

in the following assume $m = n$.

1. use Boyer-Moore voting algorithm, maintain the information (number, count) by segment tree. $O(n \log n)$.
2. the information we maintain forms an associative semigroup, so it's mergeable. we need to query range sum on a static array, which needs $O(n\alpha(n))$ by Yao [4]. [1] also gives a $\Theta(n\lambda(k, n))$ ($= O(n\alpha(n))$ for our purpose) time and space algorithm, where $\lambda(k, \cdot)$ is the inverse of a certain function at the $\lfloor \frac{k}{2} \rfloor$ -th level of the primitive recursive hierarchy. then check by computing the number of occurrence of the number we find in the query interval, using persistent array (or vEB tree) in $O(\log \log n)$. We can also solve this in $O(\frac{\log \log n}{\log \log \log n})$ by reducing to the static predecessor problem https://en.wikipedia.org/wiki/Predecessor_problem [2]. In conclusion we get $O(n \frac{\log \log n}{\log \log \log n})$.
3. $O(n \log n)$ preprocessing, $O(1)$ per query [3].

References

- [1] Noga Alon and Baruch Schieber. *Optimal preprocessing for answering on-line product queries*. Citeseer, 1987.
- [2] Paul Beame and Faith E Fich. Optimal bounds for the predecessor problem. In *STOC*, volume 99, pages 295–304. Citeseer, 1999.
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- [4] Andrew C Yao. Space-time tradeoff for answering range queries. In *Proceedings of the fourteenth annual ACM symposium on Theory of computing*, pages 128–136. ACM, 1982.