

1. mincost flow with upper and lower flow bounds. $O(\text{mincost-flow}(n, n^2))$.
2. This is the minimum cost edge cover problem for bipartite graph (with non-negative weights), which can be reduced to minimum cost bipartite perfect matching.
<https://cstheory.stackexchange.com/questions/14690/reducing-a-minimum-cost-edge-cover-problem-to-minimum-cost-weighted-bipartite-perfect-matching>
 $O(n^3)$ using KM, $O(m\sqrt{n}\log(nW))$ [2], $\tilde{O}(m^{4/3+o(1)}\log W)$ [1] for sparse graphs, or $\tilde{O}((m+n^{1.5})\log^2 W)$ [3].

References

- [1] Kyriakos Axiotis, Aleksander Madry, and Adrian Vladu. Circulation control for faster minimum cost flow in unit-capacity graphs. *arXiv preprint arXiv:2003.04863*, 2020.
- [2] Harold N Gabow and Robert E Tarjan. Faster scaling algorithms for network problems. *SIAM Journal on Computing*, 18(5):1013–1036, 1989.
- [3] Jan van den Brand, Yin-Tat Lee, Danupon Nanongkai, Richard Peng, Thatchaphol Saranurak, Aaron Sidford, Zhao Song, and Di Wang. Bipartite matching in nearly-linear time on moderately dense graphs. *arXiv e-prints*, pages arXiv–2009, 2020.