

1. DP, let $f[i][j][\ell]$ denote whether $s1[i, \dots, j]$ can be matched with $s2[\ell, \dots, \ell + j - i]$. Enumerate the breakpoint in $O(n)$ time. In total $O(n^4)$.
2. Use bit packing. $O(\frac{n^4}{w})$.
3. The idea is to use boolean matrix multiplication (BMM). Wlog consider the case that we don't flip the string (the flip case is similar). Fix $i - \ell = d$, let $g[i][j] = f[i][j][i - d]$, we simultaneously consider all such $g[i][j]$'s. The transition function is $g[i][j] = \bigvee (g[i][k] \wedge g[k + 1][j])$, which is exactly BMM. But we successfully compute $g[i][j]$ only when the values of $g[i][k]$ and $g[k + 1][j]$ have been already computed, and the dependency chain can have depth $O(n)$. To fix this, let α be a parameter to be fixed later, if $\min\{k - i + 1, j - k\} \leq n^\alpha$ then we compute $g[i][k]$ by brute force, in $O(n^\alpha)$ time. Now we only need to perform $O(n^{1-\alpha})$ rounds of BMMs, each round consists of $O(n)$ BMMs (because there are $O(n)$ possible d 's). Set $\alpha = \frac{\omega-1}{2}$, the total running time is $O(n^3 \cdot n^\alpha + n^{1-\alpha} \cdot n \cdot n^\omega) = O(n^{\frac{5+\omega}{2}}) \approx O(n^{3.69})$.

The transition function is

$$f[i][j][\ell] = \bigvee (f[i][k][\ell] \wedge f[k + 1][j][k + 1 - i + \ell]) \bigvee (f[i][k][\ell + j - k] \wedge f[k + 1][j][\ell + j - k - 1]).$$

For the second part, fix $\ell + j = d$, we can also use BMM.

References