

let n be the number of rows and m be the number of columns, and wlog assume $n \geq m$.

1. $O(m^2n)$.

2. This can be reduced to the problem of counting C_4 in a graph (i.e. cycles with length 4). The vertices are the rows and the columns, and the edges are the grid entries with value 1.

$O(n^\omega)$ using matrix multiplication [1], or $O(|E|^{\frac{4}{3}}) = O(n^{\frac{8}{3}})$ [2] (combinatorial algorithm; need the counting version).

<https://chaoxuprime.com/posts/2015-02-02-rectangle-in-point-set.html>

remark. for arbitrary n points, finding axis-parallel rectangles takes $O(n^{2-1/d}) = O(n^{\frac{3}{2}})$ time [3], and also $O(n^{4/3})$ [2].

References

- [1] Noga Alon, Raphael Yuster, and Uri Zwick. Color-coding: a new method for finding simple paths, cycles and other small subgraphs within large graphs. In *STOC*, volume 94, pages 326–335, 1994.
- [2] Noga Alon, Raphael Yuster, and Uri Zwick. Finding and counting given length cycles. *Algorithmica*, 17(3):209–223, 1997.
- [3] Marc J Van Kreveld and Mark T De Berg. Finding squares and rectangles in sets of points. *BIT Numerical Mathematics*, 31(2):202–219, 1991.