

1. bitmask DP. let $f[i][j]$ denote the number of ways to allocate the first i hats to subset j of people. $O(2^n \cdot nm)$.

The problem is equivalent to counting the number of perfect matchings in a bipartite graph, which is #P-Complete [1].

References

- [1] Leslie G Valiant. The complexity of enumeration and reliability problems. *SIAM Journal on Computing*, 8(3):410–421, 1979.