1. DP. let $f[i][j]$ denote the number of permutations of $1, \ldots, i$ with k inverse pairs. $f[i][j] = \sum_{0 \le k \le \min\{i-1, j\}} f[i-1][j-k]$, use prefix sum to optimize. $O(nk)$.
2. in the worst case, $k \le O(n^2)$. the array $f[i]$ can be computed by convolutions of arrays $(1) \star (1,1) \star \cdots \star (\underbrace{1, \ldots, 1}_{n \text{ elements}})$, with indices starting from 0. using the associative law of convolution, FFT and divide and conquer, $O(n^2 \log^2 n)$ in the worst case. (divide and conquer is not very good when k is small)
3. The i-th array in the convolution can be written as $\sum_{j=0}^{i} x^j = \frac{1-x^{i+1}}{1-x}$, and we want to compute the product of them. $O(k \log k)$ using the technique of polynomial ln & exp in [1].

References

[1] Ce Jin and Hongxun Wu. A simple near-linear pseudopolynomial time randomized algorithm for subset sum. In *2nd Symposium on Simplicity in Algorithms (SOSA 2019)*, 2018.