1. DP along the antidiagonal, let the state encode the two current endpoints of the path. $O(n^3)$.
2. min-cost flow. the graph is planar, and min-cost flow takes $\tilde{O}((|V||E|)^{2/3} \log C) = \tilde{O}(n^{8/3})$ [1]. we actually only need to compute a shortest path on a planar graph (after the first augmentation), and this takes $O(|V| \log^2 |V| / \log \log |V|) = O(n^2 \log^2 n / \log \log n)$ [2].
the graph is very special, so there should be an $O(n^2)$ time algorithm...

References

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