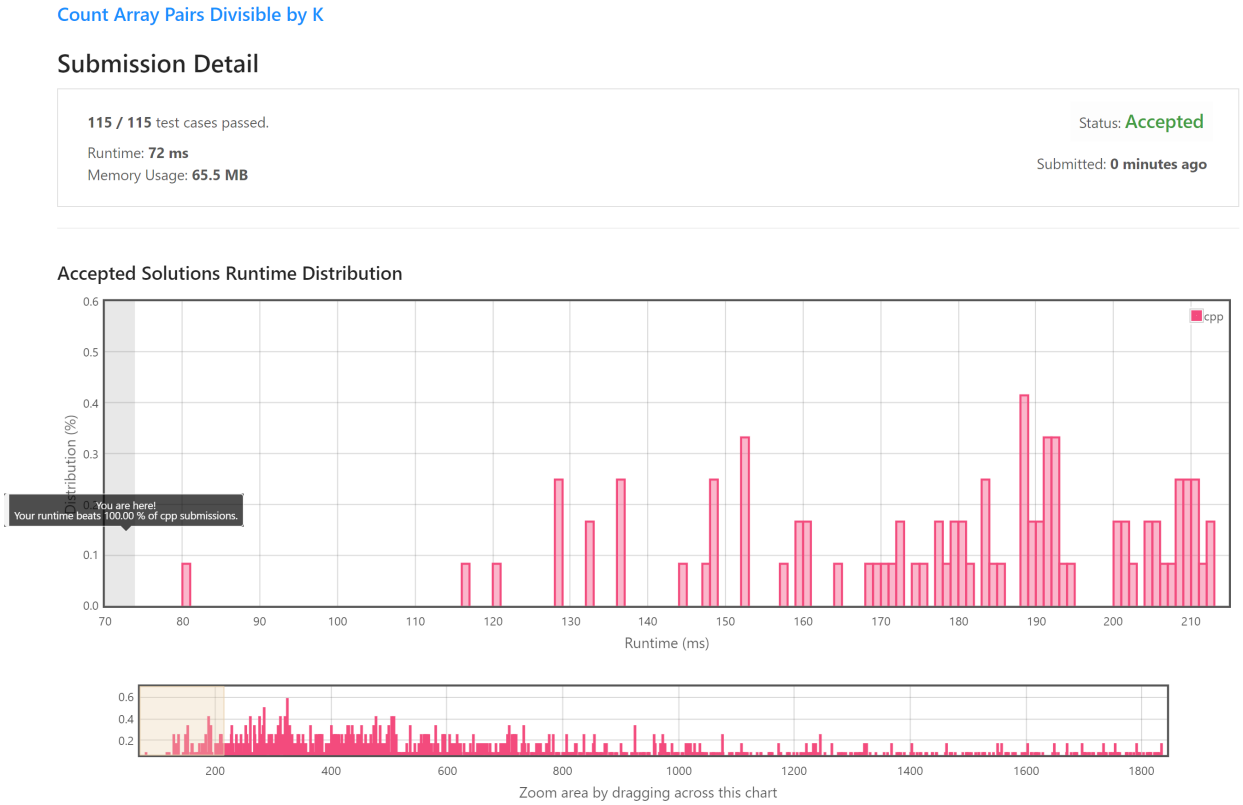
1. Let $f[i]$ denote the number of input numbers that are divisible by i. We only need to compute $f[i]$ for each divisor i of k, which takes $\sum_{i:i|k} \frac{U}{i} = \sigma(k)$ time. We can compute gcd in $O(1)$ time after $O(U)$ preprocessing. $O(U \log \log U)$.
2. Put input number x into bucket $\gcd(x, k)$, then enumerate all pairs of buckets in $d^2(k) = o(U)$ time. $O(U)$.



References