

1. DP, let  $f[i][j][\ell]$  denote whether  $s1[i, \dots, j]$  can be matched with  $s2[\ell, \dots, \ell + j - i]$ . Enumerate the breakpoint in  $O(n)$  time. In total  $O(n^4)$ .
2. Use bit packing.  $O(\frac{n^4}{w})$ .
3. The idea is to use boolean matrix multiplication (BMM). Wlog consider the case that we don't flip the string (the flip case is similar). Fix  $i - \ell = d$ , let  $g[i][j] = f[i][j][i - d]$ , we simultaneously consider all such  $g[i][j]$ 's. The transition function is  $g[i][j] = \bigvee (g[i][k] \wedge g[k + 1][j])$ , which is exactly BMM. But we successfully compute  $g[i][j]$  only when the values of  $g[i][k]$  and  $g[k + 1][j]$  have been already computed, and the dependency chain can have depth  $O(n)$ . To fix this, let  $\alpha$  be a parameter to be fixed later, if  $\min\{k - i + 1, j - k\} \leq n^\alpha$  then we compute  $g[i][k]$  by brute force, in  $O(n^\alpha)$  time. Now we only need to perform  $O(n^{1-\alpha})$  rounds of BMMs, each round consists of  $O(n)$  BMMs (because there are  $O(n)$  possible  $d$ 's). Set  $\alpha = \frac{\omega-1}{2}$ , the total running time is  $O(n^3 \cdot n^\alpha + n^{1-\alpha} \cdot n \cdot n^\omega) = O(n^{\frac{5+\omega}{2}}) \approx O(n^{3.69})$ .

## References