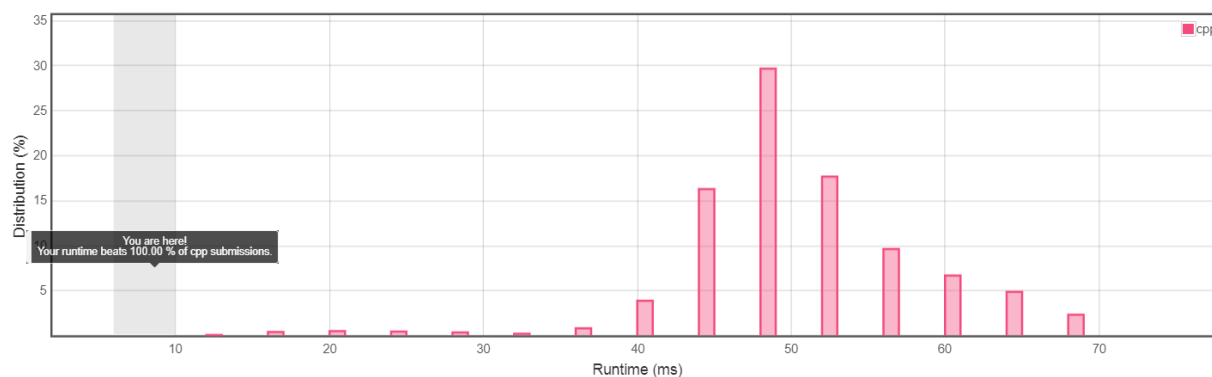


1. binary search. $O(n \log W)$.
2. Let d denote the divisor, we know $f(d) \triangleq \sum_{i=1}^n \lceil \frac{a_i}{d} \rceil \geq \sum_{i=1}^n \frac{a_i}{d}$. each $\lceil \cdot \rceil$ has additive error at most 1, so if we set $d = \frac{\sum_{i=1}^n a_i}{t}$, $f(d)$ is an approximation for t with additive error $O(n)$. if we maintain for each a_i the next value d s.t. $\lceil \frac{a_i}{d} \rceil$ will change (which can be computed in $O(1)$), then we need to test only $O(n)$ subsequent d 's to make sure $f(d) \leq t$. this can be maintained by heap in $O(n \log n)$ time.
For a fixed d , we can estimate $f(d) - \sum_{i=1}^n \frac{a_i}{d}$ by sampling $O(n^c)$ elements to get an $\tilde{O}(n^{1-\frac{c}{2}})$ additive approximation, by Chernoff bound (since $0 \leq \frac{a_i \bmod d}{d} < 1$). Set $c = \frac{2}{3}$ suffices. Then first perform $O(\log W)$ binary search steps (each step in sublinear time), then use heap to test a sublinear number of subsequent d 's (notice that build a heap only takes $O(n)$ time). $O(n + \log W \cdot n^{\frac{2}{3}})$, which is $O(n)$ if we reduce W to $\text{poly}(n)$ by standard techniques.
3. We can directly use the results for finding the m -th largest element in the union of n sorted arrays [1, 2, 3], and also get $O(n + \min\{n, m\} \log \frac{m}{\min\{n, m\}}) = O(n)$ time (since $m = O(n)$). see 004. Median of Two Sorted Arrays.
There's a simpler algorithm that finds the m -th largest element in the union of n sorted array in $O(n + m)$ time, using linear-time median selection. Each round using $O(n)$ time, we either reduce n by half, or decrease m by at least $\frac{n}{2}$.

Remark.

1. it's possible to perform only $O(\log n)$ binary search steps, by known techniques. (extract the first n largest elements in the union of n sorted arrays, after using the observation of Alg. 2, and then binary search on those n values.)
2. with clever implementation we don't need long long.

Accepted Solutions Runtime Distribution



References

- [1] Greg N Frederickson and Donald B Johnson. Generalized selection and ranking (preliminary version). In *Proceedings of the twelfth annual ACM symposium on Theory of computing*, pages 420–428, 1980.
- [2] Greg N Frederickson and Donald B Johnson. The complexity of selection and ranking in $X + Y$ and matrices with sorted columns. *Journal of Computer and System Sciences*, 24(2):197–208, 1982.
- [3] Greg N Frederickson and Donald B Johnson. Generalized selection and ranking: sorted matrices. *SIAM Journal on computing*, 13(1):14–30, 1984.