let m be the length of the expression and n be the length of the string.
1. DP. $O(nm)$.
2. $O(\frac{nm \log\log n}{\log^{3/2} n} + n + m)$ for general regular expression matching [2]. for a fixed size alphabet, we can shave the $\log\log n$.

this is the $\circ|*$-membership problem. conditional lower bound based on SETH: $\Omega((nm)^{1-o(1)})$ [1, 3].

note. for deterministic regular expressions, the running time can be improved [4].

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