

let the amount be  $t$ .

this is the change-making problem [https://en.wikipedia.org/wiki/Change-making\\_problem](https://en.wikipedia.org/wiki/Change-making_problem), which is NPC.

1. dfs.

2. knapsack, DP.  $O(nt)$ .

3. set a threshold  $b$ . for coins with values  $> b$ , we will use at most  $\frac{t}{b}$  (with multiplicity) coins, use FFT and recursion to compute subset sum with cardinality information, in  $O(t \cdot \frac{t}{b} \log^2 t)$ . for coins with value  $\leq b$ , there are at most  $b$  distinct coin values, perform DP for knapsack in  $O(bt)$ . set  $b = \sqrt{t} \log t$ , the total time is  $O(\frac{t}{b} \cdot t \log^2 t + bt) = O(t\sqrt{t} \log t)$ .

4. FFT+doubling, each FFT takes  $O(t \log t)$ , need to perform  $O(\log t)$  doubling steps.  $O(t \log^2 t)$ .

5. deterministic  $O(t \log t \log \log t)$  and randomized  $O(t \log t)$ . to be published.

## References