

1. store the numbers in a Trie. for each number  $i$ , query  $\max_j a[j] \text{ xor } a[i]$  takes  $O(w)$ , by walking down the Trie. total time  $O(nw)$ .
2. determine the result bit by bit. We can verify whether we can get an xor result with prefix  $t$  in  $O(n)$  by hashing. total time  $O(nw)$ .
3.  $w$ -ary Trie with depth  $O(\frac{w}{\log w})$ , use (nonstandard?) word operations to walk down.  $O(\frac{nw}{\log w})$ .  
The space complexity can be reduced to  $O(\frac{nw}{\log w})$  without using hashing (the naïve implementation needs  $O(\frac{nw^2}{\log w})$  space), by 2-level indexing technique. So this algorithm is deterministic.
4. construct a patricia tree of the array in  $O(n)$  after sorting, by word operations (e.g. `clz`). the tree has  $O(n)$  nodes and edges. recursively find the maximum xor by a function  $f(T, e)$ , which returns the maximum xor between an element in  $T$  and an element in the subtree below  $e$ , where  $T$  is a subtree in the patricia tree and  $e$  is an edge. during each recursion we will either remove a node or an edge, so by amortized analysis, the total running time after sorting is  $O(n)$ .

Q: Could you do this in  $O(n)$  runtime?

A: No I can't.

## References