

This is the Klee's measure problem.

1. segment tree. $O(n \log n)$.

$\Omega(n \log n)$ lower bound (under the usual model of computation) [1].

References

- [1] Michael L Fredman and Bruce Weide. On the complexity of computing the measure of $[a_i, b_i]$. *Communications of the ACM*, 21(7):540–544, 1978.