

For simplicity assume $m = n$.

$O^*(n^{2-\frac{2}{d+1}}) = O^*(n^{\frac{3}{2}})$ by reducing to halfspace range counting in 3D and combining partition and cutting [2], or $O(n\sqrt{n}\log^2 n)$ [1] with near-linear space, which is optimal within polylog factors.

References

- [1] Bernard Chazelle and Emo Welzl. Quasi-optimal range searching in spaces of finite vc-dimension. *Discrete & Computational Geometry*, 4(5):467–489, 1989.
- [2] Jiří Matoušek. Efficient partition trees. *Discrete & Computational Geometry*, 8(3):315–334, 1992.