SATQ PART — A

1. $K = 3$

2. 23

3. $\frac{2}{5}$

4. 1

5. -3

6. 3449

7. a $\rightarrow f'(x_0)$

8. 15 ways

9.

10. $\frac{1}{\sqrt{2}}$

B 5

a

$$\lim_{K\to\infty} \sqrt{K^2a^2+2pK} - \sqrt{K^2a^2}$$

$$= \lim_{K\to\infty} \frac{\left(\sqrt{K^2a^2+2pK}+\sqrt{K^2a^2}\right)\left(\sqrt{K^2a^2+2pK}-\sqrt{K^2a^2}\right)}{\left(\sqrt{K^2a^2+2pK}+\sqrt{K^2a^2}\right)}$$

$$= \lim_{K\to\infty} \frac{\cancel{K^2a^2}+2pK-\cancel{K^2a^2}}{\sqrt{K^2a^2+2pK}+\sqrt{K^2a^2}}$$

$$= \lim_{K\to\infty} \frac{2pK}{Ka+K\sqrt{a^2+\frac{2p}{K}}}$$

$$= \lim_{K\to\infty} \frac{2p\cancel{K}}{\cancel{K}\left[a+\sqrt{a^2+\frac{2p}{K}}\right]}$$

$$= \lim_{K\to\infty} \frac{2p}{\sqrt{a^2+\frac{2p}{K}}+a}$$

$$= \frac{2p}{2a} = \boxed{\frac{p}{a}}$$ (ans)

(b) $\lim_{x \to \infty} \frac{1}{x} \int_0^x \frac{dt}{3+\cos t}$

let's calculate $\int_0^x \frac{dt}{3+\cos t}$

$$\int \frac{dt}{3+\cos t} = \int \frac{dt}{3(\sin^2 t/2 + \cos^2 t/2) + \cos^2 t/2 - \sin^2 t/2}$$

$$= \int \frac{\sec^2 t/2 \, dt}{3\left(1+\tan^2 \frac{t}{2}\right) + 1 - \tan^2 t/2}$$

$$= \int \frac{\sec^2 t/2 \, dt}{4 + 2\tan^2 \frac{t}{2}}$$

$$= \frac{1}{2} \int \frac{\sec^2 \frac{t}{2} \, dt}{2 + \tan^2 \frac{t}{2}}$$

let $\tan \frac{t}{2} = z$

$\Rightarrow \frac{1}{2} \sec^2 \frac{t}{2} \, dt = dz$

$\Rightarrow \sec^2 \frac{t}{2} \, dt = 2dz$

$$= \frac{1}{2}\int \frac{2\,dz}{2+z^2}$$

$$= \int \frac{dz}{2+z^2}$$

$$= \frac{1}{\sqrt{2}}\tan^{-1}\frac{z}{\sqrt{2}} + C.$$

So $\displaystyle\int_0^x \frac{dt}{3+\cos t}$

$$= \left[\frac{1}{\sqrt{2}}\tan^{-1}\left(\frac{\tan\frac{t}{2}}{\sqrt{2}}\right)\right]_0^x$$

$$= \frac{1}{\sqrt{2}}\tan^{-1}\left(\frac{\tan\frac{x}{2}}{\sqrt{2}}\right)$$

$$\lim_{x\to\infty} \frac{1}{x}\frac{1}{\sqrt{2}}\tan^{-1}\left(\frac{\tan\frac{x}{2}}{\sqrt{2}}\right)$$

$$= \lim_{x\to\infty} \frac{1}{x}\frac{1}{\sqrt{2}} \frac{\tan^{-1}\left(\frac{\tan\frac{x}{2}}{\sqrt{2}}\right)}{\frac{\tan\frac{x}{2}}{\sqrt{2}}} \cdot \frac{\tan\frac{x}{2}}{\sqrt{2}}$$

$$= \lim_{x\to\infty} \frac{1}{2}\frac{\tan\frac{x}{2}}{x}.$$

$$= \lim_{x\to\infty} \frac{1}{2}\frac{\tan\frac{x}{2}}{\frac{x}{2}} \cdot \frac{1}{2} = \boxed{\frac{1}{4}} \text{ (ans)}$$

(B1) The given quadratic function is

$f(x) = x^2 + bx + c$

p, q are two distinct primes where

p and q are the 2 roots of $f(x)$

$p + q = -b$ [sum of roots]

$qp = c$ [product of roots]

$$f(p-q) = (p-q)^2 + b(p-q) + c$$

$$= (p-q)^2 + -(p+q)(p-q) + pq.$$

$$= (p-q)^2 - (p^2 - q^2) + pq$$

$$= \cancel{p^2} + q^2 - 2pq - \cancel{p^2} + q^2 + pq$$

$$= 2q^2 - qp. \longrightarrow (1)$$

given, $f(p-q) = 6pq \longrightarrow (2)$

$$2q^2 - pq = 6pq$$

$$\Rightarrow 2q^2 = 7pq$$

$$\Rightarrow 2q^2 - 7pq = 0$$

$$\Rightarrow q(2q - 7p) = 0$$

$q \neq 0$ [as it is a prime number]

$2q - 7p = 0$

$\Rightarrow 2q = 7p$

$\Rightarrow \frac{p}{q} = \frac{2}{7}$

$\boxed{p = 2, \ q = 7}$ came 1)

here 2 is a prime 7 is a prime

$f(x) = x^2 - (2+7)x + (2 \cdot 7)$

$= \boxed{x^2 - 9x + 14}$ came 2)

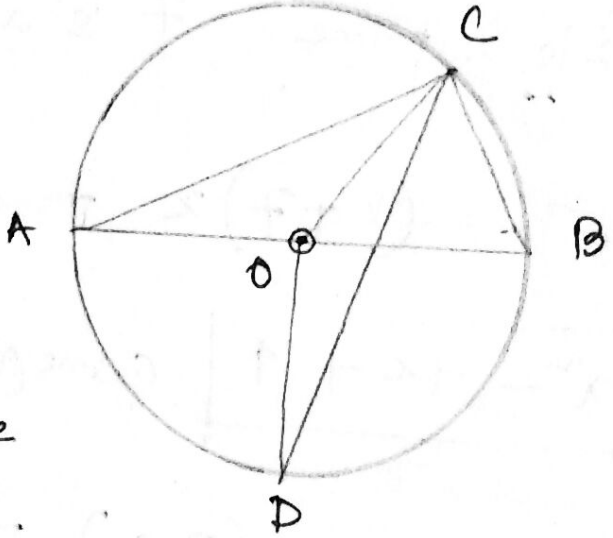
Ans 1 The prime $(p, q) = (2, 7)$

2 function $f(x) = x^2 - 9x + 14$.

(B2)

Given, AB is a diameter of a circle of radius 10 units. C is a point on the circle such that the length of the arc BC = $\frac{5\pi}{3}$. The bisector of angle $\angle ACB$ cuts the circle at D. ~~Find length of OD~~

Diagram



here O is the centre

$$\overset{\frown}{BC} = \frac{5\pi}{3}$$

we know $s = r\theta$ [here s is arc length θ is the angle subtended by the arc towards centre at O and r is radius]

$$\frac{5\pi}{3} = 10\theta$$

$$\Rightarrow \theta = \frac{\cancel{5}\pi}{3} \cdot \frac{1}{\cancel{10}_{2}}$$

$\Rightarrow \theta = \frac{\pi}{6}$

~~By Sm~~ $\angle DOB = 90^\circ$

So $\angle DOC = 90^\circ + 30^\circ$

$= 120^\circ$

<u>by cosine rule</u>,

$$\cos \angle DOC = \frac{DO^2 + OC^2 - CD^2}{2\, DO \cdot OC}$$

$$\Rightarrow \cos 120^\circ = \frac{10^2 + 10^2 - CD^2}{2 \cdot 10 \cdot 10}$$

$$\Rightarrow -\frac{1}{2} = \frac{100 + 100 - CD^2}{200}$$

$$\Rightarrow -100 = 200 - CD^2$$

$$\Rightarrow CD^2 = 300$$

$$\Rightarrow CD = \boxed{10\sqrt{3}}$$

<u>Ans</u> length of CD is $10\sqrt{3}$ units.

(B4)

(a) Checking of injectivity

$f: \mathbb{R} \to \mathbb{R}$

let x_1 and x_2 be 2 elements from domain set.

$$f(f(x_1)) = f(f(x_2))$$

$$\Rightarrow -x_1 = -x_2$$

$$\Rightarrow x_1 = x_2.$$

as $x_1 = x_2$ so the function is Injective

(c) let's check continuity

RHL

$$\lim_{x \to a^+} f(f(x)) = -a$$

LHL $$\lim_{x \to a^-} f(f(x)) = -(-a)$$

$$= +a$$

RHL $\neq$ LHL so not continuous.