DP, let f[i][j] denote the length of the longest fibonacci sequence starting from i and j. $O(n^2)$. We can reduce 3sum to this problem, so the conditional lower bound is $\Omega(n^{2-o(1)})$.

Remark. can we shave log factors?

References