divide and conquer, compare the middle elements of two arrays, and recurse. $O(\log(n+m))$.

in general, finding the t-th largest element in the union of k sorted array with respective sizes n_1, \ldots, n_k takes time: 1. $O(\sum_{i=1}^{k} \log n_i)$ [1].

output sensitive version: $O(k + \sum_{i=1}^{k} \log(t_i + 1))$, where t_i is the number of items of the *i*-th list within the t-th largest elements [2].

https://cstheory.stackexchange.com/questions/20944/select-in-union-of-sorted-arrays-already-known/ 20955#20955.

2. let $p = \min\{k, t\}$, the running time is $\Theta(k + p \log \frac{t}{n})$ [3]. i.e. if $t \ge k$, $O(k \log \frac{t}{k})$. if t < k, O(k).

References

- [1] Greg N Frederickson and Donald B Johnson. Generalized selection and ranking: sorted matrices. SIAM Journal on computing, 13(1):14-30, 1984.
- [2] Haim Kaplan, László Kozma, Or Zamir, and Uri Zwick. Selection from heaps, row-sorted matrices and x + y using soft heaps. arXiv preprint arXiv:1802.07041, 2018.
- [3] Andranik Mirzaian and Eshrat Arjomandi. Selection in x+ y and matrices with sorted rows and columns. Information processing letters, 20(1):13–17, 1985.