

1. $O(n)$ sieve algorithm.
2. $O(\frac{n^{2/3}}{\log^2 n})$ time, $O(n^{1/3} \log^3 n \log \log n)$ space [1].
[https://en.wikipedia.org/wiki/Prime-counting_function#Algorithms_for_evaluating_%CF%80\(x\)](https://en.wikipedia.org/wiki/Prime-counting_function#Algorithms_for_evaluating_%CF%80(x))
 The space complexity is later improved to $O(n^{1/3} \log^2 n)$ [2].

References

- [1] Marc Deléglise and Joël Rivat. Computing $\pi(x)$: the meissel, lehmer, lagarias, miller, odlyzko method. *Mathematics of Computation of the American Mathematical Society*, 65(213):235–245, 1996.
- [2] Douglas B. Staple. The combinatorial algorithm for computing $\pi(x)$. *arXiv e-prints*, page arXiv:1503.01839, Mar 2015.