

assume an ugly number can be stored in a word.

1. heap+hashing.  $O(n \log n)$ .

2. for each prime 2/3/5, maintain a pointer pointing to the ugly number which can multiply with the prime to generate the next ugly number.  $O(n)$ .

3. there are  $O(\log^3 x)$  ugly numbers that  $\leq x$ , and here  $n = O(\log^3 x)$ . implicitly construct an  $O(\log x) \times O(\log x) \times O(\log x)$  3D sorted matrix, where entry  $(i, j, k)$  denote  $2^i \cdot 3^j \cdot 5^k$ . we can find the  $n$ -th ugly number in  $O(n^{\frac{2}{3}} \log n)$ , by finding the  $n$ -th smallest element in  $O(\log x) \times O(\log x)$  sorted lists. (we can possibly remove the  $\log n$  factor)

see 378. Kth Smallest Element in a Sorted Matrix and 4. Median of Two Sorted Arrays.

## References