We can solve prime counting using analytic algorithms in $O(n^{1/2+\epsilon})$ time [1, 2, 3]. Also, $n!$ mod p can be computed in $\tilde{O}(\sqrt{n})$ time by FFT, or $O(\sqrt{n}\log n)$ time https://www.cnblogs.com/zzqsblog/p/8408691.html. In total $O(n^{1/2+\epsilon})$ time.

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