

1. online, reduce to the partial sums problem, in the special case that the number of bits in an update is  $O(1)$ .  $O(\frac{\log n}{\log \log n})$  [2], or reduce to the dynamic integer set problem,  $O(\frac{\log n}{\log w})$  [3].
2. offline, reduce to offline dominance (or orthogonal range) counting in 2D.  $O(n\sqrt{\log n})$  ( $O(n \log^{d-2+1/d} n)$  when  $d = 2$ ) [1].

## References

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- [2] Paul F Dietz. Optimal algorithms for list indexing and subset rank. In *Workshop on Algorithms and Data Structures*, pages 39–46. Springer, 1989.
- [3] Mihai Patrascu and Mikkel Thorup. Dynamic integer sets with optimal rank, select, and predecessor search. In *2014 IEEE 55th Annual Symposium on Foundations of Computer Science*, pages 166–175. IEEE, 2014.