

Use the (existential) marked ancestor problem to decide whether there's any marked ancestor in $O(\frac{\log n}{\log \log n})$ time [1], and use vEB + dfs sequence to find the locked descendants in amortized $O(\log \log n)$ time. Amortized $O(\frac{\log n}{\log \log n})$ per operation.

References

- [1] Stephen Alstrup, Thore Husfeldt, and Theis Rauhe. Marked ancestor problems. In *Proceedings 39th Annual Symposium on Foundations of Computer Science (Cat. no. 98CB36280)*, pages 534–543. IEEE, 1998.