

This is the Klee's measure problem.

1. segment tree. $O(n \log n)$.

higher dimensions: $O((n^{d/2} / \log^{d/2-2} n)(\log \log n)^{O(1)})$, for $d \geq 5$ [1].

$\Omega(n \log n)$ lower bound (under the usual model of computation) [2].

References

- [1] Timothy M Chan. Klee's measure problem made easy. In *2013 IEEE 54th annual symposium on foundations of computer science*, pages 410–419. IEEE, 2013.
- [2] Michael L Fredman and Bruce Weide. On the complexity of computing the measure of [ai, bi]. *Communications of the ACM*, 21(7):540–544, 1978.