

This is the Klee's measure problem.

1. segment tree.  $O(n \log n)$ .

higher dimensions:  $O((n^{d/2}/\log^{d/2-2} n)(\log \log n)^{O(1)})$ , for  $d \geq 5$  [1].

$\Omega(n \log n)$  lower bound (under the usual model of computation) [2].

## References

- [1] Timothy M. Chan. Klee's measure problem made easy. In *2013 IEEE 54th annual symposium on foundations of computer science*, pages 410–419. IEEE, 2013.
- [2] Michael L Fredman and Bruce Weide. On the complexity of computing the measure of  $\cup[\text{ai}, \text{bi}]$ . *Communications of the ACM*, 21(7):540–544, 1978.