# 1. algorithm

#include <algorithm> #include <numeric>

Algo	Params	Funcion
sort, stable_sort	f, 1	ordena el intervalo
nth_element	f, nth, l	void ordena el n-esimo, y
		particiona el resto
fill, fill_n	f, l / n, elem	void llena [f, l) o [f,
		f+n) con elem
lower_bound, upper_bound	f, l, elem	it al primer / ultimo donde se
		puede insertar elem para que
		quede ordenada
binary_search	f, l, elem	bool esta elem en [f, l)
copy	f, l, resul	hace resul+ $i$ =f+ $i$ $\forall i$
find, find_if, find_first_of	f, l, elem	$it$ encuentra $i \in [f,l)$ tq. $i=elem$ ,
	/ pred / f2, l2	$\operatorname{pred}(i), i \in [f2,l2)$
count, count_if	f, l, elem/pred	cuenta elem, pred(i)
search	f, l, f2, l2	busca $[f2,l2) \in [f,l)$
replace_if	f, l, old	cambia old / pred(i) por new
	/ pred, new	
reverse	f, 1	da vuelta
partition, stable_partition	f, l, pred	pred(i) ad, !pred(i) atras
min_element, max_element	f, l, [comp]	it min, max de [f,l]
lexicographical_compare	f1,l1,f2,l2	bool  con  [f1,l1]; $[f2,l2]$
next/prev_permutation	f,l	deja en [f,l) la perm sig, ant
set_intersection,	f1, l1, f2, l2, res	[res,) la op. de conj
set_difference, set_union,		
set_symmetric_difference,		
push_heap, pop_heap,	f, l, e / e /	mete/saca e en heap [f,l),
make_heap		hace un heap de [f,l)
is_heap	f,l	bool es [f,l) un heap
accumulate	f,l,i,[op]	$T = \sum /\text{oper de [f,l)}$
inner_product	f1, l1, f2, i	$T = i + [f1, l1) \cdot [f2,)$
partial_sum	f, l, r, [op]	$r+i = \sum /oper de [f,f+i] \forall i \in [f,l)$
builtin_ffs	unsigned int	Pos. del primer 1 desde la derecha
builtin_clz	unsigned int	Cant. de ceros desde la izquierda.
_builtin_ctz	unsigned int	Cant. de ceros desde la derecha.
_builtin_popcount	unsigned int	Cant. de 1's en x.
_builtin_parity	unsigned int	1 si x es par, 0 si es impar.
builtin_XXXXXXII	unsigned ll	= pero para long long's.

### 2. Estructuras

# 2.1. RMQ (static)

Dado un arreglo y una operacion asociativa *idempotente*, get(i, j) opera sobre el rango [i, j). Restriccion: LVL  $\geq$  ceil(logn); Usar [] para llenar arreglo y luego build().

```
struct RMQ{
    #define LVL 10

tipo vec[LVL][1<<(LVL+1)];

tipo &operator[](int p){return vec[0][p];}

tipo get(int i, int j) {//intervalo [i,j)
    int p = 31-__builtin_clz(j-i);
    return min(vec[p][i],vec[p][j-(1<<p)]);
}

void build(int n) {//O(nlogn)
    int mp = 31-__builtin_clz(n);
    forn(p, mp) forn(x, n-(1<<p))
    vec[p+1][x] = min(vec[p][x], vec[p][x+(1<<p)]);
};
};</pre>
```

## 2.2. RMQ (dynamic and lazy)

```
1 //Dado un arreglo y una operacion asociativa con neutro, get(i, j) opera
        sobre el rango [i, j).
typedef int Elem;//Elem de los elementos del arreglo
   typedef int Alt;//Elem de la alteracion//*
   #define operacion(x,y) x+y
   const Elem neutro=0; const Alt neutro2=0;//*
   #define MAXN 100000
   struct RMQ{
     int sz;
     Elem t[4*MAXN];
     Alt dirty[4*MAXN];//las alteraciones pueden ser de distinto Elem//*
     Elem &operator[](int p){return t[sz+p];}
11
     void init(int n){//O(nlgn)
12
       sz = 1 << (32-__builtin_clz(n));</pre>
13
       forn(i, 2*sz) t[i]=neutro;
14
       forn(i, 2*sz) dirty[i]=neutro2;//*
15
16
     void updall(){//O(n)//para lazy limpiar dirty!
17
       dforn(i, sz) t[i]=operacion(t[2*i], t[2*i+1]);}
18
19
     void push(int n, int a, int b){//propaga el dirty a sus hijos//*
```

```
if(dirty[n]!=0){
20
         t[n]+=dirty[n]*(b-a);//altera el nodo
^{21}
         if(n<sz){
^{22}
           dirty[2*n]+=dirty[n];
23
           dirty[2*n+1]+=dirty[n];
24
25
         dirty[n]=0;
26
27
28
     Elem get(int i, int j, int n, int a, int b){\frac{1}{0}}
29
       if(j<=a || i>=b) return neutro;
30
       push(n, a, b);//corrige el valor antes de usarlo//*
31
       if(i<=a && b<=j) return t[n];
32
       int c=(a+b)/2:
33
       return operacion(get(i, j, 2*n, a, c), get(i, j, 2*n+1, c, b));
34
35
     Elem get(int i, int j){return get(i,j,1,0,sz);}
36
     //altera los valores en [i, j) con una alteración de val
37
     void alterar(Alt val, int i, int j, int n, int a, int b)\frac{1}{0}\frac{1}{2}
38
       push(n, a, b);
39
       if(j<=a || i>=b) return;
40
       if(i<=a && b<=j){
41
         dirty[n]+=val;
42
         push(n, a, b);
43
         return;
44
       }
45
       int c=(a+b)/2;
46
       alterar(val, i, j, 2*n, a, c), alterar(val, i, j, 2*n+1, c, b);
47
       t[n]=operacion(t[2*n], t[2*n+1]);//por esto es el push de arriba
48
     }
49
     void alterar(Alt val, int i, int j){alterar(val,i,j,1,0,sz);}//*
50
     void set(int p, tipo val){//O(lgn)//No usar con lazy!!
51
       for(p+=sz; p>0 && t[p]!=val;){
52
         t[p]=val;
53
         p/=2;
54
         val=operacion(t[p*2], t[p*2+1]);
55
56
     }
57
   }rmq;
```

#### 2.3. Fenwick Tree

```
1 //For 2D threat each column as a Fenwick tree, by adding a nested for in
        each operation
2 | struct Fenwick{
     static const int sz=1000001;
     tipo t[sz];
     void adjust(int p, tipo v){//valid with p in [1, sz), O(lgn)
       for(; p<sz; p+=(p&-p)) t[p]+=v; }</pre>
     tipo sum(int p){//cumulative sum in [1, p], O(lgn)
       tipo s=0;
       for(; p; p-=(p&-p)) s+=t[p];
       return s;
10
    }
11
     tipo sum(int a, int b){return sum(b)-sum(a-1);}
12
     //get largest value with cumulative sum less than or equal to x;
13
     //for smallest, pass x-1 and add 1 to result
14
     int getind(tipo x) {//O(lgn)
15
         int idx = 0, mask = N;
16
         while(mask && idx < N) {</pre>
17
           int t = idx + mask:
18
         if(x \ge tree[t])
             idx = t, x -= tree[t];
20
           mask >>= 1;
21
         }
22
         return idx;
23
    }};
24
2.4. Union Find
1 struct UnionFind{
     vector<int> f;//the array contains the parent of each node
2
     void init(int n){f.clear(); f.insert(f.begin(), n, -1);}
     int comp(int x){return (f[x]==-1?x:f[x]=comp(f[x]));}//O(1)
     bool join(int i, int j) {
5
       bool con=comp(i)==comp(j);
6
       if(!con) f[comp(i)] = comp(j);
7
       return con:
8
    }};
9
2.5. Disjoint Intervals
| bool operator< (const ii &a, const ii &b) {return a.fst<b.fst;}
2 //Stores intervals as [first, second]
```

3 //in case of a collision it joins them in a single interval

4 | struct disjoint\_intervals {

```
29 };
     set<ii>> segs;
5
     void insert(ii v) {//O(lgn)
                                                                                       //Example to initialize a grid of M rows and N columns:
6
       if(v.snd-v.fst==0.) return;//0J0
                                                                                        RMQ2D rmq;
       set<ii>>::iterator it,at;
                                                                                    32 forn(i, M)
8
       at = it = segs.lower_bound(v);
                                                                                        forn(j, N)
9
       if (at!=segs.begin() && (--at)->snd >= v.fst)
                                                                                            cin >> rmq[i][j];
10
         v.fst = at->fst, --it;
                                                                                     35 rmq.build(N, M);
11
       for(; it!=segs.end() && it->fst <= v.snd; segs.erase(it++))</pre>
12
                                                                                     2.7. Big Int
         v.snd=max(v.snd, it->snd);
13
       segs.insert(v);
14
     }
                                                                                     1 #define BASEXP 6
15
<sub>16</sub> | };
                                                                                        #define BASE 1000000
                                                                                        #define LMAX 1000
2.6. RMQ (2D)
                                                                                        struct bint{
                                                                                            int 1;
  struct RMQ2D{
                                                                                            11 n[LMAX];
     static const int sz=1024;
                                                                                            bint(11 x=0){
                                                                                     7
     RMQ t[sz];
                                                                                                1=1;
                                                                                     8
3
     RMQ & operator[](int p){return t[sz/2+p];}
                                                                                                forn(i, LMAX){
     void build(int n, int m){\frac{}{\frac{1}{0}}}
                                                                                                     if (x) l=i+1;
       forr(y, sz/2, sz/2+m)
                                                                                                     n[i]=x %BASE;
         t[y].build(m);
                                                                                                     x/=BASE;
       forr(y, sz/2+m, sz)
8
         forn(x, sz)
                                                                                                }
9
                                                                                     14
                                                                                            }
           t[y].t[x]=0;
10
                                                                                     15
       dforn(y, sz/2)
                                                                                            bint(string x){
                                                                                     16
11
         forn(x, sz)
                                                                                            l=(x.size()-1)/BASEXP+1;
                                                                                    17
12
           t[y].t[x]=max(t[y*2].t[x], t[y*2+1].t[x]);
                                                                                                fill(n, n+LMAX, 0);
                                                                                     18
13
                                                                                                ll r=1;
                                                                                     19
14
     void set(int x, int y, tipo v){\frac{1}{0(\text{lgm.lgn})}}
                                                                                                forn(i, sz(x)){
15
                                                                                    20
                                                                                                     n[i / BASEXP] += r * (x[x.size()-1-i]-'0');
       v + = sz/2;
                                                                                    21
16
                                                                                                     r*=10; if(r==BASE)r=1;
       t[y].set(x, v);
                                                                                    22
17
       while(v/=2)
                                                                                                }
                                                                                     23
18
         t[y].set(x, max(t[y*2][x], t[y*2+1][x]));
                                                                                            }
                                                                                    24
19
     }
                                                                                            void out(){
20
                                                                                    25
     //O(lgm.lgn)
                                                                                            cout << n[l-1];
                                                                                    26
^{21}
     int get(int x1, int y1, int x2, int y2, int n=1, int a=0, int b=sz/2){
                                                                                            dforn(i, l-1) printf("%6.61lu", n[i]);//6=BASEXP!
                                                                                    27
^{22}
       if(y2<=a || y1>=b) return 0;
                                                                                    28
23
       if(y1<=a && b<=y2) return t[n].get(x1, x2);
                                                                                    29
                                                                                          void invar(){
24
       int c=(a+b)/2:
                                                                                            fill(n+l, n+LMAX, 0);
                                                                                    30
25
       return max(get(x1, y1, x2, y2, 2*n, a, c),
                                                                                            while(1>1 && !n[1-1]) 1--;
26
                                                                                    31
             get(x1, y1, x2, y2, 2*n+1, c, b));
                                                                                    32
                                                                                         }
27
     }
                                                                                    33 };
28
```

```
bint operator+(const bint&a, const bint&b){
                                                                                                 c.n[i+b.1] = q;
                                                                                     73
     bint c:
                                                                                     74
35
       c.1 = max(a.1, b.1);
                                                                                            c.invar();
                                                                                     75
36
       11 q = 0;
                                                                                            return c;
                                                                                     76
37
       forn(i, c.1) q += a.n[i]+b.n[i], c.n[i]=q %BASE, q/=BASE;
                                                                                        }
                                                                                     77
38
       if(q) c.n[c.l++] = q;
39
       c.invar();
                                                                                          bint c;
40
                                                                                          11 \text{ rm} = 0;
       return c;
41
                                                                                     80
                                                                                          dforn(i, a.1){
42
   pair bint, bool lresta(const bint& a, const bint& b) // c = a - b
                                                                                                     rm = rm * BASE + a.n[i];
                                                                                                     c.n[i] = rm / b;
44
                                                                                     83
     bint c;
                                                                                                     rm % b;
45
                                                                                     84
                                                                                            }
       c.1 = max(a.1, b.1):
                                                                                     85
       11 q = 0;
                                                                                            c.1 = a.1:
47
       forn(i, c.1) q += a.n[i]-b.n[i], c.n[i]=(q+BASE) %BASE, q=(q+BASE)/
                                                                                            c.invar();
48
                                                                                            return make_pair(c, rm);
           BASE-1:
                                                                                     88
       c.invar();
                                                                                        }
49
                                                                                     89
       return make_pair(c, !q);
50
51
   bint& operator-= (bint& a, const bint& b){return a=lresta(a, b).first;}
                                                                                        pair < bint, bint > ldiv(const bint & a, const bint & b) {
   bint operator- (const bint&a, const bint&b){return lresta(a, b).first;}
                                                                                          bint c;
                                                                                     93
   bool operator< (const bint&a, const bint&b){return !lresta(a, b).second
                                                                                            bint rm = 0;
                                                                                     94
       ;}
                                                                                            dforn(i, a.l){
                                                                                     95
   bool operator <= (const bint&a, const bint&b) {return lresta(b, a).second
                                                                                                 if (rm.l==1 && !rm.n[0])
55
                                                                                     96
                                                                                                     rm.n[0] = a.n[i];
                                                                                     97
   bool operator==(const bint&a, const bint&b){return a <= b && b <= a;}
                                                                                                 else{
                                                                                     98
   bint operator*(const bint&a, ll b){
                                                                                                     dforn(j, rm.l) rm.n[j+1] = rm.n[j];
                                                                                     99
57
                                                                                                     rm.n[0] = a.n[i];
       bint c;
                                                                                    100
58
       11 q = 0;
                                                                                                     rm.l++;
                                                                                    101
59
       forn(i, a.l) q += a.n[i]*b, c.n[i] = q \text{BASE}, q/=BASE};
                                                                                                 }
                                                                                    102
60
                                                                                                 ll q = rm.n[b.1] * BASE + rm.n[b.1-1];
       c.1 = a.1;
                                                                                    103
61
       while(q) c.n[c.l++] = q \text{ $\beta$ASE}, q/=BASE;
                                                                                                 ll u = q / (b.n[b.l-1] + 1);
                                                                                    104
62
       c.invar():
                                                                                                 ll v = q / b.n[b.l-1] + 1;
                                                                                    105
63
                                                                                                 while (u < v-1){
       return c:
                                                                                    106
64
                                                                                                     11 m = (u+v)/2:
                                                                                    107
65
   bint operator*(const bint&a, const bint&b){
                                                                                                     if (b*m \le rm) u = m:
                                                                                    108
66
                                                                                                     else v = m;
       bint c:
67
                                                                                    109
       c.1 = a.1+b.1:
                                                                                    110
68
       fill(c.n, c.n+b.1, 0);
                                                                                                 c.n[i]=u;
                                                                                    111
69
       forn(i, a.1){
                                                                                                 rm-=b*u;
                                                                                    112
70
           11 q = 0;
                                                                                    113
71
           forn(j, b.l) q += a.n[i]*b.n[j]+c.n[i+j], c.n[i+j] = q BASE, q
                                                                                          c.l=a.l;
                                                                                    114
72
                /=BASE;
                                                                                            c.invar();
                                                                                    115
```

```
pair<br/>
\frac{b}{c} = a / b; rm = a % b
bint operator/(const bint&a, ll b){return ldiv(a, b).first;}
11 operator %(const bint&a, ll b) {return ldiv(a, b).second;}
```

```
return make_pair(c, rm);
                                                                                               push(1), push(r);
116
                                                                                    23
                                                                                               pnode t;
117
                                                                                    24
   bint operator/(const bint&a, const bint&b){return ldiv(a, b).first;}
                                                                                               if (1->prior < r->prior) 1->r=merge(1->r, r), t = 1;
                                                                                    25
118
| bint operator %(const bint&a, const bint&b) {return ldiv(a, b).second;}
                                                                                               else r\rightarrow l=merge(l, r\rightarrow l), t = r;
                                                                                    26
                                                                                               pull(t);
                                                                                    27
2.8. Modnum
                                                                                               return t;
                                                                                    28
                                                                                           }//opcional:
                                                                                    29
  struct mnum{
                                                                                           void merge(treap t) {root = merge(root, t.root), t.root=0;}
      static const tipo mod=12582917;
 2
                                                                                           //*****KEY OPERATIONS*****//
                                                                                    31
      tipo v:
 3
                                                                                           void splitKey(pnode t, Key key, pnode &1, pnode &r) {
     mnum(tipo v=0): v(v mod) {}
 4
                                                                                               if (!t) return void(1 = r = 0);
                                                                                    33
     mnum operator+(mnum b){return v+b.v;}
                                                                                               push(t);
                                                                                    34
     mnum operator-(mnum b){return v>=b.v? v-b.v : mod-b.v+v;}
 6
                                                                                               if (\text{key} \leftarrow \text{t->key}) splitKey(t->1, key, 1, t->1), r = t;
                                                                                    35
     mnum operator*(mnum b){return v*b.v;}
                                                                                               else splitKey(t->r, key, t->r, r), l = t;
                                                                                    36
     mnum operator^(int n){
 8
                                                                                               pull(t);
                                                                                    37
       if(!n) return 1:
 9
                                                                                           }
                                                                                    38
       return n \% ? (*this)^(n/2) * (this) : (*this)^(n/2) : }
10
                                                                                           void insertKey(Key key) {
                                                                                    39
11 | };
                                                                                               pnode elem = new node(key, rand());
                                                                                    40
       Treap
                                                                                               pnode t1, t2; splitKey(root, key, t1, t2);
                                                                                    41
                                                                                               t1=merge(t1,elem);
                                                                                               root=merge(t1,t2);
                                                                                    43
 1 typedef int Key;
   typedef struct node *pnode;
                                                                                    44
                                                                                           void eraseKeys(Key key1, Key key2) {
   struct node{
                                                                                    45
                                                                                               pnode t1,t2,t3;
       Kev kev:
                                                                                    46
                                                                                               splitKey(root,key1,t1,t2);
       int prior, size;
                                                                                    47
 5
                                                                                               splitKey(t2,key2, t2, t3);
                                                                                    48
       pnode l,r;
                                                                                               root=merge(t1,t3);
       node(Key key=0, int prior=0): key(key), prior(prior), size(1), 1(0),
                                                                                    49
             r(0) {}
                                                                                    50
                                                                                           void eraseKey(pnode &t, Key key) {
   };
                                                                                    51
 8
                                                                                               if (!t) return;
                                                                                    52
   struct treap {
 9
                                                                                               push(t);
        pnode root;
                                                                                    53
10
                                                                                               if (key == t->key) t=merge(t->1, t->r);
        treap(): root(0) {}
                                                                                    54
11
                                                                                                else if (key < t->key) eraseKey(t->1, key);
       int size(pnode p) { return p ? p->size : 0; }
                                                                                    55
12
                                                                                                else eraseKey(t->r, key);
        int size() { return size(root); }
                                                                                    56
13
                                                                                               pull(t);
        void push(pnode p) {
                                                                                    57
14
           // modificar y propagar el dirty a los hijos aca(para lazy)
                                                                                    58
15
                                                                                           void eraseKey(Key key) {eraseKey(root, key);}
       }
                                                                                    59
16
                                                                                           pnode findKey(pnode t, Key key) {
       // Update function and size from children's values
                                                                                    60
17
                                                                                               if (!t) return 0;
        void pull(pnode p) {//recalcular valor del nodo aca (para rmg)
                                                                                    61
18
                                                                                               if (key == t->key) return t;
            p->size = 1 + size(p->1) + size(p->r);
                                                                                    62
19
                                                                                               if (key < t->key) return findKey(t->1, key);
                                                                                    63
20
                                                                                               return findKey(t->r, key);
       pnode merge(pnode 1, pnode r) {
                                                                                    64
21
            if (!1 || !r) return 1 ? 1 : r;
                                                                                    65
22
```

```
pnode findKey(Key key) { return findKey(root, key); }
66
       //****POS OPERATIONS*****// No mezclar con las funciones Key
67
       //(No funciona con pos:)
68
       void splitSize(pnode t, int sz, pnode &l, pnode &r) {
69
           if (!t) return void(l = r = 0);
70
           push(t);
71
           if (sz \le size(t->1)) splitSize(t->1, sz, 1, t->1), r = t;
72
           else splitSize(t->r, sz - 1 - size(t->l), t->r, r), l = t;
73
           pull(t);
74
       }
75
       void insertPos(int pos, Key key) {
76
           pnode elem = new node(key, rand());
77
           pnode t1,t2; splitSize(root, pos, t1, t2);
78
           t1=merge(t1,elem);
79
           root=merge(t1,t2);
80
       }
81
       void erasePos(int pos1, int pos2=-1) {
82
       if(pos2==-1) pos2=pos1+1;
83
           pnode t1,t2,t3;
84
           splitSize(root,pos1,t1,t2);
85
           splitSize(t2,pos2-pos1,t2,t3);
86
           root=merge(t1, t2);
87
88
       pnode findPos(pnode t, int pos) {
89
           if(!t) return 0;
90
           if(pos <= size(t->1)) return findPos(t->1, pos);
91
           return findPos(t->r, pos - 1 - size(t->l));
92
93
       Key &operator[](int pos){return findPos(root, pos)->key;}//ojito
94
95 };
```

## 2.10. Convex Hull Trick

```
const ll is_query = -(1LL<<62);</pre>
  struct Line {
       ll m, b;
3
       mutable multiset<Line>::iterator it;
4
       const Line *succ(multiset<Line>::iterator it) const:
5
       bool operator<(const Line& rhs) const {</pre>
6
           if (rhs.b != is_query) return m < rhs.m;</pre>
           const Line *s=succ(it);
           if(!s) return 0;
9
           11 x = rhs.m;
10
```

```
return b - s->b < (s->m - m) * x:
11
       }
12
   };
13
   struct HullDynamic : public multiset<Line>{ // will maintain upper hull
       for maximum
       bool bad(iterator y) {
15
           iterator z = next(y);
16
           if (v == begin()) {
17
               if (z == end()) return 0;
18
               return y->m == z->m && y->b <= z->b;
19
20
           iterator x = prev(y);
21
           if (z == end()) return y->m == x->m && y->b <= x->b;
22
           return (x->b - y->b)*(z->m - y->m) >= (y->b - z->b)*(y->m - x->m)
23
               );
24
       iterator next(iterator y){return ++y;}
       iterator prev(iterator y){return --y;}
       void insert line(ll m. ll b) {
27
           iterator y = insert((Line) { m, b });
           v->it=v;
29
           if (bad(y)) { erase(y); return; }
           while (next(y) != end() && bad(next(y))) erase(next(y));
31
           while (y != begin() && bad(prev(y))) erase(prev(y));
32
       }
33
       ll eval(ll x) {
           Line l = *lower_bound((Line) { x, is_query });
35
           return 1.m * x + 1.b;
36
       }
37
   }h:
38
   const Line *Line::succ(multiset<Line>::iterator it) const{
       return (++it==h.end()? NULL : &*it);}
```

# 3. Algos

# 3.1. Longest Increasing Subsecuence

```
//Para non-increasing, cambiar comparaciones y revisar busq binaria
//Given an array, paint it in the least number of colors so that each
color turns to a non-increasing subsequence.
//Solution:Min number of colors=Length of the longest increasing
subsequence
int N, a[MAXN];//secuencia y su longitud
```

```
5 | ii d[MAXN+1];//d[i]=ultimo valor de la subsecuencia de tamanio i
  int p[MAXN];//padres
   vector<int> R;//respuesta
   void rec(int i){
     if(i==-1) return;
     R.push_back(a[i]);
     rec(p[i]);
11
12
   int lis(){//O(nlogn)
13
     d[0] = ii(-INF, -1); forn(i, N) d[i+1]=ii(INF, -1);
14
     forn(i, N){
15
       int j = upper_bound(d, d+N+1, ii(a[i], INF))-d;
16
       if (d[j-1].first < a[i]&&a[i] < d[j].first){</pre>
17
         p[i]=d[j-1].second;
18
         d[i] = ii(a[i], i);
19
       }
20
     }
21
     R.clear();
22
     dforn(i, N+1) if(d[i].first!=INF){
23
       rec(d[i].second);//reconstruir
24
       reverse(R.begin(), R.end());
25
       return i;//longitud
26
     }
27
     return 0;
28
29
```

#### 3.2. Manacher

```
int d1[MAXN];//d1[i]=long del maximo palindromo impar con centro en i
  int d2[MAXN];//d2[i]=analogo pero para longitud par
  //0 1 2 3 4
3
  //a a b c c <--d1[2]=3
   //a a b b <--d2[2]=2 (estan uno antes)
   void manacher(){
6
     int l=0, r=-1, n=sz(s);
     forn(i, n){
8
       int k=(i>r? 1 : min(d1[l+r-i], r-i));
9
       while(i+k \le k i-k >= 0 & s[i+k] == s[i-k]) ++k:
10
       d1[i] = k--;
11
       if(i+k > r) l=i-k, r=i+k;
12
     }
13
     1=0, r=-1;
14
     forn(i, n){
```

```
int k=(i>r? 0 : min(d2[1+r-i+1], r-i+1))+1;
while(i+k-1<n && i-k>=0 && s[i+k-1]==s[i-k]) k++;
d2[i] = --k;
if(i+k-1 > r) l=i-k, r=i+k-1;
}
```

## 3.3. Alpha-Beta prunning

```
1 | 11 alphabeta(State &s, bool player = true, int depth = 1e9, 11 alpha = -
       INF, 11 beta = INF) { //player = true -> Maximiza
       if(s.isFinal()) return s.score;
2
     //~ if (!depth) return s.heuristic();
3
       vector<State> children;
4
       s.expand(player, children);
5
       int n = children.size();
6
       forn(i, n) {
           ll v = alphabeta(children[i], !player, depth-1, alpha, beta);
           if(!player) alpha = max(alpha, v);
9
           else beta = min(beta, v);
10
           if(beta <= alpha) break;</pre>
11
       }
12
       return !player ? alpha : beta;}
13
```

# 4. Strings

### 4.1. KMP

```
string T;//cadena donde buscar(where)
   string P;//cadena a buscar(what)
   int b[MAXLEN];//back table
   void kmppre(){//by gabina with love
       int i =0, j=-1; b[0]=-1;
5
       while(i<sz(P)){</pre>
6
           while(j>=0 && P[i] != P[j]) j=b[j];
           i++, j++;
8
9
           b[i] = j;
       }
10
   }
11
12
   void kmp(){
       int i=0, j=0;
14
15
       while(i<sz(T)){</pre>
           while(j>=0 && T[i]!=P[j]) j=b[j];
16
```

19

forn(i, n) sa[i]=i, r[i]=s[i];

```
for(int k=1; k<n; k<<=1){</pre>
           i++, j++;
17
                                                                                  21
           if(j==sz(P)){
                                                                                          countingSort(k), countingSort(0);
                                                                                  22
18
               printf("Puisufounduatuindexu %duinuT\n", i-j);
                                                                                          int rank, tmpr[MAX_N];
                                                                                  23
19
                                                                                          tmpr[sa[0]]=rank=0;
               j=b[j];
20
                                                                                  24
           }
                                                                                          forr(i, 1, n)
                                                                                  25
21
                                                                                            tmpr[sa[i]] = r[sa[i-1]] \&\& r[sa[i]+k] = r[sa[i-1]+k])?
       }
22
                                                                                  26
  |}
                                                                                                rank: ++rank;
23
                                                                                          memcpy(r, tmpr, sizeof(r));
                                                                                  27
       Trie
                                                                                          if(r[sa[n-1]]==n-1) break;
                                                                                       }
                                                                                  29
  struct trie{
                                                                                  30
     map<char, trie> m;
2
                                                                                      void print(){//for debug
     void add(const string &s, int p=0){
3
                                                                                       forn(i, n)
                                                                                  32
       if(s[p]) m[s[p]].add(s, p+1);
4
                                                                                          cout << i << ''' <<
                                                                                  33
5
                                                                                          s.substr(sa[i], s.find( '$', sa[i])-sa[i]) << endl;}
                                                                                  34
     void dfs(){
6
      //Do stuff
                                                                                         String Matching With Suffix Array
7
       forall(it, m)
8
         it->second.dfs();
9
                                                                                   1 //returns (lowerbound, upperbound) of the search
     }
10
                                                                                     ii stringMatching(string P){ //O(sz(P)lgn)
  |};
11
                                                                                        int lo=0, hi=n-1, mid=lo;
                                                                                        while(lo<hi){</pre>
      Suffix Array (largo, nlogn)
                                                                                          mid=(lo+hi)/2:
                                                                                          int res=s.compare(sa[mid], sz(P), P);
                                                                                   6
  #define MAX N 1000
                                                                                          if(res>=0) hi=mid;
   #define rBOUND(x) (x<n? r[x] : 0)
                                                                                          else lo=mid+1;
   //sa will hold the suffixes in order.
                                                                                   8
   int sa[MAX_N], r[MAX_N], n;
                                                                                   9
                                                                                        if(s.compare(sa[lo], sz(P), P)!=0) return ii(-1, -1);
   string s; //input string, n=sz(s)
                                                                                  10
                                                                                        ii ans; ans.fst=lo;
                                                                                  11
   int f[MAX_N], tmpsa[MAX_N];
                                                                                       lo=0, hi=n-1, mid;
                                                                                        while(lo<hi){</pre>
                                                                                  13
   void countingSort(int k){
                                                                                          mid=(lo+hi)/2;
     zero(f);
9
                                                                                          int res=s.compare(sa[mid], sz(P), P);
     forn(i, n) f[rBOUND(i+k)]++;
10
                                                                                          if(res>0) hi=mid;
     int sum=0;
11
                                                                                          else lo=mid+1:
     forn(i, max(255, n)){
                                                                                  17
12
                                                                                       }
                                                                                  18
       int t=f[i]; f[i]=sum; sum+=t;}
13
                                                                                        if(s.compare(sa[hi], sz(P), P)!=0) hi--;
                                                                                  19
14
                                                                                        ans.snd=hi:
       tmpsa[f[rBOUND(sa[i]+k)]++]=sa[i];
15
                                                                                        return ans;
     memcpy(sa, tmpsa, sizeof(sa));
                                                                                  21
16
                                                                                  22 }
17
   void constructsa(){\frac{1}{0}} n log n
18
     n=sz(s);
                                                                                   1 //Calculates the LCP between consecutives suffixes in the Suffix Array.
```

2 //LCP[i] is the length of the LCP between sa[i] and sa[i-1]

```
3 | int LCP[MAX_N], phi[MAX_N], PLCP[MAX_N];
   void computeLCP(){\frac{}{0}}
     phi[sa[0]]=-1;
     forr(i, 1, n) phi[sa[i]]=sa[i-1];
     int L=0;
     forn(i, n){
8
       if(phi[i]==-1) {PLCP[i]=0; continue;}
9
       while(s[i+L]==s[phi[i]+L]) L++;
       PLCP[i]=L;
11
       L=max(L-1, 0);
12
13
     forn(i, n) LCP[i]=PLCP[sa[i]];
14
  | }
15
```

### 4.5. Corasick

```
1
   struct trie{
     map<char, trie> next;
     trie* tran[256];//transiciones del automata
     int idhoja, szhoja;//id de la hoja o O si no lo es
     //link lleva al sufijo mas largo, nxthoja lleva al mas largo pero que
         es hoja
     trie *padre, *link, *nxthoja;
7
     char pch;//caracter que conecta con padre
8
     trie(): tran(), idhoja(), padre(), link() {}
9
     void insert(const string &s, int id=1, int p=0){//id>0!!!
10
       if(p<sz(s)){</pre>
11
         trie &ch=next[s[p]];
12
         tran[(int)s[p]]=&ch;
13
         ch.padre=this, ch.pch=s[p];
14
         ch.insert(s, id, p+1);
15
16
       else idhoja=id, szhoja=sz(s);
17
18
     trie* get_link() {
19
       if(!link){
20
         if(!padre) link=this;//es la raiz
21
         else if(!padre->padre) link=padre;//hijo de la raiz
22
         else link=padre->get_link()->get_tran(pch);
23
       }
24
       return link;
25
26
```

```
trie* get_tran(int c) {
27
       if(!tran[c])
28
         tran[c] = !padre? this : this->get_link()->get_tran(c);
29
       return tran[c];
30
     }
31
     trie *get_nxthoja(){
32
       if(!nxthoja) nxthoja = get_link()->idhoja? link : link->nxthoja;
33
       return nxthoja;
34
     }
35
     void print(int p){
36
       if(idhoja)
37
         cout << "found," << idhoja << ",,,at,,position,," << p-szhoja << endl
38
       if(get_nxthoja()) get_nxthoja()->print(p);
39
40
     void matching(const string &s, int p=0){
41
       print(p);
42
       if(p<sz(s)) get_tran(s[p])->matching(s, p+1);
43
```

### 5. Geometria

### 5.1. Punto

```
1 struct pto{
     tipo x, y;
     pto(tipo x=0, tipo y=0):x(x),y(y){}
    pto operator+(pto a){return pto(x+a.x, y+a.y);}
     pto operator-(pto a){return pto(x-a.x, y-a.y);}
5
    pto operator+(tipo a){return pto(x+a, y+a);}
6
     pto operator*(tipo a){return pto(x*a, y*a);}
    pto operator/(tipo a){return pto(x/a, y/a);}
    //dot product, producto interno:
9
     tipo operator*(pto a){return x*a.x+y*a.y;}
10
     //module of the cross product or vectorial product:
11
     //if a is less than 180 clockwise from b, a^b>0
12
     tipo operator^(pto a){return x*a.y-y*a.x;}
13
     //returns true if this is at the left side of line gr
14
     bool left(pto q, pto r){return ((q-*this)^(r-*this))>0;}
15
     bool operator<(const pto &a) const{return x<a.x || (abs(x-a.x)<EPS &&
16
         v<a.y);}</pre>
   bool operator==(pto a){return abs(x-a.x)<EPS && abs(y-a.y)<EPS;}
     double norm(){return sqrt(x*x+y*y);}
     tipo norm_sq(){return x*x+y*y;}
```

```
20 };
   double dist(pto a, pto b){return (b-a).norm();}
   typedef pto vec;
   double angle(pto a, pto o, pto b){
     pto oa=a-o, ob=b-o;
     return atan2(oa^ob, oa*ob);}
    //rotate p by theta rads CCW w.r.t. origin (0,0)
28
   pto rotate(pto p, double theta){
     return pto(p.x*cos(theta)-p.y*sin(theta),
        p.x*sin(theta)+p.y*cos(theta));
31
32
33
    //orden total de puntos alrededor de un punto r
   struct Cmp{
     pto r;
36
     Cmp(pto _r)\{r = _r;\}
37
     int cuad(const pto &a) const{
38
       if(a.x > 0 \&\& a.y >= 0)return 0;
39
       if(a.x \le 0 \&\& a.y > 0)return 1;
40
       if(a.x < 0 && a.y <= 0)return 2;
41
       if(a.x >= 0 \&\& a.y < 0)return 3;
42
       assert(a.x ==0 && a.v==0);
43
       return -1;
44
45
     bool cmp(const pto&p1, const pto&p2)const{
46
       int c1 = cuad(p1), c2 = cuad(p2);
47
       if(c1==c2){
48
         return p1.y*p2.x<p1.x*p2.y;
49
       }else{
50
         return c1 < c2;
51
52
   bool operator()(const pto&p1, const pto&p2) const{
   return cmp(pto(p1.x-r.x,p1.y-r.y),pto(p2.x-r.x,p2.y-r.y));
   }
55
56 };
```

### 5.2. Line

```
struct line{
line() {}
double a,b,c;//Ax+By=C
```

```
4 //pto MUST store float coordinates!
    line(double a, double b, double c):a(a),b(b),c(c){}
     line(pto p, pto q): a(q.y-p.y), b(p.x-q.x), c(a*p.x+b*p.y) {}
   |};
   bool parallels(line 11, line 12){return abs(11.a*12.b-12.a*11.b) < EPS;}
   pto inter(line 11, line 12){//intersection
     double det=11.a*12.b-12.a*11.b;
     if(abs(det) < EPS) return pto(INF, INF); //parallels</pre>
     return pto(12.b*11.c-11.b*12.c, 11.a*12.c-12.a*11.c)/det;
13 }
5.3. Segment
1 struct segm{
     pto s,f;
     segm(pto s, pto f):s(s), f(f) {}
     pto closest(pto p) {//use for dist to point
        double 12 = dist_sq(s, f);
5
        if(12==0.) return s;
6
        double t = ((p-s)*(f-s))/12;
        if (t<0.) return s;//not write if is a line
        else if(t>1.)return f;//not write if is a line
        return s+((f-s)*t);
10
    }
11
     bool inside(pto p){
   return ((s-p)^(f-p))==0 \&\& min(s, f)<*this&&*this<max(s, f);}
14
15
   bool insidebox(pto a, pto b, pto p) {
     return (a.x-p.x)*(p.x-b.x)>-EPS && (a.y-p.y)*(p.y-b.y)>-EPS;
17
18
   pto inter(segm s1, segm s2){
     pto r=inter(line(s1.s, s1.f), line(s2.s, s2.f));
     if(insidebox(s1.s,s1.f,p) && insidebox(s2.s,s2.f,p))
21
         return r;
22
     return pto(INF, INF);
23
24 }
5.4. Polygon Area
double area(vector<pto> &p){//O(sz(p))
     double area=0:
    forn(i, sz(p)) area+=p[i]^p[(i+1) %z(p)];
    //if points are in clockwise order then area is negative
```

```
return abs(area)/2;
5
  |}
6
  //Area ellipse = M_PI*a*b where a and b are the semi axis lengths
8 //Area triangle = sqrt(s*(s-a)(s-b)(s-c)) where s=(a+b+c)/2
5.5. Circle
   vec perp(vec v){return vec(-v.y, v.x);}
  line bisector(pto x, pto y){
     line l=line(x, y); pto m=(x+y)/2;
     return line(-1.b, 1.a, -1.b*m.x+1.a*m.y);
5
   struct Circle{
     pto o;
7
     double r;
8
     Circle(pto x, pto y, pto z){
9
       o=inter(bisector(x, y), bisector(y, z));
10
       r=dist(o, x);
11
12
     pair<pto, pto> ptosTang(pto p){
13
       pto m=(p+o)/2;
14
       tipo d=dist(o, m);
15
       tipo a=r*r/(2*d);
16
       tipo h=sqrt(r*r-a*a);
17
       pto m2=o+(m-o)*a/d;
18
       vec per=perp(m-o)/d;
19
       return mkp(m2-per*h, m2+per*h);
20
21
^{22}
   //finds the center of the circle containing p1 and p2 with radius r
   //as there may be two solutions swap p1, p2 to get the other
   bool circle2PtsRad(pto p1, pto p2, double r, pto &c){
25
           double d2=(p1-p2).norm_sq(), det=r*r/d2-0.25;
26
           if(det<0) return false;</pre>
27
           c=(p1+p2)/2+perp(p2-p1)*sqrt(det);
28
           return true;
^{29}
30
   #define sqr(a) ((a)*(a))
   #define feq(a,b) (fabs((a)-(b))<EPS)</pre>
```

pair<tipo, tipo > ecCuad(tipo a, tipo b, tipo c){//a\*x\*x+b\*x+c=0

return make\_pair((-b + dx)/(2.0\*a), (-b - dx)/(2.0\*a));

tipo dx = sqrt(b\*b-4.0\*a\*c);

34

35 36 }

```
pair<pto, pto> interCL(Circle c, line 1){
     bool sw=false;
38
     if((sw=feq(0,1.b))){}
39
     swap(1.a, 1.b);
40
     swap(c.o.x, c.o.y);
41
42
     pair<tipo, tipo> rc = ecCuad(
43
     sqr(1.a)+sqr(1.b),
     2.0*1.a*1.b*c.o.y-2.0*(sqr(1.b)*c.o.x+1.c*1.a),
45
     sqr(1.b)*(sqr(c.o.x)+sqr(c.o.y)-sqr(c.r))+sqr(1.c)-2.0*1.c*1.b*c.o.y
     );
47
     pair<pto, pto> p( pto(rc.first, (1.c - 1.a * rc.first) / 1.b),
48
               pto(rc.second, (l.c - l.a * rc.second) / l.b) );
49
     if(sw){
50
     swap(p.first.x, p.first.y);
     swap(p.second.x, p.second.y);
53
     return p;
54
55
   pair<pto, pto> interCC(Circle c1, Circle c2){
    line 1;
    1.a = c1.o.x-c2.o.x;
    1.b = c1.o.y-c2.o.y;
     1.c = (sqr(c2.r) - sqr(c1.r) + sqr(c1.o.x) - sqr(c2.o.x) + sqr(c1.o.y)
     -sgr(c2.o.v))/2.0;
61
     return interCL(c1, 1);
63 }
5.6. Point in Poly
 1 //checks if v is inside of P, using ray casting
  //works with convex and concave.
   //excludes boundaries, handle it separately using segment.inside()
   bool inPolygon(pto v, vector<pto>& P) {
     bool c = false;
    forn(i, sz(P)){
6
       int j=(i+1) %z(P);
       if((P[j].y>v.y) != (P[i].y > v.y) &&
8
     (v.x < (P[i].x - P[j].x) * (v.y-P[j].y) / (P[i].y - P[j].y) + P[j].x))
9
         c = !c:
10
     }
11
12
     return c;
13 | }
```

#### 5.7. Convex Check CHECK

```
bool isConvex(vector<int> &p){//O(N)
int N=sz(p);
if(N<3) return false;
bool isLeft=p[0].left(p[1], p[2]);
forr(i, 1, N)
if(p[i].left(p[(i+1) M], p[(i+2) M])!=isLeft)
return false;
return true; }</pre>
```

#### 5.8. Convex Hull

```
//stores convex hull of P in S, CCW order
   void CH(vector<pto>& P, vector<pto> &S){
     S.clear():
3
     sort(P.begin(), P.end());
     forn(i, sz(P)){
       while(sz(S) \ge 2 \&\& S[sz(S)-1].left(S[sz(S)-2], P[i])) S.pop_back();
6
       S.pb(P[i]);
7
     }
8
     S.pop_back();
9
     int k=sz(S);
10
     dforn(i, sz(P)){
11
       while(sz(S) \ge k+2 \&\& S[sz(S)-1].left(S[sz(S)-2], P[i])) S.pop_back
12
           ();
       S.pb(P[i]);
13
14
     S.pop_back();
16
```

# 5.9. Cut Polygon

```
1 //cuts polygon Q along the line ab
  //stores the left side (swap a, b for the right one) in P
  |void cutPolygon(pto a, pto b, vector<pto> Q, vector<pto> &P){
    P.clear();
4
    forn(i, sz(Q)){
5
       double left1=(b-a)^(Q[i]-a), left2=(b-a)^(Q[(i+1) \%z(Q)]-a);
6
      if(left1>=0) P.pb(Q[i]);
7
       if(left1*left2<0)
         P.pb(inter(line(Q[i], Q[(i+1) \slashz(Q)]), line(a, b)));
9
    }
10
11 }
```

#### 5.10. Bresenham

```
1 //plot a line approximation in a 2d map
  void bresenham(pto a, pto b){
    pto d=b-a; d.x=abs(d.x), d.y=abs(d.y);
    pto s(a.x<b.x? 1: -1, a.y<b.y? 1: -1);
    int err=d.x-d.y;
    while(1){
      m[a.x][a.y]=1;//plot
      if(a==b) break;
      int e2=2*err;
      if(e2 > -d.y){
        err-=d.y, a.x+=s.x;
      if(e2 < d.x)
         err+= d.x, a.y+= s.y;
14
    }
15 }
```

### 5.11. Interseccion de Circulos en n3log(n)

```
1 struct event {
       double x; int t;
       event(double xx, int tt) : x(xx), t(tt) {}
       bool operator <(const event &o) const { return x < o.x; }</pre>
   }:
5
   typedef vector<Circle> VC;
   typedef vector<event> VE;
   int n;
   double cuenta(VE &v, double A,double B) {
       sort(v.begin(), v.end());
       double res = 0.0, lx = ((v.empty())?0.0:v[0].x);
11
       int contador = 0;
12
       forn(i,sz(v)) {
13
           // interseccion de todos (contador == n), union de todos (
14
                contador > 0),
           // conjunto de puntos cubierto por exacta k Circulos (contador
15
           if (contador == n) res += v[i].x - lx;
16
           contador += v[i].t:
17
           lx = v[i].x:
18
       }
19
       return res;
20
21 |}
```

```
// Primitiva de sqrt(r*r - x*x) como funcion double de una variable x.
   inline double primitiva(double x,double r) {
23
       if (x \ge r) return r*r*M_PI/4.0;
^{24}
       if (x \le -r) return -r*r*M_PI/4.0;
25
       double raiz = sqrt(r*r-x*x);
26
       return 0.5 * (x * raiz + r*r*atan(x/raiz));
27
28
   double interCircle(VC &v) {
29
       vector<double> p; p.reserve(v.size() * (v.size() + 2));
30
       forn(i,sz(v)) {
31
           p.push_back(v[i].c.x + v[i].r);
32
           p.push_back(v[i].c.x - v[i].r);
33
       }
34
       forn(i,sz(v)) forn(j,i) {
35
           Circle &a = v[i], b = v[j];
36
           double d = (a.c - b.c).norm();
37
           if (fabs(a.r - b.r) < d \&\& d < a.r + b.r) {
38
               double alfa = acos((sqr(a.r) + sqr(d) - sqr(b.r)) / (2.0 * d)
39
                     * a.r)):
               pto vec = (b.c - a.c) * (a.r / d);
40
               p.pb((a.c + rotate(vec, alfa)).x);
41
               p.pb((a.c + rotate(vec, -alfa)).x);
42
43
       }
44
       sort(p.begin(), p.end());
45
       double res = 0.0;
46
       forn(i,sz(p)-1) {
47
           const double A = p[i], B = p[i+1];
48
           VE ve; ve.reserve(2 * v.size());
49
           forn(j,sz(v)) {
50
               const Circle &c = v[j];
51
               double arco = primitiva(B-c.c.x,c.r) - primitiva(A-c.c.x,c.r
52
               double base = c.c.y * (B-A);
53
               ve.push_back(event(base + arco,-1));
54
               ve.push_back(event(base - arco, 1));
55
56
           res += cuenta(ve,A,B);
57
58
       return res;
59
60 }
```

### 6. Math

#### 6.1. Identidades

```
\begin{split} \sum_{i=0}^{n} \binom{n}{i} &= 2^{n} \\ \sum_{i=0}^{n} i \binom{n}{i} &= n * 2^{n-1} \\ \sum_{i=m}^{n} i &= \frac{n(n+1)}{2} - \frac{m(m-1)}{2} = \frac{(n+1-m)(n+m)}{2} \\ \sum_{i=0}^{n} i &= \sum_{i=1}^{n} i = \frac{n(n+1)}{2} \\ \sum_{i=0}^{n} i^{2} &= \frac{n(n+1)(2n+1)}{6} = \frac{n^{3}}{3} + \frac{n^{2}}{2} + \frac{n}{6} \\ \sum_{i=0}^{n} i(i-1) &= \frac{8}{6} (\frac{n}{2})(\frac{n}{2} + 1)(n+1) \text{ (doubles)} \rightarrow \text{Sino ver caso impar y par} \\ \sum_{i=0}^{n} i^{3} &= \left(\frac{n(n+1)}{2}\right)^{2} = \frac{n^{4}}{4} + \frac{n^{3}}{2} + \frac{n^{2}}{4} = \left[\sum_{i=1}^{n} i\right]^{2} \\ \sum_{i=0}^{n} i^{4} &= \frac{n(n+1)(2n+1)(3n^{2}+3n-1)}{30} &= \frac{n^{5}}{5} + \frac{n^{4}}{2} + \frac{n^{3}}{3} - \frac{n}{30} \\ \sum_{i=0}^{n} i^{p} &= \frac{(n+1)^{p+1}}{p+1} + \sum_{k=1}^{p} \frac{B_{k}}{p-k+1} \binom{p}{k} (n+1)^{p-k+1} \\ r &= e - v + k + 1 \end{split} Teorema de Pick: (Area, puntos interiores y puntos en el borde) A = I + \frac{B}{2} - 1
```

#### 6.2. Ec. Caracteristica

```
a_0T(n) + a_1T(n-1) + \ldots + a_kT(n-k) = 0
p(x) = a_0x^k + a_1x^{k-1} + \ldots + a_k
Sean r_1, r_2, \ldots, r_q las raíces distintas, de mult. m_1, m_2, \ldots, m_q
T(n) = \sum_{i=1}^q \sum_{j=0}^{m_i-1} c_{ij} n^j r_i^n
Las constantes c_{ij} se determinan por los casos base.
```

### 6.3. Combinatorio

```
forn(i, MAXN+1){//comb[i][k]=i tomados de a k
     comb[i][0]=comb[i][i]=1;
     forr(k, 1, i) comb[i][k]=(comb[i-1][k]+comb[i-1][k-1]) MOD;
4
5
   ll lucas (ll n, ll k, int p){ //Calcula (n,k) % teniendo comb[p][p]
       precalculado.
     11 \text{ aux} = 1;
     while (n + k)
       aux = (aux * comb[n\%][k\%]) \%;
       n/=p, k/=p;
10
11
     return aux;
12
13 | }
```

# 6.4. Exp. de Numeros Mod.

```
1 | 11 expmod (11 b, 11 e, 11 m){//0(log b)
2 | if(!e) return 1;
3 | 11 q= expmod(b,e/2,m); q=(q*q) %m;
4 | return e%2? (b * q) %m : q;
5 |}
```

# 6.5. Exp. de Matrices y Fibonacci en log(n)

### 6.6. Teorema Chino del Resto

$$y = \sum_{j=1}^{n} (x_j * (\prod_{i=1, i \neq j}^{n} m_i)_{m_j}^{-1} * \prod_{i=1, i \neq j}^{n} m_i)$$

# 6.7. Funciones de primos

```
forall(it, f) rta += 1;
12
     return rta;
13
14
   11 sumPrimeFactors (ll n){
    ll rta = 0;
     map<ll,ll> f=fact(n);
     forall(it, f) rta += it->first;
     return rta;
19
20
21
   11 numDiv (ll n){
    ll rta = 1:
23
     map<ll,ll> f=fact(n);
     forall(it, f) rta *= (it->second + 1);
25
     return rta;
26
   }
27
   ll sumDiv (ll n){
    ll rta = 1;
     map<ll,11> f=fact(n);
     forall(it, f) rta *= ((ll)pow((double)it->first, it->second + 1.0)-1)
         / (it->first-1);
     return rta;
33
34
   ll eulerPhi (ll n){ // con criba: O(lg n)
    11 \text{ rta} = n;
     map<ll,ll> f=fact(n);
     forall(it, f) rta -= rta / it->first;
     return rta;
40
41
   11 eulerPhi2 (11 n){ // 0 (sqrt n)
    11 r = n;
    forr (i,2,n+1){
      if ((ll)i*i > n)
         break:
    if (n \% i == 0){
       while (n\%i == 0) n/=i;
         r = r/i;
       }}
     if (n != 1)
```

### 6.8. Phollard's Rho (rolando)

#### 6.9. Criba

```
#define MAXP 80000 //no necesariamente primo
int criba[MAXP+1];
vector<int> primos;

void buscarprimos(){
   int sq=sqrt(MAXP)+1;
   forr(p, 2, MAXP+1) if(!criba[p]){
      primos.push_back(p);
      if(p<=sq)
      for(int m=p*p; m<=MAXP; m+=p)//borro los multiplos de p
      if(!criba[m])criba[m]=p;
}</pre>
```

### 6.10. Factorizacion

Sea  $n = \prod p_i^{k_i}$ , fact(n) genera un map donde a cada  $p_i$  le asocia su  $k_i$ 

```
//factoriza bien numeros hasta MAXP^2
map<11,11> fact(11 n){ //0 (cant primos)
map<11,11> ret;
forall(p, primos){
while(!(n%p)){
ret[*p]++;//divisor found
```

```
n/=*p;
8
    }
     if(n>1) ret[n]++;
     return ret;
11
12
13
   //factoriza bien numeros hasta MAXP
   map<11,11> fact2(11 n){ //0 (lg n)
     map<ll,ll> ret;
     while (criba[n]){
17
      ret[criba[n]]++;
18
      n/=criba[n];
19
    }
20
    if(n>1) ret[n]++;
21
     return ret;
22
23 }
6.11. GCD
1 | tipo gcd(tipo a, tipo b){return a?gcd(b %a, a):b;}
6.12. Extended Euclid
void extendedEuclid (ll a, ll b) \{ //a * x + b * y = d \}
     if (!b) { x = 1; y = 0; d = a; return;}
    extendedEuclid (b, a%);
    11 x1 = y;
    11 y1 = x - (a/b) * y;
     x = x1; y = y1;
6
7 }
6.13. LCM
tipo lcm(tipo a, tipo b){return a / gcd(a,b) * b;}
6.14. Inversos
1 #define MAXMOD 15485867
1 ll inv[MAXMOD];//inv[i]*i=1 mod MOD
```

forr(i, 2, p) inv[i] = p-((p/i)\*inv[p%i])%p;

void calc(int p) $\{//0(p)$ 

inv[1]=1;

```
6 | }
                                                                                   1
  int inverso(int x){\frac{1}{0}}
                                                                                   2
     return expmod(x, eulerphi(MOD)-2);//si mod no es primo(sacar a mano)
     return expmod(x, MOD-2);//si mod es primo
  |}
10
                                                                                   5
6.15. Simpson
                                                                                   8
  double integral(double a, double b, int n=10000) {//O(n), n=cantdiv
     double area=0, h=(b-a)/n, fa=f(a), fb;
                                                                                  10
     forn(i, n){
3
                                                                                  11
       fb=f(a+h*(i+1));
                                                                                  12
       area+=fa+ 4*f(a+h*(i+0.5)) +fb, fa=fb;
5
                                                                                  13
6
                                                                                  14
     return area*h/6.;}
                                                                                  15
6.16. Fraction
                                                                                  16
                                                                                  17
   tipo mcd(tipo a, tipo b){return a?mcd(b%, a):b;}
   struct frac{
2
                                                                                  19
     tipo p,q;
     frac(tipo p=0, tipo q=1):p(p),q(q) {norm();}
                                                                                  21
     void norm(){
       tipo a = mcd(p,q);
                                                                                  23
       if(a) p/=a, q/=a;
                                                                                  24
       else q=1;
                                                                                  25
       if (q<0) q=-q, p=-p;}
     frac operator+(const frac& o){
                                                                                  27
10
       tipo a = mcd(q, o.q);
                                                                                  28
11
       return frac(p*(o.q/a)+o.p*(q/a), q*(o.q/a));}
                                                                                  29
12
     frac operator-(const frac& o){
13
       tipo a = mcd(q, o.q);
14
       return frac(p*(o.q/a)-o.p*(q/a), q*(o.q/a));}
15
     frac operator*(frac o){
16
       tipo a = mcd(q, o.p), b = mcd(o.q, p);
17
       return frac((p/b)*(o.p/a), (q/a)*(o.q/b));}
18
     frac operator/(frac o){
19
       tipo a = mcd(q,o.q), b = mcd(o.p,p);
20
                                                                                   4
       return frac((p/b)*(o.q/a),(q/a)*(o.p/b));}
21
                                                                                   5
     bool operator<(const frac &o) const{return p*o.q < o.p*q;}</pre>
     bool operator==(frac o){return p==o.p&kq==o.q;}
23
24 };
                                                                                   7
```

#### 6.17. Polinomio

```
//poly contains only a vector<int> c (the coeficients)
    //the following function generates the roots of the polynomial
  //it can be easily modified to return float roots
     set<tipo> roots(){
       set<tipo> roots;
      tipo a0 = abs(c[0]), an = abs(c[sz(c)-1]);
       vector<tipo> ps,qs;
       forr(p,1,sqrt(a0)+1) if (a0\%p==0) ps.pb(p),ps.pb(a0/p);
       forr(q,1,sqrt(an)+1) if (an)q==0) qs.pb(q),qs.pb(an/q);
      forall(pt,ps)
        forall(qt,qs) if ( (*pt) % (*qt)==0 ) {
          tipo root = abs((*pt) / (*qt));
          if (eval(root)==0) roots.insert(root):
      return roots;
    }
  };
   poly interpolate(const vector<tipo> &x, const vector<tipo> &y) {
       int n = sz(x);
       poly p;
       vector<tipo> aux(2);
      forn(i, n) {
         double a = y[i] - p.eval(x[i]);
         forn(j, i) a /= x[i] - x[j];
         poly add(vector<tipo>(1, a));
         forn(j, i) aux[0]=-x[j], aux[1]=1, add = add*aux;
         p = p + add;
       return p;
30 }
6.18. Ec. Lineales
bool resolver_ev(Mat a, Vec y, Vec &x, Mat &ev){
    int n = a.size(), m = n?a[0].size():0, rw = min(n, m);
    vector<int> p; forn(i,m) p.push_back(i);
    forn(i, rw) {
      int uc=i. uf=i:
      forr(f, i, n) forr(c, i, m) if(fabs(a[f][c])>fabs(a[uf][uc])) {uf=f;
          uc=c:}
       if (feg(a[uf][uc], 0)) { rw = i; break; }
       forn(j, n) swap(a[j][i], a[j][uc]);
8
      swap(a[i], a[uf]); swap(y[i], y[uf]); swap(p[i], p[uc]);
```

```
tipo inv = 1 / a[i][i]; //aca divide
10
       forr(j, i+1, n) {
11
         tipo v = a[j][i] * inv;
12
         forr(k, i, m) a[j][k]-=v * a[i][k];
13
         y[j] = v*y[i];
14
15
     } // rw = rango(a), aca la matriz esta triangulada
16
     forr(i, rw, n) if (!feq(y[i],0)) return false; // checkeo de
         compatibilidad
     x = vector < tipo > (m, 0);
18
     dforn(i, rw){
19
       tipo s = y[i];
20
       forr(j, i+1, rw) s -= a[i][j]*x[p[j]];
       x[p[i]] = s / a[i][i]; //aca divide
22
23
     ev = Mat(m-rw, Vec(m, 0)); // Esta parte va SOLO si se necesita el ev
24
     forn(k, m-rw) {
25
       ev[k][p[k+rw]] = 1;
26
       dforn(i, rw){
27
         tipo s = -a[i][k+rw];
28
         forr(j, i+1, rw) s -= a[i][j]*ev[k][p[j]];
29
         ev[k][p[i]] = s / a[i][i]; //aca divide
30
       }
31
     }
32
     return true;
33
34
```

# 7. Grafos

## 7.1. Dijkstra

```
#define INF 1e9
  int N;
2
  #define MAX_V 250001
   vector<ii> G[MAX_V];
   //To add an edge use
   #define add(a, b, w) G[a].pb(mkp(w, b))
   ll dijkstra(int s, int t){\frac{}{0(|E| \log |V|)}}
     priority_queue<ii, vector<ii>, greater<ii> > Q;
9
     vector<ll> dist(N, INF); vector<int> dad(N, -1);
10
     Q.push(mkp(0, s)); dist[s] = 0;
11
     while(sz(Q)){
12
```

```
ii p = Q.top(); Q.pop();
       if(p.snd == t) break;
14
       forall(it, G[p.snd])
15
         if(dist[p.snd]+it->first < dist[it->snd]){
16
           dist[it->snd] = dist[p.snd] + it->fst;
17
           dad[it->snd] = p.snd;
18
           Q.push(mkp(dist[it->snd], it->snd));
19
20
     }
21
     return dist[t];
     if(dist[t]<INF)//path generator</pre>
23
       for(int i=t; i!=-1; i=dad[i])
24
         printf("%d%c", i, (i==s?'\n':','));
25
26 }
7.2. Bellman-Ford
vector<ii> G[MAX_N];//ady. list with pairs (weight, dst)
  int dist[MAX N]:
  void bford(int src){//O(VE)
     dist[src]=0;
     forn(i, N-1) forn(j, N) if(dist[j]!=INF) forall(it, G[j])
       dist[it->snd]=min(dist[it->snd], dist[j]+it->fst);
6
   }
7
8
   bool hasNegCycle(){
    forn(j, N) if(dist[j]!=INF) forall(it, G[j])
       if(dist[it->snd]>dist[j]+it->fst) return true;
     //inside if: all points reachable from it->snd will have -INF distance
12
         (do bfs)
     return false;
13
14 }
7.3. Floyd-Warshall
1 //G[i][j] contains weight of edge (i, j) or INF
2 //G[i][i]=0
3 int G[MAX_N][MAX_N];
  void floyd(){//0(N^3)}
5 | forn(k, N) forn(i, N) if(G[i][k]!=INF) forn(j, N) if(G[k][j]!=INF)
    G[i][j]=min(G[i][j], G[i][k]+G[k][j]);
6
  1
7
  bool inNegCycle(int v){
```

return G[v][v]<0;}

18 }

```
//checks if there's a neg. cycle in path from a to b bool hasNegCycle(int a, int b){
forn(i, N) if(G[a][i]!=INF && G[i][i]<0 && G[i][b]!=INF)
return true;
return false;
}
```

#### 7.4. Kruskal

```
struct Ar{int a,b,w;};
   bool operator (const Ar& a, const Ar &b) {return a.w <b.w;}
   vector<Ar> E;
   11 kruskal(){
       11 cost=0;
5
       sort(E.begin(), E.end());//ordenar aristas de menor a mayor
6
       uf.init(n):
       forall(it, E){
8
           if(uf.comp(it->a)!=uf.comp(it->b)){//si no estan conectados
9
               uf.unir(it->a, it->b);//conectar
10
               cost+=it->w:
11
           }
12
       }
13
       return cost;
14
  | }
15
```

### 7.5. Prim

```
bool taken[MAXN];
  priority_queue<ii, vector<ii>, greater<ii> > pq;//min heap
   void process(int v){
       taken[v]=true;
4
       forall(e, G[v])
5
           if(!taken[e->second]) pq.push(*e);
6
7
8
   11 prim(){
9
       zero(taken);
10
       process(0);
11
       11 cost=0:
12
       while(sz(pq)){
13
           ii e=pq.top(); pq.pop();
14
           if(!taken[e.second]) cost+=e.first, process(e.second);
15
       }
16
       return cost;
17
```

# 7.6. 2-SAT + Tarjan SCC

```
1 //We have a vertex representing a var and other for his negation.
2 //Every edge stored in G represents an implication. To add an equation
       of the form a | |b, use addor(a, b)
3 //MAX=max cant var, n=cant var
   #define addor(a, b) (G[neg(a)].pb(b), G[neg(b)].pb(a))
   vector<int> G[MAX*2];
   //idx[i]=index assigned in the dfs
   //lw[i]=lowest index(closer from the root) reachable from i
   int lw[MAX*2], idx[MAX*2], qidx;
   stack<int> q;
   int qcmp, cmp[MAX*2];
   //verdad[cmp[i]]=valor de la variable i
   bool verdad[MAX*2+1];
13
   int neg(int x) { return x>=n? x-n : x+n;}
   void tjn(int v){
     lw[v]=idx[v]=++qidx;
     q.push(v), cmp[v]=-2;
     forall(it, G[v]){
       if(!idx[*it] || cmp[*it]==-2){
         if(!idx[*it]) tjn(*it);
20
         lw[v]=min(lw[v], lw[*it]);
21
       }
22
23
     if(lw[v]==idx[v]){
24
       qcmp++;
25
       int x;
26
       do{x=q.top(); q.pop(); cmp[x]=qcmp;}while(x!=v);
27
       verdad[qcmp] = (cmp[neg(v)] < 0);</pre>
28
29
30
   //remember to CLEAR G!!!
   bool satisf(){\frac{}{0}}
     memset(idx, 0, sizeof(idx)), qidx=0;
33
     memset(cmp, -1, sizeof(cmp)), qcmp=0;
34
     forn(i, n){
35
       if(!idx[i]) tjn(i);
36
       if(!idx[neg(i)]) tjn(neg(i));
37
    }
38
```

```
Caloventor en Dos - Universidad Nacional de Rosario
     forn(i, n) if(cmp[i] == cmp[neg(i)]) return false;
     return true;
40
41 }
      Articulation Points
1 | int N;
   vector<int> G[1000000];
   //V[i]=node number(if visited), L[i]= lowest V[i] reachable from i
   int qV, V[1000000], L[1000000], P[1000000];
   void dfs(int v, int f){
     L[v]=V[v]=++qV;
6
    forall(it, G[v])
       if(!V[*it]){
8
         dfs(*it, v);
9
```

# Comp. Biconexas y Puentas

L[v] = min(L[v], L[\*it]);

L[v]=min(L[v], V[\*it]);

P[v] += L[\*it] >= V[v];

else if(\*it!=f)

int cantart() { //0(n)

zero(V), zero(P);

dfs(1, 0); P[1]--;

forn(i, N) if(P[i]) q++;

qV=0;

return q;

int q=0;

10

11

12

13

14

15

16

17

19

20

21

22

23

```
struct edge {
     int u, v, comp;
     bool bridge;
3
   };
4
   vector<edge> e;
   void addEdge(int u, int v) {
    G[u].pb(sz(e)), G[v].pb(sz(e));
     e.pb((edge)\{u,v,-1,false\});
8
9
   //d[i]=id de la dfs
   //b[i]=lowest id reachable from i
12 | int d[MAXN], b[MAXN], t;
```

```
int nbc;//cant componentes
   int comp[MAXN];//comp[i]=cant comp biconexas a la cual pertenece i
   void initDfs(int n) {
     zero(G), zero(comp);
     e.clear();
     forn(i,n) d[i]=-1;
18
     nbc = t = 0;
19
   }
20
   stack<int> st;
^{21}
   void dfs(int u, int pe) \{//0(n + m)\}
     b[u] = d[u] = t++;
23
     comp[u] = (pe != -1);
24
     forall(ne, G[u]) if (*ne != pe){
25
       int v = e[*ne].u ^e[*ne].v ^u;
26
       if (d[v] == -1) {
27
         st.push(*ne);
         dfs(v,*ne);
29
         if (b[v] > d[u]){
           e[*ne].bridge = true; // bridge
31
32
         if (b[v] >= d[u]) \{ // art \}
33
           int last;
34
           do {
35
             last = st.top(); st.pop();
             e[last].comp = nbc;
37
           } while (last != *ne);
38
           nbc++;
39
           comp[u]++;
40
41
         b[u] = min(b[u], b[v]);
42
43
       else if (d[v] < d[u]) \{ // back edge
44
         st.push(*ne);
45
         b[u] = min(b[u], d[v]);
46
47
     }
48
49 }
7.9. LCA + Climb
1 //f[v][k] holds the 2^k father of v
2 //L[v] holds the level of v
```

3 int N, f[100001][20], L[100001];

```
void build(){//f[i][0] must be filled previously, O(nlgn)
     forn(k, 20-1) forn(i, N) f[i][k+1]=f[f[i][k]][k];}
5
   #define lg(x) (31-__builtin_clz(x))//=floor(log2(x))
   int climb(int a, int d){\frac{1}{0(lgn)}}
     if(!d) return a;
10
     dforn(i, lg(L[a])+1)
11
       if(1<<i<=d)
^{12}
         a=f[a][i], d-=1<<i;
13
       return a;
14
15
   int lca(int a, int b){\frac{1}{0}}
     if(L[a]<L[b]) swap(a, b);</pre>
     a=climb(a, L[a]-L[b]);
18
     if(a==b) return a;
19
     dforn(i, lg(L[a])+1)
20
       if(f[a][i]!=f[b][i])
21
         a=f[a][i], b=f[b][i];
22
     return f[a][0];
23
24 }
```

### 7.10. Heavy Light Decomposition

```
int treesz[MAXN]://cantidad de nodos en el subarbol del nodo v
  int dad[MAXN];//dad[v]=padre del nodo v
   void dfs1(int v, int p=-1){//pre-dfs
     dad[v]=p;
4
     treesz[v]=1;
5
     forall(it, G[v]) if(*it!=p){
6
       dfs1(*it, v);
7
       treesz[v]+=treesz[*it];
8
9
10
   int pos[MAXN], q;//pos[v]=posicion del nodo v en el recorrido de la dfs
   //Las cadenas aparecen continuas en el recorrido!
12
   int homecad[MAXN]://dada una cadena devuelve su nodo inicial
   int cad[MAXN];//cad[v]=cadena a la que pertenece el nodo
   void heavylight(int v, int cur=-1){
     if(cur==-1) homecad[cur=cantcad++]=v;
17
    pos[v]=q++;
18
     cad[v]=cur;
```

```
int mx=-1:
20
     forn(i, sz(G[v])) if(G[v][i]!=dad[v])
21
       if(mx==-1 || treesz[G[v][mx]]<treesz[G[v][i]]) mx=i;</pre>
22
     if(mx!=-1) heavylight(G[v][mx], cur);
23
     forn(i, sz(G[v])) if(i!=mx && G[v][i]!=dad[v])
24
       heavylight(G[v][i], -1);
25
26
   //ejemplo de obtener el maximo numero en el camino entre dos nodos
   //RTA: max(query(low, u), query(low, v)), con low=lca(u, v)
   //esta funcion va trepando por las cadenas
   int query(int an, int v){//O(logn)
    //si estan en la misma cadena:
     if(cad[an]==cad[v]) return rmq.get(pos[an], pos[v]+1);
     return max(query(an, dad[homecad[cad[v]]]),
            rmq.get(pos[homecad[cad[v]]], pos[v]+1));
34
35 }
```

# 7.11. Euler Cycle

```
int n,m,ars[MAXE], eq;
vector<int> G[MAXN];//fill G,n,m,ars,eq
  list<int> path;
   int used[MAXN];
   bool usede[MAXE]:
   queue<list<int>::iterator> q;
   int get(int v){
     while(used[v]\leq z(G[v]) && usede[G[v][used[v]]]) used[v]++;
     return used[v];
10
   void explore(int v, int r, list<int>::iterator it){
     int ar=G[v][get(v)]; int u=v^ars[ar];
12
     usede[ar]=true;
13
     list<int>::iterator it2=path.insert(it, u);
14
     if(u!=r) explore(u, r, it2);
15
     if(get(v)<sz(G[v])) q.push(it);</pre>
16
   }
17
   void euler(){
18
     zero(used), zero(usede);
19
     path.clear();
20
     q=queue<list<int>::iterator>();
21
     path.push_back(0); q.push(path.begin());
22
     while(sz(q)){
23
       list<int>::iterator it=q.front(); q.pop();
24
```

```
Caloventor en Dos - Universidad Nacional de Rosario
       if(used[*it]<sz(G[*it])) explore(*it, *it, it);</pre>
25
26
     reverse(path.begin(), path.end());
27
28
   void addEdge(int u, int v){
29
     G[u].pb(eq), G[v].pb(eq);
30
     ars[eq++]=u^v;
32 }
7.12. Chu-liu
   void visit(graph &h, int v, int s, int r,
     vector<int> &no, vector< vector<int> > &comp,
     vector<int> &mark, weight &cost, bool &found) {
     if (mark[v]) {
5
       vector<int> temp = no;
6
       found = true;
       do {
8
```

```
vector<int> &prev, vector< vector<int> > &next, vector<weight> &mcost,
         cost += mcost[v];
         v = prev[v];
         if (v != s) {
           while (comp[v].size() > 0) {
12
             no[comp[v].back()] = s;
13
             comp[s].push_back(comp[v].back());
14
             comp[v].pop_back();
15
16
17
       } while (v != s);
18
       forall(j,comp[s]) if (*j != r) forall(e,h[*j])
19
         if (no[e->src] != s) e->w -= mcost[ temp[*i] ];
20
     }
^{21}
     mark[v] = true;
^{22}
     forall(i,next[v]) if (no[*i] != no[v] && prev[no[*i]] == v)
23
       if (!mark[no[*i]] || *i == s)
24
         visit(h, *i, s, r, no, comp, prev, next, mcost, mark, cost, found)
25
26
   weight minimumSpanningArborescence(const graph &g, int r) {
       const int n=sz(g);
28
     graph h(n);
29
     forn(u,n) forall(e,g[u]) h[e->dst].pb(*e);
30
     vector<int> no(n);
31
```

```
vector<vector<int> > comp(n);
     forn(u, n) comp[u].pb(no[u] = u);
33
     for (weight cost = 0; ;) {
34
       vector<int> prev(n, -1);
35
       vector<weight> mcost(n, INF);
36
       forn(j,n) if (j != r) forall(e,h[j])
         if (no[e->src] != no[i])
           if (e->w < mcost[ no[j] ])</pre>
             mcost[ no[j] ] = e->w, prev[ no[j] ] = no[e->src];
       vector< vector<int> > next(n);
       forn(u,n) if (prev[u] >= 0)
42
         next[ prev[u] ].push_back(u);
43
       bool stop = true;
44
       vector<int> mark(n);
       forn(u,n) if (u != r && !mark[u] && !comp[u].empty()) {
46
         bool found = false;
         visit(h, u, u, r, no, comp, prev, next, mcost, mark, cost, found);
         if (found) stop = false;
       }
50
       if (stop) {
         forn(u,n) if (prev[u] >= 0) cost += mcost[u];
         return cost;
       }
54
    }
55
56 }
```

# 7.13. Hungarian

```
1 #define MAXN 256
   #define INFTO 0x7f7f7f7f
   int n;
   int mt[MAXN] [MAXN]; // Matriz de costos (X * Y)
   int xy[MAXN], yx[MAXN]; // Matching resultante (X->Y, Y->X)
   int lx[MAXN], ly[MAXN], slk[MAXN], slkx[MAXN], prv[MAXN];
   char S[MAXN], T[MAXN];
   void updtree(int x) {
    forn(y, n) if (lx[x] + ly[y] - mt[x][y] < slk[y]) {
       slk[y] = lx[x] + ly[y] - mt[x][y];
10
       slkx[y] = x;
11
  1 }
12
   int hungar(){//Matching maximo de mayor costo en grafos dirigidos (N^3)
     forn(i, n) {
14
       ly[i] = 0;
15
```

```
lx[i] = *max_element(mt[i], mt[i]+n); }
16
     memset(xy, -1, sizeof(xy));
17
     memset(yx, -1, sizeof(yx));
18
     forn(m, n) {
19
       memset(S, 0, sizeof(S));
20
       memset(T, 0, sizeof(T));
21
       memset(prv, -1, sizeof(prv));
22
       memset(slk, 0x7f, sizeof(slk));
23
       queue<int> q;
^{24}
   #define bpone(e, p) { q.push(e); prv[e] = p; S[e] = 1; updtree(e); }
       forn(i, n) if (xy[i] == -1) { bpone(i, -2); break; }
26
       int x=0, y=-1;
27
       while (v==-1) {
28
         while (!q.empty() && y==-1) {
29
           x = q.front(); q.pop();
30
           forn(j, n) if (mt[x][j] == lx[x] + ly[j] && !T[j]) {
31
             if (yx[j] == -1) \{ y = j; break; \}
32
             T[i] = 1;
33
             bpone(yx[j], x);
34
35
         }
36
         if (v!=-1) break;
37
         int dlt = INFTO;
38
         forn(j, n) if (!T[j]) dlt = min(dlt, slk[j]);
39
         forn(k, n) {
40
           if (S[k]) lx[k] -= dlt;
41
           if (T[k]) ly [k] += dlt;
42
           if (!T[k]) slk[k] -= dlt;
43
         }
44
         forn(j, n) if (!T[j] && !slk[j]) {
45
           if (yx[j] == -1) {
46
             x = slkx[j]; y = j; break;
47
           } else {
48
             T[i] = 1;
49
             if (!S[yx[j]]) bpone(yx[j], slkx[j]);
50
           }
51
         }
52
       }
53
       if (v!=-1) {
54
         for(int p = x; p != -2; p = prv[p]) {
55
           yx[y] = p;
56
           int ty = xy[p]; xy[p] = y; y = ty;
57
58
```

### 8. Network Flow

#### 8.1. Dinic

```
int nodes, src, dest;
   int dist[MAX], q[MAX], work[MAX];
   struct Edge {
     int to, rev;
     11 f, cap;
     Edge(int to, int rev, ll f, ll cap) : to(to), rev(rev), f(f), cap(cap)
          {}
   };
8
   vector<Edge> G[MAX];
11
   // Adds bidirectional edge
   void addEdge(int s, int t, ll cap){
     G[s].push_back(Edge(t, G[t].size(), 0, cap));
     G[t].push_back(Edge(s, G[s].size()-1, 0, 0));
15
16
17
   bool dinic_bfs() {
     fill(dist, dist + nodes, -1);
19
     dist[src] = 0;
20
     int qt = 0;
21
     q[qt++] = src;
22
     for (int qh = 0; qh < qt; qh++) {
23
       int u = q[qh];
24
       forall(e, G[u]){
25
         int v = e \rightarrow to;
26
         if(dist[v]<0 \&\& e->f < e->cap){
27
           dist[v]=dist[u]+1;
28
           q[qt++]=v;
29
         }
30
31
```

```
}
32
     return dist[dest] >= 0;
33
   }
34
35
   ll dinic_dfs(int u, ll f) {
36
     if (u == dest) return f;
37
     for (int &i = work[u]; i < (int) G[u].size(); i++) {</pre>
38
       Edge &e = G[u][i];
39
       if (e.cap <= e.f) continue;</pre>
40
       int v = e.to;
41
       if (dist[v] == dist[u] + 1) {
42
         11 df = dinic_dfs(v, min(f, e.cap - e.f));
43
         if (df > 0) {
44
            e.f += df:
45
            G[v][e.rev].f -= df;
46
            return df;
47
         }
48
       }
49
     }
50
     return 0;
51
52
53
   11 maxFlow(int _src, int _dest) {//O(V^2 E)
54
     src = _src;
55
     dest = _dest;
56
     11 result = 0;
57
     while (dinic_bfs()) {
58
       fill(work, work + nodes, 0);
59
       while(ll delta = dinic_dfs(src, INF))
60
         result += delta;
61
     }
62
63
     // todos los nodos con dist[v]!=-1 vs los que tienen dist[v]==-1
64
          forman el min cut
65
     return result;
66
67 }
```

## 8.2. Konig

```
1 // asume que el dinic YA ESTA tirado
2 // asume que nodes-1 y nodes-2 son la fuente y destino
```

```
3 | int match[maxnodes]; // match[v]=u si u-v esta en el matching, -1 si v
       no esta matcheado
int s[maxnodes]; // numero de la bfs del koning
   queue<int> kq;
  // s[e] %2==1 o si e esta en V1 y s[e] ==-1-> lo agarras
   void koning() {//O(n)
     forn(v,nodes-2) s[v] = match[v] = -1;
     forn(v,nodes-2) forall(it,g[v]) if (it->to < nodes-2 && it->f>0)
       { match[v]=it->to; match[it->to]=v;}
     forn(v,nodes-2) if (match[v]==-1) \{s[v]=0; kq.push(v);\}
11
     while(!kq.empty()) {
12
       int e = kq.front(); kq.pop();
13
       if (s[e] %2==1) {
14
         s[match[e]] = s[e]+1;
         kq.push(match[e]);
      } else {
18
         forall(it,g[e]) if (it->to < nodes-2 && s[it->to]==-1) {
           s[it->to] = s[e]+1:
20
           kq.push(it->to);
21
22
24
```

# 8.3. Edmonds Karp's

```
1 #define MAX_V 1000
   #define INF 1e9
   //special nodes
   #define SRC 0
   #define SNK 1
   map<int, int> G[MAX_V];//limpiar esto
   //To add an edge use
   #define add(a, b, w) G[a][b]=w
   int f, p[MAX_V];
   void augment(int v, int minE){
     if(v==SRC) f=minE;
11
     else if(p[v]!=-1){
12
       augment(p[v], min(minE, G[p[v]][v]));
       G[p[v]][v]-=f, G[v][p[v]]+=f;
14
    }
15
16 | }
```

```
17 | 11 maxflow(){//O(VE^2)
     11 Mf=0;
18
     do{
19
       f=0;
20
       char used[MAX_V]; queue<int> q; q.push(SRC);
21
       zero(used), memset(p, -1, sizeof(p));
22
       while(sz(q)){
23
         int u=q.front(); q.pop();
24
         if(u==SNK) break;
25
         forall(it, G[u])
26
           if(it->snd>0 && !used[it->fst])
27
              used[it->fst]=true, q.push(it->fst), p[it->fst]=u;
28
       }
29
       augment(SNK, INF);
30
       Mf+=f:
31
     }while(f);
32
     return Mf;
33
34 | }
```

# 8.4. Push-Relabel O(N3)

```
1 #define MAX_V 1000
  int N://valid nodes are [0...N-1]
  #define INF 1e9
   //special nodes
  #define SRC 0
   #define SNK 1
  map<int, int> G[MAX_V];
   //To add an edge use
   #define add(a, b, w) G[a][b]=w
   11 excess[MAX_V];
   int height[MAX_V], active[MAX_V], count[2*MAX_V+1];
   queue<int> Q;void enqueue(int v) { if (!active[v] && excess[v] > 0)
       active[v]=true, Q.push(v); }
  void push(int a,int b){int amt=min(excess[a],11(G[a][b]));if(height[a]<=</pre>
      height[b]||amt==0)return;G[a][b]-=amt,G[b][a]+=amt;excess[b]+=amt,
       excess[a] -= amt; enqueue(b);}
void gap(int k){forn(v,N){if(height[v]<k) continue;count[height[v]]--;
       height[v]=max(height[v],N+1);count[height[v]]++;enqueue(v);}}
  void relabel(int v){count[height[v]]--;height[v]=2*N;forall(it,G[v])if(
      it->snd)height[v]=min(height[v],height[it->fst]+1);count[height[v]
      ]]++;enqueue(v);}
```

```
16 ll maxflow(){zero(height),zero(active),zero(count),zero(excess);count
        [0]=N-1;count[N]=1;height[SRC]=N;active[SRC]=active[SNK]=true;forall
        (it,G[SRC]){excess[SRC]+=it->snd;push(SRC,it->fst);}while(sz(Q)){int
        v=Q.front();Q.pop();active[v]=false;forall(it,G[v])push(v,it->fst);
        if(excess[v]>0)count[height[v]]==1?gap(height[v]):relabel(v);}ll mf
        =0;forall(it,G[SRC])mf+=G[it->fst][SRC];return mf;}
```

#### 8.5. Min-cost Max-flow

```
1 struct edge {
     int u, v;
     11 cap, cost, flow;
     ll rem() { return cap - flow; }
   };
5
   int n;//numero de nodos
   vector<int> G[MAXN];
   vector<edge> e;
   void addEdge(int u, int v, ll cap, ll cost) {
     G[u].pb(si(e)); e.pb((edge)\{u,v,cap,cost,0\});
     G[v].pb(si(e)); e.pb((edge)\{v,u,0,-cost,0\});
12
   11 pot[MAXN], dist[MAXN], pre[MAXN], cap[MAXN];
   11 mxFlow, mnCost;
   void flow(int s. int t) {
     fill(pot, pot+n, 0);
     mxFlow=mnCost=0;
17
     while(1){
18
       fill(dist, dist+n, INF); dist[s] = 0;
19
       fill(pre, pre+n, -1); pre[s]=0;
20
       fill(cap, cap+n, 0); cap[s] = INF;
21
       priority_queue<pair<ll,int> > q; q.push(mkp(0,s));
22
       while (!q.empty()) {
23
         pair<ll,int> top = q.top(); q.pop();
24
         int u = top.second, d = -top.first;
25
         if (u == t) break;
26
         if (d > dist[u]) continue;
27
         forn(i,si(G[u])) {
28
           edge E = e[G[u][i]];
29
           int c = E.cost + pot[u] - pot[E.v];
30
           if (E.rem() && dist[E.v] > dist[u] + c) {
31
             dist[E.v] = dist[u] + c;
32
             pre[E.v] = G[u][i];
33
             cap[E.v] = min(cap[u], E.rem());
34
```

```
q.push(mkp(-dist[E.v], E.v));
35
36
         }
37
       }
38
       if (pre[t] == -1) break;
39
       forn(u,n)
40
         if (dist[u] == INF) pot[u] = INF;
41
         else pot[u] += dist[u];
^{42}
       mxFlow +=cap[t];
43
       mnCost +=cap[t]*pot[t];
44
       for (int v = t; v != s; v = e[pre[v]].u) {
45
         e[pre[v]].flow += cap[t];
         e[pre[v]^1].flow -= cap[t];
47
48
     }
49
50
```

# 9. Ayudamemoria

### Leer hasta fin de linea

```
#include <sstream>
//hacer cin.ignore() antes de getline()

while(getline(cin, line)){
   istringstream is(line);
   while(is >> X)
        cout << X << """;
   cout << endl;
}</pre>
```

# Doubles Comp.

```
const double EPS = 1e-9;
x == y <=> fabs(x-y) < EPS
x > y <=> x > y + EPS
x >= y <=> x > y - EPS
```

#### Limites

```
#include <limits>
numeric_limits<T>::max() ::min() ::epsilon()
```

#### Muahaha

```
#include <signal.h>
void divzero(int p){
    while(true);}
  void segm(int p){
    exit(0);}
  //in main
signal(SIGFPE, divzero);
8 signal(SIGSEGV, segm);
Mejorar velocidad
ios::sync_with_stdio(0);
Mejorar velocidad 2
1 //Solo para enteros positivos
  inline void Scanf(int& a){
    char c = 0;
    while(c<33) c = getc(stdin);</pre>
    a = 0:
    while(c>33) a = a*10 + c - '0', c = getc(stdin);
Leer del teclado
freopen("/dev/tty", "a", stdin);
File setup
1 //tambien se pueden usar comas: {a, x, m, l}
touch {a..l}.in; tee {a..l}.cpp < template.cpp
```