

It suffices to partition the people into maximum number of components each with sum 0. For a component of size d with sum 0, we can use $d - 1$ transactions to clear the debt of each person within it.

1. Let $f[i]$ denote the maximum number of subsets with sum 0 that we can partition the subset i into. $O(2^n \cdot n)$.

2. The answer will not exceed $n - 1$. Use Fast Walsh Transform (\mid operation), the running time is $\tilde{O}(2^n)$.

lower bound: we can reduce subset sum to this problem ($n - 2$ person have negative balance and 2 person have positive balance), so this is NP-hard [1].

References

- [1] Tom Verhoeff et al. Settling multiple debts efficiently: An invitation to computing science. *Informatics in Education-An International Journal*, 3(1):105–126, 2004.