By the theorem on the sum of four squares, the answer is at most 4. https://en.wikipedia.org/wiki/Lagrange%27s_four-square_theorem
By Legendre's three-square theorem, the answer is at most 3 iff n is not of the form $n = 4^a(8b + 7)$ for integers a and b.
determine whether n is a perfect square is $O(1)$. so we only need to determine whether n is the sum of 2 perfect squares. this needs $O(\sqrt{n})$.

notes.
[1] gives a randomized $O(\log^2 n)$ algorithm to represent n as the sum of 4 perfect squares.
Fermat's theorem on sums of two squares: https://en.wikipedia.org/wiki/Fermat%27s_theorem_on_sums_of_two_squares
Brahmagupta-Fibonacci identity: https://en.wikipedia.org/wiki/Brahmagupta%E2%80%93Fibonacci_identity

References

[1] Michael O Rabin and Jeffery O Shallit. *Randomized algorithms in number theory.* 1985.