1. online, reduce to the partial sums problem, in the special case that the number of bits in an update is $O(1)$. $O(\frac{\log n}{\log \log n})$ [2].

2. offline, reduce to offline dominance (or orthogonal range) counting in 2D. $O(n\sqrt{\log n})$ ($O(n \log^{d-2+1/d} n)$) when $d = 2$) [1].

References

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