

1. Let $a[i][j] = (\# \text{occurrences of character } j \text{ in } s[1..i]) \bmod 2$. $s[i..j]$ can be rearranged to become a palindrome iff $a[i-1]$ and $a[j]$ differ at at most 1 coordinate. Enumerate that coordinate and use bitmask hashing. $O(\min\{n, 2^{|\Sigma|}\} \cdot |\Sigma|)$.
2. let t be a parameter to be set later. We first consider the case that the different coordinate is within $1, \dots, t$. Group the vector $a[i]$'s using the coordinates $t+1, \dots, |\Sigma|$, and solve each group separately. For each group, we can (pre)sort the indices in that group, and there are $n_i \leq 2^t$ indices, so there are at most $(2^t)!$ possible orders. So we can precompute the solution in $\text{poly}((2^t)!) = e^{O(t2^t)}$ time, and query in $O(n_i)$ time. Set $t = O(\log \log n)$, the total running time is $O(\min\{n, 2^{|\Sigma|}\} \cdot \frac{|\Sigma|}{\log \log n})$.

References