

1. mincost flow with upper and lower flow bounds.  $O(\text{mincost} - \text{flow}(n, n^2))$ .
2. This is the minimum cost edge cover problem for bipartite graph (with non-negative weights), which can be reduced to minimum cost bipartite perfect matching.  
[https://cstheory.stackexchange.com/questions/14690/reducing-a-minimum-cost-edge-cover-problem-to-minimum-cost-weighted-bipartite-per](https://cstheory.stackexchange.com/questions/14690/reducing-a-minimum-cost-edge-cover-problem-to-minimum-cost-weighted-bipartite-perfect-matching)  
 $O(n^3)$  using KM, or  $\tilde{O}(m^{10/7} \log W)$  [1].

## References

- [1] Michael B Cohen, Aleksander Madry, Piotr Sankowski, and Adrian Vladu. Negative-weight shortest paths and unit capacity minimum cost flow in  $\tilde{O}(m^{10/7} \log W)$  time. In *Proceedings of the Twenty-Eighth Annual ACM-SIAM Symposium on Discrete Algorithms*, pages 752–771. SIAM, 2017.