

let  $n$  be the number of rows and  $m$  be the number of columns, and wlog assume  $n \geq m$ .

1.  $O(m^2n)$ .
2. This can be reduced to the problem of counting  $C_4$  in a graph (i.e. cycles with length 4). The vertices are the rows and the columns, and the edges are the grid entries with value 1.  $O(n^\omega)$  using matrix multiplication [1], or  $O(|E|^{\frac{4}{3}}) = O(n^{\frac{8}{3}})$  [2] (combinatorial algorithm).  
<https://chaoxuprime.com/posts/2015-02-02-rectangle-in-point-set.html>

remark. for arbitrary  $n$  points, finding axis-parallel rectangles takes  $O(n^{2-1/d}) = O(n^{\frac{3}{2}})$  time [3], and also  $O(n^{4/3})$  [2].

## References

- [1] Noga Alon, Raphael Yuster, and Uri Zwick. Color-coding: a new method for finding simple paths, cycles and other small subgraphs within large graphs. In *STOC*, volume 94, pages 326–335, 1994.
- [2] Noga Alon, Raphael Yuster, and Uri Zwick. Finding and counting given length cycles. *Algorithmica*, 17(3):209–223, 1997.
- [3] Marc J Van Kreveld and Mark T De Berg. Finding squares and rectangles in sets of points. *BIT Numerical Mathematics*, 31(2):202–219, 1991.