Let $U$ be the range of $a[i]$, and $t = \sum_{i=1}^{n} a[i]$. $S \leq t \leq nU$.
1. DP. $f[i][j]$ denote the number of ways for the first $i$ integers sum up to $j$. $O(n^2U)$.
2. reduce to integer subset sum (counting version). assume the output fits in a word, we do not need mod $p$. $O(n + t \log t)$ [1].

# References


[1] Ce Jin and Hongxun Wu. A simple near-linear pseudopolynomial time randomized algorithm for subset sum. *arXiv preprint arXiv:1807.11597*, 2018.