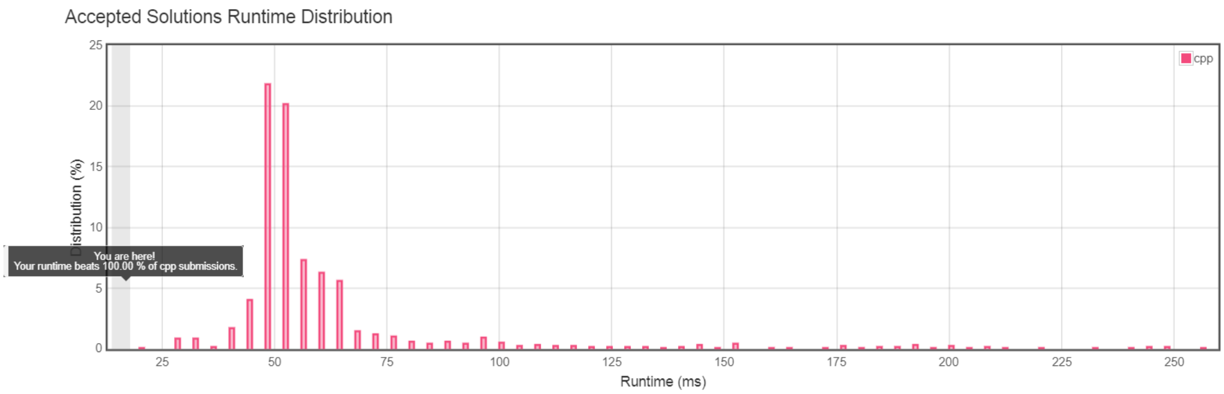
our non-theory assumption is $\log n \le |\Sigma| \le w$.
1. we can represent all characters in a string by a word. testing whether two strings share any common letter is $O(1)$. in total $O(n^2)$.
2. first sort all strings in decreasing order. let $C(s)$ denote the set of letters in string s, and let $f[S]$ be a bitset with n bits representing all strings with no letter appear in S, where S is a set of letters. let $d = \Theta(\log n)$, we precompute $f[S]$ for each subset S of the set of the first d letters $\{'a', \ldots, 'a' + d - 1\}$, this takes $O(2^d \frac{n}{w})$ time. Also do this for the next d letters $\{'a' + d, \ldots, 'a' + 2d - 1\}$ and so on. Fix a string s, all strings that share some common letter with s can be computed by taking the union of $O(\frac{|\Sigma|}{d})$ bitsets according to $C(s)$. finally we need to find the first 0 in the bitset. $O(\frac{|\Sigma|}{d} 2^d \frac{n}{w} + n \cdot \frac{|\Sigma|}{d} \cdot \frac{n}{w}) = O(\frac{n^2 |\Sigma|}{w \log n})$.



We can reduce the Orthogonal Vectors problem to this, so the conditional lower bound is $\Omega(n^{2-o(1)})$.