

1. Hashing. Expected  $O(n)$  time.
2. Reduce to integer sorting. After sorting, only need deterministic  $O(n)$  time by monotone pointers.  
For integer sorting:  
 $O(n\sqrt{\log \log n})$  randomized [3].  
 $O(n \log \log n)$  deterministic [2], and  $O(n \log \log_w n)$  deterministic [4].
3. For the lower bound,  $\Omega(n \log n)$  in the degree- $d$  algebraic decision tree model [1]. We can also reduce the set intersection problem to this, i.e. deciding whether two sets  $\{x_1, \dots, x_n\}$  and  $\{y_1, \dots, y_n\}$  intersect, by negating one of the sets.

## References

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- [4] Mikkel Thorup, Or Zamir, and Uri Zwick. Dynamic ordered sets with approximate queries, approximate heaps and soft heaps. In *46th International Colloquium on Automata, Languages, and Programming (ICALP 2019)*. Schloss Dagstuhl-Leibniz-Zentrum fuer Informatik, 2019.