1. DP, $f[i][j]$ denote whether $s1[1\ldots i]$ and $s2[1\ldots j]$ can interleave to get $s3[1\ldots i+j]$.
$f[i][j] = (s1[i] == s3[i+j])\&\&f[i-1][j] || (s2[j] == s3[i+j])\&\&f[i][j-1]$. $O(n^2)$.

This DP function is similar to the one for LCS, so some advanced algorithms for LCS can be applied. For the LCS algorithms, see 516. Longest Palindromic Subsequence.

2. $O(\frac{n^2}{\log n})$ by method of four russians. divide the $n \times n$ DP matrix into blocks of size $t \times t$, where $t = O(\log n)$.

3. $O(\frac{n^2}{w})$ by bit packing.

References