

1. counting for each bit.  $O(nw)$ .

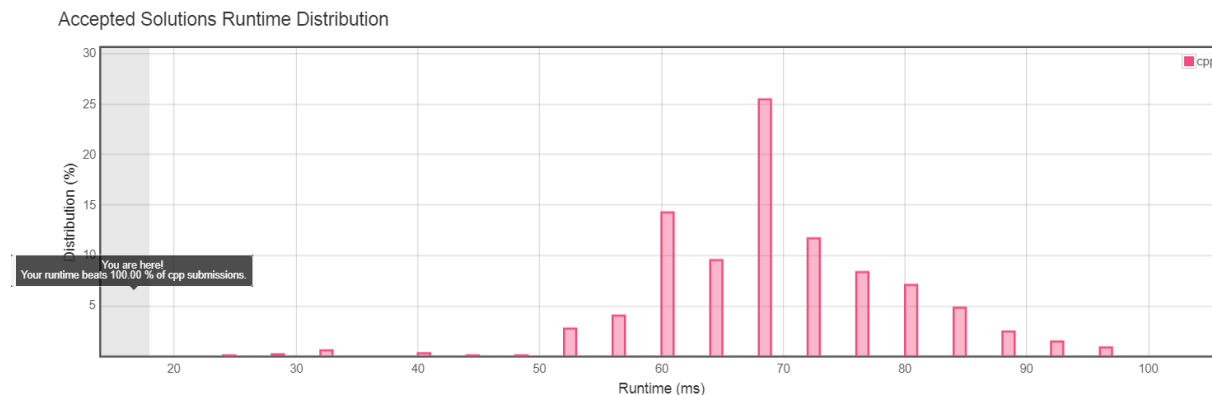
2. for each bit, count the number of 0s and 1s in the array, which needs  $O(\log n)$  bits of space to store if the array has length  $n$ . assume word operation is  $O(1)$  after preprocessing (the word operations we want: (1) add two  $w_1$ -bit numbers, (2) padding, i.e. pad  $w_1$  bits with leading 0 to get  $w_2$  bits), the counting for each bit can be performed in parallel by word operations.

Use divide and conquer to perform counting, let  $f(n)$  denote the running time to get the count for each bit when the array has length  $n$ . merge the results for two arrays with length  $\frac{n}{2}$  needs  $O(w \cdot \log n)$  bits, i.e.  $O(\log n)$  word operations.

$f(n) = 2f(n/2) + O(\log n)$ , i.e.  $f(n) = \sum_{i=0}^{\log n} 2^{\log n - i} \cdot i = O(n)$ .

total time  $O(n)$ .

actually if we represent the counters by  $O(\log n)$  words, where the  $j$ -th bit of the  $i$ -th word represent the  $i$ -th bit of the  $j$ -th counter, we can construct the word operations we want by  $O(1)$  basic word operations. The following C++ implementation uses the logarithmic method [1] to reduce the space complexity to  $O(\log^2 n)$ .



## References

- [1] Jon Louis Bentley and James B Saxe. Decomposable searching problems i. static-to-dynamic transformation. *Journal of Algorithms*, 1(4):301–358, 1980.