

reduce to integer subset sum (counting version).

Let U be the range of $a[i]$, $\sigma = \sum_{i=1}^n a[i]$, and the subset sum we want is $t = \frac{\sigma - S}{2}$. $S \leq \sigma \leq nU$.

1. DP. $f[i][j]$ denote the number of ways for the first i integers sum up to j . $O(nt) = O(n^2U)$.

2. $O(n + t \log t)$ randomized and succeed w.h.p. [1]. (assume the output fits in a word, we do not need mod p in that paper.)

deterministic: $\tilde{O}(\sqrt{nt})$ [3], $\tilde{O}(\min\{\sqrt{nt}, t^{4/3}, \sigma\})$ [2].

for the decision version of subset sum, $O(nU)$ [4], based on balancing.

References

- [1] Ce Jin and Hongxun Wu. A simple near-linear pseudopolynomial time randomized algorithm for subset sum. *arXiv preprint arXiv:1807.11597*, 2018.
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- [3] Konstantinos Koiliaris and Chao Xu. Subset sum made simple. 2018.
- [4] David Pisinger. Linear time algorithms for knapsack problems with bounded weights. *Journal of Algorithms*, 33(1):1–14, 1999.