Numerical Methods for Finance

Second Order Discretization Schemes for CIR Processes

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Outline

Introduction

- What is the problem
- What does it matter?
- How we solved it = What we are going tell you

Problem setting

Noting X_t^x the solution of the CIR SDE:

$$dX_t^x = a - kX_t^x + \sigma\sqrt{X_t^x}dW_t$$
$$X_0^x = x, x \in \mathbb{R}_+$$

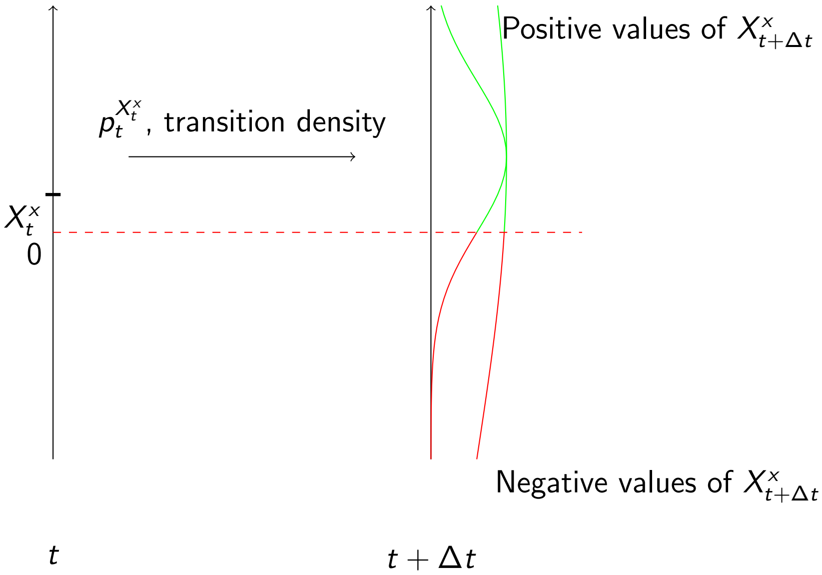
Qualitative analysis: when X is very close to 0, then the SDE becomes approximately

$$dX_t^x \approx a + \sigma\sqrt{X_t^x}dW_t$$

There are two possible regimes:

- If $\sigma << a$: X will mostly stay positive
- If $\sigma >> a$: X may become negative!

How usual numerical schemes can fail



Regime $\sigma << a$ ($\frac{\sigma^2}{4} \leq a$)

How do we keep the process positive and the scheme precise? Replace the original distribution by another with compact support. The higher the degree of precision, the more this variables has to account for the tail behaviour of the subsituted one.
Alfonsi shows [CITE] that we keep the scheme of order ν, if we subsitute the random variable with one that matches the first $2\nu + 1$ moments.
Using discrete random variables, we can control the tail probability and ensure the process stays positive.

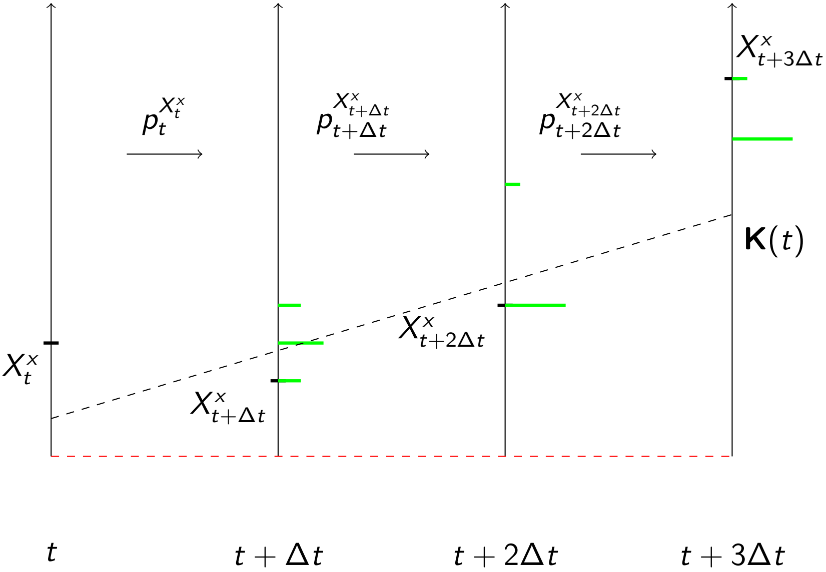
Regime $\sigma >> a$ ($\frac{\sigma^2}{4} > a$)

Alonfsi proves [CITE]:

- When X_t^x is far away from 0, then the scheme in the case $\sigma << a$ is also valid
- When X_t^x is close to 0, we can approximate the process by a *positive* discrete random variable that matches the first two moments and still have a second-order scheme

And we know when to switch between the two, via a threshold **K**.

Algorithm in practice



Dealing with Higher Order Schemes

Dealing with Higher Order Schemes

Simulation Results

Simulation Results

Using the Heston Model

Using the Heston Model

Conclusion

- First thing we did
- Second thing we did
- Third thing we did

Thank you!

github.com/tjespel/discretization-cir-processes