1. fractional programming, binary search on the average, each verify step needs $O(n)$ by prefix sum. $O(n \log W)$.
2. let $s[i]$ denote the prefix sum of $a[1 \dots i]$, view $(i, s[i])$ as 2D points, maintain convex hull. $O(n \log n)$.
3. optimal $O(n)$ [1] (which also works for the weighted case).

References

[1] Kai-min Chung and Hsueh-I Lu. An optimal algorithm for the maximum-density segment problem. *SIAM Journal on Computing*, 34(2):373–387, 2005.