

let the amount be t .

this is the change-making problem, which is NPC.

1. dfs.

2. knapsack, DP. $O(nt)$.

3. set a threshold b . for coins with values $> b$, we will use at most $\frac{t}{b}$ (with multiplicity) coins, use FFT and recursion to compute subset sum with cardinality information, in $O(t \cdot \frac{t}{b} \log^2 t)$. for coins with value $\leq b$, there are at most b distinct coin values, perform DP for knapsack in $O(bt)$. set $b = \sqrt{t} \log t$, the total time is $O(\frac{t}{b} \cdot t \log^2 t + bt) = O(t\sqrt{t} \log t)$.

4. perform $O(\log \log t)$ binary searches to get 2-approximation.
 $O(t \log t \log \log t)$.

References