Diet design

Diet design

Can linear programming save money on the food budget of the U.S. Army without damaging the nutritional health of members of the armed forces?

This example solves a simple variation of the well-known diet problem posed by George Stigler and George Dantzig: how to choose foods satisfying nutritional requirements while minimizing costs or maximizing satiety.

Stigler solved his model "by hand" because technology at the time did not yet support more sophisticated methods. However, in 1947, Jack Laderman, of the U.S. National Bureau of Standards, applied the **simplex method** (an algorithm recently proposed by George Dantzig) to Stigler's model. Laderman and his team of nine linear programmers, working on desk calculators, showed that Stigler's heuristic approximation was very close to optimal (only 24 cents per year over the optimum found by the simplex method) and thus demonstrated the practicality of the simplex method on large-scale, real-world problems.

Problem

The original diet problem posed by George Stigler in 1939 was one of the earliest practical applications of linear programming in operations research. Stigler used a heuristic method to approximate a solution of a problem from the U.S. Army: to feed its soldiers an adequate diet within minimum cost. Stigler's model included minimal daily requirements of nine nutrients from 77 foods, based on 1939 prices. The data of the problem assumed a moderately active male of 154 pounds, who required the recommended daily allowance of calories, protein, calcium, iron, vitamin A, thiamine, riboflavin, niacin, and ascorbic acid. The solution proposed a diet costing 11 cents per day or $39.93 per year in 1939 U.S. dollars.

Model

This model of the diet problem represents nutritional requirements as linear constraints and the cost of food as an objective to minimize. Specifically, variables represent the foods, and linear constraints on these variables represent the nutritional value of foods. Coefficients in the objective function represent the price of foods. The objective in this model minimizes the cost of foods satisfying the nutritional requirements. Bounds on the variables represent the minimum requirements and maximum limits.

This formulation of the model makes it possible for you to add more foods (for example, based on what is available at your market) by adding a new column for each food. Likewise, you can add more nutritional requirements by adding more rows to the model. For more about this idea of modifying the model by adding more rows or by adding more columns, see the online documentation of CPLEX, especially Example: optimizing the diet problem in IBM Knowledge Center.

Objective

George Dantzig also applied the simplex method to solving his own variation of the diet problem when he was actually attempting to lose weight. On the advice of his doctor, he limited the daily intake of calories to 1500. He collected the official price of various foods from the U.S. Department of Labor Statistics. He gathered the nutritional value of those foods from the Home Economics Department of the U.S. Department of Agriculture. He changed the objective from Stigler's original minimization of cost. By that time, Dantzig was no longer a poorly paid government worker in the Depression era, but instead, a well-paid employee of a thriving corporation in post-war America. As Dantzig said, "I need to maximize the feeling of feeling full" rather than to minimize cost.

Modifications

Dantzig's change in the objective required modifications in the model. To assign a fullness coefficient to a food, Dantzig subtracted the weight of the water content of a given food from the weight of the food. This representation of fullness led to an amusing result in Dantzig's first solution of the problem. His initial computation recommended 500 gallons of vinegar per day because the nutritional table he was using assumed no water content in vinegar. Consequently, the computation concluded that the more vinegar Dantzig drank, the more full he would feel. On reflection, Dantzig adjusted his model to eliminate vinegar as a food.

Bounds

After eliminating vinegar, Dantzig found that the next solution recommended 200 bouillon cubes per day. Dantzig gamely tried three cubes in a cup of hot water for breakfast, but quickly concluded that the model seriously needed a limit on daily intake of salt. Indeed, Dantzig later credited this part of the problem with introducing the idea of upper bounds on variables in linear programs generally.

Solution

In the end, Dantzig accepted the dietary advice of his spouse, who agreed that the computations suggested some intriguing solutions, but that the model was not sufficiently nuanced for human consumption. The linear model of the diet problem, however, did demonstrate even in its earliest forms that optimization in linear programming offered feasible solutions to practical problems.