

1. hashing.  $O(n)$  expected.
2. reduce to integer sorting. after sorting, only need  $O(n)$  time by monotone pointers.  
for integer sorting:  
 $O(n\sqrt{\log \log n})$  randomized [2].  
 $O(n \log \log n)$  deterministic [3].
3. for the lower bound,  $\Omega(n \log n)$  in the degree- $d$  algebraic decision tree model [1]. we can also reduce the set intersection problem to this, i.e. deciding whether two sets  $\{x_1, \dots, x_n\}$  and  $\{y_1, \dots, y_n\}$  intersect, by negating one of the sets.

## References

- [1] Alexander C. L. Chan, William I. Gasarch, and Clyde P. Kruskal. Refined upper and lower bounds for 2-sum. 2004.
- [2] Y Han and M Thorup. Sorting integers in  $o(n \log \log n)$  expected time and linear space. In *IEEE Symposium on Foundations of Computer Science (FOCS02)*, 2002.
- [3] Yijie Han. Deterministic sorting in  $o(n \log \log n)$  time and linear space. In *Proceedings of the thirty-fourth annual ACM symposium on Theory of computing*, pages 602–608. ACM, 2002.