

1. mincost flow with upper and lower flow bounds.  $O(\text{mincost-flow}(n, n^2))$ .
2. This is the minimum cost edge cover problem for bipartite graph (with non-negative weights), which can be reduced to minimum cost bipartite perfect matching.  
<https://cstheory.stackexchange.com/questions/14690/reducing-a-minimum-cost-edge-cover-problem-to-minimum-cost-weighted-bipartite-perfect-matching>  
 $O(n^3)$  using KM,  $O(m\sqrt{n}\log(nW))$  [2],  $\tilde{O}(m^{4/3+o(1)}\log W)$  [1] for sparse graphs, or  $\tilde{O}((m+n^{1.5})\log^2 W)$  [3].

## References

- [1] Kyriakos Axiotis, Aleksander Madry, and Adrian Vladu. Circulation control for faster minimum cost flow in unit-capacity graphs. *arXiv preprint arXiv:2003.04863*, 2020.
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- [3] Jan van den Brand, Yin-Tat Lee, Danupon Nanongkai, Richard Peng, Thatchaphol Saranurak, Aaron Sidford, Zhao Song, and Di Wang. Bipartite matching in nearly-linear time on moderately dense graphs. *arXiv e-prints*, pages arXiv–2009, 2020.