let m be the length of the expression and n be the length of the string.
1. DP. $O(nm)$.
2. $O(\frac{nm \log\log n}{\log^{3/2} n} + n + m)$ for general regular expression matching [1]. for a fixed size alphabet, we can shave the $\log\log n$.
it seems that this is still an open problem.

note. for deterministic regular expressions, the running time can be improved [2].

References

[1] Philip Bille and Mikkel Thorup. Faster regular expression matching. In *International Colloquium on Automata, Languages, and Programming*, pages 171–182. Springer, 2009.

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