bit operation. if without preprocessing:
1. deal with the carry bit by bit, $O(\log W)$.
2. partition into blocks with size $O(\sqrt{\log W})$, deal with the carry between blocks and within blocks separately, use divide and conquer+parallel add. $O(\sqrt{\log W})$.
3. use multiplication, $(a \cdot 2^w + 1) \cdot (b \cdot 2^w + 1) = ab \cdot 2^{2w} + (a + b) \cdot 2^w + 1$. $O(1)$.
for analysis, see my article https://zhuanlan.zhihu.com/p/72730434.

References