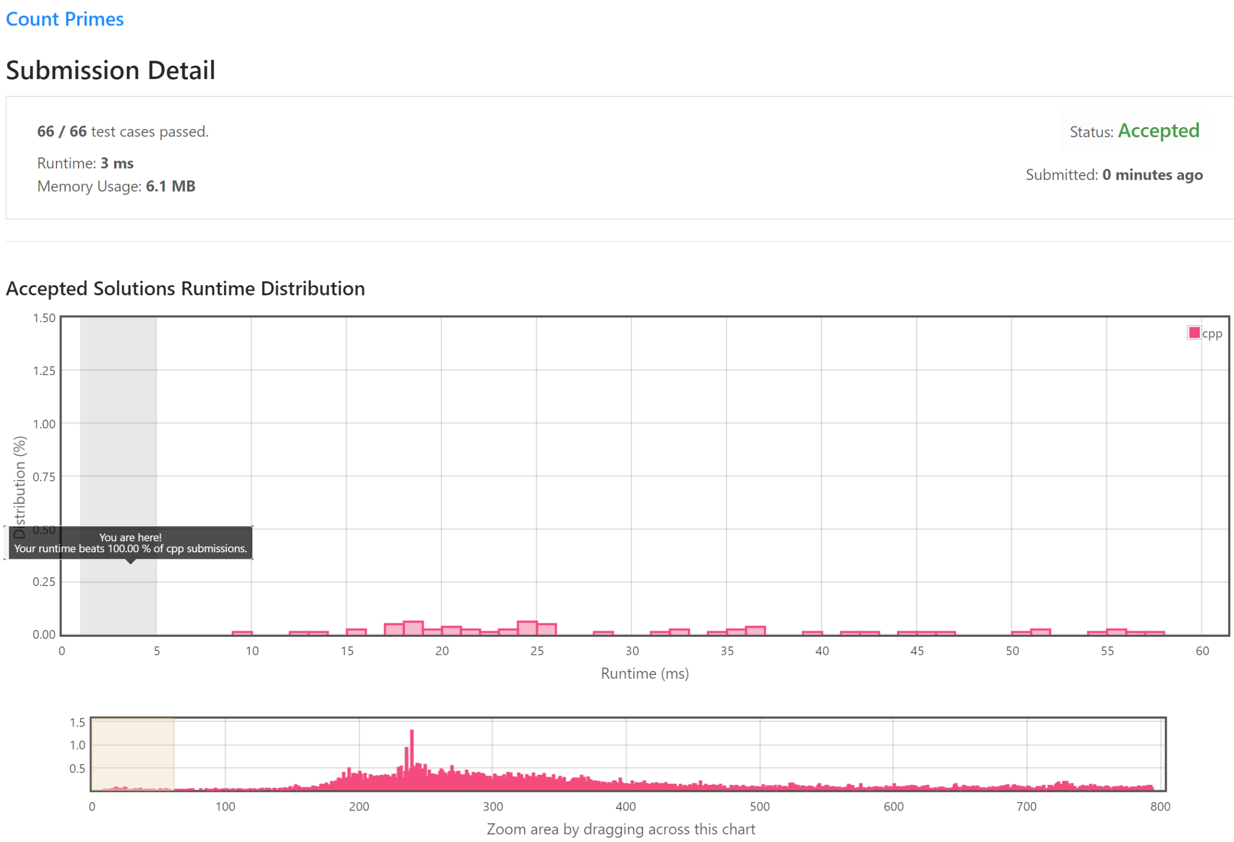
1. $O(n)$ sieve algorithm.
2. $O(\frac{n^{2/3}}{\log^2 n})$ time, $O(n^{1/3} \log^3 n \log\log n)$ space [1].
https://en.wikipedia.org/wiki/Prime-counting_function#Algorithms_for_evaluating_%CF%80(x)
The space complexity is later improved to $O(n^{1/3} \log^2 n)$ [5].
3. analytic algorithms with $O(n^{1/2+\epsilon})$ time [4, 3, 2].



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