

This is a special case of the min-weight triangle finding problem (on dense undirected graphs with node weights in $[n]$).

It is no harder than the unweighted triangle detection problem, which can be solved in $O(n^\omega) \approx O(n^{2.373})$ time [1]. (see [2]. By binary search we can reduce to node-weighted negative weight triangle detection.)

References

- [1] Alon Itai and Michael Rodeh. Finding a minimum circuit in a graph. *SIAM Journal on Computing*, 7(4):413–423, 1978.
- [2] Virginia Vassilevska Williams and Ryan Williams. Subcubic equivalences between path, matrix and triangle problems. In *2010 IEEE 51st Annual Symposium on Foundations of Computer Science*, pages 645–654. IEEE, 2010.