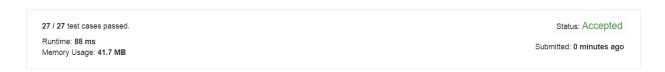
in the following assume m = n.

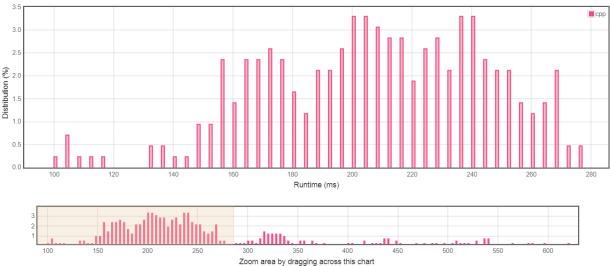
- 1. use Boyer-Moore voting algorithm, maintain the information (number, count) by segment tree. $O(n \log n)$.
- 2. the information we maintain forms an (associative) semigroup, so it's mergeable. we need to query range sum on a static array, which needs $O(n\alpha(n))$ by Yao [5]. [1] also gives a $\Theta(n\lambda(k,n))$ (= $O(n\alpha(n))$ for our purpose) time and space algorithm, where $\lambda(k,\cdot)$ is the inverse of a certain function at the $\lfloor \frac{k}{2} \rfloor$ -th level of the primitive recursive hierarchy. Then check whether the number we find is valid, by computing the number of occurrence of it in the query interval, using persistent array (or vEB tree) in $O(\log \log n)$. We can also solve this in $O(\frac{\log \log n}{\log \log \log n})$ per query by reducing to the static predecessor problem https://en.wikipedia.org/wiki/Predecessor_problem [2] (but with $O(n^4)$ preprocessing time; in our case the universe N=n). In conclusion we get $O(n \log \log n)$.

We can also get O(n) preprocessing and O(1) per query, see my article here: https://zhuanlan.zhihu.com/p/79423299.

- 3. $O(n \log n)$ preprocessing, O(1) per query [3, 4].
- 4. If we randomly select $O(\log n)$ elements in the array and check the count in $O(\log \log n)$, we can get w.h.p. $O(n \log n \log \log n)$.







Runtime: 88 ms, faster than 100.00% of C++ online submissions for Online Majority Element In Subarray.

Memory Usage: $41.7\,$ MB, less than 100.00% of C++ online submissions for Online Majority Element In Subarray.

References

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