1 algorithm

#include <algorithm> #include <numeric>

Algo	Params	Funcion
sort, stable_sort	f, l	ordena el intervalo
nth_element	f, nth, l	*void* ordena el n-esimo, y particiona el resto
fill, fill_n	f, l / n, elem	*void* llena [f, l) o [f, f+n) con elem
lower_bound, upper_bound	f, l, elem	*it* al primer / ultimo donde se puede insertar elem para que quede ordenada
binary_search	f, l, elem	*bool* esta elem en [f, l)
copy	f, l, resul	hace resul+i=f+i $\forall i$
find, find_if, find_first_of	f, l, elem / pred / f2, l2	*it* encuentra i $\in$[f,l) tq. i=elem, pred(i), i$\in$[f2,l2)
count, count_if	f, l, elem/pred	cuenta elem, pred(i)
search	f, l, f2, l2	busca [f2,l2) $\in$ [f,l)
replace, replace_if	f, l, old / pred, new	cambia old / pred(i) por new
reverse	f, l	da vuelta
partition, stable_partition	f, l, pred	pred(i) ad, !pred(i) atras
min_element, max_element	f, l, [comp]	*it* min, max de [f,l]
lexicographical_compare	f1,l1,f2,l2	*bool* con [f1,l1]¡[f2,l2]
next/prev_permutation	f,l	deja en [f,l) la perm sig, ant
set_intersection, set_difference, set_union, set_symmetric_difference,	f1, l1, f2, l2, res	[res, ...) la op. de conj
push_heap, pop_heap, make_heap	f, l, e / e /	mete/saca e en heap [f,l), hace un heap de [f,l)
is_heap	f,l	*bool* es [f,l) un heap
accumulate	f,l,i,[op]	$T = \sum$/oper de [f,l)
inner_product	f1, l1, f2, i	T = i + [f1, l1) . [f2, ...)
partial_sum	f, l, r, [op]	r+i = $\sum$/oper de [f,f+i] $\forall i \in$[f,l)
__builtin_ffs	unsigned int	Pos. del primer 1 desde la derecha
__builtin_clz	unsigned int	Cant. de ceros desde la izquierda.
__builtin_ctz	unsigned int	Cant. de ceros desde la derecha.
__builtin_popcount	unsigned int	Cant. de 1's en x.
__builtin_parity	unsigned int	1 si x es par, 0 si es impar.
__builtin_XXXXXXll	unsigned ll	= pero para long long's.

2 Estructuras

2.1 RMQ (static)

Dado un arreglo y una operacion asociativa *idempotente*, get(i, j) opera sobre el rango [i, j). Restriccion: LVL $\geq$ ceil(logn); Usar [ ] para llenar arreglo y luego build().

```
struct RMQ{
  #define LVL 10
  tipo vec[LVL][1<<(LVL+1)];
  tipo &operator[](int p){return vec[0][p];}
  tipo get(int i, int j) {//intervalo [i,j)
    int p = 31-__builtin_clz(j-i);
    return min(vec[p][i],vec[p][j-(1<<p)]);
  }
  void build(int n) {//O(nlogn)
    int mp = 31-__builtin_clz(n);
    forn(p, mp) forn(x, n-(1<<p))
      vec[p+1][x] = min(vec[p][x], vec[p][x+(1<<p)]);
  }};
```

2.2 RMQ (dynamic)

```
//Dado un arreglo y una operacion asociativa con neutro, get(i, j) opera sobre
    el rango [i, j).
#define MAXN 100000
#define operacion(x, y) max(x, y)
const int neutro=0;
struct RMQ{
  int sz;
  tipo t[4*MAXN];
  tipo &operator[](int p){return t[sz+p];}
  void init(int n){//O(nlgn)
    sz = 1 << (32-__builtin_clz(n));
    forn(i, 2*sz) t[i]=neutro;
  }
  void updall(){//O(n)
    dforn(i, sz) t[i]=operacion(t[2*i], t[2*i+1]);}
  tipo get(int i, int j){return get(i,j,1,0,sz);}
  tipo get(int i, int j, int n, int a, int b){//O(lgn)
    if(j<=a || i>=b) return neutro;
    if(i<=a && b<=j) return t[n];
    int c=(a+b)/2;
    return operacion(get(i, j, 2*n, a, c), get(i, j, 2*n+1, c, b));
  }
  void set(int p, tipo val){//O(lgn)
    for(p+=sz; p>0 && t[p]!=val;){
      t[p]=val;
```

```
      p/=2;
      val=operacion(t[p*2], t[p*2+1]);
    }
  }
}rmq;
//Usage:
cin >> n; rmq.init(n); forn(i, n) cin >> rmq[i]; rmq.updall();
```

2.3 RMQ (lazy)

```
//Dado un arreglo y una operacion asociativa con neutro, get(i, j) opera sobre el rango [i, j).
typedef int Elem;//Elem de los elementos del arreglo
typedef int Alt;//Elem de la alteracion
#define operacion(x,y) x+y
const Elem neutro=0; const Alt neutro2=0;
#define MAXN 100000
struct RMQ{
  int sz;
  Elem t[4*MAXN];
  Alt dirty[4*MAXN];//las alteraciones pueden ser de distinto Elem
  Elem &operator[](int p){return t[sz+p];}
  void init(int n){//O(nlgn)
    sz = 1 << (32-__builtin_clz(n));
    forn(i, 2*sz) t[i]=neutro;
    forn(i, 2*sz) dirty[i]=neutro2;
  }
  void push(int n, int a, int b){//propaga el dirty a sus hijos
    if(dirty[n]!=0){
      t[n]+=dirty[n]*(b-a);//altera el nodo
      if(n<sz){
        dirty[2*n]+=dirty[n];
        dirty[2*n+1]+=dirty[n];
      }
      dirty[n]=0;
    }
  }
  Elem get(int i, int j, int n, int a, int b){//O(lgn)
    if(j<=a || i>=b) return neutro;
    push(n, a, b);//corrige el valor antes de usarlo
    if(i<=a && b<=j) return t[n];
    int c=(a+b)/2;
    return operacion(get(i, j, 2*n, a, c), get(i, j, 2*n+1, c, b));
  }
  Elem get(int i, int j){return get(i,j,1,0,sz);}
  //altera los valores en [i, j) con una alteracion de val
  void alterar(Alt val, int i, int j, int n, int a, int b){//O(lgn)
    push(n, a, b);
    if(j<=a || i>=b) return;
    if(i<=a && b<=j){
      dirty[n]+=val;
      push(n, a, b);
      return;
    }
    int c=(a+b)/2;
    alterar(val, i, j, 2*n, a, c), alterar(val, i, j, 2*n+1, c, b);
    t[n]=operacion(t[2*n], t[2*n+1]);//por esto es el push de arriba
  }
  void alterar(Alt val, int i, int j){alterar(val,i,j,1,0,sz);}
}rmq;
```

2.4 RMQ (persistente)

```
typedef int tipo;
tipo oper(const tipo &a, const tipo &b){
    return a+b;
}
struct node{
  tipo v; node *l,*r;
  node(tipo v):v(v), l(NULL), r(NULL) {}
    node(node *l, node *r) : l(l), r(r){
        if(!l) v=r->v;
        else if(!r) v=l->v;
        else v=oper(l->v, r->v);
    }
};
node *build (tipo *a, int tl, int tr) {//modificar para que tome tipo a
  if (tl+1==tr) return new node(a[tl]);
  int tm=(tl + tr)>>1;
  return new node(build(a, tl, tm), build(a, tm, tr));
}
node *update(int pos, int new_val, node *t, int tl, int tr){
  if (tl+1==tr) return new node(new_val);
  int tm=(tl+tr)>>1;
  if(pos < tm) return new node(update(pos, new_val, t->l, tl, tm), t->r);
  else return new node(t->l, update(pos, new_val, t->r, tm, tr));
}
tipo get(int l, int r, node *t, int tl, int tr){
    if(l==tl && tr==r) return t->v;
  int tm=(tl + tr)>>1;
    if(r<=tm) return get(l, r, t->l, tl, tm);
    else if(l>=tm) return get(l, r, t->r, tm, tr);
  return oper(get(l, tm, t->l, tl, tm), get(tm, r, t->r, tm, tr));
}
```

2.5 Fenwick Tree

```
//For 2D threat each column as a Fenwick tree, by adding a nested for in each operation
struct Fenwick{
  static const int sz=1000001;
  tipo t[sz];
  void adjust(int p, tipo v){//valid with p in [1, sz), O(lgn)
    for(int i=p; i<sz; i+=(i&-i)) t[i]+=v; }
  tipo sum(int p){//cumulative sum in [1, p], O(lgn)
    tipo s=0;
    for(int i=p; i; i-=(i&-i)) s+=t[i];
    return s;
  }
  tipo sum(int a, int b){return sum(b)-sum(a-1);}
  //get largest value with cumulative sum less than or equal to x;
  //for smallest, pass x-1 and add 1 to result
  int getind(tipo x) {//O(lgn)
      int idx = 0, mask = N;
      while(mask && idx < N) {
        int t = idx + mask;
      if(x >= tree[t])
          idx = t, x -= tree[t];
        mask >>= 1;
      }
      return idx;
  }};
```

2.6 Union Find

```
struct UnionFind{
  vector<int> f;//the array contains the parent of each node
  void init(int n){f.clear(); f.insert(f.begin(), n, -1);}
  int comp(int x){return (f[x]==-1?x:f[x]=comp(f[x]));}//O(1)
  bool join(int i, int j) {
    bool con=comp(i)==comp(j);
    if(!con) f[comp(i)] = comp(j);
    return con;
  }};
```

2.7 Disjoint Intervals

```
bool operator< (const ii &a, const ii &b) {return a.fst<b.fst;}
//Stores intervals as [first, second]
//in case of a collision it joins them in a single interval
struct disjoint_intervals {
  set<ii> segs;
  void insert(ii v) {//O(lgn)
    if(v.snd-v.fst==0.) return;//OJO
    set<ii>::iterator it,at;
    at = it = segs.lower_bound(v);
    if (at!=segs.begin() && (--at)->snd >= v.fst)
      v.fst = at->fst, --it;
    for(; it!=segs.end() && it->fst <= v.snd; segs.erase(it++))
      v.snd=max(v.snd, it->snd);
    segs.insert(v);
  }
};
```

2.8 RMQ (2D)

```
struct RMQ2D{//n filas x m columnas
  int sz;
  RMQ t[4*MAXN];
  RMQ &operator[](int p){return t[sz/2+p];}//t[i][j]=i fila, j col
  void init(int n, int m){//O(n*m)
    sz = 1 << (32-__builtin_clz(n));
    forn(i, 2*sz) t[i].init(m); }
  void set(int i, int j, tipo val){//O(lgm.lgn)
    for(i+=sz; i>0;){
      t[i].set(j, val);
      i/=2;
      val=operacion(t[i*2][j], t[i*2+1][j]);
    } }
  tipo get(int i1, int j1, int i2, int j2){return get(i1,j1,i2,j2,1,0,sz);}
  //O(lgm.lgn), rangos cerrado abierto
  int get(int i1, int j1, int i2, int j2, int n, int a, int b){
    if(i2<=a || i1>=b) return 0;
    if(i1<=a && b<=i2) return t[n].get(j1, j2);
    int c=(a+b)/2;
    return operacion(get(i1, j1, i2, j2, 2*n, a, c),
         get(i1, j1, i2, j2, 2*n+1, c, b));
  }
} rmq;
//Example to initialize a grid of M rows and N columns:
RMQ2D rmq; rmq.init(n,m);
forn(i, n) forn(j, m){
  int v; cin >> v; rmq.set(i, j, v);}
```

2.9 Big Int

```
#define BASEXP 6
#define BASE 1000000
#define LMAX 1000
struct bint{int l;ll n[LMAX];bint(ll x=0){l=1;forn(i,LMAX){if(x)l=i+1;n[i]=x%BASE;x/=BASE;}}bint(string x){l=(x.size()-1)/BASEXP+1;fill(n,n+LMAX,0);ll
```

```
    r=1;forn(i,sz(x)){n[i/BASEXP]+=r*(x[x.size()-1-i]-'0');r*=10;if(r==BASE)r
    =1;}}void out(){cout<<n[l-1];dforn(i,l-1)printf("%6.6llu",n[i]);}void
    invar(){fill(n+l,n+LMAX,0);while(l>1&&!n[l-1])l--;}};bint operator+(const
    bint&a,const bint&b){bint c;c.l=max(a.l,b.l);ll q=0;forn(i,c.l)q+=a.n[i]+b
    .n[i],c.n[i]=q%BASE,q/=BASE;if(q)c.n[c.l++]=q;c.invar();return c;}pair<
    bint,bool>lresta(const bint&a,const bint&b){bint c;c.l=max(a.l,b.l);ll q
    =0;forn(i,c.l)q+=a.n[i]-b.n[i],c.n[i]=(q+BASE)%BASE,q=(q+BASE)/BASE-1;c.
    invar();return make_pair(c,!q);}bint&operator-=(bint&a,const bint&b){
    return a=lresta(a,b).first;}bint operator-(const bint&a,const bint&b){
    return lresta(a,b).first;}bool operator<(const bint&a,const bint&b){return
    !lresta(a,b).second;}bool operator<=(const bint&a,const bint&b){return
    lresta(b,a).second;}bool operator==(const bint&a,const bint&b){return a<=b
    &&b<=a;}bint operator*(const bint&a,ll b){bint c;ll q=0;forn(i,a.l)q+=a.n[
    i]*b,c.n[i]=q%BASE,q/=BASE;c.l=a.l;while(q)c.n[c.l++]=q%BASE,q/=BASE;c.
    invar();return c;}bint operator*(const bint&a,const bint&b){bint c;c.l=a.l
    +b.l;fill(c.n,c.n+b.l,0);forn(i,a.l){ll q=0;forn(j,b.l)q+=a.n[i]*b.n[j]+c.
    n[i+j],c.n[i+j]=q%BASE,q/=BASE;c.n[i+b.l]=q;}c.invar();return c;}pair<bint
    ,ll>ldiv(const bint&a,ll b){bint c;ll rm=0;dforn(i,a.l){rm=rm*BASE+a.n[i];
    c.n[i]=rm/b;rm%=b;}c.l=a.l;c.invar();return make_pair(c,rm);}bint operator
    /(const bint&a,ll b){return ldiv(a,b).first;}ll operator%(const bint&a,ll
    b){return ldiv(a,b).second;}pair<bint,bint>ldiv(const bint&a,const bint&b)
    {bint c;bint rm=0;dforn(i,a.l){if(rm.l==1&&!rm.n[0])rm.n[0]=a.n[i];else{
    dforn(j,rm.l)rm.n[j+1]=rm.n[j];rm.n[0]=a.n[i];rm.l++;}ll q=rm.n[b.l]*BASE+
    rm.n[b.l-1];ll u=q/(b.n[b.l-1]+1);ll v=q/b.n[b.l-1]+1;while(u<v-1){ll m=(u
    +v)/2;if(b*m<=rm)u=m;else v=m;}c.n[i]=u;rm-=b*u;}c.l=a.l;c.invar();return
    make_pair(c,rm);}bint operator/(const bint&a,const bint&b){return ldiv(a,b
    ).first;}bint operator%(const bint&a,const bint&b){return ldiv(a,b).second
    ;}
```

2.10 HashTables

```
//Compilar: g++ --std=c++11
struct Hash{
  size_t operator()(const ii &a)const{
    size_t s=hash<int>()(a.fst);
    return hash<int>()(a.snd)+0x9e3779b9+(s<<6)+(s>>2);
  }
  size_t operator()(const vector<int> &v)const{
    size_t s=0;
    for(auto &e : v)
      s ^= hash<int>()(e)+0x9e3779b9+(s<<6)+(s>>2);
    return s;
  }
};
unordered_set<ii, Hash> s;
unordered_map<ii, int, Hash> m;//map<key, value, hasher>
```

2.11 Modnum

```
struct mnum{
  static const tipo mod=12582917;
  tipo v;
  mnum(tipo v=0): v(v%mod) {}
  mnum operator+(mnum b){return v+b.v;}
  mnum operator-(mnum b){return v>=b.v? v-b.v : mod-b.v+v;}
  mnum operator*(mnum b){return v*b.v;}
  mnum operator^(int n){
    if(!n) return 1;
    return n%2? (*this)^(n/2)*(this) : (*this)^(n/2);}
};
```

2.12 Treap para set

```
typedef int Key;
typedef struct node *pnode;
struct node{
    Key key;
    int prior, size;
    pnode l,r;
    node(Key key=0): key(key), prior(rand()), size(1), l(0), r(0) {}
};
static int size(pnode p) { return p ? p->size : 0; }
void push(pnode p) {
  // modificar y propagar el dirty a los hijos aca(para lazy)
}
// Update function and size from children's Value
void pull(pnode p) {//recalcular valor del nodo aca (para rmq)
  p->size = 1 + size(p->l) + size(p->r);
}
//junta dos arreglos
pnode merge(pnode l, pnode r) {
  if (!l || !r) return l ? l : r;
  push(l), push(r);
  pnode t;
  if (l->prior < r->prior) l->r=merge(l->r, r), t = l;
  else r->l=merge(l, r->l), t = r;
  pull(t);
  return t;
}
//parte el arreglo en dos, l<key<=r
void split(pnode t, Key key, pnode &l, pnode &r) {
    if (!t) return void(l = r = 0);
    push(t);
    if (key <= t->key) split(t->l, key, l, t->l), r = t;
    else split(t->r, key, t->r, r), l = t;
```

```
    pull(t);
}

void erase(pnode &t, Key key) {
    if (!t) return;
    push(t);
    if (key == t->key) t=merge(t->l, t->r);
    else if (key < t->key) erase(t->l, key);
    else erase(t->r, key);
    if(t) pull(t);
}

ostream& operator<<(ostream &out, const pnode &t) {
  if(!t) return out;
    return out << t->l << t->key << ' ' << t->r;
}
pnode find(pnode t, Key key) {
    if (!t) return 0;
    if (key == t->key) return t;
    if (key < t->key) return find(t->l, key);
    return find(t->r, key);
}
struct treap {
    pnode root;
    treap(pnode root=0): root(root) {}
    int size() { return ::size(root); }
    void insert(Key key) {
        pnode t1, t2; split(root, key, t1, t2);
        t1=::merge(t1,new node(key));
        root=::merge(t1,t2);
    }
    void erase(Key key1, Key key2) {
        pnode t1,t2,t3;
        split(root,key1,t1,t2);
        split(t2,key2, t2, t3);
        root=merge(t1,t3);
    }
    void erase(Key key) {::erase(root, key);}
    pnode find(Key key) { return ::find(root, key); }
    Key &operator[](int pos){return find(pos)->key;}//ojito
};
treap merge(treap a, treap b) {return treap(merge(a.root, b.root));}
```

2.13 Treap para arreglo

```
typedef struct node *pnode;
struct node{
    Value val, mini;
    int dirty;
    int prior, size;
    pnode l,r,parent;
    node(Value val): val(val), mini(val), dirty(0), prior(rand()), size(1), l
        (0), r(0), parent(0) {}
};
static int size(pnode p) { return p ? p->size : 0; }
void push(pnode p) {//propagar dirty a los hijos(aca para lazy)
  p->val.fst+=p->dirty;
  p->mini.fst+=p->dirty;
  if(p->l) p->l->dirty+=p->dirty;
  if(p->r) p->r->dirty+=p->dirty;
  p->dirty=0;
}
static Value mini(pnode p) { return p ? push(p), p->mini : ii(1e9, -1); }
// Update function and size from children's Value
void pull(pnode p) {//recalcular valor del nodo aca (para rmq)
  p->size = 1 + size(p->l) + size(p->r);
  p->mini = min(min(p->val, mini(p->l)), mini(p->r));//operacion del rmq!
  p->parent=0;
  if(p->l) p->l->parent=p;
  if(p->r) p->r->parent=p;
}
//junta dos arreglos
pnode merge(pnode l, pnode r) {
  if (!l || !r) return l ? l : r;
  push(l), push(r);
  pnode t;
  if (l->prior < r->prior) l->r=merge(l->r, r), t = l;
  else r->l=merge(l, r->l), t = r;
  pull(t);
  return t;
}
//parte el arreglo en dos, sz(l)==tam
void split(pnode t, int tam, pnode &l, pnode &r) {
  if (!t) return void(l = r = 0);
  push(t);
  if (tam <= size(t->l)) split(t->l, tam, l, t->l), r = t;
  else split(t->r, tam - 1 - size(t->l), t->r, r), l = t;
  pull(t);
}
pnode at(pnode t, int pos) {
  if(!t) exit(1);
  push(t);
  if(pos == size(t->l)) return t;
```

```
  if(pos < size(t->l)) return at(t->l, pos);
  return at(t->r, pos - 1 - size(t->l));
}
int getpos(pnode t){//inversa de at
  if(!t->parent) return size(t->l);
  if(t==t->parent->l) return getpos(t->parent)-size(t->r)-1;
  return getpos(t->parent)+size(t->l)+1;
}
void split(pnode t, int i, int j, pnode &l, pnode &m, pnode &r) {
  split(t, i, l, t), split(t, j-i, m, r);}
Value get(pnode &p, int i, int j){//like rmq
  pnode l,m,r;
    split(p, i, j, l, m, r);
    Value ret=mini(m);
    p=merge(l, merge(m, r));
    return ret;
}
void print(const pnode &t) {//for debugging
  if(!t) return;
    push(t);
    print(t->l);
    cout << t->val.fst << ' ';
    print(t->r);
}
```

2.14 Convex Hull Trick

```
struct Line{tipo m,h;};
tipo inter(Line a, Line b){
    tipo x=b.h-a.h, y=a.m-b.m;
    return x/y+(x%y?!((x>0)^(y>0)):0);//==ceil(x/y)
}
struct CHT {
  vector<Line> c;
  bool mx;
  int pos;
  CHT(bool mx=0):mx(mx),pos(0){}//mx=1 si las query devuelven el max
  inline Line acc(int i){return c[c[0].m>c.back().m? i : sz(c)-1-i];}
  inline bool irre(Line x, Line y, Line z){
    return c[0].m>z.m? inter(y, z) <= inter(x, y)
                     : inter(y, z) >= inter(x, y);
  }
  void add(tipo m, tipo h) {//O(1), los m tienen que entrar ordenados
       if(mx) m*=-1, h*=-1;
    Line l=(Line){m, h};
       if(sz(c) && m==c.back().m) { l.h=min(h, c.back().h), c.pop_back(); if(
           pos) pos--; }
       while(sz(c)>=2 && irre(c[sz(c)-2], c[sz(c)-1], l)) { c.pop_back(); if(
           pos) pos--; }
       c.pb(l);
  }
  inline bool fbin(tipo x, int m) {return inter(acc(m), acc(m+1))>x;}
  tipo eval(tipo x){
    int n = sz(c);
    //query con x no ordenados O(lgn)
    int a=-1, b=n-1;
    while(b-a>1) { int m = (a+b)/2;
      if(fbin(x, m)) b=m;
      else a=m;
    }
    return (acc(b).m*x+acc(b).h)*(mx?-1:1);
       //query O(1)
    while(pos>0 && fbin(x, pos-1)) pos--;
    while(pos<n-1 && !fbin(x, pos)) pos++;
    return (acc(pos).m*x+acc(pos).h)*(mx?-1:1);
  }
} ch;
```

2.15 Convex Hull Trick (Dynamic)

```
const ll is_query = -(1LL<<62);
struct Line {
    ll m, b;
    mutable multiset<Line>::iterator it;
    const Line *succ(multiset<Line>::iterator it) const;
    bool operator<(const Line& rhs) const {
        if (rhs.b != is_query) return m < rhs.m;
        const Line *s=succ(it);
        if(!s) return 0;
        ll x = rhs.m;
        return b - s->b < (s->m - m) * x;
    }
};
struct HullDynamic : public multiset<Line>{ // will maintain upper hull for
    maximum
    bool bad(iterator y) {
        iterator z = next(y);
        if (y == begin()) {
            if (z == end()) return 0;
            return y->m == z->m && y->b <= z->b;
        }
        iterator x = prev(y);
        if (z == end()) return y->m == x->m && y->b <= x->b;
        return (x->b - y->b)*(z->m - y->m) >= (y->b - z->b)*(y->m - x->m);
```

```
    }
    iterator next(iterator y){return ++y;}
    iterator prev(iterator y){return --y;}
    void insert_line(ll m, ll b) {
        iterator y = insert((Line) { m, b });
        y->it=y;
        if (bad(y)) { erase(y); return; }
        while (next(y) != end() && bad(next(y))) erase(next(y));
        while (y != begin() && bad(prev(y))) erase(prev(y));
    }
    ll eval(ll x) {
        Line l = *lower_bound((Line) { x, is_query });
        return l.m * x + l.b;
    }
}h;
const Line *Line::succ(multiset<Line>::iterator it) const{
    return (++it==h.end()? NULL : &*it);}
```

2.16 Set con busq binaria

```
#include <ext/pb_ds/assoc_container.hpp>
#include <ext/pb_ds/tree_policy.hpp>
using namespace __gnu_pbds;
typedef tree<int,null_type,less<int>,//key,mapped type, comparator
    rb_tree_tag,tree_order_statistics_node_update> set_t;
//find_by_order(i) devuelve iterador al i-esimo elemento
//order_of_key(k): devuelve la pos del lower bound de k
//Ej: 12, 100, 505, 1000, 10000.
//order_of_key(10) == 0, order_of_key(100) == 1,
//order_of_key(707) == 3, order_of_key(9999999) == 5
```

3 Algos

3.1 Longest Increasing Subsecuence

```
//Para non-increasing, cambiar comparaciones y revisar busq binaria
//Given an array, paint it in the least number of colors so that each color
    turns to a non-increasing subsequence.
//Solution:Min number of colors=Length of the longest increasing subsequence
int N, a[MAXN];//secuencia y su longitud
ii d[MAXN+1];//d[i]=ultimo valor de la subsecuencia de tamanio i
int p[MAXN];//padres
vector<int> R;//respuesta
void rec(int i){
  if(i==-1) return;
  R.push_back(a[i]);
  rec(p[i]);
}
int lis(){//O(nlogn)
  d[0] = ii(-INF, -1); forn(i, N) d[i+1]=ii(INF, -1);
  forn(i, N){
    int j = upper_bound(d, d+N+1, ii(a[i], INF))-d;
    if (d[j-1].first < a[i]&&a[i] < d[j].first){
      p[i]=d[j-1].second;
      d[j] = ii(a[i], i);
    }
  }
  R.clear();
  dforn(i, N+1) if(d[i].first!=INF){
    rec(d[i].second);//reconstruir
    reverse(R.begin(), R.end());
    return i;//longitud
  }
  return 0;
}
```

3.2 Alpha-Beta prunning

```
ll alphabeta(State &s, bool player = true, int depth = 1e9, ll alpha = -INF, ll
     beta = INF) { //player = true -> Maximiza
    if(s.isFinal()) return s.score;
  //~ if (!depth) return s.heuristic();
    vector<State> children;
    s.expand(player, children);
    int n = children.size();
    forn(i, n) {
        ll v = alphabeta(children[i], !player, depth-1, alpha, beta);
        if(!player) alpha = max(alpha, v);
        else beta = min(beta, v);
        if(beta <= alpha) break;
    }
    return !player ? alpha : beta;}
```

3.3 Mo's algorithm

```
int n,sq;
struct Qu{//queries [l, r]
    //intervalos cerrado abiertos !!! importante!!
    int l, r, id;
}qs[MAXN];
int ans[MAXN], curans;//ans[i]=ans to ith query
bool bymos(const Qu &a, const Qu &b){
    if(a.l/sq!=b.l/sq) return a.l<b.l;
    return (a.l/sq)&1? a.r<b.r : a.r>b.r;
}
```

```
void mos(){
    forn(i, t) qs[i].id=i;
    sort(qs, qs+t, bymos);
    int cl=0, cr=0;
    sq=sqrt(n);
    curans=0;
    forn(i, t){ //intervalos cerrado abiertos !!! importante!!
        Qu &q=qs[i];
        while(cl>q.l) add(--cl);
        while(cr<q.r) add(cr++);
        while(cl<q.l) remove(cl++);
        while(cr>q.r) remove(--cr);
        ans[q.id]=curans;
    }
}
```

4 Strings

4.1 Manacher

```
int d1[MAXN];//d1[i]=long del maximo palindromo impar con centro en i
int d2[MAXN];//d2[i]=analogo pero para longitud par
//0 1 2 3 4
//a a b c c <--d1[2]=3
//a a b b <--d2[2]=2 (estan uno antes)
void manacher(){
  int l=0, r=-1, n=sz(s);
  forn(i, n){
    int k=(i>r? 1 : min(d1[l+r-i], r-i));
    while(i+k<n && i-k>=0 && s[i+k]==s[i-k]) ++k;
    d1[i] = k--;
    if(i+k > r) l=i-k, r=i+k;
  }
  l=0, r=-1;
  forn(i, n){
    int k=(i>r? 0 : min(d2[l+r-i+1], r-i+1))+1;
    while(i+k-1<n && i-k>=0 && s[i+k-1]==s[i-k]) k++;
    d2[i] = --k;
    if(i+k-1 > r) l=i-k, r=i+k-1;
  }
```

4.2 KMP

```
string T;//cadena donde buscar(where)
string P;//cadena a buscar(what)
int b[MAXLEN];//back table b[i] maximo borde de [0..i)
void kmppre(){//by gabina with love
    int i =0, j=-1; b[0]=-1;
    while(i<sz(P)){
        while(j>=0 && P[i] != P[j]) j=b[j];
        i++, j++, b[i] = j;
    }
}
void kmp(){
    int i=0, j=0;
    while(i<sz(T)){
        while(j>=0 && T[i]!=P[j]) j=b[j];
        i++, j++;
        if(j==sz(P)) printf("P is found at index %d in T\n", i-j), j=b[j];
    }
}
```

4.3 Trie

```
struct trie{
  map<char, trie> m;
  void add(const string &s, int p=0){
    if(s[p]) m[s[p]].add(s, p+1);
  }
  void dfs(){
    //Do stuff
    forall(it, m)
      it->second.dfs();
  }
};
```

4.4 Suffix Array (largo, nlogn)

```
#define MAX_N 1000
#define rBOUND(x) (x<n? r[x] : 0)
//sa will hold the suffixes in order.
int sa[MAX_N], r[MAX_N], n;
string s; //input string, n=sz(s)

int f[MAX_N], tmpsa[MAX_N];
void countingSort(int k){
  zero(f);
  forn(i, n) f[rBOUND(i+k)]++;
  int sum=0;
  forn(i, max(255, n)){
    int t=f[i]; f[i]=sum; sum+=t;}
  forn(i, n)
    tmpsa[f[rBOUND(sa[i]+k)]++]=sa[i];
  memcpy(sa, tmpsa, sizeof(sa));
}
```

```
void constructsa(){//O(n log n)
  n=sz(s);
  forn(i, n) sa[i]=i, r[i]=s[i];
  for(int k=1; k<n; k<<=1){
    countingSort(k), countingSort(0);
    int rank, tmpr[MAX_N];
    tmpr[sa[0]]=rank=0;
    forr(i, 1, n)
      tmpr[sa[i]]=(r[sa[i]]==r[sa[i-1]] && r[sa[i]+k]==r[sa[i-1]+k] )? rank :
          ++rank;
    memcpy(r, tmpr, sizeof(r));
    if(r[sa[n-1]]==n-1) break;
  }
}
void print(){//for debug
  forn(i, n)
    cout << i << '␣' <<
    s.substr(sa[i], s.find( '$', sa[i])-sa[i]) << endl;}
```

4.5 String Matching With Suffix Array

```
//returns (lowerbound, upperbound) of the search
ii stringMatching(string P){ //O(sz(P)lgn)
  int lo=0, hi=n-1, mid=lo;
  while(lo<hi){
    mid=(lo+hi)/2;
    int res=s.compare(sa[mid], sz(P), P);
    if(res>=0) hi=mid;
    else lo=mid+1;
  }
  if(s.compare(sa[lo], sz(P), P)!=0) return ii(-1, -1);
  ii ans; ans.fst=lo;
  lo=0, hi=n-1, mid;
  while(lo<hi){
    mid=(lo+hi)/2;
    int res=s.compare(sa[mid], sz(P), P);
    if(res>0) hi=mid;
    else lo=mid+1;
  }
  if(s.compare(sa[hi], sz(P), P)!=0) hi--;
  ans.snd=hi;
  return ans;
}
```

4.6 LCP (Longest Common Prefix)

```
//Calculates the LCP between consecutives suffixes in the Suffix Array.
//LCP[i] is the length of the LCP between sa[i] and sa[i-1]
int LCP[MAX_N], phi[MAX_N], PLCP[MAX_N];
void computeLCP(){//O(n)
  phi[sa[0]]=-1;
  forr(i, 1, n) phi[sa[i]]=sa[i-1];
  int L=0;
  forn(i, n){
    if(phi[i]==-1) {PLCP[i]=0; continue;}
    while(s[i+L]==s[phi[i]+L]) L++;
    PLCP[i]=L;
    L=max(L-1, 0);
  }
  forn(i, n) LCP[i]=PLCP[sa[i]];
}
```

4.7 Corasick

```

struct trie{
  map<char, trie> next;
  trie* tran[256];//transiciones del automata
  int idhoja, szhoja;//id de la hoja o 0 si no lo es
  //link lleva al sufijo mas largo, nxthoja lleva al mas largo pero que es hoja
  trie *padre, *link, *nxthoja;
  char pch;//caracter que conecta con padre
  trie(): tran(),  idhoja(), padre(), link() {}
  void insert(const string &s, int id=1, int p=0){//id>0!!!
    if(p<sz(s)){
      trie &ch=next[s[p]];
      tran[(int)s[p]]=&ch;
      ch.padre=this, ch.pch=s[p];
      ch.insert(s, id, p+1);
    }
    else idhoja=id, szhoja=sz(s);
  }
  trie* get_link() {
    if(!link){
      if(!padre) link=this;//es la raiz
      else if(!padre->padre) link=padre;//hijo de la raiz
      else link=padre->get_link()->get_tran(pch);
    }
    return link; }
  trie* get_tran(int c) {
    if(!tran[c]) tran[c] = !padre? this : this->get_link()->get_tran(c);
    return tran[c]; }
  trie *get_nxthoja(){
    if(!nxthoja) nxthoja = get_link()->idhoja? link : link->nxthoja;
    return nxthoja; }
```

```
  void print(int p){
    if(idhoja) cout << "found " << idhoja << "  at position " << p-szhoja <<
        endl;
    if(get_nxthoja()) get_nxthoja()->print(p); }
  void matching(const string &s, int p=0){
    print(p); if(p<sz(s)) get_tran(s[p])->matching(s, p+1); }
}tri;
```

4.8 Suffix Automaton

```
struct state {
  int len, link;
  map<char,int> next;
  state() { }
};
const int MAXLEN = 10010;
state st[MAXLEN*2];
int sz, last;
void sa_init() {
  forn(i,sz) st[i].next.clear();
  sz = last = 0;
  st[0].len = 0;
  st[0].link = -1;
  ++sz;
}
// Es un DAG de una sola fuente y una sola hoja
// cantidad de endpos = cantidad de apariciones = cantidad de caminos de la
    clase al nodo terminal
// cantidad de miembros de la clase = st[v].len-st[st[v].link].len (v>0) =
    caminos del inicio a la clase
// El arbol de los suffix links es el suffix tree de la cadena invertida. La
    string de la arista link(v)->v son los caracteres que difieren
void sa_extend (char c) {
  int cur = sz++;
  st[cur].len = st[last].len + 1;
  // en cur agregamos la posicion que estamos extendiendo
  //podria agregar tambien un identificador de las cadenas a las cuales
      pertenece (si hay varias)
  int p;
  for (p=last; p!=-1 && !st[p].next.count(c); p=st[p].link) // modificar esta
      linea para hacer separadores unicos entre varias cadenas (c=='$')
    st[p].next[c] = cur;
  if (p == -1)
    st[cur].link = 0;
  else {
    int q = st[p].next[c];
    if (st[p].len + 1 == st[q].len)
      st[cur].link = q;
    else {
      int clone = sz++;
      // no le ponemos la posicion actual a clone sino indirectamente por el
          link de cur
      st[clone].len = st[p].len + 1;
      st[clone].next = st[q].next;
      st[clone].link = st[q].link;
      for (; p!=-1 && st[p].next.count(c) && st[p].next[c]==q; p=st[p].link)
        st[p].next[c] = clone;
      st[q].link = st[cur].link = clone;
    }
  }
  last = cur;
}
```

4.9 Z Function

```
char s[MAXN];
int z[MAXN]; // z[i] = i==0 ? 0 : max k tq s[0,k) match with s[i,i+k)
void z_function(char s[],int z[]) {
    int n = strlen(s);
    forn(i, n) z[i]=0;
    for (int i = 1, l = 0, r = 0; i < n; ++i) {
        if (i <= r) z[i] = min (r - i + 1, z[i - l]);
        while (i + z[i] < n && s[z[i]] == s[i + z[i]])  ++z[i];
        if (i + z[i] - 1 > r) l = i, r = i + z[i] - 1;
    }
}
```

5 Geometria

5.1 Punto

```
struct pto{
  double x, y;
  pto(double x=0, double y=0):x(x),y(y){}
  pto operator+(pto a){return pto(x+a.x, y+a.y);}
  pto operator-(pto a){return pto(x-a.x, y-a.y);}
  pto operator+(double a){return pto(x+a, y+a);}
  pto operator*(double a){return pto(x*a, y*a);}
  pto operator/(double a){return pto(x/a, y/a);}
  //dot product, producto interno:
  double operator*(pto a){return x*a.x+y*a.y;}
  //module of the cross product or vectorial product:
  //if a is less than 180 clockwise from b, a^b>0
  double operator^(pto a){return x*a.y-y*a.x;}
```

```
  //returns true if this is at the left side of line qr
  bool left(pto q, pto r){return ((q-*this)^(r-*this))>0;}
  bool operator<(const pto &a) const{return x<a.x-EPS || (abs(x-a.x)<EPS && y<a
      .y-EPS);}
bool operator==(pto a){return abs(x-a.x)<EPS && abs(y-a.y)<EPS;}
  double norm(){return sqrt(x*x+y*y);}
  double norm_sq(){return x*x+y*y;}
};
double dist(pto a, pto b){return (b-a).norm();}
typedef pto vec;

double angle(pto a, pto o, pto b){
  pto oa=a-o, ob=b-o;
  return atan2(oa^ob, oa*ob);}

//rotate p by theta rads CCW w.r.t. origin (0,0)
pto rotate(pto p, double theta){
  return pto(p.x*cos(theta)-p.y*sin(theta),
     p.x*sin(theta)+p.y*cos(theta));
}
```

5.2 Orden radial de puntos

```
struct Cmp{//orden total de puntos alrededor de un punto r
  pto r;
  Cmp(pto r):r(r) {}
  int cuad(const pto &a) const{
    if(a.x > 0 && a.y >= 0)return 0;
    if(a.x <= 0 && a.y > 0)return 1;
    if(a.x < 0 && a.y <= 0)return 2;
    if(a.x >= 0 && a.y < 0)return 3;
    assert(a.x ==0 && a.y==0);
    return -1;
  }
  bool cmp(const pto&p1, const pto&p2)const{
    int c1 = cuad(p1), c2 = cuad(p2);
    if(c1==c2) return p1.y*p2.x<p1.x*p2.y;
        else return c1 < c2;
  }
    bool operator()(const pto&p1, const pto&p2) const{
    return cmp(pto(p1.x-r.x,p1.y-r.y),pto(p2.x-r.x,p2.y-r.y));
    }
};
```

5.3 Line

```
int sgn(ll x){return x<0? -1 : !!x;}
struct line{
  line() {}
  double a,b,c;//Ax+By=C
//pto MUST store float coordinates!
  line(double a, double b, double c):a(a),b(b),c(c){}
  line(pto p, pto q): a(q.y-p.y), b(p.x-q.x), c(a*p.x+b*p.y) {}
  int side(pto p){return sgn(ll(a) * p.x + ll(b) * p.y - c);}
};
bool parallels(line l1, line l2){return abs(l1.a*l2.b-l2.a*l1.b)<EPS;}
pto inter(line l1, line l2){//intersection
  double det=l1.a*l2.b-l2.a*l1.b;
  if(abs(det)<EPS) return pto(INF, INF);//parallels
  return pto(l2.b*l1.c-l1.b*l2.c, l1.a*l2.c-l2.a*l1.c)/det;
}
```

5.4 Segment

```
struct segm{
  pto s,f;
  segm(pto s, pto f):s(s), f(f) {}
  pto closest(pto p) {//use for dist to point
     double l2 = dist_sq(s, f);
     if(l2==0.) return s;
     double t =((p-s)*(f-s))/l2;
     if (t<0.) return s;//not write if is a line
     else if(t>1.)return f;//not write if is a line
     return s+((f-s)*t);
  }
    bool inside(pto p){return abs(dist(s, p)+dist(p, f)-dist(s, f))<EPS;}
};

pto inter(segm s1, segm s2){
  pto r=inter(line(s1.s, s1.f), line(s2.s, s2.f));
    if(s1.inside(r) && s2.inside(r)) return r;
  return pto(INF, INF);
}
```

5.5 Polygon Area

```
double area(vector<pto> &p){//O(sz(p))
  double area=0;
  forn(i, sz(p)) area+=p[i]^p[(i+1)%sz(p)];
  //if points are in clockwise order then area is negative
  return abs(area)/2;
}
//Area ellipse = M_PI*a*b where a and b are the semi axis lengths
//Area triangle = sqrt(s*(s-a)(s-b)(s-c)) where s=(a+b+c)/2
```

5.6 Circle

```
vec perp(vec v){return vec(-v.y, v.x);}
line bisector(pto x, pto y){
  line l=line(x, y); pto m=(x+y)/2;
  return line(-l.b, l.a, -l.b*m.x+l.a*m.y);
}
struct Circle{
  pto o;
  double r;
  Circle(pto x, pto y, pto z){
    o=inter(bisector(x, y), bisector(y, z));
    r=dist(o, x);
  }
  pair<pto, pto> ptosTang(pto p){
    pto m=(p+o)/2;
    tipo d=dist(o, m);
    tipo a=r*r/(2*d);
    tipo h=sqrt(r*r-a*a);
    pto m2=o+(m-o)*a/d;
    vec per=perp(m-o)/d;
    return make_pair(m2-per*h, m2+per*h);
  }
};
//finds the center of the circle containing p1 and p2 with radius r
//as there may be two solutions swap p1, p2 to get the other
bool circle2PtsRad(pto p1, pto p2, double r, pto &c){
        double d2=(p1-p2).norm_sq(), det=r*r/d2-0.25;
        if(det<0) return false;
        c=(p1+p2)/2+perp(p2-p1)*sqrt(det);
        return true;
}
#define sqr(a) ((a)*(a))
#define feq(a,b) (fabs((a)-(b))<EPS)
pair<tipo, tipo> ecCuad(tipo a, tipo b, tipo c){//a*x*x+b*x+c=0
  tipo dx = sqrt(b*b-4.0*a*c);
  return make_pair((-b + dx)/(2.0*a),(-b - dx)/(2.0*a));
}
pair<pto, pto> interCL(Circle c, line l){
  bool sw=false;
  if((sw=feq(0,l.b))){
  swap(l.a, l.b);
  swap(c.o.x, c.o.y);
  }
  pair<tipo, tipo> rc = ecCuad(
  sqr(l.a)+sqr(l.b),
  2.0*l.a*l.b*c.o.y-2.0*(sqr(l.b)*c.o.x+l.c*l.a),
  sqr(l.b)*(sqr(c.o.x)+sqr(c.o.y)-sqr(c.r))+sqr(l.c)-2.0*l.c*l.b*c.o.y
  );
  pair<pto, pto> p( pto(rc.first, (l.c - l.a * rc.first) / l.b),
            pto(rc.second, (l.c - l.a * rc.second) / l.b) );
  if(sw){
  swap(p.first.x, p.first.y);
  swap(p.second.x, p.second.y);
  }
  return p;
}
pair<pto, pto> interCC(Circle c1, Circle c2){
  line l;
  l.a = c1.o.x-c2.o.x;
  l.b = c1.o.y-c2.o.y;
  l.c = (sqr(c2.r)-sqr(c1.r)+sqr(c1.o.x)-sqr(c2.o.x)+sqr(c1.o.y)
  -sqr(c2.o.y))/2.0;
  return interCL(c1, l);
}
```

5.7 Point in Poly

```
//checks if v is inside of P, using ray casting
//works with convex and concave.
//excludes boundaries, handle it separately using segment.inside()
bool inPolygon(pto v, vector<pto>& P) {
  bool c = false;
  forn(i, sz(P)){
    int j=(i+1)%sz(P);
    if((P[j].y>v.y) != (P[i].y > v.y) &&
  (v.x < (P[i].x - P[j].x) * (v.y-P[j].y) / (P[i].y - P[j].y) + P[j].x))
      c = !c;
  }
  return c;
}
```

5.8 Point in Convex Poly log(n)

```
void normalize(vector<pto> &pt){//delete collinear points first!
  //this makes it clockwise:
    if(pt[2].left(pt[0], pt[1])) reverse(pt.begin(), pt.end());
  int n=sz(pt), pi=0;
  forn(i, n)
    if(pt[i].x<pt[pi].x || (pt[i].x==pt[pi].x && pt[i].y<pt[pi].y))
      pi=i;
  vector<pto> shift(n);//puts pi as first point
    forn(i, n) shift[i]=pt[(pi+i)%n];
    pt.swap(shift);
}
bool inPolygon(pto p, const vector<pto> &pt){
```

```
  //call normalize first!
  if(p.left(pt[0], pt[1]) || p.left(pt[sz(pt)-1], pt[0])) return false;
  int a=1, b=sz(pt)-1;
  while(b-a>1){
    int c=(a+b)/2;
    if(!p.left(pt[0], pt[c])) a=c;
    else b=c;
  }
  return !p.left(pt[a], pt[a+1]);
}
```

5.9 Convex Hull

```
//stores convex hull of P in S, CCW order
//left must return >=0 to delete collinear points!
void CH(vector<pto>& P, vector<pto> &S){
  S.clear();
  sort(P.begin(), P.end());//first x, then y
  forn(i, sz(P)){//lower hull
    while(sz(S)>= 2 && S[sz(S)-1].left(S[sz(S)-2], P[i])) S.pop_back();
    S.pb(P[i]);
  }
  S.pop_back();
  int k=sz(S);
  dforn(i, sz(P)){//upper hull
    while(sz(S) >= k+2 && S[sz(S)-1].left(S[sz(S)-2], P[i])) S.pop_back();
    S.pb(P[i]);
  }
  S.pop_back();
}
```

5.10 Cut Polygon

```
//cuts polygon Q along the line ab
//stores the left side (swap a, b for the right one) in P
void cutPolygon(pto a, pto b, vector<pto> Q, vector<pto> &P){
  P.clear();
  forn(i, sz(Q)){
    double left1=(b-a)^(Q[i]-a), left2=(b-a)^(Q[(i+1)%sz(Q)]-a);
    if(left1>=0) P.pb(Q[i]);
    if(left1*left2<0)
      P.pb(inter(line(Q[i], Q[(i+1)%sz(Q)]), line(a, b)));
  }
}
```

5.11 Bresenham

```
//plot a line approximation in a 2d map
void bresenham(pto a, pto b){
  pto d=b-a; d.x=abs(d.x), d.y=abs(d.y);
  pto s(a.x<b.x? 1: -1, a.y<b.y? 1: -1);
  int err=d.x-d.y;
  while(1){
    m[a.x][a.y]=1;//plot
    if(a==b) break;
    int e2=err;
    if(e2 >= 0) err-=2*d.y, a.x+=s.x;
    if(e2 <= 0) err+= 2*d.x, a.y+= s.y;
  }
}
```

5.12 Interseccion de Circulos en n3log(n)

```
struct event {
    double x; int t;
    event(double xx, int tt) : x(xx), t(tt) {}
    bool operator <(const event &o) const { return x < o.x; }
};
typedef vector<Circle> VC;
typedef vector<event> VE;
int n;
double cuenta(VE &v, double A,double B) {
    sort(v.begin(), v.end());
    double res = 0.0, lx = ((v.empty())?0.0:v[0].x);
    int contador = 0;
    forn(i,sz(v)) {
        //interseccion de todos (contador == n), union de todos (contador > 0)
        //conjunto de puntos cubierto por exacta k Circulos (contador == k)
        if (contador == n) res += v[i].x - lx;
        contador += v[i].t, lx = v[i].x;
    }
    return res;
}
// Primitiva de sqrt(r*r - x*x) como funcion double de una variable x.
inline double primitiva(double x,double r) {
    if (x >= r) return r*r*M_PI/4.0;
    if (x <= -r) return -r*r*M_PI/4.0;
    double raiz = sqrt(r*r-x*x);
    return 0.5 * (x * raiz + r*r*atan(x/raiz));
}
double interCircle(VC &v) {
    vector<double> p; p.reserve(v.size() * (v.size() + 2));
    forn(i,sz(v))  p.push_back(v[i].c.x + v[i].r), p.push_back(v[i].c.x - v[i].
        r);
    forn(i,sz(v)) forn(j,i) {
        Circle &a = v[i], b = v[j];
```

```
        double d = (a.c - b.c).norm();
        if (fabs(a.r - b.r) < d && d < a.r + b.r) {
            double alfa = acos((sqr(a.r) + sqr(d) - sqr(b.r)) / (2.0 * d * a.r)
                );
            pto vec = (b.c - a.c) * (a.r / d);
            p.pb((a.c + rotate(vec, alfa)).x), p.pb((a.c + rotate(vec, -alfa)).
                x);
        }
    }
    sort(p.begin(), p.end());
    double res = 0.0;
    forn(i,sz(p)-1) {
        const double A = p[i], B = p[i+1];
        VE ve; ve.reserve(2 * v.size());
        forn(j,sz(v)) {
            const Circle &c = v[j];
            double arco = primitiva(B-c.c.x,c.r) - primitiva(A-c.c.x,c.r);
            double base = c.c.y * (B-A);
            ve.push_back(event(base + arco,-1));
            ve.push_back(event(base - arco, 1));
        }
        res += cuenta(ve,A,B);
    }
    return res;
}
```

6 Math

6.1 Identidades

$\sum_{i=0}^{n} \binom{n}{i} = 2^n$
$\sum_{i=0}^{n} i\binom{n}{i} = n * 2^{n-1}$
$\sum_{i=m}^{n} i = \frac{n(n+1)}{2} - \frac{m(m-1)}{2} = \frac{(n+1-m)(n+m)}{2}$
$\sum_{i=0}^{n} i = \sum_{i=1}^{n} i = \frac{n(n+1)}{2}$
$\sum_{i=0}^{n} i^2 = \frac{n(n+1)(2n+1)}{6} = \frac{n^3}{3} + \frac{n^2}{2} + \frac{n}{6}$
$\sum_{i=0}^{n} i(i-1) = \frac{8}{6}(\frac{n}{2})(\frac{n}{2}+1)(n+1)$ (doubles) $\rightarrow$ Sino ver caso impar y par
$\sum_{i=0}^{n} i^3 = \left(\frac{n(n+1)}{2}\right)^2 = \frac{n^4}{4} + \frac{n^3}{2} + \frac{n^2}{4} = [\sum_{i=1}^{n} i]^2$
$\sum_{i=0}^{n} i^4 = \frac{n(n+1)(2n+1)(3n^2+3n-1)}{30} = \frac{n^5}{5} + \frac{n^4}{2} + \frac{n^3}{3} - \frac{n}{30}$
$\sum_{i=0}^{n} i^p = \frac{(n+1)^{p+1}}{p+1} + \sum_{k=1}^{p} \frac{B_k}{p-k+1}\binom{p}{k}(n+1)^{p-k+1}$
$r = e - v + k + 1$
Teorema de Pick: (Area, puntos interiores y puntos en el borde)
$A = I + \frac{B}{2} - 1$

6.2 Ec. Caracteristica

$a_0T(n) + a_1T(n-1) + ... + a_kT(n-k) = 0$
$p(x) = a_0x^k + a_1x^{k-1} + ... + a_k$

Sean $r_1, r_2, ..., r_q$ las raíces distintas, de mult. $m_1, m_2, ..., m_q$
$T(n) = \sum_{i=1}^{q} \sum_{j=0}^{m_i-1} c_{ij} n^j r_i^n$
Las constantes c_{ij} se determinan por los casos base.

6.3 Combinatorio

```
forn(i, MAXN+1){//comb[i][k]=i tomados de a k
  comb[i][0]=comb[i][i]=1;
  forr(k, 1, i) comb[i][k]=(comb[i-1][k]+comb[i-1][k-1])%MOD;
}
ll lucas (ll n, ll k, int p){ //Calcula (n,k)%p teniendo comb[p][p]
    precalculado.
  ll aux = 1;
  while (n + k) aux = (aux * comb[n%p][k%p]) %p, n/=p, k/=p;
  return aux;
}
```

6.4 Exp. de Numeros Mod.

```
ll expmod (ll b, ll e, ll m){//O(log b)
  if(!e) return 1;
  ll q= expmod(b,e/2,m); q=(q*q)%m;
  return e%2? (b * q)%m : q;
}
```

6.5 Exp. de Matrices

```
#define SIZE 350
int NN;
double tmp[SIZE][SIZE];
void mul(double a[SIZE][SIZE], double b[SIZE][SIZE]){ zero(tmp);
    forn(i, NN) forn(j, NN) forn(k, NN) res[i][j]+=a[i][k]*b[k][j];
    forn(i, NN) forn(j, NN) a[i][j]=res[i][j];
}
void powmat(double a[SIZE][SIZE], int n, double res[SIZE][SIZE]){
    forn(i, NN) forn(j, NN) res[i][j]=(i==j);
    while(n){
        if(n&1) mul(res, a), n--;
        else mul(a, a), n/=2;
    } }
```

6.6 Matrices y determinante $O(n^3)$

```
struct Mat {
    vector<vector<double> > vec;
    Mat(int n): vec(n, vector<double>(n) ) {}
    Mat(int n, int m): vec(n, vector<double>(m) ) {}
    vector<double> &operator[](int f){return vec[f];}
    const vector<double> &operator[](int f) const {return vec[f];}
    int size() const {return sz(vec);}
```

```
    Mat operator+(Mat &b) { ///this de n x m entonces b de n x m
        Mat m(sz(b),sz(b[0]));
        forn(i,sz(vec)) forn(j,sz(vec[0])) m[i][j] = vec[i][j] + b[i][j];
        return m;    }
    Mat operator*(const Mat &b) { ///this de n x m entonces b de m x t
        int n = sz(vec), m = sz(vec[0]), t = sz(b[0]);
        Mat mat(n,t);
        forn(i,n) forn(j,t) forn(k,m) mat[i][j] += vec[i][k] * b[k][j];
        return mat;    }
    double determinant(){//sacado de e maxx ru
        double det = 1;
        int n = sz(vec);
        Mat m(*this);
        forn(i, n){//para cada columna
            int k = i;
            forr(j, i+1, n)//busco la fila con mayor val abs
                if(abs(m[j][i])>abs(m[k][i])) k = j;
            if(abs(m[k][i])<1e-9) return 0;
            m[i].swap(m[k]);//la swapeo
            if(i!=k) det = -det;
            det *= m[i][i];
            forr(j, i+1, n) m[i][j] /= m[i][i];
            //hago 0 todas las otras filas
            forn(j, n) if (j!= i && abs(m[j][i])>1e-9)
                forr(k, i+1, n) m[j][k]-=m[i][k]*m[j][i];
        }
        return det;
    }
};
```

6.7 Teorema Chino del Resto

$$y = \sum_{j=1}^{n} (x_j * (\prod_{i=1, i\neq j}^{n} m_i)^{-1}_{m_j} * \prod_{i=1, i\neq j}^{n} m_i)$$

6.8 Criba

```
#define MAXP 100000 //no necesariamente primo
int criba[MAXP+1];
void crearcriba(){
  int w[] = {4,2,4,2,4,6,2,6};
  for(int p=25;p<=MAXP;p+=10) criba[p]=5;
  for(int p=9;p<=MAXP;p+=6) criba[p]=3;
  for(int p=4;p<=MAXP;p+=2) criba[p]=2;
  for(int p=7,cur=0;p*p<=MAXP;p+=w[cur++&7]) if (!criba[p])
    for(int j=p*p;j<=MAXP;j+=(p<<1)) if(!criba[j]) criba[j]=p;
}
```

```
vector<int> primos;
void buscarprimos(){
  crearcriba();
  forr (i,2,MAXP+1) if (!criba[i]) primos.push_back(i);
}
//~ Useful for bit trick: #define SET(i) ( criba[(i)>>5]|=1<<((i)&31) ), #
    define INDEX(i) ( (criba[i>>5]>>((i)&31))&1 ), unsigned int criba[MAXP
    /32+1];
```

6.9 Funciones de primos

Sea $n = \prod p_i^{k_i}$, fact(n) genera un map donde a cada p_i le asocia su k_i

```
//factoriza bien numeros hasta MAXP^2
map<ll,ll> fact(ll n){ //O (cant primos)
  map<ll,ll> ret;
  forall(p, primos){
    while(!(n%*p)){
      ret[*p]++;//divisor found
      n/=*p;
    }
  }
  if(n>1) ret[n]++;
  return ret;
}
//factoriza bien numeros hasta MAXP
map<ll,ll> fact2(ll n){ //O (lg n)
  map<ll,ll> ret;
  while (criba[n]){
    ret[criba[n]]++;
    n/=criba[n];
  }
  if(n>1) ret[n]++;
  return ret;
}
//Usar asi:  divisores(fac, divs, fac.begin()); NO ESTA ORDENADO
void divisores(const map<ll,ll> &f, vector<ll> &divs, map<ll,ll>::iterator it,
    ll n=1){
    if(it==f.begin()) divs.clear();
    if(it==f.end()) { divs.pb(n);  return; }
    ll p=it->fst, k=it->snd; ++it;
    forn(_, k+1) divisores(f, divs, it, n), n*=p;
}
ll sumDiv (ll n){
  ll rta = 1;
  map<ll,ll> f=fact(n);
  forall(it, f) {
  ll pot = 1, aux = 0;
```

```
  forn(i, it->snd+1) aux += pot, pot *= it->fst;
  rta*=aux;
  }
  return rta;
}
ll eulerPhi (ll n){ // con criba: O(lg n)
  ll rta = n;
  map<ll,ll> f=fact(n);
  forall(it, f) rta -= rta / it->first;
  return rta;
}
ll eulerPhi2 (ll n){ // O (sqrt n)
  ll r = n;
  forr (i,2,n+1){
    if ((ll)i*i > n) break;
    if (n % i == 0){
      while (n%i == 0) n/=i;
      r -= r/i; }
  }
  if (n != 1) r-= r/n;
  return r;
}
```

6.10 Phollard's Rho (rolando)

```
ll mulmod(ll a,ll b,ll c){ll x=0,y=a%c;while(b>0){if(b%2==1)x=(x+y)%c;y=(y*2)%c
    ;b/=2;}return x%c;}ll expmod(ll b,ll e,ll m){if(!e)return 1;ll q=expmod(b,
    e/2,m);q=mulmod(q,q,m);return e%2?mulmod(b,q,m):q;}bool es_primo_prob(ll n
    ,int a){if(n==a)return true;ll s=0,d=n-1;while(d%2==0)s++,d/=2;ll x=expmod
    (a,d,n);if((x==1)||(x+1==n))return true;forn(i,s-1){x=mulmod(x,x,n);if(x
    ==1)return false;if(x+1==n)return true;}return false;}bool rabin(ll n){if(
    n==1)return false;const int ar[]={2,3,5,7,11,13,17,19,23};forn(j,9)if(!
    es_primo_prob(n,ar[j]))return false;return true;}ll rho(ll n){if((n&1)==0)
    return 2;ll x=2,y=2,d=1;ll c=rand()%n+1;while(d==1){x=(mulmod(x,x,n)+c)%n;
    y=(mulmod(y,y,n)+c)%n;y=(mulmod(y,y,n)+c)%n;if(x-y>=0)d=gcd(x-y,n);else d=
    gcd(y-x,n);}return d==n?rho(n):d;}map<ll,ll>prim;void factRho(ll n){if(n
    ==1)return;if(rabin(n)){prim[n]++;return;}ll factor=rho(n);factRho(factor)
    ;factRho(n/factor);}
```

6.11 GCD

```
tipo gcd(tipo a, tipo b){return a?gcd(b%a, a):b;}
```

6.12 Extended Euclid

```
void extendedEuclid (ll a, ll b){ //a * x + b * y = d
  if (!b) { x = 1; y = 0; d = a; return;}
  extendedEuclid (b, a%b);
  ll x1 = y;
  ll y1 = x - (a/b) * y;
  x = x1; y = y1;
}
```

6.13 LCM

```
tipo lcm(tipo a, tipo b){return a / gcd(a,b) * b;}
```

6.14 Inversos

```
#define MAXMOD 15485867
ll inv[MAXMOD];//inv[i]*i=1 mod MOD
void calc(int p){//O(p)
  inv[1]=1;
  forr(i, 2, p) inv[i]= p-((p/i)*inv[p%i])%p;
}
int inverso(int x){//O(log x)
  return expmod(x, eulerphi(MOD)-2);//si mod no es primo(sacar a mano)
  return expmod(x, MOD-2);//si mod es primo
}
```

6.15 Simpson

```
double integral(double a, double b, int n=10000) {//O(n), n=cantdiv
  double area=0, h=(b-a)/n, fa=f(a), fb;
  forn(i, n){
    fb=f(a+h*(i+1));
    area+=fa+ 4*f(a+h*(i+0.5)) +fb, fa=fb;
  }
  return area*h/6.;}
```

6.16 Fraction

```
tipo mcd(tipo a, tipo b){return a?mcd(b%a, a):b;}
struct frac{
  tipo p,q;
  frac(tipo p=0, tipo q=1):p(p),q(q) {norm();}
  void norm(){
    tipo a = mcd(p,q);
    if(a) p/=a, q/=a;
    else q=1;
    if (q<0) q=-q, p=-p;}
  frac operator+(const frac& o){
    tipo a = mcd(q,o.q);
    return frac(p*(o.q/a)+o.p*(q/a), q*(o.q/a));}
  frac operator-(const frac& o){
    tipo a = mcd(q,o.q);
    return frac(p*(o.q/a)-o.p*(q/a), q*(o.q/a));}
  frac operator*(frac o){
```

```
    tipo a = mcd(q,o.p), b = mcd(o.q,p);
    return frac((p/b)*(o.p/a), (q/a)*(o.q/b));}
  frac operator/(frac o){
    tipo a = mcd(q,o.q), b = mcd(o.p,p);
    return frac((p/b)*(o.q/a),(q/a)*(o.p/b));}
  bool operator<(const frac &o) const{return p*o.q < o.p*q;}
  bool operator==(frac o){return p==o.p&&q==o.q;}
};
```

6.17 Polinomio

```
struct poly {
    vector<tipo> c;//guarda los coeficientes del polinomio
    poly(const vector<tipo> &c): c(c) {}
    poly() {}
  bool isnull() {return c.empty();}
    poly operator+(const poly &o) const {
        int m = sz(c), n = sz(o.c);
        vector<tipo> res(max(m,n));
        forn(i, m) res[i] += c[i];
        forn(i, n) res[i] += o.c[i];
        return poly(res);    }
    poly operator*(const tipo cons) const {
    vector<tipo> res(sz(c));
        forn(i, sz(c)) res[i]=c[i]*cons;
        return poly(res);    }
    poly operator*(const poly &o) const {
        int m = sz(c), n = sz(o.c);
        vector<tipo> res(m+n-1);
        forn(i, m) forn(j, n) res[i+j]+=c[i]*o.c[j];
        return poly(res);    }
  tipo eval(tipo v) {
    tipo sum = 0;
    dforn(i, sz(c)) sum=sum*v + c[i];
    return sum; }
    //poly contains only a vector<int> c (the coeficients)
  //the following function generates the roots of the polynomial
//it can be easily modified to return float roots
  set<tipo> roots(){
    set<tipo> roots;
    tipo a0 = abs(c[0]), an = abs(c[sz(c)-1]);
    vector<tipo> ps,qs;
    forr(p,1,sqrt(a0)+1) if (a0%p==0) ps.pb(p),ps.pb(a0/p);
    forr(q,1,sqrt(an)+1) if (an%q==0) qs.pb(q),qs.pb(an/q);
    forall(pt,ps)
      forall(qt,qs) if ( (*pt) % (*qt)==0 ) {
        tipo root = abs((*pt) / (*qt));
        if (eval(root)==0) roots.insert(root);
      }
    return roots; }
};
pair<poly,tipo> ruffini(const poly p, tipo r) {
  int n = sz(p.c) - 1 ;
  vector<tipo> b(n);
  b[n-1] = p.c[n];
  dforn(k,n-1) b[k] = p.c[k+1] + r*b[k+1];
  tipo resto = p.c[0] + r*b[0];
  poly result(b);
  return make_pair(result,resto);
}
poly interpolate(const vector<tipo>& x,const vector<tipo>& y) {
    poly A; A.c.pb(1);
    forn(i,sz(x)) { poly aux; aux.c.pb(-x[i]), aux.c.pb(1), A = A * aux; }
  poly S; S.c.pb(0);
  forn(i,sz(x)) { poly Li;
    Li = ruffini(A,x[i]).fst;
    Li = Li * (1.0 / Li.eval(x[i])); // here put a multiple of the coefficients
         instead of 1.0 to avoid using double
    S = S + Li * y[i];  }
  return S;
}
```

6.18 Ec. Lineales

```
bool resolver_ev(Mat a, Vec y, Vec &x, Mat &ev){
  int n = a.size(), m = n?a[0].size():0, rw = min(n, m);
  vector<int> p; forn(i,m) p.push_back(i);
  forn(i, rw) {
    int uc=i, uf=i;
    forr(f, i, n) forr(c, i, m) if(fabs(a[f][c])>fabs(a[uf][uc])) {uf=f;uc=c;}
    if (feq(a[uf][uc], 0)) { rw = i; break; }
    forn(j, n) swap(a[j][i], a[j][uc]);
    swap(a[i], a[uf]); swap(y[i], y[uf]); swap(p[i], p[uc]);
    tipo inv = 1 / a[i][i]; //aca divide
    forr(j, i+1, n) {
      tipo v = a[j][i] * inv;
      forr(k, i, m) a[j][k]-=v * a[i][k];
      y[j] -= v*y[i];
    }
  } // rw = rango(a), aca la matriz esta triangulada
  forr(i, rw, n) if (!feq(y[i],0)) return false; // checkeo de compatibilidad
  x = vector<tipo>(m, 0);
  dforn(i, rw){
    tipo s = y[i];
```

```
    forr(j, i+1, rw) s -= a[i][j]*x[p[j]];
    x[p[i]] = s / a[i][i]; //aca divide
  }
  ev = Mat(m-rw, Vec(m, 0)); // Esta parte va SOLO si se necesita el ev
  forn(k, m-rw) {
    ev[k][p[k+rw]] = 1;
    dforn(i, rw){
      tipo s = -a[i][k+rw];
      forr(j, i+1, rw) s -= a[i][j]*ev[k][p[j]];
      ev[k][p[i]] = s / a[i][i]; //aca divide
    }
  }
  return true;
}
```

6.19 FFT

```
//~ typedef complex<double> base; //menos codigo, pero mas lento
//elegir si usar complejos de c (lento) o estos
struct base{
    double r,i;
    base(double r=0, double i=0):r(r), i(i){}
    double real()const{return r;}
    void operator/=(const int c){r/=c, i/=c;}
};
base operator*(const base &a, const base &b){
    return base(a.r*b.r-a.i*b.i, a.r*b.i+a.i*b.r);}
base operator+(const base &a, const base &b){
    return base(a.r+b.r, a.i+b.i);}
base operator-(const base &a, const base &b){
    return base(a.r-b.r, a.i-b.i);}
vector<int> rev; vector<base> wlen_pw;
inline static void fft(base a[], int n, bool invert) {
    forn(i, n) if(i<rev[i]) swap(a[i], a[rev[i]]);
  for (int len=2; len<=n; len<<=1) {
    double ang = 2*M_PI/len * (invert?-1:+1);
    int len2 = len>>1;
    base wlen (cos(ang), sin(ang));
    wlen_pw[0] = base (1, 0);
        forr(i, 1, len2) wlen_pw[i] = wlen_pw[i-1] * wlen;
    for (int i=0; i<n; i+=len) {
      base t, *pu = a+i, *pv = a+i+len2,  *pu_end = a+i+len2, *pw = &wlen_pw
          [0];
      for (; pu!=pu_end; ++pu, ++pv, ++pw)
        t = *pv * *pw, *pv = *pu - t,*pu = *pu + t;
    }
  }
  if (invert) forn(i, n) a[i]/= n;}
inline static void calc_rev(int n){//precalculo: llamar antes de fft!!
    wlen_pw.resize(n), rev.resize(n);
    int lg=31-__builtin_clz(n);
    forn(i, n){
    rev[i] = 0;
        forn(k, lg) if(i&(1<<k)) rev[i]|=1<<(lg-1-k);
    }}
inline static void multiply(const vector<int> &a, const vector<int> &b, vector<
    int> &res) {
  vector<base> fa (a.begin(), a.end()),  fb (b.begin(), b.end());
    int n=1; while(n < max(sz(a), sz(b))) n <<= 1; n <<= 1;
    calc_rev(n);
  fa.resize (n),  fb.resize (n);
  fft (&fa[0], n, false),  fft (&fb[0], n, false);
  forn(i, n) fa[i] = fa[i] * fb[i];
  fft (&fa[0], n, true);
  res.resize(n);
    forn(i, n) res[i] = int (fa[i].real() + 0.5); }
void toPoly(const string &s, vector<int> &P){//convierte un numero a polinomio
    P.clear();
    dforn(i, sz(s)) P.pb(s[i]-'0');}
```

6.20 Tablas y cotas (Primos, Divisores, Factoriales, etc)

Cantidad de primos menores que 10^n

$\pi(10^1) = 4$; $\pi(10^2) = 25$; $\pi(10^3) = 168$; $\pi(10^4) = 1229$; $\pi(10^5) = 9592$
$\pi(10^6) = 78.498$; $\pi(10^7) = 664.579$; $\pi(10^8) = 5.761.455$; $\pi(10^9) = 50.847.534$
$\pi(10^{10}) = 455.052{,}511$; $\pi(10^{11}) = 4.118.054.813$; $\pi(10^{12}) = 37.607.912.018$

Divisores

Cantidad de divisores (σ_0) para *algunos* $n / \neg\exists n' < n, \sigma_0(n') \geqslant \sigma_0(n)$
$\sigma_0(60) = 12$; $\sigma_0(120) = 16$; $\sigma_0(180) = 18$; $\sigma_0(240) = 20$; $\sigma_0(360) = 24$
$\sigma_0(720) = 30$; $\sigma_0(840) = 32$; $\sigma_0(1260) = 36$; $\sigma_0(1680) = 40$; $\sigma_0(10080) = 72$
$\sigma_0(15120) = 80$; $\sigma_0(50400) = 108$; $\sigma_0(83160) = 128$; $\sigma_0(110880) = 144$
$\sigma_0(498960) = 200$; $\sigma_0(554400) = 216$; $\sigma_0(1081080) = 256$; $\sigma_0(1441440) = 288$ $\sigma_0(4324320) = 384$; $\sigma_0(8648640) = 448$

7 Grafos

7.1 Dijkstra

```
#define add(a, b, w) G[a].pb(make_pair(w, b))
ll dijkstra(int s, int t){//O(|E| log |V|)
  priority_queue<ii, vector<ii>, greater<ii> > Q;
  vector<ll> dist(N, INF); vector<int> dad(N, -1);
  Q.push(make_pair(0, s)); dist[s] = 0;
```

```
  while(sz(Q)){
    ii p = Q.top(); Q.pop();
    if(p.snd == t) break;
    forall(it, G[p.snd])
      if(dist[p.snd]+it->first < dist[it->snd]){
        dist[it->snd] = dist[p.snd] + it->fst;
        dad[it->snd] = p.snd;
        Q.push(make_pair(dist[it->snd], it->snd));  }
  }
  return dist[t];
  if(dist[t]<INF)//path generator
    for(int i=t; i!=-1; i=dad[i])
      printf("%d%c", i, (i==s?'\n':' '));}
```

7.2 Bellman-Ford

```
vector<ii> G[MAX_N];//ady. list with pairs (weight, dst)
int dist[MAX_N];
void bford(int src){//O(VE)
  dist[src]=0;
  forn(i, N-1) forn(j, N) if(dist[j]!=INF) forall(it, G[j])
    dist[it->snd]=min(dist[it->snd], dist[j]+it->fst);
}

bool hasNegCycle(){
  forn(j, N) if(dist[j]!=INF) forall(it, G[j])
    if(dist[it->snd]>dist[j]+it->fst) return true;
  //inside if: all points reachable from it->snd will have -INF distance(do bfs
      )
  return false;
}
```

7.3 Floyd-Warshall

```
//G[i][j] contains weight of edge (i, j) or INF
//G[i][i]=0
int G[MAX_N][MAX_N];
void floyd(){//O(N^3)
forn(k, N) forn(i, N) if(G[i][k]!=INF) forn(j, N) if(G[k][j]!=INF)
  G[i][j]=min(G[i][j], G[i][k]+G[k][j]);
}
bool inNegCycle(int v){
  return G[v][v]<0;}
//checks if there's a neg. cycle in path from a to b
bool hasNegCycle(int a, int b){
  forn(i, N) if(G[a][i]!=INF && G[i][i]<0 && G[i][b]!=INF)
    return true;
  return false;
}
```

7.4 Kruskal

```
struct Ar{int a,b,w;};
bool operator<(const Ar& a, const Ar &b){return a.w<b.w;}
vector<Ar> E;
ll kruskal(){
    ll cost=0;
    sort(E.begin(), E.end());//ordenar aristas de menor a mayor
    uf.init(n);
    forall(it, E){
        if(uf.comp(it->a)!=uf.comp(it->b)){//si no estan conectados
            uf.unir(it->a, it->b);//conectar
            cost+=it->w;
        }
    }
    return cost;
}
```

7.5 Prim

```
bool taken[MAXN];
priority_queue<ii, vector<ii>, greater<ii> > pq;//min heap
void process(int v){
    taken[v]=true;
    forall(e, G[v])
        if(!taken[e->second]) pq.push(*e);
}

ll prim(){
    zero(taken);
    process(0);
    ll cost=0;
    while(sz(pq)){
        ii e=pq.top(); pq.pop();
        if(!taken[e.second]) cost+=e.first, process(e.second);
    }
    return cost;
}
```

7.6 2-SAT + Tarjan SCC

```
//We have a vertex representing a var and other for his negation.
//Every edge stored in G represents an implication. To add an equation of the
    form a||b, use addor(a, b)
//MAX=max cant var, n=cant var
#define addor(a, b) (G[neg(a)].pb(b), G[neg(b)].pb(a))
vector<int> G[MAX*2];
```

```cpp
//idx[i]=index assigned in the dfs
//lw[i]=lowest index(closer from the root) reachable from i
int lw[MAX*2], idx[MAX*2], qidx;
stack<int> q;
int qcmp, cmp[MAX*2];
//verdad[cmp[i]]=valor de la variable i
bool verdad[MAX*2+1];

int neg(int x) { return x>=n? x-n : x+n;}
void tjn(int v){
  lw[v]=idx[v]=++qidx;
  q.push(v), cmp[v]=-2;
  forall(it, G[v]){
    if(!idx[*it] || cmp[*it]==-2){
      if(!idx[*it]) tjn(*it);
      lw[v]=min(lw[v], lw[*it]);
    }
  }
  if(lw[v]==idx[v]){
    int x;
    do{x=q.top(); q.pop(); cmp[x]=qcmp;}while(x!=v);
    verdad[qcmp]=(cmp[neg(v)]<0);
    qcmp++;
  }
}
//remember to CLEAR G!!!
bool satisf(){//O(n)
  memset(idx, 0, sizeof(idx)), qidx=0;
  memset(cmp, -1, sizeof(cmp)), qcmp=0;
  forn(i, n){
    if(!idx[i]) tjn(i);
    if(!idx[neg(i)]) tjn(neg(i));
  }
  forn(i, n) if(cmp[i]==cmp[neg(i)]) return false;
  return true;
}
```

7.7 Articulation Points

```cpp
int N;
vector<int> G[1000000];
//V[i]=node number(if visited), L[i]= lowest V[i] reachable from i
int qV, V[1000000], L[1000000], P[1000000];
void dfs(int v, int f){
  L[v]=V[v]=++qV;
  forall(it, G[v])
    if(!V[*it]){
      dfs(*it, v);
      L[v] = min(L[v], L[*it]);
      P[v]+= L[*it]>=V[v];
    }
    else if(*it!=f)
      L[v]=min(L[v], V[*it]);
}
int cantart(){ //O(n)
  qV=0;
  zero(V), zero(P);
  dfs(1, 0); P[1]--;
  int q=0;
  forn(i, N) if(P[i]) q++;
return q;
}
```

7.8 Comp. Biconexas y Puentes

```cpp
struct edge {
  int u,v, comp;
  bool bridge;
};
vector<edge> e;
void addEdge(int u, int v) {
  G[u].pb(sz(e)), G[v].pb(sz(e));
  e.pb((edge){u,v,-1,false});
}
//d[i]=id de la dfs
//b[i]=lowest id reachable from i
int d[MAXN], b[MAXN], t;
int nbc;//cant componentes
int comp[MAXN];//comp[i]=cant comp biconexas a la cual pertenece i
void initDfs(int n) {
  zero(G), zero(comp);
  e.clear();
  forn(i,n) d[i]=-1;
  nbc = t = 0;
}
stack<int> st;
void dfs(int u, int pe) {//O(n + m)
  b[u] = d[u] = t++;
  comp[u] = (pe != -1);
  forall(ne, G[u]) if (*ne != pe){
    int v = e[*ne].u ^ e[*ne].v ^ u;
    if (d[v] == -1) {
      st.push(*ne);
      dfs(v,*ne);
```

```
      if (b[v] > d[u]){
        e[*ne].bridge = true; // bridge
      }
      if (b[v] >= d[u]){ // art
        int last;
        do {
          last = st.top(); st.pop();
          e[last].comp = nbc;
        } while (last != *ne);
        nbc++;
        comp[u]++;
      }
      b[u] = min(b[u], b[v]);
    }
    else if (d[v] < d[u]) { // back edge
      st.push(*ne);
      b[u] = min(b[u], d[v]);
    }
  }
}
```

7.9 LCA + Climb

```
const int MAXN=100001;
const int LOGN=20;
//f[v][k] holds the 2^k father of v
//L[v] holds the level of v
int N, f[MAXN][LOGN], L[MAXN];
//call before build:
void dfs(int v, int fa=-1, int lvl=0){//generate required data
  f[v][0]=fa, L[v]=lvl;
  forall(it, G[v])if(*it!=fa) dfs(*it, v, lvl+1); }
void build(){//f[i][0] must be filled previously, O(nlgn)
  forn(k, LOGN-1) forn(i, N) f[i][k+1]=f[f[i][k]][k];}
#define lg(x) (31-__builtin_clz(x))//=floor(log2(x))
int climb(int a, int d){//O(lgn)
  if(!d) return a;
  dforn(i, lg(L[a])+1) if(1<<i<=d) a=f[a][i], d-=1<<i;
    return a;}
int lca(int a, int b){//O(lgn)
  if(L[a]<L[b]) swap(a, b);
  a=climb(a, L[a]-L[b]);
  if(a==b) return a;
  dforn(i, lg(L[a])+1) if(f[a][i]!=f[b][i]) a=f[a][i], b=f[b][i];
  return f[a][0]; }
int dist(int a, int b) {//returns distance between nodes
  return L[a]+L[b]-2*L[lca(a, b)];}
```

7.10 Heavy Light Decomposition

```
int treesz[MAXN];//cantidad de nodos en el subarbol del nodo v
int dad[MAXN];//dad[v]=padre del nodo v
void dfs1(int v, int p=-1){//pre-dfs
  dad[v]=p;
  treesz[v]=1;
  forall(it, G[v]) if(*it!=p){
    dfs1(*it, v);
    treesz[v]+=treesz[*it];
  }
}
//PONER Q EN 0  !!!!!
int pos[MAXN], q;//pos[v]=posicion del nodo v en el recorrido de la dfs
//Las cadenas aparecen continuas en el recorrido!
int cantcad;
int homecad[MAXN];//dada una cadena devuelve su nodo inicial
int cad[MAXN];//cad[v]=cadena a la que pertenece el nodo
void heavylight(int v, int cur=-1){
  if(cur==-1) homecad[cur=cantcad++]=v;
  pos[v]=q++;
  cad[v]=cur;
  int mx=-1;
  forn(i, sz(G[v])) if(G[v][i]!=dad[v])
    if(mx==-1 || treesz[G[v][mx]]<treesz[G[v][i]]) mx=i;
  if(mx!=-1) heavylight(G[v][mx], cur);
  forn(i, sz(G[v])) if(i!=mx && G[v][i]!=dad[v])
    heavylight(G[v][i], -1);
}
//ejemplo de obtener el maximo numero en el camino entre dos nodos
//RTA: max(query(low, u), query(low, v)), con low=lca(u, v)
//esta funcion va trepando por las cadenas
int query(int an, int v){//O(logn)
  //si estan en la misma cadena:
  if(cad[an]==cad[v]) return rmq.get(pos[an], pos[v]+1);
  return max(query(an, dad[homecad[cad[v]]]),
         rmq.get(pos[homecad[cad[v]]], pos[v]+1));
}
```

7.11 Centroid Decomposition

```
int n;
vector<int> G[MAXN];
bool taken[MAXN];//poner todos en FALSE al principio!!
int padre[MAXN];//padre de cada nodo en el centroid tree

int szt[MAXN];
void calcsz(int v, int p) {
```

```
  szt[v] = 1;
  forall(it,G[v]) if (*it!=p && !taken[*it])
    calcsz(*it,v), szt[v]+=szt[*it];
}
void centroid(int v=0, int f=-1, int lvl=0, int tam=-1) {//O(nlogn)
  if(tam==-1) calcsz(v, -1), tam=szt[v];
  forall(it, G[v]) if(!taken[*it] && szt[*it]>=tam/2)
    {szt[v]=0; centroid(*it, f, lvl, tam); return;}
  taken[v]=true;
  padre[v]=f;
  forall(it, G[v]) if(!taken[*it])
    centroid(*it, v, lvl+1, -1);
}
```

7.12 Euler Cycle

```
int n,m,ars[MAXE], eq;
vector<int> G[MAXN];//fill G,n,m,ars,eq
list<int> path;
int used[MAXN];
bool usede[MAXE];
queue<list<int>::iterator> q;
int get(int v){
  while(used[v]<sz(G[v]) && usede[ G[v][used[v]] ]) used[v]++;
  return used[v];
}
void explore(int v, int r, list<int>::iterator it){
  int ar=G[v][get(v)]; int u=v^ars[ar];
  usede[ar]=true;
  list<int>::iterator it2=path.insert(it, u);
  if(u!=r) explore(u, r, it2);
  if(get(v)<sz(G[v])) q.push(it);
}
void euler(){
  zero(used), zero(usede);
  path.clear();
  q=queue<list<int>::iterator>();
  path.push_back(0); q.push(path.begin());
  while(sz(q)){
    list<int>::iterator it=q.front(); q.pop();
    if(used[*it]<sz(G[*it])) explore(*it, *it, it);
  }
  reverse(path.begin(), path.end());
}
void addEdge(int u, int v){
  G[u].pb(eq), G[v].pb(eq);
  ars[eq++]=u^v;
}
```

```
}
```

7.13 Diametro árbol

```
vector<int> G[MAXN]; int n,m,p[MAXN],d[MAXN],d2[MAXN];
int bfs(int r, int *d) {
  queue<int> q;
  d[r]=0; q.push(r);
  int v;
  while(sz(q)) { v=q.front(); q.pop();
    forall(it,G[v]) if (d[*it]==-1)
      d[*it]=d[v]+1, p[*it]=v, q.push(*it);
  }
  return v;//ultimo nodo visitado
}
vector<int> diams; vector<ii> centros;
void diametros(){
  memset(d,-1,sizeof(d));
  memset(d2,-1,sizeof(d2));
  diams.clear(), centros.clear();
  forn(i, n) if(d[i]==-1){
    int v,c;
    c=v=bfs(bfs(i, d2), d);
    forn(_,d[v]/2) c=p[c];
    diams.pb(d[v]);
    if(d[v]&1) centros.pb(ii(c, p[c]));
    else centros.pb(ii(c, c));
  }
}
```

7.14 Chu-liu

```
void visit(graph&h,int v,int s,int r,vector<int>&no,vector<vector<int>>&comp,
    vector<int>&prev,vector<vector<int>>&next,vector<weight>&mcost,vector<int
    >&mark,weight&cost,bool&found){if(mark[v]){vector<int>temp=no;found=true;
    do{cost+=mcost[v];v=prev[v];if(v!=s){while(comp[v].size()>0){no[comp[v].
    back()]=s;comp[s].push_back(comp[v].back());comp[v].pop_back();}}}while(v
    !=s);forall(j,comp[s])if(*j!=r)forall(e,h[*j])if(no[e->src]!=s)e->w-=mcost
    [ temp[*j] ];}mark[v]=true;forall(i,next[v])if(no[*i]!=no[v]&&prev[no[*i
    ]]==v)if(!mark[no[*i]]||*i==s)visit(h,*i,s,r,no,comp,prev,next,mcost,mark,
    cost,found);}weight minimumSpanningArborescence(const graph&g,int r){const
     int n=sz(g);graph h(n);forn(u,n)forall(e,g[u])h[e->dst].pb(*e);vector<int
    >no(n);vector<vector<int>>comp(n);forn(u,n)comp[u].pb(no[u]=u);for(weight
    cost=0;;){vector<int>prev(n,-1);vector<weight>mcost(n,INF);forn(j,n)if(j!=
    r)forall(e,h[j])if(no[e->src]!=no[j])if(e->w<mcost[ no[j] ])mcost[ no[j]
    ]=e->w,prev[ no[j] ]=no[e->src];vector<vector<int>>next(n);forn(u,n)if(
    prev[u]>=0)next[ prev[u] ].push_back(u);bool stop=true;vector<int>mark(n);
    forn(u,n)if(u!=r&&!mark[u]&&!comp[u].empty()){bool found=false;visit(h,u,u
```

```
    ,r,no,comp,prev,next,mcost,mark,cost,found);if(found)stop=false;}if(stop){
    forn(u,n)if(prev[u]>=0)cost+=mcost[u];return cost;}}}
```

7.15 Hungarian

```
//Dado un grafo bipartito completo con costos no negativos, encuentra el
    matching perfecto de minimo costo.
const tipo EPS=1e-9;const tipo INF=1e14;
#define N 502
tipo cost[N][N],lx[N],ly[N],slack[N];int n,max_match,xy[N],yx[N],slackx[N],
    prev2[N];bool S[N],T[N];void add_to_tree(int x,int prevx){S[x]=true,prev2[
    x]=prevx;forn(y,n)if(lx[x]+ly[y]-cost[x][y]<slack[y]-EPS)slack[y]=lx[x]+ly
    [y]-cost[x][y],slackx[y]=x;}void update_labels(){tipo delta=INF;forn(y,n)
    if(!T[y])delta=min(delta,slack[y]);forn(x,n)if(S[x])lx[x]-=delta;forn(y,n)
    if(T[y])ly[y]+=delta;else slack[y]-=delta;}void init_labels(){zero(lx),
    zero(ly);forn(x,n)forn(y,n)lx[x]=max(lx[x],cost[x][y]);}void augment(){if(
    max_match==n)return;int x,y,root,q[N],wr=0,rd=0;memset(S,false,sizeof(S)),
    memset(T,false,sizeof(T));memset(prev2,-1,sizeof(prev2));forn(x,n)if(xy[x
    ]==-1){q[wr++]=root=x,prev2[x]=-2;S[x]=true;break;}forn(y,n)slack[y]=lx[
    root]+ly[y]-cost[root][y],slackx[y]=root;while(true){while(rd<wr){x=q[rd
    ++];for(y=0;y<n;y++)if(cost[x][y]==lx[x]+ly[y]&&!T[y]){if(yx[y]==-1)break;
    T[y]=true;q[wr++]=yx[y],add_to_tree(yx[y],x);}if(y<n)break;}if(y<n)break;
    update_labels(),wr=rd=0;for(y=0;y<n;y++)if(!T[y]&&slack[y]==0){if(yx[y
    ]==-1){x=slackx[y];break;}else{T[y]=true;if(!S[yx[y]])q[wr++]=yx[y],
    add_to_tree(yx[y],slackx[y]);}}if(y<n)break;}if(y<n){max_match++;for(int
    cx=x,cy=y,ty;cx!=-2;cx=prev2[cx],cy=ty)ty=xy[cx],yx[cy]=cx,xy[cx]=cy;
    augment();}}tipo hungarian(){tipo ret=0;max_match=0,memset(xy,-1,sizeof(xy
    ));memset(yx,-1,sizeof(yx)),init_labels(),augment();forn(x,n)ret+=cost[x][
    xy[x]];return ret;}
```

7.16 Dynamic Conectivity

```
struct UnionFind {
    int n, comp;
    vector<int> pre,si,c;
    UnionFind(int n=0):n(n), comp(n), pre(n), si(n, 1) {
        forn(i,n) pre[i] = i; }
    int find(int u){return u==pre[u]?u:find(pre[u]);}
    bool merge(int u, int v) {
        if((u=find(u))==(v=find(v))) return false;
        if(si[u]<si[v]) swap(u, v);
        si[u]+=si[v], pre[v]=u, comp--, c.pb(v);
        return true;
    }
    int snap(){return sz(c);}
    void rollback(int snap){
        while(sz(c)>snap){
            int v = c.back(); c.pop_back();
            si[pre[v]] -= si[v], pre[v] = v, comp++;
        }
    }
};
enum {ADD,DEL,QUERY};
struct Query {int type,u,v;};
struct DynCon {
    vector<Query> q;
    UnionFind dsu;
    vector<int> match,res;
    map<ii,int> last;//se puede no usar cuando hay identificador para cada
        arista (mejora poco)
    DynCon(int n=0):dsu(n){}
    void add(int u, int v) {
        if(u>v) swap(u,v);
        q.pb((Query){ADD, u, v}), match.pb(-1);
        last[ii(u,v)] = sz(q)-1;
    }
    void remove(int u, int v) {
        if(u>v) swap(u,v);
        q.pb((Query){DEL, u, v});
        int prev = last[ii(u,v)];
        match[prev] = sz(q)-1;
        match.pb(prev);
    }
    void query() {//podria pasarle un puntero donde guardar la respuesta
        q.pb((Query){QUERY, -1, -1}), match.pb(-1);}
    void process() {
        forn(i,sz(q)) if (q[i].type == ADD && match[i] == -1) match[i] = sz(q);
        go(0,sz(q));
    }
    void go(int l, int r) {
        if(l+1==r){
            if (q[l].type == QUERY)//Aqui responder la query usando el dsu!
                res.pb(dsu.comp);//aqui query=cantidad de componentes conexas
            return;
        }
        int s=dsu.snap(), m = (l+r) / 2;
        forr(i,m,r) if(match[i]!=-1 && match[i]<l) dsu.merge(q[i].u, q[i].v);
        go(l,m);
        dsu.rollback(s);
        s = dsu.snap();
        forr(i,l,m) if(match[i]!=-1 && match[i]>=r) dsu.merge(q[i].u, q[i].v);
        go(m,r);
        dsu.rollback(s);
    }
```

```
}dc;
```

8 Network Flow

8.1 Dinic

```
// Corte minimo: vertices con dist[v]>=0 (del lado de src) VS.  dist[v]==-1 (del lado del dst)
// Para el caso de la red de Bipartite Matching (Sean V1 y V2 los conjuntos mas proximos a src y dst respectivamente):
// Reconstruir matching: para todo v1 en V1 ver las aristas a vertices de V2 con it->f>0, es arista del Matching
// Min Vertex Cover: vertices de V1 con dist[v]==-1 + vertices de V2 con dist[v]>0
// Max Independent Set: tomar los vertices NO tomados por el Min Vertex Cover
// Max Clique: construir la red de G complemento (debe ser bipartito!) y encontrar un Max Independet Set
// Min Edge Cover: tomar las aristas del matching + para todo vertices no cubierto hasta el momento, tomar cualquier arista de el
int nodes, src, dst;
int dist[MAX], q[MAX], work[MAX];
struct Edge {
    int to, rev;
    ll f, cap;
    Edge(int to, int rev, ll f, ll cap) : to(to), rev(rev), f(f), cap(cap) {}
};
vector<Edge> G[MAX];
void addEdge(int s, int t, ll cap){
    G[s].pb(Edge(t, sz(G[t]), 0, cap)), G[t].pb(Edge(s, sz(G[s])-1, 0, 0));}
bool dinic_bfs(){
    fill(dist, dist+nodes, -1), dist[src]=0;
    int qt=0; q[qt++]=src;
    for(int qh=0; qh<qt; qh++){
        int u =q[qh];
        forall(e, G[u]){
            int v=e->to;
            if(dist[v]<0 && e->f < e->cap)
                dist[v]=dist[u]+1, q[qt++]=v;
        }
    }
    return dist[dst]>=0;
}
ll dinic_dfs(int u, ll f){
    if(u==dst) return f;
    for(int &i=work[u]; i<sz(G[u]); i++){
        Edge &e = G[u][i];
        if(e.cap<=e.f) continue;
        int v=e.to;
        if(dist[v]==dist[u]+1){
                ll df=dinic_dfs(v, min(f, e.cap-e.f));
                if(df>0){
                        e.f+=df, G[v][e.rev].f-= df;
                        return df;  }
        }
    }
    return 0;
}
ll maxFlow(int _src, int _dst){
    src=_src, dst=_dst;
    ll result=0;
    while(dinic_bfs()){
        fill(work, work+nodes, 0);
        while(ll delta=dinic_dfs(src,INF))
            result+=delta;
    }
    // todos los nodos con dist[v]!=-1 vs los que tienen dist[v]==-1 forman el min-cut
    return result; }
```

8.2 Min-cost Max-flow

```
const int MAXN=10000;
typedef ll tf;
typedef ll tc;
const tf INFFLUJO = 1e14;
const tc INFCOSTO = 1e14;
struct edge {
  int u, v;
  tf cap, flow;
  tc cost;
  tf rem() { return cap - flow; }
};
int nodes; //numero de nodos
vector<int> G[MAXN]; // limpiar!
vector<edge> e;  // limpiar!
void addEdge(int u, int v, tf cap, tc cost) {
  G[u].pb(sz(e)); e.pb((edge){u,v,cap,0,cost});
  G[v].pb(sz(e)); e.pb((edge){v,u,0,0,-cost});
}
tc dist[MAXN], mnCost;
int pre[MAXN];
tf cap[MAXN], mxFlow;
bool in_queue[MAXN];
```

```
void flow(int s, int t) {
  zero(in_queue);
  mxFlow=mnCost=0;
  while(1){
    fill(dist, dist+nodes, INFCOSTO); dist[s] = 0;
    memset(pre, -1, sizeof(pre)); pre[s]=0;
    zero(cap); cap[s] = INFFLUJO;
    queue<int> q; q.push(s); in_queue[s]=1;
    while(sz(q)){
      int u=q.front(); q.pop(); in_queue[u]=0;
      for(auto it:G[u]) {
        edge &E = e[it];
        if(E.rem() && dist[E.v] > dist[u] + E.cost + 1e-9){ // ojo EPS
          dist[E.v]=dist[u]+E.cost;
          pre[E.v] = it;
          cap[E.v] = min(cap[u], E.rem());
          if(!in_queue[E.v]) q.push(E.v), in_queue[E.v]=1;
        }
      }
    }
    if (pre[t] == -1) break;
    mxFlow +=cap[t];
    mnCost +=cap[t]*dist[t];
    for (int v = t; v != s; v = e[pre[v]].u) {
      e[pre[v]].flow += cap[t];
      e[pre[v]^1].flow -= cap[t];
    }
  }
}
```

9 Template

```
//touch {a..m}.in; tee {a..m}.cpp < template.cpp
#include <bits/stdc++.h>
using namespace std;
#define forr(i,a,b) for(int i=(a); i<(b); i++)
#define forn(i,n) forr(i,0,n)
#define sz(c) ((int)c.size())
#define zero(v) memset(v, 0, sizeof(v))
#define forall(it,v) for(auto it=v.begin();it!=v.end();++it)
#define pb push_back
#define fst first
#define snd second
typedef long long ll;
typedef pair<int,int> ii;
#define dforn(i,n) for(int i=n-1; i>=0; i--)
#define dprint(v) cout << #v"=" << v << endl //;)

const int MAXN=100100;
int n;

int main() {
    freopen("input.in", "r", stdin);
    ios::sync_with_stdio(0);
    while(cin >> n){

    }
    return 0;
}
```

10 Ayudamemoria

Leer hasta fin de linea

```
#include <sstream>
//hacer cin.ignore() antes de getline()
while(getline(cin, line)){
     istringstream is(line);
     while(is >> X)
       cout << X << "␣";
     cout << endl;
}
```

Expandir pila

```
#include <sys/resource.h>
rlimit rl;
getrlimit(RLIMIT_STACK, &rl);
rl.rlim_cur=1024L*1024L*256L;//256mb
setrlimit(RLIMIT_STACK, &rl);
```

Iterar subconjunto

```
for(int sbm=bm; sbm; sbm=(sbm-1)&bm)
```