This problem is NPC, because we can reduce hamiltonian path on grids to it.
1. dfs. A naive upper bound is $O^*(3^k)$, where $k$ is the number of nonempty cells. We should be able to get more refined bounds, and some related papers are asymptotic number of hamiltonian paths on planar graphs [1], and on grid graphs [2]. An upper bound is the number of self-avoiding walks on the square graph $\mathbb{Z}^2$, which is estimated to be $O^*(2.639^k)$ [3, 4].
other related papers: [5].
2. We can use bitmask DP, or planar separator theorem + meet in the middle.

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