This is the Stirling number of the second kind. $O(n \log n)$ using FFT.

$$\left\{ n \atop k \right\} = \frac{1}{k!} \sum_{i=0}^{k} (-1)^i \frac{k!}{i!(k-i)!} (k-i)^n = \sum_{i=0}^{k} \frac{(-1)^i}{i!} \frac{(k-i)^n}{(k-i)!}.$$

Let $A(x) = \sum_{i=0}^{n} \frac{(-1)^i}{i!} x^i$, $B(x) = \sum_{i=0}^{n} \frac{i^n}{i!} x^i$, then $\left\{ n \atop k \right\} = [x^k] A(x) B(x)$.

# References