let the amount be t.
this is the change-making problem https://en.wikipedia.org/wiki/Change-making_problem, which is NPC.
1. dfs.
2. knapsack, DP. $O(nt)$.
3. set a threshold b. for coins with values $> b$, we will use at most $\frac{t}{b}$ (with multiplicity) coins, use FFT and recursion to compute subset sum with cardinality information, in $O(t \cdot \frac{t}{b} \log^2 t)$. for coins with value $\leq b$, there are at most b distinct coin values, perform DP for knapsack in $O(bt)$. set $b = \sqrt{t} \log t$, the total time is $O(\frac{t}{b} \cdot t \log^2 t + bt) = O(t\sqrt{t} \log t)$.
4. FFT+doubling, each FFT takes $O(t \log t)$, need to perform $O(\log t)$ doubling steps. $O(t \log^2 t)$.
5. $O(t \log t \log \log t)$. to be published.

References