1. prefix sum+binary search. $O(n \log n)$.

2. offline, generate all samples in a batch. reduce to sorting.

3. use the alias method https://en.wikipedia.org/wiki/Alias_method. $O(n)$ preprocessing, $O(1)$ per query.

4. after normalizing the weights, divide the cdf into n buckets each with length $\frac{1}{n}$. when we perform a query, first randomly pick a bucket, then randomly select an element in the bucket by brute force, in time proportional to the number of elements in that bucket. in expectation there are $O(1)$ elements in the picked bucket, so $O(1)$ per query in expectation.

remark. some related papers for more complicated query ranges: [2], [1].

References

[1] Peyman Afshani and Jeff M Phillips. Independent range sampling, revisited again. *arXiv preprint arXiv:1903.08014*, 2019.

[2] Peyman Afshani and Zhewei Wei. Independent range sampling, revisited. In *25th Annual European Symposium on Algorithms (ESA 2017)*. Schloss Dagstuhl-Leibniz-Zentrum fuer Informatik, 2017.