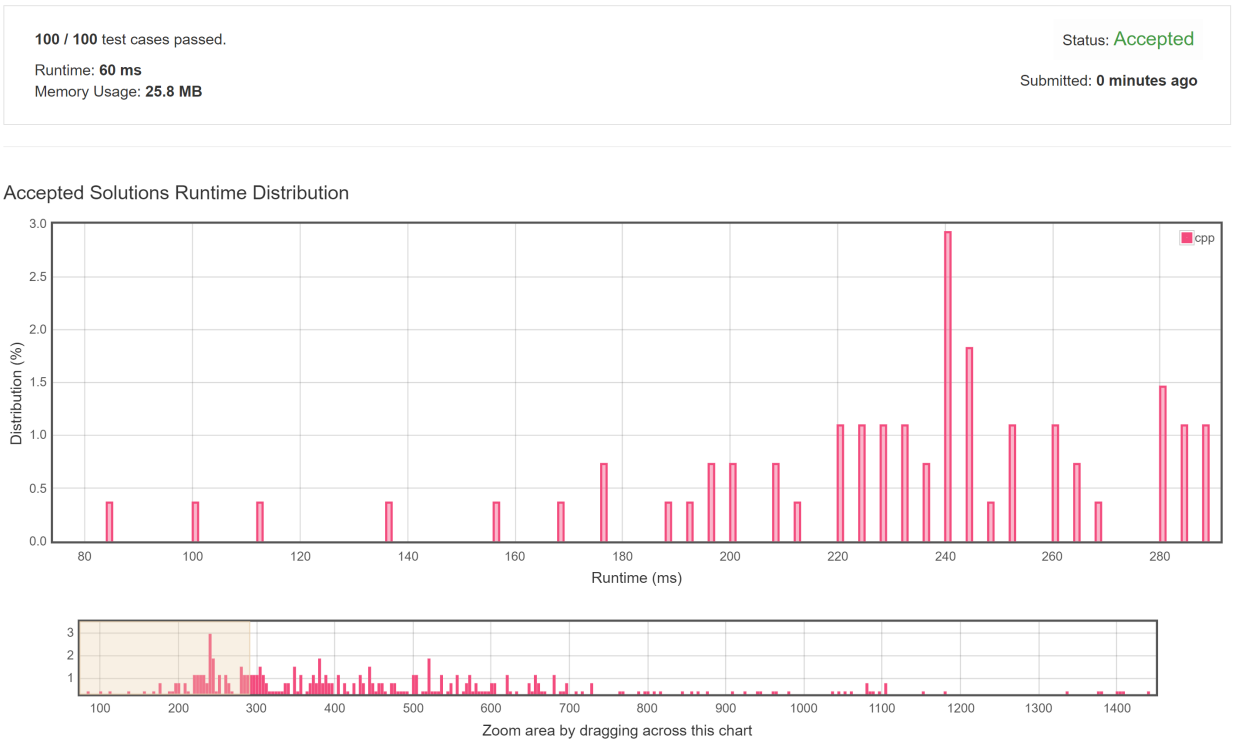
1. Connect each number with its prime factors. $O(n \cdot \text{factor}(U))$.
2. Preprocess an arbitrary prime factor for each number in $1, \ldots, U$ using sieve. For each input number, connect all its prime factors. $O(U + n \cdot \frac{\log U}{\log \log U})$. (we can use sieve with sublinear time complexity, e.g. Wheel factorization [2] or Sieve of Atkin [1] in $O(\frac{U}{\log \log U})$ time.)

Remark. There could be other running time tradeoffs between n and U.



References

[1] Arthur Atkin and Daniel Bernstein. Prime sieves using binary quadratic forms. *Mathematics of Computation*, 73(246):1023–1030, 2004.

[2] Paul Pritchard. Explaining the wheel sieve. *Acta Informatica*, 17:477–485, 1982.