

Results are obtained with h_0^P estimated

| CALIBRATED PARAMETERS ON WEDNESDAYS USING OPTIONS LIKELIHOOD, $h_0^Q = h_t^P$ | | | | | | | | | |
|---|--------------------|--------------------|--------------------|--------------------|--------------------|--------------------|--------------------|--------------------|--------------------|
| θ | 2010 | 2011 | 2012 | 2013 | 2014 | 2015 | 2016 | 2017 | 2018 |
| ω | $2.9373e-07$ | $8.3428e-06$ | $2.0557e-09$ | $1.4603e-06$ | $1.9938e-06$ | $4.7113e-07$ | $6.9592e-07$ | $3.5608e-07$ | $2.8373e-07$ |
| ci | $(\pm 4.3822e-07)$ | $(\pm 7.4910e-06)$ | $(\pm 1.1272e-09)$ | $(\pm 1.4013e-06)$ | $(\pm 1.8706e-06)$ | $(\pm 7.0501e-07)$ | $(\pm 8.3824e-07)$ | $(\pm 5.5763e-07)$ | $(\pm 5.2647e-07)$ |
| median | $4.4080e-10$ | $2.1679e-09$ | $1.1257e-09$ | $1.5347e-09$ | $1.3127e-09$ | $1.3856e-09$ | $7.3148e-10$ | $2.7972e-10$ | $4.9914e-10$ |
| α | $2.6506e-05$ | $2.2808e-05$ | $2.0436e-05$ | $1.5988e-05$ | $1.4776e-05$ | $1.3678e-05$ | $1.3866e-05$ | $9.2340e-06$ | $1.6279e-05$ |
| ci | $(\pm 5.9068e-06)$ | $(\pm 6.1876e-06)$ | $(\pm 4.9583e-06)$ | $(\pm 3.4444e-06)$ | $(\pm 2.5542e-06)$ | $(\pm 1.8989e-06)$ | $(\pm 2.4092e-06)$ | $(\pm 1.3797e-06)$ | $(\pm 3.0725e-06)$ |
| median | $2.1958e-05$ | $2.0325e-05$ | $1.4954e-05$ | $1.5884e-05$ | $1.4270e-05$ | $1.2722e-05$ | $1.2912e-05$ | $9.1517e-06$ | $1.5918e-05$ |
| β | 0.4708 | 0.3164 | 0.4553 | 0.3331 | 0.1703 | 0.1908 | 0.2374 | 0.1474 | 0.2896 |
| ci | (± 0.0902) | (± 0.0888) | (± 0.1018) | (± 0.1060) | (± 0.0776) | (± 0.0647) | (± 0.0857) | (± 0.0816) | (± 0.0940) |
| median | 0.5549 | 0.3131 | 0.6192 | 0.0023 | 0.0002 | 0.0090 | 0.0007 | 0.0001 | 0.0010 |
| γ^* | 155.5027 | 256.7574 | 176.9894 | 247.3834 | 225.9295 | 224.0573 | 257.7868 | 275.5854 | 191.7579 |
| ci | (± 41.3407) | (± 79.8138) | (± 33.8088) | (± 75.2711) | (± 55.6646) | (± 13.4744) | (± 68.1243) | (± 52.2661) | (± 30.0767) |
| median | 115.7838 | 148.3374 | 137.7486 | 166.2098 | 189.4759 | 226.1581 | 205.5256 | 237.1314 | 159.7501 |
| $h_0^Q = h_t^P$ | $1.2843e-04$ | $1.5885e-04$ | $8.8858e-05$ | $6.0313e-05$ | $6.5265e-05$ | $1.1085e-04$ | $9.9075e-05$ | $4.0828e-05$ | $1.1258e-04$ |
| ci | $(\pm 2.4166e-05)$ | $(\pm 2.8191e-05)$ | $(\pm 1.1827e-05)$ | $(\pm 8.7213e-06)$ | $(\pm 1.0436e-05)$ | $(\pm 1.8145e-05)$ | $(\pm 2.0030e-05)$ | $(\pm 6.5382e-06)$ | $(\pm 2.4678e-05)$ |
| median | $1.1288e-04$ | $1.3446e-04$ | $8.4289e-05$ | $4.8973e-05$ | $5.5260e-05$ | $9.2823e-05$ | $7.8758e-05$ | $3.3053e-05$ | $9.1614e-05$ |
| MSE | 1.3115 | 4.7861 | 2.6162 | 4.2244 | 8.4450 | 6.3652 | 10.9788 | 23.0601 | 13.4936 |
| IVRMSE | 0.0639 | 0.0955 | 0.0867 | 0.0890 | 0.0933 | 0.0939 | 0.1111 | 0.1248 | 0.0897 |
| MAPE | 0.0741 | 0.0936 | 0.1184 | 0.1292 | 0.1568 | 0.1523 | 0.1709 | 0.2464 | 0.1414 |
| OptLL | 215.4291 | 208.3681 | 251.0076 | 333.0039 | 351.3072 | 436.8099 | 513.2066 | 555.4006 | 684.7143 |

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| CALIBRATED PARAMETERS ON WEDNESDAYS, $h_0^Q = \frac{\omega_0 + \alpha_0}{1 - \beta_0 - \alpha_0 \gamma_0^{*2}}$, WITH $\omega_0, \alpha_0, \beta_0, \gamma_0^{*2}$ FROM MLE UNDER P AND UPDATED UNDER Q | | | | | | | | | |
|--|--------------------|--------------------|--------------------|--------------------|--------------------|--------------------|--------------------|--------------------|--------------------|
| θ | 2010 | 2011 | 2012 | 2013 | 2014 | 2015 | 2016 | 2017 | 2018 |
| ω | $8.5029e-08$ | $9.2714e-06$ | $1.9246e-07$ | $2.2129e-06$ | $1.9389e-06$ | $4.1610e-07$ | $5.9987e-07$ | $3.5299e-07$ | $5.5812e-07$ |
| ci | $(\pm 1.2370e-07)$ | $(\pm 7.7090e-06)$ | $(\pm 3.0224e-07)$ | $(\pm 1.8052e-06)$ | $(\pm 1.8258e-06)$ | $(\pm 6.8777e-07)$ | $(\pm 8.0455e-07)$ | $(\pm 5.6074e-07)$ | $(\pm 7.5576e-07)$ |
| median | $4.8556e-10$ | $1.1932e-09$ | $9.6775e-10$ | $1.6296e-09$ | $1.4218e-09$ | $1.6699e-09$ | $8.8906e-10$ | $3.4979e-10$ | $6.2288e-10$ |
| α | $2.5394e-05$ | $2.1003e-05$ | $1.8778e-05$ | $1.3908e-05$ | $1.3646e-05$ | $1.3883e-05$ | $1.3858e-05$ | $8.2692e-06$ | $1.5954e-05$ |
| ci | $(\pm 6.0720e-06)$ | $(\pm 5.7736e-06)$ | $(\pm 4.5685e-06)$ | $(\pm 3.2313e-06)$ | $(\pm 2.3532e-06)$ | $(\pm 1.6292e-06)$ | $(\pm 2.2149e-06)$ | $(\pm 1.3559e-06)$ | $(\pm 2.6286e-06)$ |
| median | $1.7658e-05$ | $1.9181e-05$ | $1.2068e-05$ | $1.2723e-05$ | $1.3239e-05$ | $1.3217e-05$ | $1.3228e-05$ | $8.3302e-06$ | $1.4242e-05$ |
| β | 0.5032 | 0.3363 | 0.4882 | 0.3724 | 0.1836 | 0.1643 | 0.2466 | 0.1768 | 0.2450 |
| ci | (± 0.0879) | (± 0.0885) | (± 0.0950) | (± 0.1069) | (± 0.0799) | (± 0.0627) | (± 0.0871) | (± 0.0910) | (± 0.0889) |
| median | 0.5759 | 0.3823 | 0.5857 | 0.3025 | 0.0003 | 0.0007 | 0.0018 | 0.0001 | 0.0023 |
| γ^* | 152.7405 | 213.9027 | 178.3425 | 268.5595 | 254.9716 | 221.9130 | 209.9787 | 301.8938 | 202.9867 |
| ci | (± 37.6445) | (± 46.4971) | (± 39.1533) | (± 83.1723) | (± 66.0837) | (± 11.4391) | (± 20.3795) | (± 52.8764) | (± 36.8218) |
| median | 112.0207 | 155.9251 | 147.8898 | 169.4020 | 202.0041 | 228.8470 | 208.6253 | 261.8796 | 167.7543 |
| $h_0^Q = h_t^P$ | $1.2504e-04$ | $1.6094e-04$ | $8.8020e-05$ | $6.3516e-05$ | $6.4968e-05$ | $1.0677e-04$ | $9.4593e-05$ | $4.2065e-05$ | $1.2042e-04$ |
| ci | $(\pm 2.3250e-05)$ | $(\pm 2.7914e-05)$ | $(\pm 1.1134e-05)$ | $(\pm 8.4851e-06)$ | $(\pm 1.0419e-05)$ | $(\pm 1.4866e-05)$ | $(\pm 1.8237e-05)$ | $(\pm 7.1338e-06)$ | $(\pm 2.5752e-05)$ |
| median | $1.0398e-04$ | $1.3887e-04$ | $7.9893e-05$ | $5.2671e-05$ | $5.4472e-05$ | $8.9209e-05$ | $6.9330e-05$ | $3.6036e-05$ | $1.0226e-04$ |
| MSE | 1.1660 | 4.6442 | 2.4437 | 4.3159 | 7.5939 | 6.1701 | 10.7231 | 20.7106 | 13.3130 |
| IVRMSE | 0.0633 | 0.0921 | 0.0863 | 0.0894 | 0.0927 | 0.0927 | 0.1089 | 0.1237 | 0.0887 |
| MAPE | 0.0734 | 0.0906 | 0.1179 | 0.1315 | 0.1531 | 0.1484 | 0.1669 | 0.2416 | 0.1395 |
| OptLL | 216.3430 | 211.5388 | 252.2146 | 334.4711 | 356.0208 | 438.7128 | 515.4908 | 559.3221 | 688.0683 |

Results are obtained with h_0^P estimated

| CALIBRATED PARAMETERS ON WEDNESDAYS, h_0^Q IS CALIBRATED WITH RESPECT TO OPTIONS LIKELIHOOD | | | | | | | | | |
|---|--------------------|--------------------|--------------------|--------------------|--------------------|--------------------|--------------------|--------------------|--------------------|
| θ | 2010 | 2011 | 2012 | 2013 | 2014 | 2015 | 2016 | 2017 | 2018 |
| ω | $1.2384e-08$ | $4.0086e-07$ | $3.1333e-08$ | $1.3755e-07$ | $6.9525e-08$ | $2.3119e-08$ | $4.5524e-08$ | $1.5096e-08$ | $1.8178e-08$ |
| ci | $(\pm 1.9830e-08)$ | $(\pm 4.3891e-07)$ | $(\pm 4.5975e-08)$ | $(\pm 1.3389e-07)$ | $(\pm 1.0399e-07)$ | $(\pm 3.3431e-08)$ | $(\pm 6.5163e-08)$ | $(\pm 2.0662e-08)$ | $(\pm 2.3115e-08)$ |
| median | $4.2614e-10$ | $1.2486e-09$ | $7.9886e-10$ | $1.2631e-09$ | $7.7874e-10$ | $1.1199e-09$ | $8.6919e-10$ | $5.7281e-10$ | $7.1012e-10$ |
| α | $1.8162e-05$ | $1.4839e-05$ | $8.9367e-06$ | $6.2985e-06$ | $8.1050e-06$ | $7.2957e-06$ | $4.6588e-06$ | $2.9109e-06$ | $1.3426e-05$ |
| ci | $(\pm 5.3349e-06)$ | $(\pm 5.5222e-06)$ | $(\pm 3.4023e-06)$ | $(\pm 2.2367e-06)$ | $(\pm 2.7073e-06)$ | $(\pm 2.0235e-06)$ | $(\pm 1.1279e-06)$ | $(\pm 1.1222e-06)$ | $(\pm 4.6039e-06)$ |
| median | $1.0695e-05$ | $7.6836e-06$ | $4.7572e-06$ | $3.3952e-06$ | $2.8549e-06$ | $4.3648e-06$ | $2.9570e-06$ | $1.4933e-06$ | $4.5110e-06$ |
| β | 0.6465 | 0.5668 | 0.7271 | 0.7325 | 0.6222 | 0.5594 | 0.6426 | 0.7106 | 0.5391 |
| ci | (± 0.0728) | (± 0.0805) | (± 0.0603) | (± 0.0703) | (± 0.0847) | (± 0.0689) | (± 0.0573) | (± 0.0781) | (± 0.1047) |
| median | 0.7430 | 0.6600 | 0.8060 | 0.8158 | 0.7748 | 0.6585 | 0.6903 | 0.8071 | 0.6888 |
| γ^* | 134.3603 | 195.1009 | 191.4698 | 217.4109 | 237.1588 | 270.9957 | 276.1619 | 324.0345 | 227.4457 |
| ci | (± 13.3942) | (± 27.1816) | (± 26.7061) | (± 41.1883) | (± 30.6111) | (± 33.9784) | (± 20.7518) | (± 31.8355) | (± 30.7136) |
| median | 127.2052 | 175.8919 | 171.5645 | 181.2201 | 221.1372 | 254.0407 | 294.1570 | 327.0867 | 198.2446 |
| h_0^Q | $1.2662e-04$ | $2.2087e-04$ | $8.4211e-05$ | $4.9742e-05$ | $4.9380e-05$ | 0.0001 | $6.8390e-05$ | $1.8939e-05$ | $1.3543e-04$ |
| ci | $(\pm 3.5965e-05)$ | $(\pm 6.3341e-05)$ | $(\pm 1.6174e-05)$ | $(\pm 1.2877e-05)$ | $(\pm 1.6179e-05)$ | $(\pm 3.1241e-05)$ | $(\pm 2.1089e-05)$ | $(\pm 5.3916e-06)$ | $(\pm 4.7933e-05)$ |
| median | $9.2058e-05$ | $1.1459e-04$ | $6.0478e-05$ | $3.5047e-05$ | $2.7422e-05$ | $5.5033e-05$ | $3.8411e-05$ | $1.3835e-05$ | $4.6850e-05$ |
| MSE | 0.6622 | 1.0575 | 1.0914 | 0.6991 | 1.0554 | 1.3990 | 1.6195 | 2.2744 | 4.8658 |
| IVRMSE | 0.0559 | 0.0659 | 0.0806 | 0.0776 | 0.0798 | 0.0917 | 0.0983 | 0.1006 | 0.0792 |
| MAPE | 0.0662 | 0.0726 | 0.1098 | 0.1032 | 0.1205 | 0.1355 | 0.1307 | 0.1651 | 0.1233 |
| OptLL | 226.0306 | 234.8200 | 265.2162 | 363.1728 | 389.5383 | 469.0620 | 572.8691 | 650.3873 | 729.6044 |