Vehicle routing: eil33-2

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When a company dispatches vehicles from a depot to deliver goods or services to customers, the capacity of the vehicle, the route that the vehicle travels, and the demand of the customer must be taken into account. How are these issues decided optimally? Many experts in operations and logistics formulate a vehicle routing problem (VRP) to support decisions in such situations.

The solution of a VRP can address a variety of goals:

- Which depots serve which locations?
- How many vehicles are needed?
- Which route minimizes the costs of delivery?

A vehicle routing problem (VRP) conventionally consists of nodes and arcs, where one node represents a depot or distribution center, and other nodes represent locations served by the depot. A value at each node represents the demand at that node, or the capacity of a depot.

The arcs between nodes represent distance or costs. For example, arcs can represent distance between the depot and a customer location or the distance between two customers. Similarly, for example, arcs can represent costs such as the tolls and mileage between the depot and a customer location. In problems where the routes of vehicles are minimized, the length of a route for a vehicle can be calculated from these values, or similarly, the cost of a route can be computed and compared to costs of other routes. In this particular model, these costs appear as coefficients of variables in the objective function.

In a **capacitated** vehicle routing problem (CVRP), vehicles of limited capacity travel between the depot and the locations that the depot serves to supply the needs of those locations.

In more formal terms, the objective of a vehicle routing problem (VRP) is to construct a set of vehicle routes that visits all customers, that has minimum cost, and that satisfies demands without violating the constraints on the capacity of the vehicles.

Model

Frequently, vehicle routing problems (such as the one in this example) are represented by a set partitioning model. Set partitioning models also often address fleet assignment and crew scheduling as well.

This model, eil33-2, from the MIPLIB and TSPLIB libraries, solves a capacitated vehicle routing problem (CVRP) by approximating the problem as a set-partitioning model. As expressed by Jeff Linderoth, the model consists of 32 rows and 4,516 columns. All 4,516 variables are binary (also known as 0-1 variables). In other words, the model consists of 32 constraints on 4,516 binary variables.

The model includes 44,243 non-zero coefficients.

Columns

When you compare the small number of rows (32) with the large number of columns (4,516), you see that the **aspect ratio** of this model is quite large, suggesting that the problem lends itself to column generation, a technique familiar in mathematical programming and well supported by CPLEX. In fact, each column in the model corresponds to a feasible assignment of vehicles to stops at locations on the route. In that way, the capacity of each vehicle is implicitly taken into account: if a vehicle does not have sufficient capacity to service a specific location, then none of the columns in the model will define a route with that vehicle servicing that location as a possibility.

Rows

The 32 rows of the model define the set to partition: `{1, 2, 3, . . . , 32}`. Each column intersects a subset of the constraints. Might those subsets define a partition of the set?

To get a clearer idea of that point, consider the variable x666. If you ask CPLEX to display problem variable x666, CPLEX displays the following table:

Table 1. Variable x666

Constraint	Coefficient of x666
obj	279.93 (objective)
c13	1
c15	1
c17	1
c19	1
c20	1
c21	1
c22	1
c23	1
c24	1
c27	1

The table shows that variable x666 corresponds to the subset identified by these intersecting constraints, that is, {13, 15, 17, 19, 20, 21, 22, 23, 24, 27}.

A feasible solution

The model declares that variables x_1 through x_{32} each occur in exactly one constraint. In other words, those columns represent the simple route where one vehicle goes from the depot to one location and returns directly to the depot. This solution is evidently feasible, but is it optimal? Clearly not optimal: CPLEX detects this "solution" and eliminates it early in the process.

Much more interesting as a feasible solution is any partitioning of the 32 constraints into subsets (like the subset for variable x666 that you saw in the table) where the union of the subsets includes all 32 with no overlap between the subsets.

The goal then is to find the partition that costs the least, based on the costs expressed as coefficients of variables in the objective function.

An optimal solution

CPLEX solves the model with an optimal objective value of 9.3400791600e+02 (a little more than 934). To solve the problem, CPLEX explores 108,000 nodes in a matter of seconds.

Four of the 4,516 binary variables have a value of 1 (one) in the solution.

- x1662
- x2299
- x2590
- x4020

(The other binary variables are 0 (zero) in the solution.) When CPLEX displays those variables, you see that they appear in the following constraints.

```
Variable x1662 coefficients
   Linear Constraint:      Coefficient:
                 obj       264.582 (objective)
                 c13             1
                 c14             1
                 c15             1
                 c17             1
                 c18             1
                 c19             1
                 c20             1
                 c21             1
                 c23             1
                 c24             1

Variable x2299 coefficients
   Linear Constraint:   Coefficient:
                 obj       184.955 (objective)
                  c2             1
                  c3             1
                  c4             1
                  c5             1
                  c7             1
                  c9             1
                 c10             1
                 c11             1
                 c32             1

Variable x2590 coefficients
   Linear Constraint:      Coefficient:
                 obj       249.772 (objective)
                  c1             1
                 c16             1
                 c27             1
                 c28             1
                 c29             1
                 c30             1
                 c31             1

Variable x4020 coefficients
   Linear Constraint:      Coefficient:
                 obj       234.699 (objective)
                  c6             1
                  c8             1
                 c12             1
                 c22             1
                 c25             1
                 c26             1
```

Those four non-zero variables in the solution suggest four routes:

```
{13, 14, 15, 17, 18, 19, 20, 21, 23, 24}
{2, 3, 4, 5, 7, 9, 10, 11, 32}
{1, 16, 27, 28, 29, 30, 31}
{6, 8, 12, 22, 25, 26}
```

In other words, one vehicle services locations 13, 14, 15, 17, 18, 19, 20, 21, 23, and 24. A second route services locations 2, 3, 4, 5, 7, 9, 10, 11, and 32. Likewise, a third route assures service to locations 1, 16, 27, 28, 29, 30, and 31. Finally, the fourth route assures service to locations 6, 8, 12, 22, 25, and 26.

Furthermore, in the objective function, the coefficients of each of those variables (x1662, x2299, x2590, and x4020) assure minimal costs of the routes overall. In other words, those four subsets partition the original set of 32 constraints into four non-overlapping routes that deliver to each location at overall minimal cost.

MIPLIB and TSPLIB

MIPLIB is a set of models used to compare the performance of mixed integer optimizers, such as IBM ILOG CPLEX. The set of models, assembled and made publicly available by a number of distinguished researchers, stimulates research and promotes better, faster solution of real-world problems.

TSPLIB is a library of traveling salesperson problems (TSP). Assembled and maintained by a host of widely known researchers, the library includes symmetric TSP, asymmetric TSP, Hamiltonian cycle problems, sequential ordering problems (SOP), capacitated vehicle routing problems (CVRP), and similar challenges. In this context, a capacitated vehicle routing problem, such as eil33-2, represents a special type of TSP.