

We can reduce this problem to dynamic closest pair. [2] gave a technique to reduce dynamic closest pair to dynamic nearest neighbor queries, with an  $O(\log^2 n)$  factor slowdown. Also see [1]. When the distance metric is Manhattan, dynamic nearest neighbor can be done in  $O(n \text{ polylog } n)$ , by changing the coordinate system and reducing to the axis-aligned case.

## References

- [1] Timothy M Chan. Dynamic generalized closest pair: Revisiting eppstein’s technique. In *Symposium on Simplicity in Algorithms*, pages 33–37. SIAM, 2020.
- [2] David Eppstein. Dynamic euclidean minimum spanning trees and extrema of binary functions. *Discrete & Computational Geometry*, 13(1):111–122, 1995.