

divide and conquer, compare the middle elements of two arrays, and recurse. $O(\log(n + m))$.

in general, finding the t -th largest element in the union of k sorted array with respective sizes n_1, \dots, n_k takes time:

1. $O(\sum_{i=1}^k \log n_i)$ [2] (and its erratum [3]).

output sensitive version: $O(k + \sum_{i=1}^k \log(t_i + 1))$, where t_i is the number of items of the i -th list within the t -th largest elements [4].

<https://cstheory.stackexchange.com/questions/20944/select-in-union-of-sorted-arrays-already-known/20955#20955>.

2. let $p = \min\{k, t\}$, the running time is $\Theta(k + p \log \frac{t}{p})$ [1].

i.e. if $t \geq k$, $O(k \log \frac{t}{k})$. if $t < k$, $O(k)$.

References

- [1] Greg N Frederickson and Donald B Johnson. The complexity of selection and ranking in $X + Y$ and matrices with sorted columns. *Journal of Computer and System Sciences*, 24(2):197–208, 1982.
- [2] Greg N Frederickson and Donald B Johnson. Generalized selection and ranking: sorted matrices. *SIAM Journal on computing*, 13(1):14–30, 1984.
- [3] Greg N Frederickson and Donald B Johnson. Erratum: generalized selection and ranking: sorted matrices. *SIAM Journal on Computing*, 19(1):205, 1990.
- [4] Haim Kaplan, László Kozma, Or Zamir, and Uri Zwick. Selection from heaps, row-sorted matrices and $X + Y$ using soft heaps. *arXiv preprint arXiv:1802.07041*, 2018.