

Let W be the range of numbers, and assume 3-sum for n numbers can be solved in $f(n)$.

1. $O(n^2)$ by sorting+monotone pointers.

2. We can reduce the decision version of this problem (whether there exist 3-sum in range $[l, r]$) to $O(\log W)$ exact 3-sum calls [1, Theorem 1 (Shrinking intervals)], replace the n in the proof by 3. To solve this problem, we need $O(\log W)$ exponential searches, so the total running time is $O(\log^2 W \cdot f(n))$ (can probably combine the two steps and shave a log).

lower bound: $\Omega(f(n))$, because this problem is 3sum-hard. see 015. 3Sum.

References

- [1] Jesper Nederlof, Erik Jan van Leeuwen, and Ruben van der Zwaan. Reducing a target interval to a few exact queries. In *International Symposium on Mathematical Foundations of Computer Science*, pages 718–727. Springer, 2012.