

By the law of diminishing returns, greedily assign the brilliant students to the class with the current smallest pass ratio. $O(n)$ or $O(n + k)$ using median selection (similar to 1283. Find the Smallest Divisor Given a Threshold).

Let the pass ratio of a class be $\frac{a}{b}$, let $d = b - a$, and let k denote the number of extra students. Suppose the marginal gain by adding one student is y . Solve $\frac{x+1}{x+d+1} - \frac{x}{x+d} = y$, we get the number of brilliant students needed is $\lfloor x - a + 1 \rfloor = \lfloor -b + \frac{1}{2} + \sqrt{\frac{d}{y} + \frac{1}{4}} \rfloor$ when $y > \frac{x}{x+d}$. To find an initial approximation for y with $O(n)$ additive error, such that $\sum_i \lfloor x_i - a_i + 1 \rfloor < k$, set y to be the solution of $\sum_{i: x_i \geq a} \sqrt{\frac{d_i}{y}} - b_i + 1 = k - 1$, which again can be found using median finding.

References