Numerical Methods for Finance

Second Order Discretization Schemes for CIR Processes

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Outline

1. Discretizing CIR Processes
2. Dealing with Higher Order Schemes
3. Simulation Results
4. Using the Heston Model

Introduction

- What is the problem
- What does it matter?
- How we solved it = What we are going tell you

1 Discretizing CIR Processes

2 Dealing with Higher Order Schemes

3 Simulation Results

4 Using the Heston Model

Problem setting

Noting X_t^x the solution of the CIR SDE:

$$dX_t^x = a - kX_t^x + \sigma\sqrt{X_t^x}dW_t$$
$$X_0^x = x, x \in \mathbb{R}_+$$

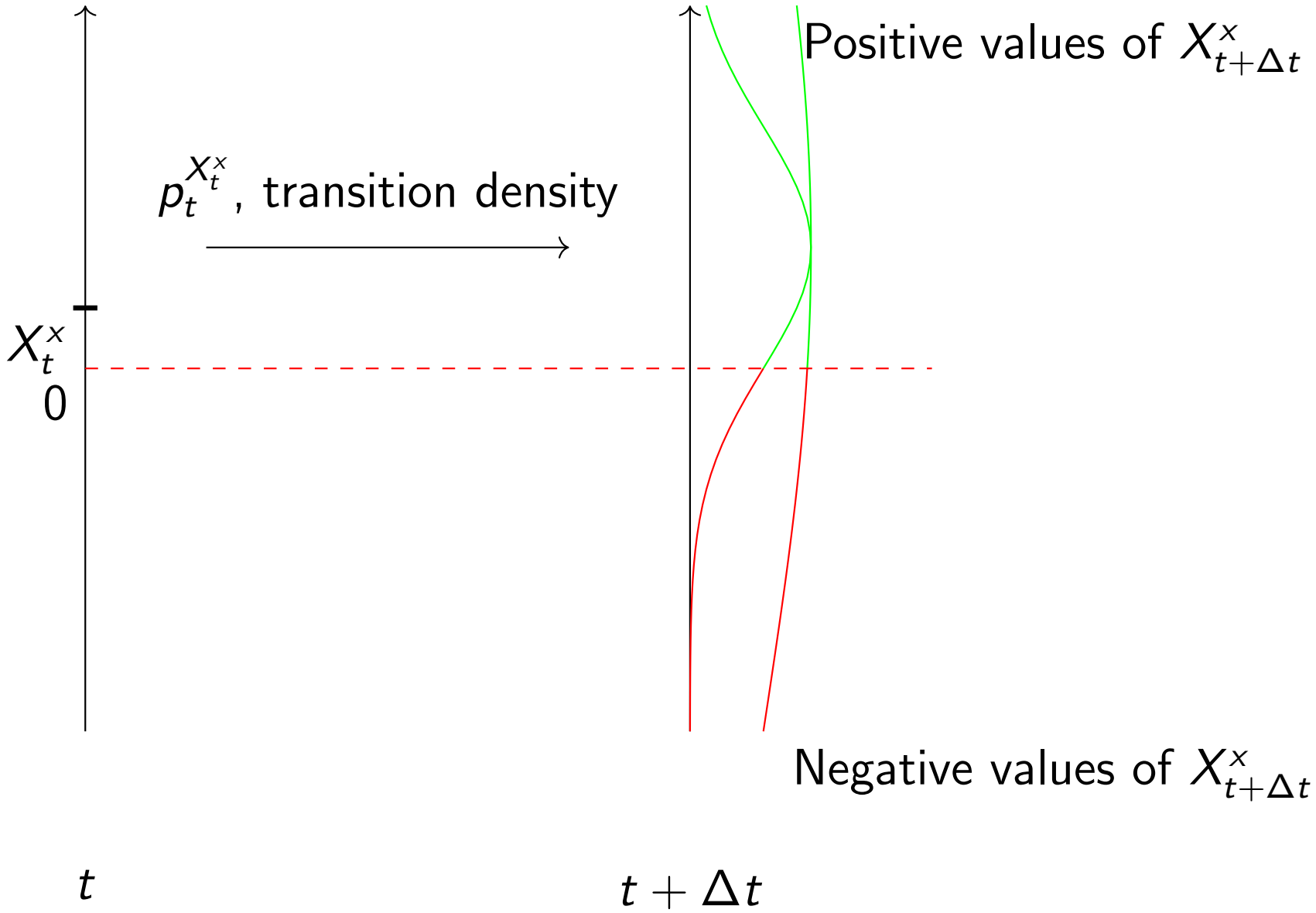
Qualitative analysis: when X is very close to 0, then the SDE becomes approximately

$$dX_t^x \approx a + \sigma\sqrt{X_t^x}dW_t$$

There are two possible regimes:

- If $\sigma << a$: X will mostly stay positive
- If $\sigma >> a$: X may become negative!

How usual numerical schemes can fail



Regime $\sigma << a$ ($\frac{\sigma^2}{4} \leq a$)

How do we keep the process positive and the scheme precise?
Replace the original distribution by another with compact support. The higher the degree of precision, the more this variables has to account for the tail behaviour of the subsituted one.
Alfonsi shows [CITE] that we keep the scheme of order ν, if we subsitute the random variable with one that matches the first $2\nu + 1$ moments.
Using discrete random variables, we can control the tail probability and ensure the process stays positive.

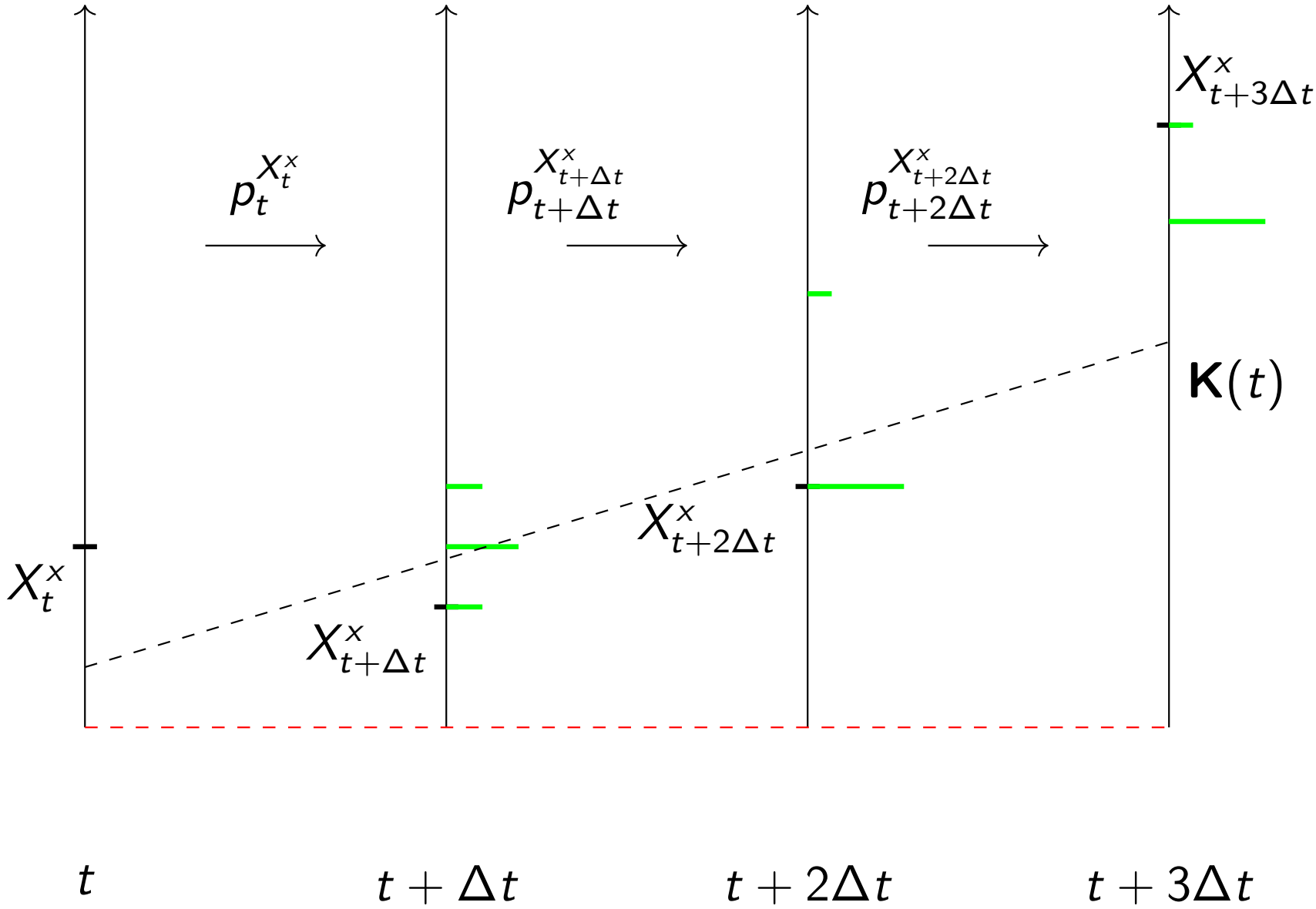
Regime $\sigma >> a$ ($\frac{\sigma^2}{4} > a$)

Alonfsi proves [CITE]:

- When X_t^x is far away from 0, then the scheme in the case $\sigma << a$ is also valid
- When X_t^x is close to 0, we can approximate the process by a *positive* discrete random variable that matches the first two moments and still have a second-order scheme

And we know when to switch between the two, via a threshold **K**.

Algorithm in practice



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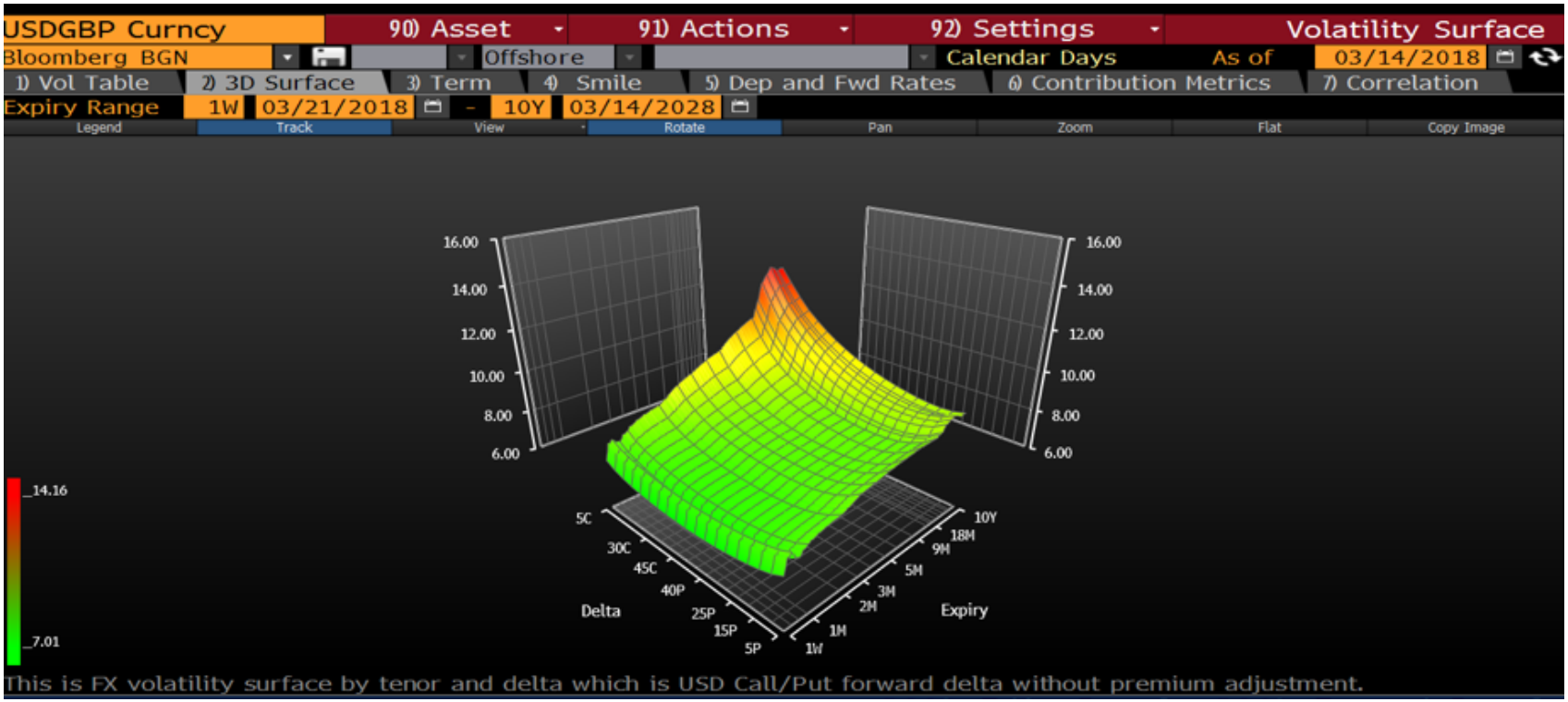
Using the Heston Model - Calibration

- In order to test the efficiency of the second order scheme, we calibrate the Heston Model using real data. In detail, we use a USDGBP volatility surface which was downloaded in Bloomberg.
- To calibrate the Heston model, we use the most liquid option in this market.

Using the Heston Model - Calibration



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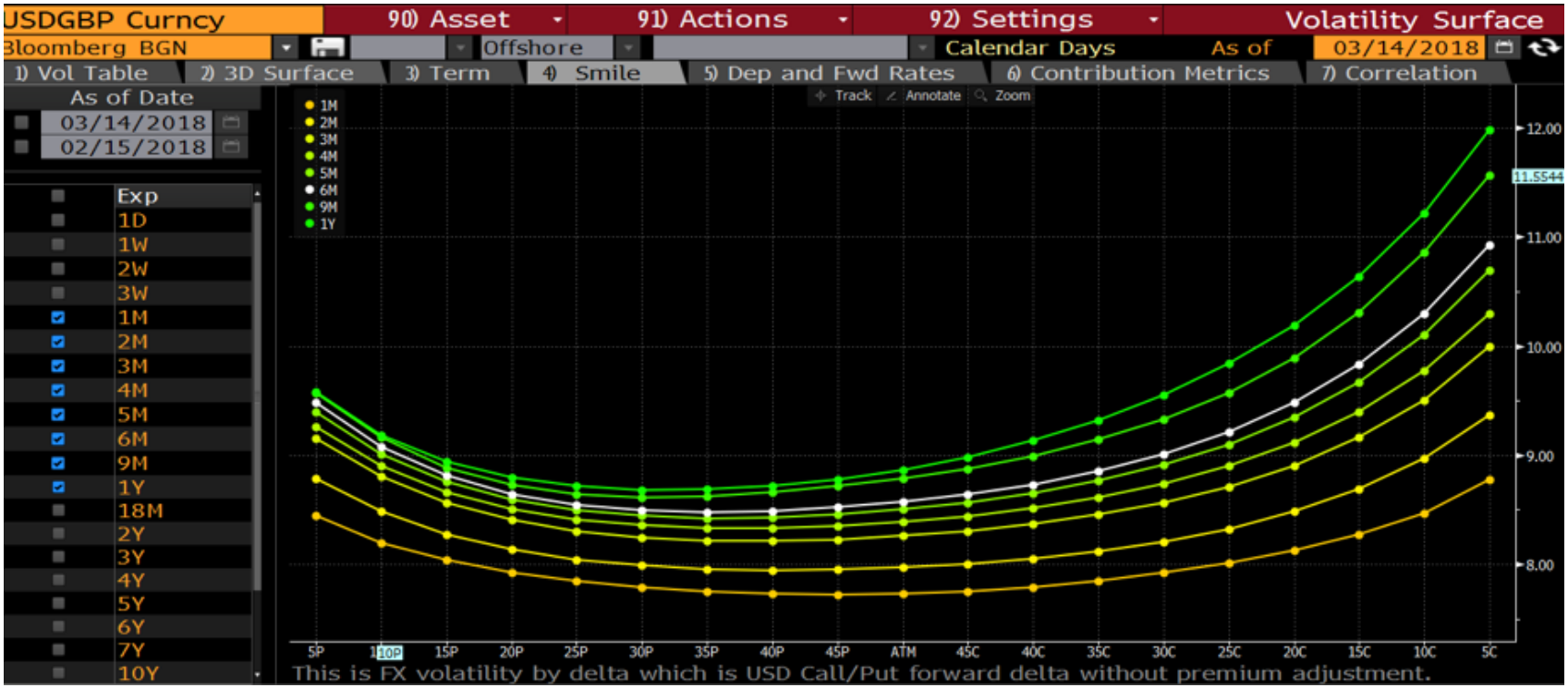

Using the Heston Model - Calibration

- 3) In the currency option market, prices are quoted for moneyness levels for different time to expiry periods. These moneyness levels are:
  - At the money level or at the money forward (50 delta dual),
  - Out of the money level at 25 delta dual,
  - In the money level (75 delta dual).
- Delta dual is the first derivative with respect to the strike price.

$$\Delta_{dual} = \frac{\partial BS_{call}}{\partial k} = e^{-r_d}\Phi\left(\frac{ln(S_0/K) + (r_d - r_f - \sigma^2(0.5))T}{\sigma\sqrt{T}}\right)$$

Using the Heston Model - Calibration

Basically, it is necessary to compute the associated strike for each option in order to compute the following equation:

$$\min_{\theta,\sigma,\rho,\kappa,\eta,\mu} = (BS(\sigma, r, K, S_0, T) - P_{heston}(\theta, \sigma, \rho, \kappa, \eta, \mu, r, K, S_0, T))^2$$

When the option is traded in the OTC market, each smile has an associated level of strike. When the option is traded in an exchange market all smiles have the same level of strike.

Using the Heston Model

Conclusion

- First thing we did
- Second thing we did
- Third thing we did

Thank you!

`github.com/tjespel/discretization-cir-processes`