dy$ by changing order of integration.

(b) Apply Dirchlet's integral to find volume and mass of solid generated by ellipsoid $\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$ in the first octane if density at any point is $kxyz$.

15. Attempt any one.

(a) Verify Gauss divergence theorem for the function $\vec{\mathbf{F}}=(x^2-yz)\hat{\mathbf{i}}+(y^2-zx)\hat{\mathbf{j}}+(z^2-xy)\hat{\mathbf{k}}$ taken over the square $0\le x\le 1, 0\le y\le 1$ and $0\le z\le 1$.

(b) Verify Green's theorem $\oint((x-y)dx+(x+y)dy)$, where C is the boundary described counter clockwise of triangle with vertices $(0,0)$, $(1,0)$ and $(1,1)$.

16. Attempt any one.

(a) To show that $\frac{\vec{r}}{|\vec{r}|^3}$ is Solenoidal as well as irrotational. Also find scalar potential.

(b) Find the directional derivative of $\vec{V}^2$, where $\vec{V}=xy^2\hat{\mathbf{i}}+y^2z\hat{\mathbf{j}}+z^2x\hat{\mathbf{k}}$ at point $(2,0,3)$ in the direction of the outward normal to the surface $x^2+y^2+z^2=14$ at the point $(3,2,1)$.

17. Attempt any one.

(a) By using Lagrange's multipliers, find the maximum and minimum distance of the point $(3,4,12)$ from the sphere whose center is origin and radius 1.

(b) If $u^3+v+w=x+y^2+z^2$, $u+v^3+w=x^2+y+z^2$ and $u+v+w^3=x^2+y^2+z$, then show that,

$$\frac{\partial(u,v,w)}{\partial(x,y,z)}=\frac{1-4(xy+yz+zx)+16xyz}{2-3(u^2+v^2+w^2)+27u^2v^2w^2}.$$

Faculty Sign **HoD Sign**