1. This is a special case of the min-weight triangle finding problem (on dense undirected graphs with node weights in $[n]$).
It is no harder than the unweighted triangle detection problem, which can be solved in $O(n^{\omega}) \approx O(n^{2.373})$ time [3]. (see [4, 5]. By binary search we can reduce to node-weighted negative weight triangle detection.)
We can also solve in $O(n^{\frac{3+\omega}{2}}) \approx O(n^{2.69})$ time by dividing into blocks of size $b = O(n^{\frac{\omega-1}{2}})$ and using boolean matrix multiplication. Let b be a parameter, the total running time is $O(\frac{n}{b} \cdot \mathcal{M}(n, b, n) + n^2 \cdot \frac{b}{w})$ [2, 1].
If we use the $O(\frac{n^3}{w \log n})$-time combinatorial BMM algorithm, then the total running time is $O(\frac{n^3}{w \log n})$ when we set $b = \frac{n}{\log n}$.
2. For each pair (i, j) of vertices, we can detect whether there exists k that together form a triangle with less weight than the current best solution, using bitset in $O(\frac{n}{w})$ time. The solution can change $O(n)$ times, and each time we use $O(n)$ time to find such k. $O(\frac{n^3}{w})$.

68 / 68 test cases passed.

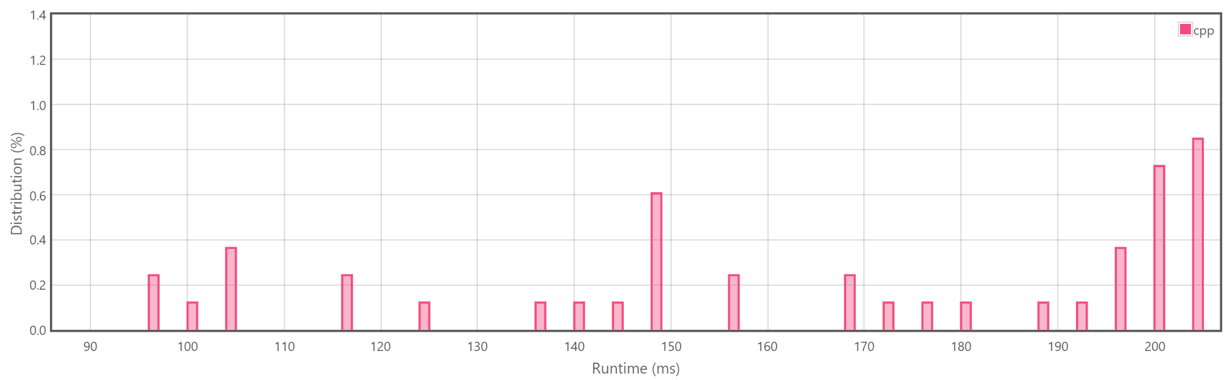
Runtime: **64 ms**

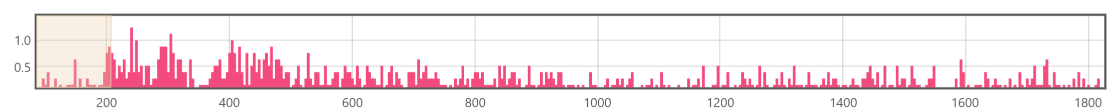
Memory Usage: **39.9 MB**

Status: Accepted

Submitted: **0 minutes ago**

Accepted Solutions Runtime Distribution





Zoom area by dragging across this chart

Runtime: 64 ms, faster than 100.00% of C++ online submissions for Minimum Degree of a Connected Trio in a Graph.

Memory Usage: 39.9 MB, less than 85.09% of C++ online submissions for Minimum Degree of a Connected Trio in a Graph.

References

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[4] Virginia Vassilevska and Ryan Williams. Finding, minimizing, and counting weighted subgraphs. In *Proceedings of the forty-first annual ACM symposium on Theory of computing*, pages 455–464, 2009.

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