This problem is called the geometric median, or (unweighted) Euclidean facility location. The function is convex (a sum of convex functions), so we can use standard optimization techniques, e.g. using 2D ternary search in $O(n \log^2 \frac{1}{\epsilon})$ time, where ϵ denote the error.
We can also use Weiszfeld's algorithm, which is a form of iteratively re-weighted least squares [2]:

$$y_{i+1} = \left(\sum_{j=1}^{m} \frac{x_j}{\|x_j - y_i\|} \right) \Bigg/ \left(\sum_{j=1}^{m} \frac{1}{\|x_j - y_i\|} \right).$$

In higher dimensions, the problem can still be solved in near linear time ($O(nd \log^3 \frac{n}{\epsilon})$) [1].

References

[1] Michael B Cohen, Yin Tat Lee, Gary Miller, Jakub Pachocki, and Aaron Sidford. Geometric median in nearly linear time. In *Proceedings of the forty-eighth annual ACM symposium on Theory of Computing*, pages 9–21, 2016.

[2] Endre Weiszfeld. Sur le point pour lequel la somme des distances de n points donnés est minimum. *Tohoku Mathematical Journal, First Series*, 43:355–386, 1937.