

1. DP, let  $n_i$  denote the number of occurrences of character  $i$  in the input string, and let  $f[i][j]$  denote the number of possibilities when the length of the sequence is  $i$  and we only use the first  $j$  characters ( $1 \leq j \leq |\Sigma|$ ).  $f[i][j] = \sum_{0 \leq k \leq \min\{i, n_j\}} \binom{i}{k} f[i-k][j-1]$ . running time  $\sum_{j=1}^{|\Sigma|} O(n) \cdot O(n_j) = O(n^2)$  (independent of  $|\Sigma|$ ).
  2. use EGF. let  $F_m(x) = \sum_{i=0}^m \frac{x^i}{i!}$ , we only need to compute  $\prod_{j=1}^{|\Sigma|} F_{n_j}(x)$ . use divide and conquer & FFT,  $O(n \log^2 n)$  or  $O(n \log n \log |\Sigma|)$  using our partition lemma or Huffman tree (can improve to  $O(n \log n)$ ?).
- references: 国家集训队2015论文集：金策《生成函数的运算与组合计数问题》.

## References