Maximum independent set, reduce to maximum matching on bipartite planar graphs. Hopcroft-Karp, or $O(nm\log^3(nm))$ [1].

References

[1] Glencora Borradaile, Philip N Klein, Shay Mozes, Yahav Nussbaum, and Christian Wulff-Nilsen. Multiple-source multiple-sink maximum flow in directed planar graphs in near-linear time. SIAM Journal on Computing, 46(4):1280–1303, 2017.