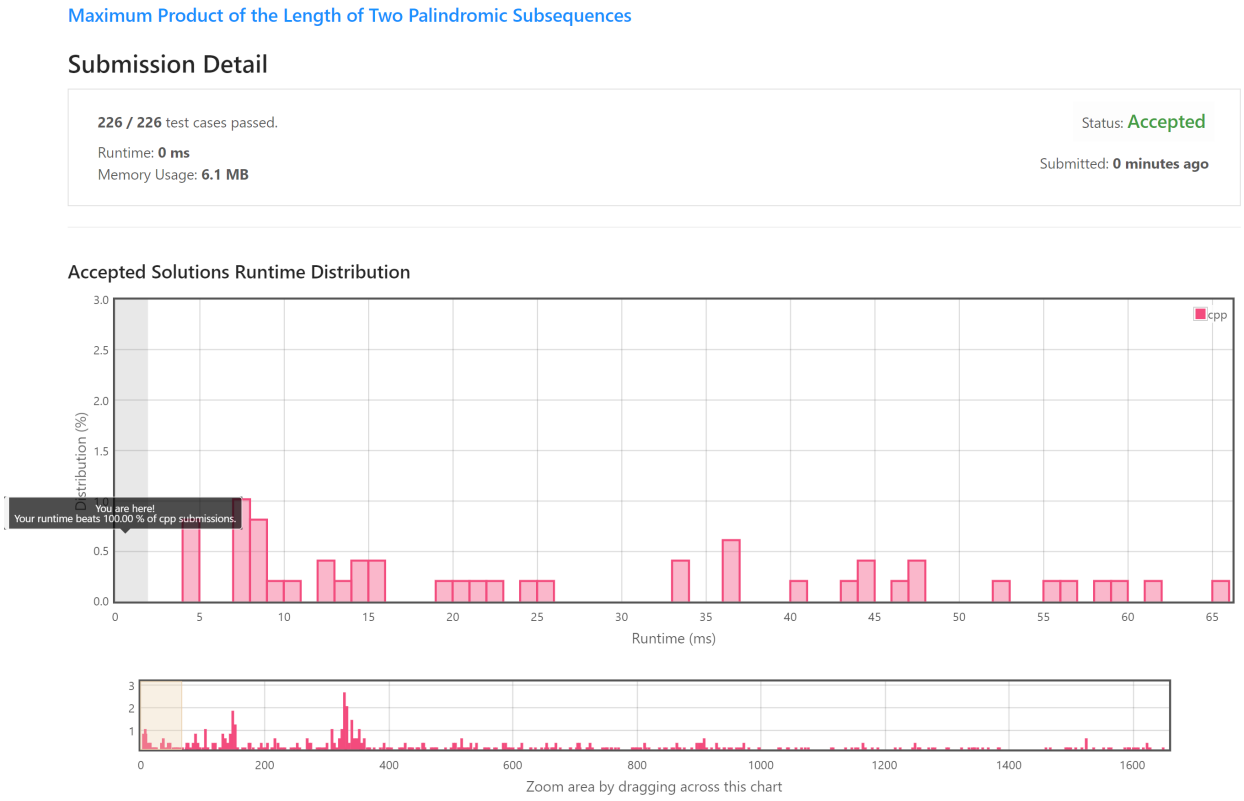
1. dfs. $O(3^n)$.
2. bitmask DP, let $f[i]$ denote the longest palindrome contained in bitmask i. $O(2^n)$.
3. $O^*(2^{n/2})$, or $O^*(2^{0.472n})$. see my article https://leetcode-cn.com/problems/maximum-product-of-the-length-of-two-palindromic-subsequences/solution/bi-o2ngeng-kuai-de-yi-xie-zuo-fa-by-hqzt-lg2f/.



Remark. 1. Is there a polynomial time algorithm?
2. Can we use meet in the middle?

References