The last L digits can be efficiently computed in $\text{poly}(\log n, L)$ time. See e.g. https://emathgroup.github.io/blog/factorial-tail.
It suffices to compute $n!$ (or the Gamma function) to some constant precision, and we can use Stirling's approximation in $O(\text{polylog } n)$ time.
The total running time is $O(\text{polylog } n)$.

References