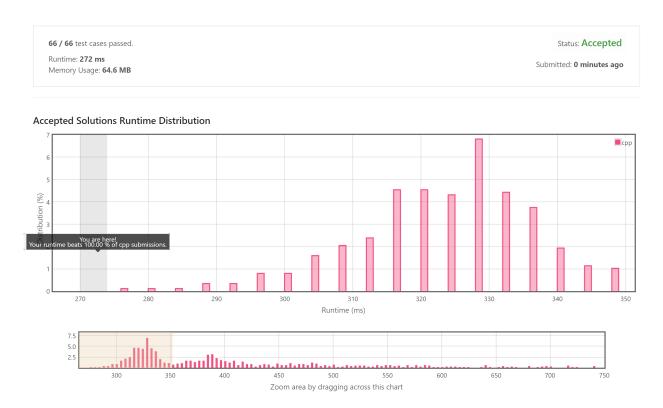
1. Enumerate the common divisor d, then connect all multiples of d. An upper bound of the running time is $O(\sum_d \frac{n}{d}) = O(n \log n)$.

For a better running time, notice that we only need to enumerate d s.t. d > threshold and all divisors of $d \le$ threshold. Let f(d) denote the smallest prime factor of d, we require that $\frac{d}{f(d)} \le$ threshold. Running time?

2. For each number $t = \prod_{1 \le i \le k} p_i^{q_i}$, connect t with each $\frac{t}{p_i}$ when $\frac{t}{p_i} >$ threshold. The running time is the total number of prime divisors of all integers in 1..n, which is $\sum_i \frac{1}{p_i} = O(n \log \log n)$.



References