dfs.

This problem is NP-Complete, because we can reduce 3-SAT to this.

- 1. Force  $|\leftrightarrow|$ , by setting the pattern to be  $|\cdots|$  and the string to be  $|\cdots|$ .
- 2. Assume we have already constructed a 1-1 mapping between k variables in the pattern and  $\{1^i|1 \le i \le k\}$  in the string. We can construct a 1-1 mapping between k' new variables in the pattern and  $\{1^i|k+1 \le i \le k\}$
- $i \le k + k'$ } in the string, using the following structure:  $|v_1 \dots v_{k'}| \leftrightarrow |1^{\sum_{i=k+1}^{k+k'}}|$ . Therefore,  $|xx'| \leftrightarrow |1^{2\ell_x+1}|$  ensures either  $x = 1^{\ell_x}$  (indicating x is false) or  $x = 1^{\ell_x+1}$  (x is true). For the i-th clause in 3-SAT, we can create a corresponding variable  $t_i$ , and force  $t_i = 1^j$ , where j satisfies  $\ell_i \le j \le \ell_i + 2$ .
- 3. Now, the *i*-th clause  $(x \vee y \vee z)$  in the 3-SAT instance is mapped to  $|xyzt_i|$  in the pattern, and set the corresponding part of the string to be  $|1^{\ell_x+\ell_y+\ell_z+\ell_i+3}|$ . The total length of the string is  $O(n^2)$ .

## References