in the following assume $m = n$.
1. use Boyer-Moore voting algorithm, maintain the information (number, count) by segment tree. $O(n \log n)$.
2. the information we maintain forms an (associative) semigroup, so it's mergeable. we need to query range sum on a static array, which needs $O(n\alpha(n))$ by Yao [4]. [1] also gives a $\Theta(n\lambda(k,n))$ ($= O(n\alpha(n))$ for our purpose) time and space algorithm, where $\lambda(k,\cdot)$ is the inverse of a certain function at the $\lfloor \frac{k}{2} \rfloor$-th level of the primitive recursive hierarchy. We can also get $O(n)$ preprocessing and $O(1)$ per query, see my article here: https://zhuanlan.zhihu.com/p/79423299. Then check whether the number we find is valid, by computing the number of occurrence of it in the query interval, using persistent array (or vEB tree) in $O(\log \log n)$. We can also solve this in $O(\frac{\log \log n}{\log \log \log n})$ per query by reducing to the static predecessor problem https://en.wikipedia.org/wiki/Predecessor_problem [2] (but with $O(n^4)$ preprocessing time; in our case the universe $N = n$). In conclusion we get $O(n \log \log n)$.
3. $O(n \log n)$ preprocessing, $O(1)$ per query [3].

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