Let n denote the length of the input string, m denote the number of strings in the dictionary, and L denote the total length of the dictionary.
1. DP. $O(nm)$.
2. let $f[i]$ denote whether the prefix $s[1..i]$ can be segmented into a sequence of dictionary words. use Aho-Corasick automation to compute f. at each position, we need to walk to the root according to the failure pointers, and there are at most $O(\min\{m, \sqrt{L}\})$ steps. $O(n \cdot \min\{m, \sqrt{L}\} + L)$.
3. $\tilde{O}(nL^{\frac{1}{3}} + L)$ [1], and there is also a matching conditional lower bound for combinatorial algorithms.

Remark. Algorithm 2 can be generalized to the min-cost version (with arbitrary weights) using the same running time. `https://chaoxuprime.com/posts/2019-09-19-word-break-with-cost.html`
Algorithm 3 can also be generalized to solve the minimum word break problem (with weight 1) using the same running time, by replacing FFT with an output-sensitive algorithm for a special case of $(\min, +)$-convolution. see my paper [2].

References

[1] Karl Bringmann, Allan Grønlund, and Kasper Green Larsen. A dichotomy for regular expression membership testing. In *2017 IEEE 58th Annual Symposium on Foundations of Computer Science (FOCS)*, pages 307–318. IEEE, 2017.

[2] Timothy M. Chan and Qizheng He. More on change-making and related problems. In *28th Annual European Symposium on Algorithms (ESA)*, 2020.