

1. reduce to distinct element.

2. reduce to integer sorting. after sorting, only need $O(n)$ by monotone pointers.

for integer sorting:

$O(n\sqrt{\log \log n})$ randomized [1].

$O(n \log \log n)$ deterministic [2].

References

- [1] Y Han and M Thorup. Sorting integers in $o(n \log \log n)$ expected time and linear space. In *IEEE Symposium on Foundations of Computer Science (FOCS02)*, 2002.
- [2] Yijie Han. Deterministic sorting in $o(n \log \log n)$ time and linear space. In *Proceedings of the thirty-fourth annual ACM symposium on Theory of computing*, pages 602–608. ACM, 2002.