TRADING TECHNIQUES

Know Thy Weapons

Moving Averages: Some Finer Properties

What is the difference between the 10-period of a five-period moving average and a 15-period moving average? Does the sum of moving averages equal the moving average of the sum? How does the smoothing of a ratio differ from the ratio of smoothing? How can you algorithmically calculate the weights of a smoothing procedure? Find the answers to such technical questions here.

All technical analysts eventually engage in creating their own indicators and methods. And they all eventually use some kind of smoothing method to filter out noise. Various moving average methods can be used to smooth a series of values. In this article, I will discuss four interesting properties of simple and exponential moving averages (hereafter referred to as SMAs and EMAs, respectively). These two averaging methods are the most popular in the technical analysis world, and their weighting scheme is simple, so they have clear and nice properties. Those who aspire to create indicators should find the concepts discussed here useful.

I will denote the n-period simple and exponential moving averages of an indicator P as

$$\mathrm{SMA}_n(P), \textit{ and}$$

$$\mathrm{EMA}_n(P), \text{ respectively.}$$

Moreover, $\mathrm{MA}_n(P)$ will denote either $\mathrm{SMA}_n(P)$ or $\mathrm{EMA}_n(P)$.

Linearity

The first property of SMAs and EMAs worth remembering has to do with the way they treat the addition of indicators and products of numbers with indicators. Here's a more precise look at the properties.

First property

If P *and* Q *are indicators and* t *is a constant number, then:*

$$\mathrm{SMA}_n\,(t \bullet P + Q) = t \bullet \mathrm{SMA}_n(P) + \mathrm{SMA}_n(Q)$$

and

$$\mathrm{EMA}_n\,(t \bullet P + Q) = t \bullet \mathrm{EMA}_n(P) + \mathrm{EMA}_n(Q)$$

This property can easily be proved with basic mathematics.

Example

Say you want to smooth the typical price (TP) using a five-period EMA. The TP has the following formula:

$$\mathrm{TP} = \frac{1}{3}\mathrm{H} + \frac{1}{3}\mathrm{L} + \frac{1}{3}\mathrm{C}$$

where H, L, and C represent the high, low, and close of a bar, respectively. It doesn't make a difference whether you take the five-period EMA of TP or *sum* the five-period EMAs of H, L, and C and then divide the sum by three. That is,

$$\mathrm{EMA}_5(\mathrm{TP}) = \frac{1}{3}\left[\mathrm{EMA}_5(\mathrm{H}) + \mathrm{EMA}_5(\mathrm{L}) + \mathrm{EMA}_5(\mathrm{C})\right]$$

Commutative property

You may not realize it, but in successive smoothing, using either an SMA or an EMA (or both) can change the order of averages without affecting the outcome.

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by Giorgos E. Siligardos, PhD

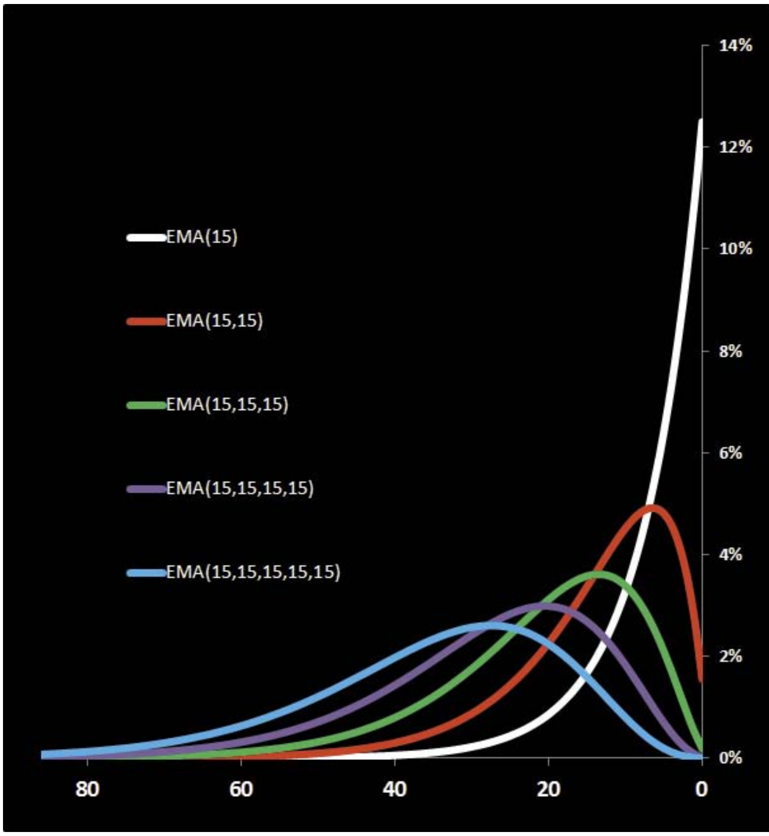


FIGURE 1: WEIGHTING SCHEMES OF SUCCESSIVE EMA SMOOTHINGS. The weighting scheme of five successive 15-period EMA smoothings is shown using different colors. EMA(15) is the weighting scheme of the 15-period EMA, EMA(15,15) is the weighting scheme of the 15-period EMA of the 15-period EMA, and so on. The weights are expressed as percentages of their total sum of 100% and they are the *y* coordinates in the graph. The *x* coordinates are the ages of the weights in ascending order from right to left.

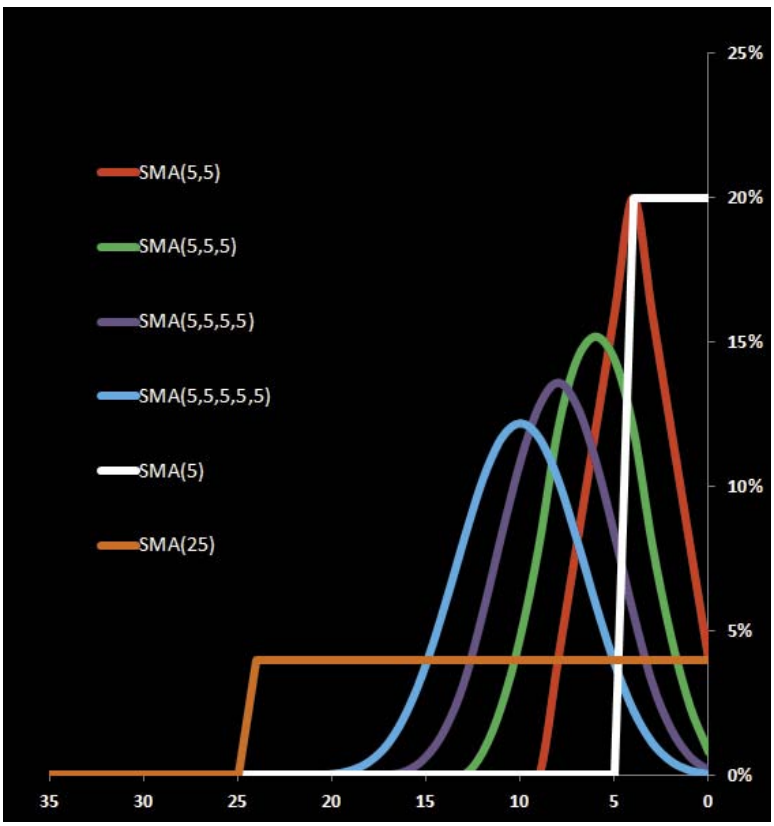


FIGURE 2: WEIGHTING SCHEMES OF SUCCESSIVE SMA SMOOTHINGS. Similarly here, this chart illustrates the percentage weighting scheme of five successive five-period SMAs as a function of age. For comparison purposes, the weighting scheme of the 25-period SMA is also shown.

Second property
If P *is an indicator, then*

$$MA1_n\,(MA2_k(P)) = MA2_k(MA1_n(P))$$

where $MA1_n$ and $MA2_k$ are SMAs or EMAs.

Example
If you want to smooth the closing price (C) three successive times using a five-period EMA, a 10-period SMA, and a 15-period SMA, then you can do it in whatever order you want, since all orders will have the same result. For example, taking the 5-EMA of the 10-SMA of the 15-SMA of C is exactly the same as taking the 15-SMA of the 10-SMA of the 5-EMA of C. That is,

$$EMA_5\big(SMA_{10}(SMA_{15}(C))\big) = SMA_{15}\big(SMA_{10}(EMA_5(C))\big)$$

Like linearity, commutative properties in successive smoothing can be proved using simple mathematics, but it is a tedious task. If you only want to grasp the underlying reasons behind why this property holds using a simple approach, try proving that:

$$SMA_2(SMA_3(P)) = SMA_3(SMA_2(P))$$

It's simple.

WEIGHTING EFFECT IN SUCCESSIVE SMOOTHING

What is the effect of successive smoothing of an indicator? The most logical answer would be that you would end up with an extremely smooth version of the indicator. Well, that's true, but how does successive smoothing differ from, say, increasing the period of single smoothing? For example, what is the difference between $EMA_5(EMA_{10}(P))$ and $EMA_{15}(P)$?

Let me cut to the chase and give you the answer in simple terms.

Third property
Successive application of MAs in an indicator creates a smoothed version of the indicator, where the percentage weighting scheme as a function of age of data resembles the shape of a bell. The more MAs applied and the higher their period, the smoother the indicator you get and the more widespread, symmetric, short, and chubby the bell-shaped weighting scheme becomes.

In Figures 1 & 2, you see examples of the weighting schemes of successive smoothing five repeated times using 15-period EMAs and five-period SMAs, respectively. The weights are expressed as percentages of their total sum, which is 100%. In Figure 1, the EMA(15) is the weighting scheme of EMA15(P), the EMA(15,15) is the weighting scheme of $\text{EMA}_{15}(\text{EMA}_{15}(P))$, the EMA(15,15,15) is the weighting scheme of $\text{EMA}_{15}(\text{EMA}_{15}(\text{EMA}_{15}(P)))$, and so on. The weights are sorted in ascending order of age from right to left so that the weight corresponding to age zero (which is the weight put to the most recent value of the indicator P) is the rightmost one. Similar notation is used for the case of SMAs in Figure 2. For comparison purposes, there is also the weighting scheme of $\text{SMA}_{25}(P)$ in Figure 2 [denoted as SMA(25)]. This general rule of bell-shaped weighting scheme also holds for successive smoothing using combinations of SMAs and EMAs.

What does the change in shape of the weighting scheme show as new smoothings are applied? Does the change in shape make sense? It does, because the basic function of moving averages is to raise the contribution (weights) of old data at the cost of the contribution (weights) of the younger data. In effect, the more EMAs or SMAs you apply on top of one another, the more the older values appear in the calculations (thus getting comparably bigger weights) and the newer data gets comparably smaller weights.

This has the effect of a bell shape in the weighting (as a function of their age), which moves like a wave to the left as new EMAs or SMAs are applied. Moreover, as the span of ages having significant percentage weights increases—due to the involvement of more and more indicator values as new MAs are applied—the bell becomes wider, relatively chubbier, and its maximum height becomes shorter.

Calculating the weights

In the cases you have seen so far, the weights depend only on the time instances in terms of age (and not on other factors like the values of some indicator or the volume of shares). For example, let $P_0, P_1, P_2\ldots$ be the values of indicator P where the subscript denotes the age of its values (P_0 is its most recent value, P_1 is its value one bar ago, P_2 is its value two bars ago, and so on). Let's consider the $\text{EMA}_3(\text{EMA}_3(P))$, which is the three-period EMA of the three-period EMA of the indicator P. It is profound that the value of $\text{EMA}_3(\text{EMA}_3(P))$ for the latest bar (that is, for the bar of age zero) eventually equals a sum of type:

$$w_0P_0 + w_1P_1 + w_2P_2 + \ldots \qquad \text{(Equation 1)}$$

where the weights $w_0, w_1, w_2\ldots$ are constant numbers independent of the values of P. These weights are not greater than 1; their total sum is 1, and they represent the contribution of the respective P's value to the creation of $\text{EMA}_3(\text{EMA}_3(P))$. For example, you can see that $w_0 = 0.25$ or 25%, $w_1 = 0.25$ (or 25%), and $w_2 = 0.1875$ (or 18.75%), so 25% of the latest value of $\text{EMA}_3(\text{EMA}_3(P))$ is attributed to the most recent value of P (which is P_0), another 25% of the latest value of $\text{EMA}_3(\text{EMA}_3(P))$ is attributed to the value of P one bar ago (which is P_1) and another 18.75% of the latest value of $\text{EMA}_3(\text{EMA}_3(P))$ is attributed to the value of P two bars ago (which is P_2).

It is when the weights of a smoothing procedure depend on only the time instances in terms of age (like in the previous example) that you can create stable charts like those of Figures 1 & 2. If you create a smoothing procedure that can be eventually formulated in a weighting scheme like the one of equation 1, where the weights depend only on the age of data, you can use a spreadsheet program like Excel to calculate and visualize the weights via charts no matter how complex your procedure is. To accomplish this, you create an artificial indicator P such that all its values are zero, and you write the formulas that dynamically calculate your smoothing function from P's values. If you now change the value of P_0 to 1, then the last value of your smoothing function will be equal to w_0. That's because, as you can see in equation 1, when P_0 equals 1 and all other older values of P equal zero, then the last value of the smoothing function equals w_0. So you copy this w_0 and paste it somewhere else in your spreadsheet. You then again set P_0=0 and proceed to set P_1 equal to 1. This will make the last value of your smoothing function equal to w_1. You copy w_1 and paste it below w_0. By setting P_1 back to zero and setting P_2 equal to 1, you can get the value of w_2 and put it below w_1. If you continue this way, you will be able to get all the weights of your smoothing procedure and chart them as a function of their age. In fact, this is the way the charts of Figures 1 & 2 were constructed. Using this technique, you can also verify the first and second properties stated earlier.

Unfortunately, since this approach requires repeated substitution of values and copying & pasting, you will need to use macros in Excel if you want to calculate a large number of weights for various cases of successive smoothing. For educational purposes, the file "succ_EMA_weights.xlsm," which I used to create the chart in Figure 1, is provided in the Subscriber Area of www.traders.com (as well as from http://traders.com/files/succ_EMA_weights.xlsm.zip). It works in Excel version 2007 and above, and you need to enable macros to make the calculations. You can find out more details in the sidebar "Using Excel To Calculate The Smoothing Weights."

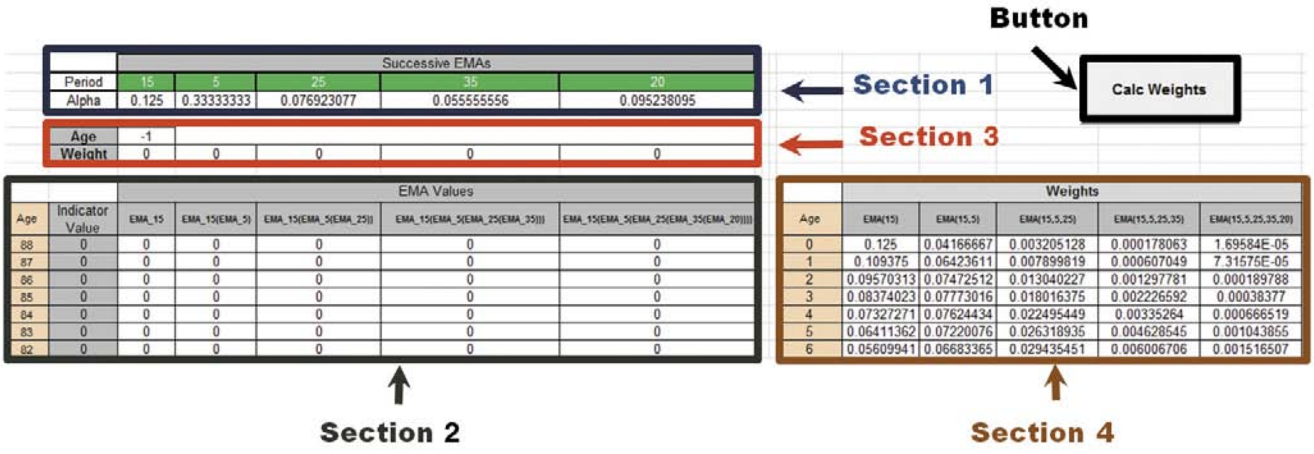
Smoothing ratios of indicators

Ratios of indicators are widely used in technical analysis, mostly as a way to produce normalized percentage values. George Lane's stochastics oscillator is such an example. I will now introduce the fourth property of MAs (which deals with smoothing indicator ratios) using a hypothetical example. Suppose you want to divide indicator P by the positive indicator Q so that you get a new indicator P/Q. This new indicator has proved to be erratic and you want a smoothed version of it using, say, a three-period SMA. You have two options:

Option 1

Calculate the three-period SMAs of P and Q separately and then divide them:

$$\frac{\text{SMA}_3(P)}{\text{SMA}_3(Q)}$$



	Successive EMAs				
Period	15	5	25	35	20
Alpha	0.125	0.33333333	0.076923077	0.055555556	0.095238095

Age	-1				
Weight	0	0	0	0	0

		EMA Values				
Age	Indicator Value	EMA_15	EMA_15(EMA_5)	EMA_15(EMA_5(EMA_25))	EMA_15(EMA_5(EMA_25(EMA_35)))	EMA_15(EMA_5(EMA_25(EMA_35(EMA_20))))
88	0	0	0	0	0	0
87	0	0	0	0	0	0
86	0	0	0	0	0	0
85	0	0	0	0	0	0
84	0	0	0	0	0	0
83	0	0	0	0	0	0
82	0	0	0	0	0	0

	Weights				
Age	EMA(15)	EMA(15,5)	EMA(15,5,25)	EMA(15,5,25,35)	EMA(15,5,25,35,20)
0	0.125	0.04166667	0.003205128	0.000178063	1.69584E-05
1	0.109375	0.06423611	0.007899819	0.000607049	7.31575E-05
2	0.09570313	0.07472512	0.013040227	0.001297781	0.000189788
3	0.08374023	0.07773016	0.018016375	0.002226592	0.00038377
4	0.07327271	0.07624434	0.022495449	0.00335264	0.000666519
5	0.06411362	0.07220076	0.026318935	0.004628545	0.001043855
6	0.05609941	0.06683365	0.029435451	0.006006706	0.001516507

SIDEBAR FIGURE 1: THE FOUR SECTIONS OF THE SPREADSHEET. The Excel spreadsheet is divided into four sections and has a button labeled *calc weights* to call the macro, which will calculate the weights for all ages.

USING EXCEL TO CALCULATE THE SMOOTHING WEIGHTS

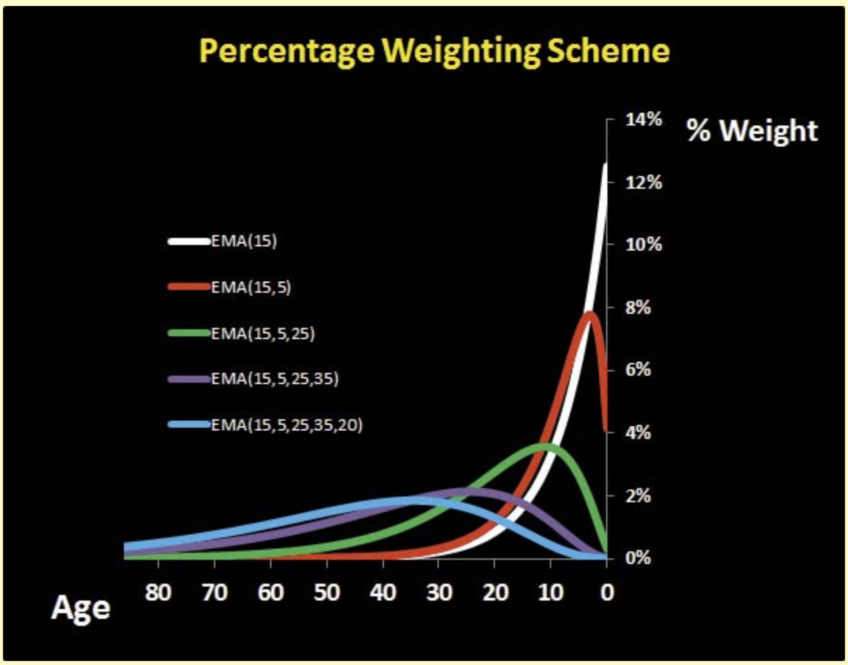
Excel is a simple and quick solution for the calculation and visualization of the distribution of weights in a smoothing method. In Sidebar Figure 1 you see a screenshot of the spreadsheet used to create the chart in the article's Figure 1. The spreadsheet is divided into four sections and offers a button to call a macro.

Section 1 is where you enter the desired periods for the five successive EMAs in the green cells. The respective alphas for the periods are automatically calculated below the periods according to the standard formula: alpha =2/(period+1).

Section 2 contains the functions that calculate the successive EMAs' values in descending order of age for an indicator whose values are in the *indicator values* column. The spreadsheet assumes 88 historical values for the indicator so that the 88th value (the one that has an age of 88) is the oldest and the 0th value is the newest. All values of the indicator are initially set to zero. When only the 0th value (in terms of age) of the indicator becomes 1, then the 0th values (in terms of age) of EMAs—that is, the newest values of EMAs—will become equal to the EMAs' weights for age zero (namely, w_0). Similarly, when only the first value of the indicator becomes 1, then the 0th values of EMAs will become equal to the EMAs' weights for age 1 (namely, w_1) and, generally, when only the k^{th} value of the indicator becomes 1, then the 0th values of EMAs will become equal to the EMAs' weights for age k (namely, w_k).

Section 3 offers a quick way to calculate the weights of EMAs for an age. When you enter the age directly into the cell labeled *age*, the value of the indicator for that age in section 2 becomes 1 automatically, and the second row of section 3 labeled *weight* is populated with the weights of the EMAs (in alignment with section 1) for that age, which are exactly those in the last row of the table of section 2.

Section 4 and button: When you click the button labeled *calc weights,* a macro runs in the background that repeatedly changes the age in section 3 starting from zero and increasing by 1 until it gets to 88. For every change in age, the macro copies the *weight* row of section 3 and pastes it in the table of section 4, populating it from the top cell and down. The table of section 4 is linked to a chart that offers a visualization of the weights like the one in Sidebar Figure 2. The labels of the chart are automatically updated according to the periods of section 1 as soon as the button is clicked. In Sidebar Figure 2, you can see the chart that corresponds to the data in Sidebar Figure 1.



SIDEBAR FIGURE 2: CHARTING THE WEIGHTS. As soon as you click the *calc weights* button, both the section 4 and its linked chart are updated. The chart shown here is based on the inputs and calculated weights of Sidebar Figure 1.

The Excel spreadsheet discussed here can be downloaded from the Subscriber Area at our website, ***www.traders.com****, in the* ***Article Code*** *area, as well as from* http://traders.com/files/succ_EMA_weights.xlsm.zip.

Option 2

Simply take the three-period SMA of P/Q; that is:

$$\text{SMA}_3\left(\frac{P}{Q}\right)$$

Technical analysts would consider the first option as a realistic solution, since it allows for occasional and isolated zero values for Q (it is more difficult for $\text{SMA}_3(Q)$ to be zero than for Q to be zero) but if Q is always nonzero, then either of the two options could be chosen. So what is the difference between these two smoothing options and how can you determine which one better suits your preferences? The answer lies in the fourth property for MAs:

Fourth property

If P and Q are indicators and Q is positive, then the formula

$$\frac{\text{SMA}_k(P)}{\text{SMA}_k(Q)}$$

results in a modification of

$$\text{SMA}_k\left(\frac{P}{Q}\right)$$

where more weight is given to the values of P/Q, where Q is larger. Similarly,

$$\frac{\text{EMA}_k(P)}{\text{EMA}_k(Q)}$$

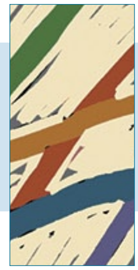
results in a modification of

$$\text{EMA}_k\left(\frac{P}{Q}\right)$$

where, again, more weight is given to the values of P/Q, where Q is larger.

The mathematically inclined might want to try and see that the underlying reason for this property is the same one that makes the harmonic mean give less significance to high-value outliers. If you don't understand this peculiar connection, don't get discouraged. Here is how you can get an idea of why the fourth property holds in this hypothetical example: If P_0, P_1, P_2 are the three most recent values of P, and Q_0, Q_1, Q_2 are the three most recent values of Q (where, as usual, the subscripts denote the age of the values), then the SMA_3 assigns equal weights to the three most recent values. More precisely, the latest values of SMA_3 for P and Q are:

All technical analysts eventually use some kind of smoothing method to filter out noise.



$$\text{SMA}_3(P) = 1/3\ P_0 + 1/3\ P_1 + 1/3\ P_2$$

and

$$\text{SMA}_3(Q) = 1/3\ Q_0 + 1/3\ Q_1 + 1/3\ Q_2$$

Using simple algebra, you can see that:

$$\frac{\text{SMA}_3(P)}{\text{SMA}_3(Q)} = w_0\left(\frac{P_0}{Q_0}\right) + w_1\left(\frac{P_1}{Q_1}\right) + w_2\left(\frac{P_2}{Q_2}\right)$$

(Equation 2)

where:

$$w_i = \frac{Q_i}{Q_0 + Q_1 + Q_2}$$

(Equation 3)

for $i = 0, 1, 2$.

It is clear from equation 2 that the latest value of $\text{SMA}_3(P)/\text{SMA}_3(Q)$ is just a weighted average of P/Q, where all three w_is have the same denominator. Consequently, the numerator Q_i in equation 3 is the one that determines the relative sizes of the w_is. As a result, the higher the Q_i (in relation to the other two values of Q), the higher the weight w_i for P/Q and the higher the contribution of the i^{th} value of P/Q (in terms of age) to the latest value of $\text{SMA}_3(P)$ / $\text{SMA}_3(Q)$.

Which of the two smoothing options best suits your purposes? By choosing $\text{SMA}_3(P/Q)$, you assign equal weights to the three most recent values of P/Q, whereas by choosing $\text{SMA}_3(P)/\text{SMA}_3(Q)$, you demand that the weights in the three most recent values of P/Q are analogous to the sizes of the respective values of Q.

Note that charts like those in Figures 1 & 2 cannot be constructed for $\text{SMA}_k(P)/\text{SMA}_k(Q)$ or $\text{EMA}_k(P)/\text{EMA}_k(Q)$, since the weights of P/Q are not determined purely from the time instances in terms of age; they are also affected by the values of Q. As new price data and new values of P and Q are introduced, the weights assigned to the values of P/Q will change according to the relative sizes of the values of Q among each other.

Smoother is easier

This article may differ from other technical analysis articles in that it doesn't promote a specific trading style or system. My main purpose here is to make the aspiring creator of new indicators familiar with four interesting properties of simple and

The basic function of moving averages is to raise the contribution (weights) of old data at the cost of the contribution (weights) of the younger data.

exponential moving averages and give you a way to calculate the distribution of weights algorithmically for some moving average cases. The first two properties may save you some time when you try different combinations of ideas, whereas the other two uncover the effects of successive smoothing and the effect of separate smoothing of numerator and denominator in ratios so you can know beforehand what kind of smoothing you can use with respect to how you want your final indicator to react.

SMAs and EMAs are widely used smoothing methods. If this article has increased your knowledge about these methods, then it has fulfilled its purpose.

Giorgos Siligardos holds a doctorate in mathematics and a market maker certificate from the Athens Exchange. He is a financial software developer, coauthor of academic books in finance, a frequent contributor to this magazine, and scientific contributor in the Department of Finance and Insurance at the Technological Institute of Crete. Material from his course writings on derivatives has been used in educational enchiridia for bank managers. His academic website is http://www.tem.uoc.gr/~siligard *and his current views on the markets can be found at* http://market-calchas.blogspot.gr/. *He may be reached at siligard@tem.uoc.gr.*

The Excel spreadsheet referenced in this article is available at the Subscriber Area at our website, ***www.traders.com****, in the* ***Article Code*** *area. The spreadsheet is also downloadable from* http://traders.com/files/succ_EMA_weights.xlsm.zip.

FURTHER READING

Siligardos, Giorgos E. [2013]. "The Average Age Of Averages," *Technical Analysis of* STOCKS & COMMODITIES, Volume 31: April.

‡Excel (Microsoft Corp.)

‡*See Editorial Resource Index*

TRADERS' GLOSSARY

Commodity Channel Index—Developed by Donald Lambert, this price momentum indicator measures the price "excursions" from the mean.

Exponential Moving Average—A variation of the moving average, the EMA places more weight on the most recent closing price.

Harmonic Mean—An average obtained by taking the reciprocal of the arithmetic mean of the reciprocals of a set of nonzero numbers. One of the three Pythagorean means, where the harmonic mean is always the least of the three means. Since it tends toward the least, compared to the arithmetic mean, it can help mitigate the impact of large outliers.

Heuristic Method—Problem-solving approached by trying out several different methods and comparing which provides the best solution. In behavioral finance, trial-and-error learning leading to the use of rules of thumb for decisions.

High-Pass Frequency Filter—A detrending filter that lets pass the high-frequency noise and rejects low-frequency trend. Implemented by first applying a low-pass filter to the data, then subtracting the filtered data from the original data.

Noisy Signal—A signal in which the effects of random influences cannot be dismissed.

Optimization—A methodology by which a system is developed with rules tailored to fit the data in question precisely.

Outlier—A value removed from the other values to such an extreme that its presence cannot be attributed to the random combination of chance causes.

Relative Strength Index (RSI)—An indicator invented by J. Welles Wilder and used to ascertain overbought/oversold and divergent situations.

Smoothing—Simply, mathematical technique that removes excess data variability while maintaining a correct appraisal of the underlying trend.

Probability Density Function—A graph showing the probability of occurrence of a particular datapoint (price).

Probability Distribution Function—A function whose integral over any set gives the probability that a random variable has values in this set. Describes the relative likelihood for a random variable to take on a given value.

Find more terms defined in the Traders' Glossary at Traders.com.