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## 1. algorithm

```
#include <algorithm> #include <numeric>
```

Algo	Funcion
sort, stable_sort	ordena el intervalo
nth_element	void ordena el n-esimo, y particiona el resto
fill, fill_n	void llena [f, l) o [f, f+n) con elem
lower_bound, upper_bound	it al primer / ultimo donde se puede insertar elem para que quede ordenada
binary_search	bool esta elem en [f, l)
copy	hace resul+i=f+i $\forall i$
find, find_if, find_first_of	it encuentra i $\in [f, l)$ tq. i=elem, pred(i), i $\in [f2, l2)$
count, count_if	cuenta elem, pred(i)
search	busca [f2, l2) $\in [f, l)$
replace, replace_if	cambia old / pred(i) por new / pred, new
reverse	da vuelta
partition, stable_partition	pred(i) ad, !pred(i) atras
min_element, max_element	it min, max de [f, l)
lexicographical_compare	bool con [f1, l1]; [f2, l2]
next/prev_permutation	deja en [f, l) la perm sig, ant
set_intersection, set_difference, set_union, set_symmetric_difference,	[res, ...) la op. de conj
push_heap, pop_heap, make_heap	mete/saca e en heap [f, l), hace un heap de [f, l)
is_heap	bool es [f, l) un heap
accumulate	$T = \sum$ /oper de [f, l)
inner_product	$T = i + [f1, l1) \cdot [f2, \dots)$
partial_sum	$r+i = \sum$ /oper de [f, f+i] $\forall i \in [f, l)$
__builtin_ffs	Pos. del primer 1 desde la derecha
__builtin_clz	Cant. de ceros desde la izquierda.
__builtin_ctz	Cant. de ceros desde la derecha.
__builtin_popcount	Cant. de 1's en x.
__builtin_parity	1 si x es par, 0 si es impar.
__builtin_XXXXXXll	= pero para long's.

## 2. Estructuras

### 2.1. Easy segment

```
1 const int N = 1e5; // limit for size
2 int n; // array size
3 int t[2 * N];
4
5 void build() {
6     for (int i = n - 1; i > 0; --i)
7         t[i] = t[i<<1] + t[i<<1|1];
8 }
9
10 void modify(int p, int value) {
11     for (t[p += n] = value; p > 1; p >>= 1)
12         t[p>>1] = t[p] + t[p^1];
13 }
14
15 int query(int l, int r) {
16     int res = 0;
17     for (l += n, r += n; l < r; l >>= 1, r >>= 1) {
18         if (l&1) res += t[l++];
19         if (r&1) res += t[--r];
20     }
21     return res;
22 }
23
24 int main() {
25     scanf("%d", &n);
26     for (int i = 0; i < n; ++i)
27         scanf("%d", t + n + i);
28     build();
29     modify(0, 1);
30     printf("%d\n", query(3, 11));
31     return 0;
32 }
```

### 2.2. RMQ (static)

Dado un arreglo y una operacion asociativa *idempotente*,  $get(i, j)$  opera sobre el rango  $[i, j)$ . Restriccion:  $LVL \geq \text{ceil}(\log n)$ ; Usar  $[]$  para llenar arreglo y luego build().

```
1 struct RMQ{
2     #define LVL 10
3     tipo vec[LVL][1<<(LVL+1)];
4     tipo &operator[] (int p){return vec[0][p];}
5     tipo get(int i, int j) { //intervalo [i,j)
6         int p = 31-__builtin_clz(j-i);
7         return min(vec[p][i], vec[p][j-(1<<p)]);
8     }
9     void build(int n) { //O(nlogn)
10         int mp = 31-__builtin_clz(n);
11         forn(p, mp) forn(x, n-(1<<p))
12             vec[p+1][x] = min(vec[p][x], vec[p][x+(1<<p)]);
13     }
14 }
```

### 2.3. RMQ (dynamic)

```

1 //Dado un arreglo y una operacion asociativa con
  neutro, get(i, j) opera sobre el rango [i, j)
  .
2 #define MAXN 100000
3 #define operacion(x, y) max(x, y)
4 const int neutro=0;
5 struct RMQ{
6     int sz;
7     tipo t[4*MAXN];
8     tipo &operator[] (int p){return t[sz+p];}
9     void init(int n){//O(nlgn)
10         sz = 1 << (32-__builtin_clz(n));
11         forn(i, 2*sz) t[i]=neutro;
12     }
13     void updall(){//O(n)
14         dforn(i, sz) t[i]=operacion(t[2*i], t[2*i+1])
15             ;}
16     tipo get(int i, int j){return get(i,j,1,0,sz);}
17     tipo get(int i, int j, int n, int a, int b){//O
18         (lgn)
19         if(j<=a || i>=b) return neutro;
20         if(i<=a && b<=j) return t[n];
21         int c=(a+b)/2;
22         return operacion(get(i, j, 2*n, a, c), get(i,
23             j, 2*n+1, c, b));
24     }
25     void set(int p, tipo val){//O(lgn)
26         for(p+=sz; p>0 && t[p]!=val;){
27             t[p]=val;
28             p/=2;
29             val=operacion(t[p*2], t[p*2+1]);
30         }
31     }
32 }rmq;
33 //Usage:
34 cin >> n; rmq.init(n); forn(i, n) cin >> rmq[i];
35 rmq.updall();

```

## 2.4. RMQ (lazy)

```

1 //Dado un arreglo y una operacion asociativa con
  neutro, get(i, j) opera sobre el rango [i, j)
  .
2 typedef int Elem;//Elem de los elementos del
  arreglo
3 typedef int Alt;//Elem de la alteracion
4 #define operacion(x,y) x+y
5 const Elem neutro=0; const Alt neutro2=0;
6 #define MAXN 100000
7 struct RMQ{
8     int sz;
9     Elem t[4*MAXN];
10    Alt dirty[4*MAXN]; //las alteraciones pueden ser
11    de distinto Elem
12    Elem &operator[] (int p){return t[sz+p];}
13    void init(int n){//O(nlgn)
14        sz = 1 << (32-__builtin_clz(n));
15        forn(i, 2*sz) t[i]=neutro;
16        forn(i, 2*sz) dirty[i]=neutro2;
17    }

```

```

17 void push(int n, int a, int b){//propaga el
18     dirty a sus hijos
19     if(dirty[n]!=0){
20         t[n]+=dirty[n]*(b-a); //altera el nodo
21         if(n<sz){
22             dirty[2*n]+=dirty[n];
23             dirty[2*n+1]+=dirty[n];
24         }
25         dirty[n]=0;
26     }
27 Elem get(int i, int j, int n, int a, int b){//O
28     (lgn)
29     if(j<=a || i>=b) return neutro;
30     push(n, a, b); //corrige el valor antes de
31     usarlo
32     if(i<=a && b<=j) return t[n];
33     int c=(a+b)/2;
34     return operacion(get(i, j, 2*n, a, c), get(i,
35         j, 2*n+1, c, b));
36 }
37 Elem get(int i, int j){return get(i,j,1,0,sz);}
38 //altera los valores en [i, j) con una
39 alteracion de val
40 void alterar(Alt val, int i, int j, int n, int
41     a, int b){//O(lgn)
42     push(n, a, b);
43     if(j<=a || i>=b) return;
44     if(i<=a && b<=j){
45         dirty[n]+=val;
46         push(n, a, b);
47         return;
48     }
49     int c=(a+b)/2;
50     alterar(val, i, j, 2*n, a, c), alterar(val, i
51         , j, 2*n+1, c, b);
52     t[n]=operacion(t[2*n], t[2*n+1]); //por esto
53     es el push de arriba
54 }
55 void alterar(Alt val, int i, int j){alterar(val
56     ,i,j,1,0,sz);}
57 }rmq;

```

## 2.5. RMQ (persistente)

```

1 typedef int tipo;
2 tipo oper(const tipo &a, const tipo &b){
3     return a+b;
4 }
5 struct node{
6     tipo v; node *l,*r;
7     node(tipo v):v(v), l(NULL), r(NULL) {}
8     node(node *l, node *r) : l(l), r(r){
9         if(!l) v=r->v;
10        else if(!r) v=l->v;
11        else v=oper(l->v, r->v);
12    }
13 };
14 node *build (tipo *a, int tl, int tr) { //
15     modificar para que tome tipo a

```

```

15     if (tl+1==tr) return new node(a[tl]);
16     int tm=(tl + tr)>>1;
17     return new node(build(a, tl, tm), build(a, tm,
18         tr));
19 }
20 node *update(int pos, int new_val, node *t, int
21     tl, int tr){
22     if (tl+1==tr) return new node(new_val);
23     int tm=(tl+tr)>>1;
24     if(pos < tm) return new node(update(pos,
25         new_val, t->l, tl, tm), t->r);
26     else return new node(t->l, update(pos, new_val,
27         t->r, tm, tr));
28 }
29 tipo get(int l, int r, node *t, int tl, int tr){
30     if(l==tl && tr==r) return t->v;
31     int tm=(tl + tr)>>1;
32     if(r<=tm) return get(l, r, t->l, tl, tm);
33     else if(l>=tm) return get(l, r, t->r, tm, tr)
34         ;
35     return oper(get(l, tm, t->l, tl, tm), get(tm, r
36         , t->r, tm, tr));
37 }

```

## 2.6. Union Find

```

1 class UnionFind {
2 private:
3     vi p, rank, setSize;
4     int numSets;
5 public:
6     UnionFind(int N) {
7         setSize.assign(N, 1); numSets = N; rank.
8             assign(N, 0);
9         p.assign(N, 0); for (int i = 0; i < N; i++) p
10             [i] = i; }
11     int findSet(int i) { return (p[i] == i) ? i : (
12         p[i] = findSet(p[i])); }
13     bool isSameSet(int i, int j) { return findSet(i
14         ) == findSet(j); }
15     void unionSet(int i, int j) {
16         if (!isSameSet(i, j)) { numSets--;
17             int x = findSet(i), y = findSet(j);
18             // rank is used to keep the tree short
19             if (rank[x] > rank[y]) { p[y] = x; setSize[x]
20                 += setSize[y]; }
21             else { p[x] = y; setSize[y]
22                 += setSize[x];
23                 if (rank[x] == rank[
24                     y]) rank[y]++; }
25             }
26     }
27     int numDisjointSets() { return numSets; }
28     int sizeOfSet(int i) { return setSize[findSet(i
29         )]; }
30 };

```

## 2.7. Disjoint Intervals

```

1 bool operator< (const ii &a, const ii &b) {return
2     a.fst<b.fst;}
3 //Stores intervals as [first, second]

```

```

3 //in case of a collision it joins them in a
4     single interval
5 struct disjoint_intervals {
6     set<ii> segs;
7     void insert(ii v) {//O(lgn)
8         if(v.snd-v.fst==0.) return;//OJO
9         set<ii>::iterator it,at;
10         at = it = segs.lower_bound(v);
11         if (at!=segs.begin() && (--at)->snd >= v.fst)
12             v.fst = at->fst, --it;
13         for(; it!=segs.end() && it->fst <= v.snd;
14             segs.erase(it++))
15             v.snd=max(v.snd, it->snd);
16         segs.insert(v);
17     }
18 };

```

## 2.8. RMQ (2D)

```

1 struct RMQ2D{//n filas x m columnas
2     int sz;
3     RMQ t[4*MAXN];
4     RMQ &operator[] (int p){return t[sz/2+p];};//t[i
5         ][j]=i fila, j col
6     void init(int n, int m){//O(n*m)
7         sz = 1 << (32-__builtin_clz(n));
8         forn(i, 2*sz) t[i].init(m); }
9     void set(int i, int j, tipo val){//O(lgm.lgn)
10         for(i+=sz; i>0;){
11             t[i].set(j, val);
12             i/=2;
13             val=operacion(t[i*2][j], t[i*2+1][j]);
14         } }
15     tipo get(int i1, int j1, int i2, int j2){return
16         get(i1,j1,i2,j2,1,0,sz);}
17     //O(lgm.lgn), rangos cerrado abierto
18     int get(int i1, int j1, int i2, int j2, int n,
19         int a, int b){
20         if(i2<=a || i1>=b) return 0;
21         if(i1<=a && b<=i2) return t[n].get(j1, j2);
22         int c=(a+b)/2;
23         return operacion(get(i1, j1, i2, j2, 2*n, a,
24             c),
25             get(i1, j1, i2, j2, 2*n+1, c, b));
26     }
27 } rmq;
28 //Example to initialize a grid of M rows and N
29     columns:
30 RMQ2D rmq; rmq.init(n,m);
31 forn(i, n) forn(j, m){
32     int v; cin >> v; rmq.set(i, j, v);}

```

## 2.9. Treap para set

Treap para set tiene un Key unico por nodo. En el split if (key <= t->key). En at, if(key == t->key) return t; en lugar de pos.

```

1 void erase(pnode &t, Key key) {
2     if (!t) return; push(t);
3     if (key == t->key) t=merge(t->l, t->r);
4     else if (key < t->key) erase(t->l, key);

```

```

5     else erase(t->r, key);
6     if(t) pull(t);}

```

## 2.10. Treap para arreglo

```

1  typedef struct node *pnode;
2  struct node{
3      Value val, mini;
4      int dirty;
5      int prior, size;
6      pnode l,r,parent;
7      node(Value val): val(val), mini(val), dirty
          (0), prior(rand()), size(1), l(0), r(0),
          parent(0) {}
8  };
9  static int size(pnode p) { return p ? p->size :
10     0; }
11 void push(pnode p) { //propagar dirty a los hijos(
12     aca para lazy)
13     p->val.fst+=p->dirty;
14     p->mini.fst+=p->dirty;
15     if(p->l) p->l->dirty+=p->dirty;
16     if(p->r) p->r->dirty+=p->dirty;
17     p->dirty=0;
18 }
19 static Value mini(pnode p) { return p ? push(p),
20     p->mini : ii(1e9, -1); }
21 // Update function and size from children's Value
22 void pull(pnode p) { //recalcular valor del nodo
23     aca (para rmq)
24     p->size = 1 + size(p->l) + size(p->r);
25     p->mini = min(min(p->val, mini(p->l)), mini(p->
26     r)); //operacion del rmq!
27     p->parent=0;
28     if(p->l) p->l->parent=p;
29     if(p->r) p->r->parent=p;
30 }
31 //junta dos arreglos
32 pnode merge(pnode l, pnode r) {
33     if (!l || !r) return l ? l : r;
34     push(l), push(r);
35     pnode t;
36     if (l->prior < r->prior) l->r=merge(l->r, r), t
37         = l;
38     else r->l=merge(l, r->l), t = r;
39     pull(t);
40     return t;
41 }
42 //parte el arreglo en dos, sz(l)==tam
43 void split(pnode t, int tam, pnode &l, pnode &r)
44 {
45     if (!t) return void(l = r = 0);
46     push(t);
47     if (tam <= size(t->l)) split(t->l, tam, l, t->l
48         ), r = t;
49     else split(t->r, tam - 1 - size(t->l), t->r, r)
50         , l = t;
51     pull(t);
52 }

```

```

44 pnode at(pnode t, int pos) {
45     if(!t) exit(1);
46     push(t);
47     if(pos == size(t->l)) return t;
48     if(pos < size(t->l)) return at(t->l, pos);
49     return at(t->r, pos - 1 - size(t->l));
50 }
51 int getpos(pnode t){ //inversa de at
52     if(!t->parent) return size(t->l);
53     if(t==t->parent->l) return getpos(t->parent)-
54         size(t->r)-1;
55     return getpos(t->parent)+size(t->l)+1;
56 }
57 void split(pnode t, int i, int j, pnode &l, pnode
58     &m, pnode &r) {
59     split(t, i, l, t), split(t, j-i, m, r);}
60 Value get(pnode &p, int i, int j){ //like rmq
61     pnode l,m,r;
62     split(p, i, j, l, m, r);
63     Value ret=mini(m);
64     p=merge(l, merge(m, r));
65     return ret;
66 }
67 //Sample program: C. LCA Online from Petrozavodsk
68     Summer-2012. Petrozavodsk SU Contest
69 //Available at http://opentrains.snarknews.info/~
70     ejudge
71 const int MAXN=300100;
72 int n;
73 pnode beg[MAXN], fin[MAXN];
74 pnode lista;

```

## 2.11. Set con busq binaria

```

1  #include <ext/pb_ds/assoc_container.hpp>
2  #include <ext/pb_ds/tree_policy.hpp>
3  using namespace __gnu_pbds;
4  typedef tree<int,null_type,less<int>, //key,mapped
5      type, comparator
6      rb_tree_tag,tree_order_statistics_node_update
7      > set_t;
8  //find_by_order(i) devuelve iterador al i-esimo
9      elemento
10 //order_of_key(k): devuelve la pos del lower
11     bound de k

```

## 3. Algos

### 3.1. Longest Increasing Subsequence

```

1
2  typedef vector<int> VI;
3  typedef pair<int,int> PII;
4  typedef vector<PII> VPII;
5
6  #define STRICTLY_INCREASNG
7
8  VI LongestIncreasingSubsequence(VI v) {
9      VPII best;

```

```

10 VI dad(v.size(), -1);
11
12 for (int i = 0; i < v.size(); i++) {
13 #ifdef STRICTLY_INCREASNG
14     PII item = make_pair(v[i], 0);
15     VPII::iterator it = lower_bound(best.begin(),
16         best.end(), item);
17     item.second = i;
18 #else
19     PII item = make_pair(v[i], i);
20     VPII::iterator it = upper_bound(best.begin(),
21         best.end(), item);
22 #endif
23     if (it == best.end()) {
24         dad[i] = (best.size() == 0 ? -1 : best.back
25             ().second);
26         best.push_back(item);
27     } else {
28         dad[i] = it == best.begin() ? -1 : prev(it)
29             ->second;
30         *it = item;
31     }
32 }

```

### 3.2. Alpha-Beta pruning

```

1 ll alphabeta(State &s, bool player = true, int
2     depth = 1e9, ll alpha = -INF, ll beta = INF)
3     { //player = true -> Maximiza
4     if(s.isFinal()) return s.score;
5     //~ if (!depth) return s.heuristic();
6     vector<State> children;
7     s.expand(player, children);
8     int n = children.size();
9     forn(i, n) {
10         ll v = alphabeta(children[i], !player,
11             depth-1, alpha, beta);
12         if(!player) alpha = max(alpha, v);
13         else beta = min(beta, v);
14         if(beta <= alpha) break;
15     }
16     return !player ? alpha : beta;
17 }

```

### 3.3. Mo's algorithm

```

1 int n,sq;
2 struct Qu{ //queries [l, r]
3     //intervalos cerrado abiertos !!! importante
4     int l, r, id;
5 }qs[MAXN];
6 int ans[MAXN], curans; //ans[i]=ans to ith query
7 bool bymos(const Qu &a, const Qu &b){
8     if(a.l/sq!=b.l/sq) return a.l<b.l;
9     return (a.l/sq)&1? a.r<b.r : a.r>b.r;
10 }
11 void mos(){
12     forn(i, t) qs[i].id=i;
13     sort(qs, qs+t, bymos);
14     int cl=0, cr=0;
15     sq=sqrt(n);

```

```

16     curans=0;
17     forn(i, t){ //intervalos cerrado abiertos !!!
18         importante!!
19         Qu &q=qs[i];
20         while(cl>q.l) add(--cl);
21         while(cr<q.r) add(cr++);
22         while(cl<q.l) remove(cl++);
23         while(cr>q.r) remove(--cr);
24         ans[q.id]=curans;
25     }

```

### 3.4. Ternary search

```

1 #include <functional>
2 //Retorna argmax de una funcion unimodal 'f' en
3 //el rango [left,right]
4 double ternarySearch(double l, double r, function
5     <double(double)> f){
6     for(int i = 0; i < 300; i++){
7         double m1 = l+(r-l)/3, m2 = r-(r-l)/3;
8         if (f(m1) < f(m2)) l = m1; else r = m2;
9     }
10     return (left + right)/2;

```

## 4. Strings

### 4.1. Manacher

```

1 int d1[MAXN]; //d1[i]=long del maximo palindromo
2 //impar con centro en i
3 int d2[MAXN]; //d2[i]=analogo pero para longitud
4 //par
5 //0 1 2 3 4
6 //a a b c c <--d1[2]=3
7 //a a b b <--d2[2]=2 (están uno antes)
8 void manacher(){
9     int l=0, r=-1, n=sz(s);
10     forn(i, n){
11         int k=(i>r? 1 : min(d1[l+r-i], r-i));
12         while(i+k<n && i-k>=0 && s[i+k]==s[i-k]) ++k;
13         d1[i] = k--;
14         if(i+k > r) l=i-k, r=i+k;
15     }
16     l=0, r=-1;
17     forn(i, n){
18         int k=(i>r? 0 : min(d2[l+r-i+1], r-i+1))+1;
19         while(i+k-1<n && i-k>=0 && s[i+k-1]==s[i-k])
20             ++k;
21         d2[i] = --k;
22         if(i+k-1 > r) l=i-k, r=i+k-1;
23     }

```

### 4.2. KMP

```

1 string T; //cadena donde buscar(what)
2 string P; //cadena a buscar(what)
3 int b[MAXLEN]; //back table b[i] maximo borde de
4 // [0..i)
5 void kmppre(){ //by gabina with love

```

```

5     int i=0, j=-1; b[0]=-1;
6     while(i<sz(P)){
7         while(j>=0 && P[i] != P[j]) j=b[j];
8         i++, j++, b[i] = j;
9     }
10 }
11 void kmp(){
12     int i=0, j=0;
13     while(i<sz(T)){
14         while(j>=0 && T[i]!=P[j]) j=b[j];
15         i++, j++;
16         if(j==sz(P)) printf("P_is_found_at_index_
17                             %d_in_T\n", i-j), j=b[j];
18     }
19 }
20 int main(){
21     cout << "T=";
22     cin >> T;
23     cout << "P=";

```

### 4.3. Trie

```

1 struct trie{ map<char, trie> m;
2     void add(const string &s, int p=0){ if(s[p]) m[
3         s[p]].add(s, p+1);}
4     void dfs(){/*Do stuff*/ forall(it, m) it->
5         second.dfs();}};

```

### 4.4. Suffix Array (largo, nlogn)

```

1 #define MAX_N 1000
2 #define rBOUND(x) (x<n? r[x] : 0)
3 //sa will hold the suffixes in order.
4 int sa[MAX_N], r[MAX_N], n;
5 string s; //input string, n=sz(s)
6
7 int f[MAX_N], tmpsa[MAX_N];
8 void countingSort(int k){
9     zero(f);
10    forn(i, n) f[rBOUND(i+k)]++;
11    int sum=0;
12    forn(i, max(255, n)){
13        int t=f[i]; f[i]=sum; sum+=t;}
14    forn(i, n)
15        tmpsa[f[rBOUND(sa[i]+k)]++] = sa[i];
16    memcpy(sa, tmpsa, sizeof(sa));
17 }
18 void constructsa(){//O(n log n)
19     n=sz(s);
20     forn(i, n) sa[i]=i, r[i]=s[i];
21     for(int k=1; k<n; k<=1){
22         countingSort(k), countingSort(0);
23         int rank, tmpr[MAX_N];
24         tmpr[sa[0]]=rank=0;
25         forr(i, 1, n)
26             tmpr[sa[i]]=(r[sa[i]]==r[sa[i-1]] && r[sa[i]
27                 +k]==r[sa[i-1]+k])? rank : ++rank;
28         memcpy(r, tmpr, sizeof(r));
29         if(r[sa[n-1]]==n-1) break;
30     }

```

```

30 }
31 //returns (lowerbound, upperbound) of the search
32 ii stringMatching(string P){ //O(sz(P)lgn)
33     int lo=0, hi=n-1, mid=lo;
34     while(lo<hi){

```

### 4.5. String Matching With Suffix Array

```

1 //returns (lowerbound, upperbound) of the search
2 ii stringMatching(string P){ //O(sz(P)lgn)
3     int lo=0, hi=n-1, mid=lo;
4     while(lo<hi){
5         mid=(lo+hi)/2;
6         int res=s.compare(sa[mid], sz(P), P);
7         if(res>=0) hi=mid;
8         else lo=mid+1;
9     }
10    if(s.compare(sa[lo], sz(P), P)!=0) return ii
11        (-1, -1);
12    ii ans; ans.fst=lo;
13    lo=0, hi=n-1, mid;
14    while(lo<hi){
15        mid=(lo+hi)/2;
16        int res=s.compare(sa[mid], sz(P), P);
17        if(res>0) hi=mid;
18        else lo=mid+1;
19    }
20    if(s.compare(sa[hi], sz(P), P)!=0) hi--;
21    ans.snd=hi;
22    return ans;

```

### 4.6. LCP (Longest Common Prefix)

```

1 //Calculates the LCP between consecutives
2 //suffixes in the Suffix Array.
3 //LCP[i] is the length of the LCP between sa[i]
4 //and sa[i-1]
5 int LCP[MAX_N], phi[MAX_N], PLCP[MAX_N];
6 void computeLCP(){//O(n)
7     phi[sa[0]]=-1;
8     forr(i, 1, n) phi[sa[i]]=sa[i-1];
9     int L=0;
10    forn(i, n){
11        if(phi[i]==-1) {PLCP[i]=0; continue;}
12        while(s[i+L]==s[phi[i]+L]) L++;
13        PLCP[i]=L;
14        L=max(L-1, 0);
15    }
16    forn(i, n) LCP[i]=PLCP[sa[i]];

```

### 4.7. Corasick

```

1 struct trie{
2     map<char, trie> next;
3     trie* tran[256]; //transiciones del automata
4     int idhoja, szhoja; //id de la hoja o 0 si no lo
5     es

```



```

6 //link lleva al sufijo mas largo, nxthoja lleva
  al mas largo pero que es hoja
7 trie *padre, *link, *nxthoja;
8 char pch;//caracter que conecta con padre
9 trie(): tran(), idhoja(), padre(), link() {}
10 void insert(const string &s, int id=1, int p=0)
    { //id>0!!!
11     if(p<sz(s)){
12         trie &ch=next[s[p]];
13         tran[(int)s[p]]=&ch;
14         ch.padre=this, ch.pch=s[p];
15         ch.insert(s, id, p+1);
16     }
17     else idhoja=id, szhoja=sz(s);
18 }
19 trie* get_link() {
20     if(!link){
21         if(!padre) link=this;//es la raiz
22         else if(!padre->padre) link=padre;//hijo de
           la raiz
23         else link=padre->get_link()->get_tran(pch);
24     }
25     return link; }
26 trie* get_tran(int c) {
27     if(!tran[c]) tran[c] = !padre? this : this->
        get_link()->get_tran(c);
28     return tran[c]; }
29 trie *get_nxthoja(){
30     if(!nxthoja) nxthoja = get_link()->idhoja?
        link : link->nxthoja;
31     return nxthoja; }
32 void print(int p){
33     if(idhoja) cout << "found_" << idhoja << "
        at_position_" << p-szhoja << endl;
34     if(get_nxthoja()) get_nxthoja()->print(p); }
35 void matching(const string &s, int p=0){
36     print(p); if(p<sz(s)) get_tran(s[p])->
        matching(s, p+1); }
37 }tri;
38
39
40 int main(){
41     tri=trie();//clear
42     tri.insert("ho", 1);
43     tri.insert("hoho", 2);

```

#### 4.8. Suffix Automaton

```

1 struct state {
2     int len, link;
3     map<char,int> next;
4     state() { }
5 };
6 const int MAXLEN = 10010;
7 state st[MAXLEN*2];
8 int sz, last;
9 void sa_init() {
10     forn(i,sz) st[i].next.clear();
11     sz = last = 0;
12     st[0].len = 0;

```

```

13     st[0].link = -1;
14     ++sz;
15 }
16 // Es un DAG de una sola fuente y una sola hoja
17 // cantidad de endpos = cantidad de apariciones =
    cantidad de caminos de la clase al nodo
    terminal
18 // cantidad de miembros de la clase = st[v].len-
    st[st[v].link].len (v>0) = caminos del inicio
    a la clase
19 // El arbol de los suffix links es el suffix tree
    de la cadena invertida. La string de la
    arista link(v)->v son los caracteres que
    difieren
20 void sa_extend (char c) {
21     int cur = sz++;
22     st[cur].len = st[last].len + 1;
23     // en cur agregamos la posicion que estamos
    extendiendo
24     //podria agregar tambien un identificador de
    las cadenas a las cuales pertenece (si hay
    varias)
25     int p;
26     for (p=last; p!=-1 && !st[p].next.count(c); p=
        st[p].link) // modificar esta linea para
        hacer separadores unicos entre varias
        cadenas (c=='$')
27         st[p].next[c] = cur;
28     if (p == -1)
29         st[cur].link = 0;
30     else {
31         int q = st[p].next[c];
32         if (st[p].len + 1 == st[q].len)
33             st[cur].link = q;
34         else {
35             int clone = sz++;
36             // no le ponemos la posicion actual a clone
            sino indirectamente por el link de cur
37             st[clone].len = st[p].len + 1;
38             st[clone].next = st[q].next;
39             st[clone].link = st[q].link;
40             for (; p!=-1 && st[p].next.count(c) && st[p]
                .next[c]==q; p=st[p].link)
41                 st[p].next[c] = clone;
42             st[q].link = st[cur].link = clone;
43         }
44     }
45     last = cur;
46 }

```

#### 4.9. Z Function

```

1 char s[MAXN];
2 int z[MAXN]; // z[i] = i==0 ? 0 : max k tq s[0,k)
    match with s[i,i+k)
3 void z_function(char s[],int z[]) {
4     int n = strlen(s);
5     forn(i, n) z[i]=0;
6     for (int i = 1, l = 0, r = 0; i < n; ++i) {
7         if (i <= r) z[i] = min (r - i + 1, z[i -

```



```

    1]);
    8 while (i + z[i] < n && s[z[i]] == s[i + z
      [i]]) ++z[i];
    9 if (i + z[i] - 1 > r) l = i, r = i + z[i]
      - 1;
    10 }
    11 }
    12
    13 int main() {
    14     ios::sync_with_stdio(0);

```

#### 4.10. Palindromic tree

```

    1 const int maxn = 10100100;
    2
    3 int len[maxn];
    4 int suffLink[maxn];
    5 int to[maxn][2];
    6 int cnt[maxn];
    7 int numV;
    8 char str[maxn];
    9
    10 int v;
    11
    12 void addLetter(int n) {
    13     while (str[n - len[v] - 1] != str[n] )
    14         v = suffLink[v];
    15     int u = suffLink[v];
    16     while (str[n - len[u] - 1] != str[n] )
    17         u = suffLink[u];
    18     int u_ = to[u][str[n] - 'a'];
    19     int v_ = to[v][str[n] - 'a'];
    20     if (v_ == -1) {
    21         v_ = to[v][str[n] - 'a'] = numV;
    22         len[numV++] = len[v] + 2;
    23         suffLink[v_] = u_;
    24     }
    25     v = v_;
    26     cnt[v]++;
    27 }
    28
    29 void init() {
    30     memset(to, -1, sizeof to);
    31     str[0] = '#'; len[0] = -1;
    32     len[1] = 0; len[2] = len[3] = 1;
    33     suffLink[0] = suffLink[1] = 0;
    34     suffLink[2] = suffLink[3] = 1;
    35     to[0][0] = 2;
    36     to[0][1] = 3;
    37     numV = 4;
    38 }
    39
    40 int main() {
    41     init();
    42     scanf("%s", str + 1);
    43     int n = strlen(str);
    44     for (int i = 1; i < n; i++) addLetter(i);
    45     long long ans = 0;
    46     for (int i = numV - 1; i > 0; i--)
        {

```

```

    47         cnt[suffLink[i] ] += cnt[i];
    48         ans = max(ans, cnt[i] * 1LL * len
      [i] );
    49 //         fprintf(stderr, "i = %d, cnt = %d
      , len = %d\n", i, cnt[i], len[i] );
    50     }
    51     printf("%lld\n", ans);
    52     return 0;
    53 }

```

#### 4.11. Rabin Karp - Distinct Substrings

```

    1 int count_unique_substrings(string const& s) {
    2     int n = s.size();
    3     //ll p = 257, M = 1000000007, inv = 70038911;
    4     //pow(257,10**9+7-2,10**9+7)
    5     const int p = 31;
    6     const int m = 1e9 + 9;
    7     vector<long long> p_pow(n);
    8     p_pow[0] = 1;
    9     for (int i = 1; i < n; i++)
    10         p_pow[i] = (p_pow[i-1] * p) % m;
    11
    12     vector<long long> h(n + 1, 0);
    13     for (int i = 0; i < n; i++)
    14         h[i+1] = (h[i] + (s[i] - 'a' + 1) * p_pow
      [i]) % m;
    15
    16     int cnt = 0;
    17     for (int l = 1; l <= n; l++) {
    18         set<long long> hs;
    19         for (int i = 0; i <= n - l; i++) {
    20             long long cur_h = (h[i + l] + m - h[i]
      ]) % m;
    21             cur_h = (cur_h * p_pow[n-i-1]) % m;
    22             hs.insert(cur_h);
    23         }
    24         cnt += hs.size();
    25     }
    26     return cnt;
    27 }

```

### 5. Geometria

#### 5.1. Punto

```

    1 struct pto{
    2     double x, y;
    3     pto(double x=0, double y=0):x(x),y(y){}
    4     pto operator+(pto a){return pto(x+a.x, y+a.y);}
    5     pto operator-(pto a){return pto(x-a.x, y-a.y);}
    6     pto operator+(double a){return pto(x+a, y+a);}
    7     pto operator*(double a){return pto(x*a, y*a);}
    8     pto operator/(double a){return pto(x/a, y/a);}
    9     //dot product, producto interno:
    10     double operator*(pto a){return x*a.x+y*a.y;}
    11     //module of the cross product or vectorial
      product:
    12     //if a is less than 180 clockwise from b, a^b>0
    13     double operator^(pto a){return x*a.y-y*a.x;}

```

```

14 //returns true if this is at the left side of
    line qr
15 bool left(pto q, pto r){return ((q-*this)^(r-*
    this))>0;}
16 bool operator<(const pto &a) const{return x<a.x
    -EPS || (abs(x-a.x)<EPS && y<a.y-EPS);}
17 bool operator==(pto a){return abs(x-a.x)<EPS &&
    abs(y-a.y)<EPS;}
18 double norm(){return sqrt(x*x+y*y);}
19 double norm_sq(){return x*x+y*y;}
20 };
21 double dist(pto a, pto b){return (b-a).norm();}
22 typedef pto vec;
23
24 double angle(pto a, pto o, pto b){
25     pto oa=a-o, ob=b-o;
26     return atan2(oa^ob, oa*ob);}
27
28 //rotate p by theta rads CCW w.r.t. origin (0,0)
29 pto rotate(pto p, double theta){
30     return pto(p.x*cos(theta)-p.y*sin(theta),
31         p.x*sin(theta)+p.y*cos(theta));
32 }

```

## 5.2. Orden radial de puntos

```

1 struct Cmp{//orden total de puntos alrededor de
    un punto r
2     pto r;
3     Cmp(pto r):r(r) {}
4     int cuad(const pto &a) const{
5         if(a.x > 0 && a.y >= 0)return 0;
6         if(a.x <= 0 && a.y > 0)return 1;
7         if(a.x < 0 && a.y <= 0)return 2;
8         if(a.x >= 0 && a.y < 0)return 3;
9         assert(a.x ==0 && a.y==0);
10        return -1;
11    }
12    bool cmp(const pto&p1, const pto&p2)const{
13        int c1 = cuad(p1), c2 = cuad(p2);
14        if(c1==c2) return p1.y*p2.x<p1.x*p2.y;
15        else return c1 < c2;
16    }
17    bool operator()(const pto&p1, const pto&p2)
        const{
18        return cmp(pto(p1.x-r.x,p1.y-r.y),pto(p2.x-r.
            x,p2.y-r.y));
19    }
20 };

```

## 5.3. Line

```

1 int sgn(ll x){return x<0? -1 : !!x;}
2 struct line{
3     line() {}
4     double a,b,c;//Ax+By=C
5     //pto MUST store float coordinates!
6     line(double a, double b, double c):a(a),b(b),c(
        c){}
7     line(pto p, pto q): a(q.y-p.y), b(p.x-q.x), c(a
        *p.x+b*p.y) {}

```

```

8     int side(pto p){return sgn(l1(a) * p.x + l1(b)
        * p.y - c);}
9 };
10 bool parallels(line l1, line l2){return abs(l1.a*
    l2.b-l2.a*l1.b)<EPS;}
11 pto inter(line l1, line l2){//intersection
12     double det=l1.a*l2.b-l2.a*l1.b;
13     if(abs(det)<EPS) return pto(INF, INF);//
        parallels
14     return pto(l2.b*l1.c-l1.b*l2.c, l1.a*l2.c-l2.a*
        l1.c)/det;
15 }

```

## 5.4. Segment

```

1 struct segm{
2     pto s,f;
3     segm(pto s, pto f):s(s), f(f) {}
4     pto closest(pto p) {//use for dist to point
5         double l2 = dist_sq(s, f);
6         if(l2==0.) return s;
7         double t=((p-s)*(f-s))/l2;
8         if (t<0.) return s;//not write if is a line
9         else if(t>1.)return f;//not write if is a
            line
10        return s+((f-s)*t);
11    }
12    bool inside(pto p){return abs(dist(s, p)+dist
        (p, f)-dist(s, f))<EPS;}
13 };
14
15 pto inter(segm s1, segm s2){
16     pto r=inter(line(s1.s, s1.f), line(s2.s, s2.f))
17     ;
18     if(s1.inside(r) && s2.inside(r)) return r;
19     return pto(INF, INF);

```

## 5.5. Polygon Area

```

1 double area(vector<pto> &p){//0(sz(p))
2     double area=0;
3     forn(i, sz(p)) area+=p[i]^p[(i+1)%sz(p)];
4     //if points are in clockwise order then area is
        negative
5     return abs(area)/2;
6 }
7 //Area ellipse = M_PI*a*b where a and b are the
    semi axis lengths
8 //Area triangle = sqrt(s*(s-a)(s-b)(s-c)) where s
    =(a+b+c)/2

```

## 5.6. Circle

```

1 vec perp(vec v){return vec(-v.y, v.x);}
2 line bisector(pto x, pto y){
3     line l=line(x, y); pto m=(x+y)/2;
4     return line(-l.b, l.a, -l.b*m.x+l.a*m.y);
5 }
6 struct Circle{
7     pto o;

```

```

8   double r;
9   Circle(pto x, pto y, pto z){
10       o=inter(bisector(x, y), bisector(y, z));
11       r=dist(o, x);
12   }
13   pair<pto, pto> ptosTang(pto p){
14       pto m=(p+o)/2;
15       tipo d=dist(o, m);
16       tipo a=r*r/(2*d);
17       tipo h=sqrt(r*r-a*a);
18       pto m2=o+(m-o)*a/d;
19       vec per=perp(m-o)/d;
20       return make_pair(m2-per*h, m2+per*h);
21   }
22 };
23 //finds the center of the circle containing p1
24 //and p2 with radius r
25 //as there may be two solutions swap p1, p2 to
26 //get the other
27 bool circle2PtsRad(pto p1, pto p2, double r, pto
28 &c){
29     double d2=(p1-p2).norm_sq(), det=r*r/d2
30     -0.25;
31     if(det<0) return false;
32     c=(p1+p2)/2+perp(p2-p1)*sqrt(det);
33     return true;
34 }
35 #define sqr(a) ((a)*(a))
36 #define feq(a,b) (fabs((a)-(b))<EPS)
37 pair<tipo, tipo> ecCuad(tipo a, tipo b, tipo c){
38     //a*x*x+b*x+c=0
39     tipo dx = sqrt(b*b-4.0*a*c);
40     return make_pair((-b + dx)/(2.0*a), (-b - dx)
41     /(2.0*a));
42 }
43 pair<pto, pto> interCL(Circle c, line l){
44     bool sw=false;
45     if((sw=feq(0,l.b))){
46         swap(l.a, l.b);
47         swap(c.o.x, c.o.y);
48     }
49     pair<tipo, tipo> rc = ecCuad(
50     sqr(l.a)+sqr(l.b),
51     2.0*l.a*l.b*c.o.y-2.0*(sqr(l.b)*c.o.x+l.c*l.a),
52     sqr(l.b)*(sqr(c.o.x)+sqr(c.o.y)-sqr(c.r))+sqr(l
53     .c)-2.0*l.c*l.b*c.o.y
54 );
55     pair<pto, pto> p( pto(rc.first, (l.c - l.a * rc
56     .first) / l.b),
57     pto(rc.second, (l.c - l.a * rc.second
58     ) / l.b) );
59     if(sw){
60         swap(p.first.x, p.first.y);
61         swap(p.second.x, p.second.y);
62     }
63     return p;
64 }
65 pair<pto, pto> interCC(Circle c1, Circle c2){
66     line l;
67     l.a = c1.o.x-c2.o.x;

```

```

59     l.b = c1.o.y-c2.o.y;
60     l.c = (sqr(c2.r)-sqr(c1.r)+sqr(c1.o.x)-sqr(c2.o
61     .x)+sqr(c1.o.y
62     -sqr(c2.o.y))/2.0;
63     return interCL(c1, l);
64 }

```

## 5.7. Point in Poly

```

1 //checks if v is inside of P, using ray casting
2 //works with convex and concave.
3 //excludes boundaries, handle it separately using
4 segment.inside()
5 bool inPolygon(pto v, vector<pto>& P) {
6     bool c = false;
7     forn(i, sz(P)){
8         int j=(i+1)%sz(P);
9         if((P[j].y>v.y) != (P[i].y > v.y) &&
10         (v.x < (P[i].x - P[j].x) * (v.y-P[j].y) / (P[i
11         ].y - P[j].y) + P[j].x))
12             c = !c;
13     }
14     return c;
15 }

```

## 5.8. Point in Convex Poly log(n)

```

1 void normalize(vector<pto> &pt){//delete
2     collinear points first!
3     //this makes it clockwise:
4     if(pt[2].left(pt[0], pt[1])) reverse(pt.begin
5     (), pt.end());
6     int n=sz(pt), pi=0;
7     forn(i, n)
8         if(pt[i].x<pt[pi].x || (pt[i].x==pt[pi].x &&
9         pt[i].y<pt[pi].y))
10             pi=i;
11     vector<pto> shift(n);//puts pi as first point
12     forn(i, n) shift[i]=pt[(pi+i)%n];
13     pt.swap(shift);
14 }
15 bool inPolygon(pto p, const vector<pto> &pt){
16     //call normalize first!
17     if(p.left(pt[0], pt[1]) || p.left(pt[sz(pt)-1],
18     pt[0])) return false;
19     int a=1, b=sz(pt)-1;
20     while(b-a>1){
21         int c=(a+b)/2;
22         if(!p.left(pt[0], pt[c])) a=c;
23         else b=c;
24     }
25     return !p.left(pt[a], pt[a+1]);
26 }

```

## 5.9. Convex Check CHECK

```

1 bool isConvex(vector<int> &p){//O(N), delete
2     collinear points!
3     int N=sz(p);
4     if(N<3) return false;
5     bool isLeft=p[0].left(p[1], p[2]);

```

```

5   forr(i, 1, N)
6       if(p[i].left(p[(i+1)%N], p[(i+2)%N])!=isLeft)
7           return false;
8   return true; }

```

## 5.10. Convex Hull

```

1   //stores convex hull of P in S, CCW order
2   //left must return >=0 to delete collinear points
3   !
4   void CH(vector<pto>& P, vector<pto> &S){
5       S.clear();
6       sort(P.begin(), P.end()); //first x, then y
7       forn(i, sz(P)){ //lower hull
8           while(sz(S)>= 2 && S[sz(S)-1].left(S[sz(S)
9               -2], P[i])) S.pop_back();
10          S.pb(P[i]);
11      }
12      S.pop_back();
13      int k=sz(S);
14      dforn(i, sz(P)){ //upper hull
15          while(sz(S) >= k+2 && S[sz(S)-1].left(S[sz(S)
16              -2], P[i])) S.pop_back();
17          S.pb(P[i]);
18      }
19      S.pop_back();
20  }

```

## 5.11. Cut Polygon

```

1   //cuts polygon Q along the line ab
2   //stores the left side (swap a, b for the right
3   one) in P
4   void cutPolygon(pto a, pto b, vector<pto> Q,
5       vector<pto> &P){
6       P.clear();
7       forn(i, sz(Q)){
8           double left1=(b-a)^(Q[i]-a), left2=(b-a)^(Q[
9               i+1]%sz(Q)]-a);
10          if(left1>=0) P.pb(Q[i]);
11          if(left1*left2<0)
12              P.pb(inter(line(Q[i], Q[(i+1)%sz(Q)]), line
13                  (a, b)));
14      }
15  }

```

## 5.12. Bresenham

```

1   //plot a line approximation in a 2d map
2   void bresenham(pto a, pto b){
3       pto d=b-a; d.x=abs(d.x), d.y=abs(d.y);
4       pto s(a.x<b.x? 1: -1, a.y<b.y? 1: -1);
5       int err=d.x-d.y;
6       while(1){
7           m[a.x][a.y]=1; //plot
8           if(a==b) break;
9           int e2=err;
10          if(e2 >= 0) err-=2*d.y, a.x+=s.x;
11          if(e2 <= 0) err+= 2*d.x, a.y+= s.y;
12      }
13  }

```

## 5.13. Interseccion de Circulos en $n^3 \log(n)$

```

1   struct event {
2       double x; int t;
3       event(double xx, int tt) : x(xx), t(tt) {}
4       bool operator <(const event &o) const {
5           return x < o.x; }
6   };
7   typedef vector<Circle> VC;
8   typedef vector<event> VE;
9   int n;
10  double cuenta(VE &v, double A, double B) {
11      sort(v.begin(), v.end());
12      double res = 0.0, lx = ((v.empty())?0.0:v[0].
13          x);
14      int contador = 0;
15      forn(i, sz(v)) {
16          //interseccion de todos (contador == n),
17          union de todos (contador > 0)
18          //conjunto de puntos cubierto por exacta
19          k Circulos (contador == k)
20          if (contador == n) res += v[i].x - lx;
21          contador += v[i].t, lx = v[i].x;
22      }
23      return res;
24  }
25  // Primitiva de sqrt(r*r - x*x) como funcion
26  double de una variable x.
27  inline double primitiva(double x, double r) {
28      if (x >= r) return r*r*M_PI/4.0;
29      if (x <= -r) return -r*r*M_PI/4.0;
30      double raiz = sqrt(r*r-x*x);
31      return 0.5 * (x * raiz + r*r*atan(x/raiz));
32  }
33  double interCircle(VC &v) {
34      vector<double> p; p.reserve(v.size() * (v.
35          size() + 2));
36      forn(i, sz(v)) p.push_back(v[i].c.x + v[i].r)
37          , p.push_back(v[i].c.x - v[i].r);
38      forn(i, sz(v)) forn(j, i) {
39          Circle &a = v[i], b = v[j];
40          double d = (a.c - b.c).norm();
41          if (fabs(a.r - b.r) < d && d < a.r + b.r)
42              {
43                  double alfa = acos((sqr(a.r) + sqr(d)
44                      - sqr(b.r)) / (2.0 * d * a.r));
45                  pto vec = (b.c - a.c) * (a.r / d);
46                  p.pb((a.c + rotate(vec, alfa)).x), p.
47                      pb((a.c + rotate(vec, -alfa)).x);
48              }
49      }
50      sort(p.begin(), p.end());
51      double res = 0.0;
52      forn(i, sz(p)-1) {
53          const double A = p[i], B = p[i+1];
54          VE ve; ve.reserve(2 * v.size());
55          forn(j, sz(v)) {
56              const Circle &c = v[j];
57              double arco = primitiva(B-c.c.x, c.r)
58                  - primitiva(A-c.c.x, c.r);

```

```

48         double base = c.c.y * (B-A);
49         ve.push_back(event(base + arco,-1));
50         ve.push_back(event(base - arco, 1));
51     }
52     res += cuenta(ve,A,B);
53 }
54 return res;
55 }

```

## 6. Math

### 6.1. Identidades

$\sum_{i=0}^n i \binom{n}{i} = n * 2^{n-1}$   
 $\binom{n}{k} = \frac{n}{k} \binom{n-1}{k-1}$   
 $\sum_{i=0}^k \binom{a}{i} \binom{b}{k-i} = \binom{a+b}{k}$   
 $\sum_{i=0}^m \binom{n+i}{i} = \binom{n+m+1}{n+1}$   
 $\sum_{i=0}^n i(i-1) = \frac{8}{6} \left(\frac{n}{2}\right) \left(\frac{n}{2}+1\right) (n+1)$  (doubles)  $\rightarrow$  Sino ver caso impar y par

$$\sum_{i=0}^n i^4 = \frac{n(n+1)(2n+1)(3n^2+3n-1)}{30} = \frac{n^5}{5} + \frac{n^4}{2} + \frac{n^3}{3} - \frac{n}{30}$$

$$\sum_{i=0}^n i^p = \frac{(n+1)^{p+1}}{p+1} + \sum_{k=1}^p \frac{B_k}{p-k+1} \binom{p}{k} (n+1)^{p-k+1}$$

$$\sum_{i=0}^n i^p = \frac{(n+1)^{p+1}}{p+1} + \sum_{k=1}^p \frac{B_k}{p-k+1} \binom{p}{k} (n+1)^{p-k+1}$$

Caras + Vertices = Aristas + 2 (Poliedros convexos y Grafos planos)

Teorema de Pick: (Area, puntos int. y puntos en el borde)

$$A = I + \frac{B}{2} - 1$$

$\left\{ \begin{smallmatrix} n+1 \\ k \end{smallmatrix} \right\} = k \left\{ \begin{smallmatrix} n \\ k \end{smallmatrix} \right\} + \left\{ \begin{smallmatrix} n \\ k-1 \end{smallmatrix} \right\}$  for  $k > 0$  with initial conditions  
 $\left\{ \begin{smallmatrix} 0 \\ 0 \end{smallmatrix} \right\} = 1$  and  $\left\{ \begin{smallmatrix} n \\ 0 \end{smallmatrix} \right\} = \left\{ \begin{smallmatrix} 0 \\ n \end{smallmatrix} \right\} = 0$  for  $n > 0$ . Same as  
 $\frac{1}{k!} \sum_{j=0}^k (-1)^{k-j} \binom{k}{j} j^n$

$\left[ \begin{smallmatrix} n+1 \\ k \end{smallmatrix} \right] = n \left[ \begin{smallmatrix} n \\ k \end{smallmatrix} \right] + \left[ \begin{smallmatrix} n \\ k-1 \end{smallmatrix} \right]$  for  $k > 0$ , with the initial conditions  
 $\left[ \begin{smallmatrix} 0 \\ 0 \end{smallmatrix} \right] = 1$  and  $\left[ \begin{smallmatrix} n \\ 0 \end{smallmatrix} \right] = \left[ \begin{smallmatrix} 0 \\ n \end{smallmatrix} \right] = 0$  for  $n > 0$ .

Markov Chain:  $\begin{pmatrix} Q & R \\ 0 & I_r \end{pmatrix}$   $Q$  transient st.,  $R$  absorb. st.

Fundamental Matrix:  $N = \sum_k Q^k = (I_t - Q)^{-1}$ .

Steps before absorption:  $t = N \cdot 1$

Absorption probability:  $B = N \cdot R$

Prob. visiting transient state:  $H = (N - I_t) N_{dg}^{-1}$

Usage Example: GaussJordan((I-Q),b), where b is a dummy column of 0's. Each row  $i$  is as if they started at state  $i$ .

Laplacian Matrix (remove one row and one column to get number of spanning trees with determinant):

$$L_{i,j} := \begin{cases} \deg(v_i) & \text{if } i = j \\ -1 & \text{if } i \neq j \text{ and } v_i \text{ is adjacent to } v_j \\ 0 & \text{otherwise} \end{cases}$$

### 6.2. Ec. Caracteristica

$$a_0 T(n) + a_1 T(n-1) + \dots + a_k T(n-k) = 0$$

$$p(x) = a_0 x^k + a_1 x^{k-1} + \dots + a_k$$

Sean  $r_1, r_2, \dots, r_q$  las raíces distintas, de mult.  $m_1, m_2, \dots, m_q$

$$T(n) = \sum_{i=1}^q \sum_{j=0}^{m_i-1} c_{ij} n^j r_i^n$$

Las constantes  $c_{ij}$  se determinan por los casos base.

### 6.3. Combinatorio

```

1  forn(i, MAXN+1){ //comb[i][k]=i tomados de a k
2      comb[i][0]=comb[i][i]=1;
3      forr(k, 1, i) comb[i][k]=(comb[i-1][k]+comb[i-1][k-1])%MOD;
4  }
5  ll lucas(ll n, ll k, int p){ //Calcula (n,k)%p
6      // teniendo comb[p][p] precalculado.
7      ll aux = 1;
8      while (n + k) aux = (aux * comb[n%p][k%p]) %p,
9          n/=p, k/=p;
10     return aux;
11 }

```

### 6.4. Gauss Jordan, Determinante

```

1  // Gauss-Jordan elimination with full pivoting.
2  //
3  // Uses:
4  // (1) solving systems of linear equations (AX=B)
5  // (2) inverting matrices (AX=I)
6  // (3) computing determinants of square matrices
7  //
8  // Running time: O(n^3)
9  //
10 // INPUT:      a[] [] = an nxn matrix
11 //             b[] [] = an nxm matrix
12 //
13 // OUTPUT:
14 // X= an nxm matrix (stored in b[] [])
15 // A^{-1} = an nxn matrix (stored in a[] [])
16 // returns determinant of a[] []
17
18 #include <iostream>
19 #include <vector>
20 #include <cmath>
21
22 using namespace std;
23
24 const double EPS = 1e-10;
25
26 typedef vector<int> VI;
27 typedef double T;
28 typedef vector<T> VT;
29 typedef vector<VT> VVT;
30
31 T GaussJordan(VVT &a, VVT &b) {
32     const int n = a.size();
33     const int m = b[0].size();
34     VI irow(n), icol(n), ipiv(n);
35     T det = 1;
36
37     for (int i = 0; i < n; i++) {
38         int pj = -1, pk = -1;
39         for (int j = 0; j < n; j++) if (!ipiv[j])
40             for (int k = 0; k < n; k++) if (!ipiv[k])
41                 if (pj == -1 || fabs(a[j][k]) > fabs(a[pj][pk]))
42                     { pj = j; pk = k; }
43         if (fabs(a[pj][pk]) < EPS) { cerr << "Matrix_
44             is_singular." << endl; exit(0); }
45         ipiv[pj]++;

```

```

44 swap(a[pj], a[pk]);
45 swap(b[pj], b[pk]);
46 if (pj != pk) det *= -1;
47 irow[i] = pj;
48 icol[i] = pk;
49
50 T c = 1.0 / a[pk][pk];
51 det *= a[pk][pk];
52 a[pk][pk] = 1.0;
53 for (int p = 0; p < n; p++) a[pk][p] *= c;
54 for (int p = 0; p < m; p++) b[pk][p] *= c;
55 for (int p = 0; p < n; p++) if (p != pk) {
56     c = a[p][pk];
57     a[p][pk] = 0;
58     for (int q = 0; q < n; q++) a[p][q] -= a[pk]
59         [q] * c;
60     for (int q = 0; q < m; q++) b[p][q] -= b[pk]
61         [q] * c;
62 }
63 }
64
65 for (int p = n-1; p >= 0; p--) if (irow[p] !=
66     icol[p]) {
67     for (int k = 0; k < n; k++) swap(a[k][irow[p]
68         ], a[k][icol[p]]);
69 }
70
71 return det;
72 }
73
74 int main() {
75     const int n = 4;
76     const int m = 2;
77     double A[n][n] = {
78         {1,2,3,4},{1,0,1,0},{5,3,2,4},{6,1,4,6} };
79     double B[n][m] = { {1,2},{4,3},{5,6},{8,7} };
80     VVT a(n), b(m);
81     for (int i = 0; i < n; i++) {
82         a[i] = VT(A[i], A[i] + n);
83         b[i] = VT(B[i], B[i] + m);
84     }
85
86     double det = GaussJordan(a, b);
87
88     // expected: 60
89     cout << "Determinant:␣" << det << endl;
90
91     // expected:
92     // -0.233333 0.166667 0.133333 0.066667
93     // 0.166667 0.166667 0.333333 -0.333333
94     // 0.233333 0.833333 -0.133333 -0.066667
95     // 0.05 -0.75 -0.1 0.2
96     cout << "Inverse:␣" << endl;
97     for (int i = 0; i < n; i++) {
98         for (int j = 0; j < n; j++)
99             cout << a[i][j] << '␣';
100         cout << endl;
101     }
102
103     // expected: 1.63333 1.3

```

```

99 // -0.166667 0.5
100 // 2.36667 1.7
101 // -1.85 -1.35
102 cout << "Solution:␣" << endl;
103 for (int i = 0; i < n; i++) {
104     for (int j = 0; j < m; j++)
105         cout << b[i][j] << '␣';
106     cout << endl;
107 }
108 }

```

## 6.5. Teorema Chino del Resto

```

1 // Chinese remainder theorem (special case): find
2   z such that
3   // z % m1 = r1, z % m2 = r2. Here, z is unique
4   modulo M = lcm(m1, m2).
5   // Return (z, M). On failure, M = -1.
6 PII chinese_remainder_theorem(int m1, int r1, int
7   m2, int r2) {
8     int s, t;
9     int g = extended_euclid(m1, m2, s, t);
10    if (r1%g != r2%g) return make_pair(0, -1);
11    return make_pair(mod(s*r2*m1 + t*r1*m2, m1*m2)
12        / g, m1*m2 / g);
13 }
14
15 // Chinese remainder theorem: find z such that
16 // z % m[i] = r[i] for all i. Note that the
17 // solution is
18 // unique modulo M = lcm_i (m[i]). Return (z, M)
19 // . On
20 // failure, M = -1. Note that we do not require
21 // the a[i]'s
22 // to be relatively prime.
23 PII chinese_remainder_theorem(const VI &m, const
24   VI &r) {
25     PII ret = make_pair(r[0], m[0]);
26     for (int i = 1; i < m.size(); i++) {
27         ret = chinese_remainder_theorem(ret.second,
28             ret.first, m[i], r[i]);
29         if (ret.second == -1) break;
30     }
31     return ret;
32 }

```

## 6.6. Funciones de primos

Iterar mientras el  $p^2 \leq N$ . Revisar que  $N! = 1$ , en este caso  $N$  es primo. **NumDiv**: Producto (exponentes+1). **SumDiv**: Product suma geom. factores. **EulerPhi** (coprimos): Inicia  $ans = N$ . Para cada primo divisor:  $ans = ans / \text{primo}$  (una vez) y dividir luego  $N$  todo lo posible por  $p$ .

## 6.7. Phollard's Rho (rolando)

```

1 ll gcd(ll a, ll b){return a?gcd(b %a, a):b;}
2
3 ll mulmod (ll a, ll b, ll c) { //returns (a*b)%c,
4     and minimize overflow
5     ll x = 0, y = a%c;

```



```

5   while (b > 0){
6       if (b % 2 == 1) x = (x+y) % c;
7       y = (y*2) % c;
8       b /= 2;
9   }
10  return x % c;
11 }

12
13 ll expmod (ll b, ll e, ll m){ //O(log b)
14     if(!e) return 1;
15     ll q= expmod(b,e/2,m); q=mulmod(q,q,m);
16     return e%2? mulmod(b,q,m) : q;
17 }

18
19 bool es_primo_prob (ll n, int a)
20 {
21     if (n == a) return true;
22     ll s = 0, d = n-1;
23     while (d % 2 == 0) s++, d/=2;
24
25     ll x = expmod(a,d,n);
26     if ((x == 1) || (x+1 == n)) return true;
27
28     forn (i, s-1){
29         x = mulmod(x, x, n);
30         if (x == 1) return false;
31         if (x+1 == n) return true;
32     }
33     return false;
34 }

35
36 bool rabin (ll n){ //devuelve true si n es primo
37     if (n == 1) return false;
38     const int ar[] = {2,3,5,7,11,13,17,19,23};
39     forn (j,9)
40         if (!es_primo_prob(n,ar[j]))
41             return false;
42     return true;
43 }

44
45 ll rho(ll n){
46     if( (n & 1) == 0 ) return 2;
47     ll x = 2 , y = 2 , d = 1;
48     ll c = rand() % n + 1;
49     while( d == 1 ){
50         x = (mulmod( x , x , n ) + c)%n;
51         y = (mulmod( y , y , n ) + c)%n;
52         y = (mulmod( y , y , n ) + c)%n;
53         if( x - y >= 0 ) d = gcd( x - y , n );
54         else d = gcd( y - x , n );
55     }
56     return d==n? rho(n):d;
57 }

58
59 map<ll,ll> prim;
60 void factRho (ll n){ //O (lg n)^3. un solo numero
61     if (n == 1) return;
62     if (rabin(n)){
63         prim[n]++;
64         return;

```

```

65     }
66     ll factor = rho(n);
67     factRho(factor);
68     factRho(n/factor);
69 }

```

## 6.8. GCD

```

1  tipo gcd(tipo a, tipo b){return a?gcd(b %a, a):b
   ;}

```

## 6.9. Extended Euclid

```

1  void extendedEuclid (ll a, ll b){ //a * x + b * y
   = d
2     if (!b) { x = 1; y = 0; d = a; return;}
3     extendedEuclid (b, a%b);
4     ll x1 = y;
5     ll y1 = x - (a/b) * y;
6     x = x1; y = y1;
7 }

```

## 6.10. Polinomio

```

1     int m = sz(c), n = sz(o.c);
2     vector<tipo> res(max(m,n));
3     forn(i, m) res[i] += c[i];
4     forn(i, n) res[i] += o.c[i];
5     return poly(res);    }
6
6 poly operator*(const tipo cons) const {
7     vector<tipo> res(sz(c));
8     forn(i, sz(c)) res[i]=c[i]*cons;
9     return poly(res);    }
10
10 poly operator*(const poly &o) const {
11     int m = sz(c), n = sz(o.c);
12     vector<tipo> res(m+n-1);
13     forn(i, m) forn(j, n) res[i+j]+=c[i]*o.c[
        j];
14     return poly(res);    }
15
15 tipo eval(tipo v) {
16     tipo sum = 0;
17     dforn(i, sz(c)) sum=sum*v + c[i];
18     return sum; }
19
19 //poly contains only a vector<int> c (the
    coeficients)
20
20 //the following function generates the roots of
    the polynomial
21
21 //it can be easily modified to return float roots
22
22 set<tipo> roots(){
23     set<tipo> roots;
24     tipo a0 = abs(c[0]), an = abs(c[sz(c)-1]);
25     vector<tipo> ps,qs;
26     forr(p,1,sqrt(a0)+1) if (a0%p==0) ps.pb(p),ps
        .pb(a0/p);
27     forr(q,1,sqrt(an)+1) if (an%q==0) qs.pb(q),qs
        .pb(an/q);
28     forall(pt,ps)
29         forall(qt,qs) if ( (*pt) % (*qt)==0 ) {
30             tipo root = abs((*pt) / (*qt));
31             if (eval(root)==0) roots.insert(root);
32         }

```

```

33     return roots; }
34 };
35 pair<poly, tipo> ruffini(const poly p, tipo r) {
36     int n = sz(p.c) - 1;
37     vector<tipo> b(n);
38     b[n-1] = p.c[n];
39     dforn(k, n-1) b[k] = p.c[k+1] + r*b[k+1];
40     tipo resto = p.c[0] + r*b[0];
41     poly result(b);
42     return make_pair(result, resto);
43 }
44 poly interpolate(const vector<tipo>& x, const
45     vector<tipo>& y) {
46     poly A; A.c.pb(1);
47     forn(i, sz(x)) { poly aux; aux.c.pb(-x[i]),
48         aux.c.pb(1), A = A * aux; }
49     poly S; S.c.pb(0);
50     forn(i, sz(x)) { poly Li;
51         Li = ruffini(A, x[i]).fst;
52         Li = Li * (1.0 / Li.eval(x[i])); // here put
53         a multiple of the coefficients instead of
54         1.0 to avoid using double
55         S = S + Li * y[i]; }
56     return S;
57 }
58
59 int main(){
60     return 0;
61 }

```

## 6.11. FFT

```

1  //~ typedef complex<double> base; //menos codigo,
2  //pero mas lento
3  //elegir si usar complejos de c (lento) o estos
4  struct base{
5      double r, i;
6      base(double r=0, double i=0):r(r), i(i){}
7      double real()const{return r;}
8      void operator/=(const int c){r/=c, i/=c;}
9  };
10 base operator*(const base &a, const base &b){
11     return base(a.r*b.r-a.i*b.i, a.r*b.i+a.i*b.r)
12     ;}
13 base operator+(const base &a, const base &b){
14     return base(a.r+b.r, a.i+b.i);}
15 base operator-(const base &a, const base &b){
16     return base(a.r-b.r, a.i-b.i);}
17 vector<int> rev; vector<base> wlen_pw;
18 inline static void fft(base a[], int n, bool
19     invert) {
20     forn(i, n) if(i<rev[i]) swap(a[i], a[rev[i]])
21     ;
22     for (int len=2; len<=n; len<=1) {
23         double ang = 2*M_PI/len * (invert?-1:+1);
24         int len2 = len>>1;
25         base wlen (cos(ang), sin(ang));
26         wlen_pw[0] = base (1, 0);
27         forr(i, 1, len2) wlen_pw[i] = wlen_pw[i
28             -1] * wlen;

```

```

24     for (int i=0; i<n; i+=len) {
25         base t, *pu = a+i, *pv = a+i+len2, *pu_end
26             = a+i+len2, *pw = &wlen_pw[0];
27         for (; pu!=pu_end; ++pu, ++pv, ++pw)
28             t = *pv * *pw, *pv = *pu - t, *pu = *pu +
29             t;
30     }
31     if (invert) forn(i, n) a[i]/= n;}
32 inline static void calc_rev(int n){//precalculo:
33     llamar antes de fft!!
34     wlen_pw.resize(n), rev.resize(n);
35     int lg=31-__builtin_clz(n);
36     forn(i, n){
37         rev[i] = 0;
38         forn(k, lg) if(i&(1<<k)) rev[i] |= 1<<(lg
39             -1-k);
40     }}
41 inline static void multiply(const vector<int> &a,
42     const vector<int> &b, vector<int> &res) {
43     vector<base> fa (a.begin(), a.end()), fb (b.
44         begin(), b.end());
45     int n=1; while(n < max(sz(a), sz(b))) n <<=
46         1; n <<= 1;
47     calc_rev(n);
48     fa.resize (n), fb.resize (n);
49     fft (&fa[0], n, false), fft (&fb[0], n, false)
50     ;
51     forn(i, n) fa[i] = fa[i] * fb[i];
52     fft (&fa[0], n, true);
53     res.resize(n);
54     forn(i, n) res[i] = int (fa[i].real() + 0.5);
55     }
56 void toPoly(const string &s, vector<int> &P){//
57     convierte un numero a polinomio
58     P.clear();
59     dforn(i, sz(s)) P.pb(s[i]-'0');}

```

## 7. Grafos

### 7.1. Bellman-Ford

```

1  vector<ii> G[MAX_N]; //ady. list with pairs (
2  weight, dst)
3  int dist[MAX_N];
4  void bford(int src){//O(VE)
5      dist[src]=0;
6      forn(i, N-1) forn(j, N) if(dist[j]!=INF) forall
7          (it, G[j])
8              dist[it->snd]=min(dist[it->snd], dist[j]+it->
9              fst);
10 }
11 bool hasNegCycle(){
12     forn(j, N) if(dist[j]!=INF) forall(it, G[j])
13         if(dist[it->snd]>dist[j]+it->fst) return true
14         ;
15     //inside if: all points reachable from it->snd
16     will have -INF distance(do bfs)
17     return false;

```

```
14 | }
```

## 7.2. 2-SAT + Tarjan SCC

```
1 //We have a vertex representing a var and other
  //for his negation.
2 //Every edge stored in G represents an
  //implication. To add an equation of the form a
  //||b, use addor(a, b)
3 //MAX=max cant var, n=cant var
4 #define addor(a, b) (G[neg(a)].pb(b), G[neg(b)].
  pb(a))
5 vector<int> G[MAX*2];
6 //idx[i]=index assigned in the dfs
7 //lw[i]=lowest index(closer from the root)
  //reachable from i
8 int lw[MAX*2], idx[MAX*2], qidx;
9 stack<int> q;
10 int qcmp, cmp[MAX*2];
11 //verdad[cmp[i]]=valor de la variable i
12 bool verdad[MAX*2+1];
13
14 int neg(int x) { return x>=n? x-n : x+n;}
15 void tjn(int v){
16     lw[v]=idx[v]=++qidx;
17     q.push(v), cmp[v]=-2;
18     forall(it, G[v]){
19         if(!idx[*it] || cmp[*it]==-2){
20             if(!idx[*it]) tjn(*it);
21             lw[v]=min(lw[v], lw[*it]);
22         }
23     }
24     if(lw[v]==idx[v]){
25         int x;
26         do{x=q.top(); q.pop(); cmp[x]=qcmp;}while(x!=
            v);
27         verdad[qcmp]=(cmp[neg(v)]<0);
28         qcmp++;
29     }
30 }
31 //remember to CLEAR G!!!
32 bool satisf(){//0(n)
33     memset(idx, 0, sizeof(idx)), qidx=0;
34     memset(cmp, -1, sizeof(cmp)), qcmp=0;
35     forn(i, n){
36         if(!idx[i]) tjn(i);
37         if(!idx[neg(i)]) tjn(neg(i));
38     }
39     forn(i, n) if(cmp[i]==cmp[neg(i)]) return false
        ;
40     return true;
41 }
```

## 7.3. Puentes y Articulation Points

```
1 int dfsNumberCounter, dfsRoot, rootChildren;
2 vi dfs_num, dfs_low, dfs_parent,
  articulation_vertex;
3
4 void articulationPointAndBridge(int u) {
5     dfs_low[u] = dfs_num[u] = dfsNumberCounter++;
```

```
6     for (int j = 0; j < (int)AdjList[u].size(); j
      ++){
7         ii v = AdjList[u][j];
8         if (dfs_num[v.first] == -1) {
9             dfs_parent[v.first] = u;
10            if (u == dfsRoot) rootChildren++;
11            articulationPointAndBridge(v.first);
12            if (dfs_low[v.first] >= dfs_num[u])
13                articulation_vertex[u] = true;
14            if (dfs_low[v.first] > dfs_num[u])
15                printf("_Edge_ %d, %d _is_a_bridge\n", u,
                        v.first);
16            dfs_low[u] = min(dfs_low[u], dfs_low[v.
                        first]);
17        }
18        else if (v.first != dfs_parent[u])
19            dfs_low[u] = min(dfs_low[u], dfs_num[v.
                        first]);
20    } }
21 // At main
22 dfsNumberCounter = 0; dfs_num.assign(V, -1);
23 dfs_low.assign(V, 0);
24 dfs_parent.assign(V, -1); articulation_vertex.
25 assign(V, 0);
26 printf("Bridges:\n");
27 for (int i = 0; i < V; i++)
28     if (dfs_num[i] == -1) {
29         dfsRoot = i; rootChildren = 0;
30         articulationPointAndBridge(i);
31         articulation_vertex[dfsRoot] = (
            rootChildren > 1); }
32 printf("Articulation_Points:\n");
33 for (int i = 0; i < V; i++)
34     if (articulation_vertex[i])
35         printf("_Vertex_ %d\n", i);
```

## 7.4. LCA + Climb

```
1 const int MAXN=100001;
2 const int LOGN=20;
3 //f[v][k] holds the 2^k father of v
4 //L[v] holds the level of v
5 int N, f[MAXN][LOGN], L[MAXN];
6 //call before build:
7 void dfs(int v, int fa=-1, int lvl=0){//generate
  required data
8     f[v][0]=fa, L[v]=lvl;
9     forall(it, G[v])if(*it!=fa) dfs(*it, v, lvl+1);
10 }
11 void build(){//f[i][0] must be filled previously,
  0(nlgn)
12     forn(k, LOGN-1) forn(i, N) f[i][k+1]=f[f[i][k]
        ][k];}
13 #define lg(x) (31-__builtin_clz(x))//=floor(log2(
  x))
14 int climb(int a, int d){//0(lgn)
15     if(!d) return a;
16     dforn(i, lg(L[a])+1) if(1<<i<=d) a=f[a][i], d
        -=1<<i;
17     return a;}
```

```

17 int lca(int a, int b){//O(lgn)
18     if(L[a]<L[b]) swap(a, b);
19     a=climb(a, L[a]-L[b]);
20     if(a==b) return a;
21     dforn(i, lg(L[a])+1) if(f[a][i]!=f[b][i]) a=f[a
        ][i], b=f[b][i];
22     return f[a][0]; }
23 int dist(int a, int b) { //returns distance
        between nodes
24     return L[a]+L[b]-2*L[lca(a, b)];}

```

## 7.5. Heavy Light Decomposition

```

1 int treesz[MAXN]; //cantidad de nodos en el
        subarbol del nodo v
2 int dad[MAXN]; //dad[v]=padre del nodo v
3 void dfs1(int v, int p=-1) { //pre-dfs
4     dad[v]=p;
5     treesz[v]=1;
6     forall(it, G[v]) if(*it!=p){
7         dfs1(*it, v);
8         treesz[v]+=treesz[*it];
9     }
10 }
11 //PONER Q EN O !!!!!
12 int pos[MAXN], q; //pos[v]=posicion del nodo v en
        el recorrido de la dfs
13 //Las cadenas aparecen continuas en el recorrido!
14 int cantcad;
15 int homecad[MAXN]; //dada una cadena devuelve su
        nodo inicial
16 int cad[MAXN]; //cad[v]=cadena a la que pertenece
        el nodo
17 void heavylight(int v, int cur=-1){
18     if(cur===-1) homecad[cur=cantcad++] = v;
19     pos[v]=q++;
20     cad[v]=cur;
21     int mx=-1;
22     forn(i, sz(G[v])) if(G[v][i]!=dad[v])
23         if(mx===-1 || treesz[G[v][mx]]<treesz[G[v][i
            ]]) mx=i;
24     if(mx!=-1) heavylight(G[v][mx], cur);
25     forn(i, sz(G[v])) if(i!=mx && G[v][i]!=dad[v])
26         heavylight(G[v][i], -1);
27 }
28 //ejemplo de obtener el maximo numero en el
        camino entre dos nodos
29 //RTA: max(query(low, u), query(low, v)), con low
        =lca(u, v)
30 //esta funcion va trepando por las cadenas
31 int query(int an, int v) { //O(logn)
32     //si estan en la misma cadena:
33     if(cad[an]==cad[v]) return rmq.get(pos[an], pos
        [v]+1);
34     return max(query(an, dad[homecad[cad[v]]]),
35         rmq.get(pos[homecad[cad[v]]], pos[v]+1))
36 }

```

## 7.6. Centroid Decomposition

```

1 int n;
2 vector<int> G[MAXN];
3 bool taken[MAXN]; //poner todos en FALSE al
        principio!!
4 int padre[MAXN]; //padre de cada nodo en el
        centroid tree
5
6 int szt[MAXN];
7 void calcsz(int v, int p) {
8     szt[v] = 1;
9     forall(it, G[v]) if (*it!=p && !taken[*it])
10         calcsz(*it, v), szt[v]+=szt[*it];
11 }
12 void centroid(int v=0, int f=-1, int lvl=0, int
        tam=-1) { //O(nlogn)
13     if(tam===-1) calcsz(v, -1), tam=szt[v];
14     forall(it, G[v]) if(!taken[*it] && szt[*it]>=
        tam/2)
15         {szt[v]=0; centroid(*it, f, lvl, tam); return
            ;}
16     taken[v]=true;
17     padre[v]=f;
18     forall(it, G[v]) if(!taken[*it])
19         centroid(*it, v, lvl+1, -1);
20 }

```

## 7.7. Euler Cycle

```

1 int n,m,ars[MAXE], eq;
2 vector<int> G[MAXN]; //fill G,n,m,ars,eq
3 list<int> path;
4 int used[MAXN];
5 bool usede[MAXE];
6 queue<list<int>::iterator> q;
7 int get(int v){
8     while(used[v]<sz(G[v]) && usede[ G[v][used[v]]
        ]) used[v]++;
9     return used[v];
10 }
11 void explore(int v, int r, list<int>::iterator it
        ){
12     int ar=G[v][get(v)]; int u=v^ars[ar];
13     usede[ar]=true;
14     list<int>::iterator it2=path.insert(it, u);
15     if(u!=r) explore(u, r, it2);
16     if(get(v)<sz(G[v])) q.push(it);
17 }
18 void euler(){
19     zero(used), zero(usede);
20     path.clear();
21     q=queue<list<int>::iterator>();
22     path.push_back(0); q.push(path.begin());
23     while(sz(q)){
24         list<int>::iterator it=q.front(); q.pop();
25         if(used[*it]<sz(G[*it])) explore(*it, *it, it
            );
26     }
27     reverse(path.begin(), path.end());
28 }
29 void addEdge(int u, int v){

```

```

30 | G[u].pb(eq), G[v].pb(eq);
31 | ars[eq++] = u ^ v;
32 | }

```

### 7.8. Chu-liu

```

1 void visit(graph &h, int v, int s, int r,
2     vector<int> &no, vector< vector<int> > &comp,
3     vector<int> &prev, vector< vector<int> > &next,
4     vector<weight> &mcost,
5     vector<int> &mark, weight &cost, bool &found) {
6     if (mark[v]) {
7         vector<int> temp = no;
8         found = true;
9         do {
10             cost += mcost[v];
11             v = prev[v];
12             if (v != s) {
13                 while (comp[v].size() > 0) {
14                     no[comp[v].back()] = s;
15                     comp[s].push_back(comp[v].back());
16                     comp[v].pop_back();
17                 }
18             } while (v != s);
19             forall(j, comp[s]) if (*j != r) forall(e, h[*j
20                 ])
21                 if (no[e->src] != s) e->w -= mcost[ temp[*j
22                     ] ];
23         }
24         mark[v] = true;
25         forall(i, next[v]) if (no[*i] != no[v] && prev[
26             no[*i]] == v)
27             if (!mark[no[*i]] || *i == s)
28                 visit(h, *i, s, r, no, comp, prev, next,
29                     mcost, mark, cost, found);
30     }
31 }
32
33 weight minimumSpanningArborescence(const graph &g
34     , int r) {
35     const int n=sz(g);
36     graph h(n);
37     forn(u,n) forall(e,g[u]) h[e->dst].pb(*e);
38     vector<int> no(n);
39     vector<vector<int> > comp(n);
40     forn(u, n) comp[u].pb(no[u] = u);
41     for (weight cost = 0; ;) {
42         vector<int> prev(n, -1);
43         vector<weight> mcost(n, INF);
44         forn(j,n) if (j != r) forall(e,h[j])
45             if (no[e->src] != no[j])
46                 if (e->w < mcost[ no[j] ])
47                     mcost[ no[j] ] = e->w, prev[ no[j] ] =
48                         no[e->src];
49         vector< vector<int> > next(n);
50         forn(u,n) if (prev[u] >= 0)
51             next[ prev[u] ].push_back(u);
52         bool stop = true;
53         vector<int> mark(n);
54         forn(u,n) if (u != r && !mark[u] && !comp[u].
55             empty()) {

```

```

47     bool found = false;
48     visit(h, u, u, r, no, comp, prev, next,
          mcost, mark, cost, found);
49     if (found) stop = false;
50 }
51 if (stop) {
52     forn(u,n) if (prev[u] >= 0) cost += mcost[u]
53         ];
54     return cost;
55 }
56 }

```

## 7.9. Hungarian

```

1 //Dado un grafo bipartito completo con costos no
    negativos, encuentra el matching perfecto de
    minimo costo.
2 tipo cost[N][N], lx[N], ly[N], slack[N]; //llenar
    : cost=matriz de adyacencia
3 int n, max_match, xy[N], yx[N], slackx[N],prev2[N]
    ;//n=cantidad de nodos
4 bool S[N], T[N]; //sets S and T in algorithm
5 void add_to_tree(int x, int prevx) {
6     S[x] = true, prev2[x] = prevx;
7     for(y, n) if (lx[x] + ly[y] - cost[x][y] <
        slack[y] - EPS)
8         slack[y] = lx[x] + ly[y] - cost[x][y], slackx
            [y] = x;
9 }
10 void update_labels(){
11     tipo delta = INF;
12     for (y, n) if (!T[y]) delta = min(delta, slack
        [y]);
13     for (x, n) if (S[x]) lx[x] -= delta;
14     for (y, n) if (T[y]) ly[y] += delta; else
        slack[y] -= delta;
15 }
16 void init_labels(){
17     zero(lx), zero(ly);
18     for (x,n) for(y,n) lx[x] = max(lx[x], cost[x
        ][y]);
19 }
20 void augment() {
21     if (max_match == n) return;
22     int x, y, root, q[N], wr = 0, rd = 0;
23     memset(S, false, sizeof(S)), memset(T, false,
        sizeof(T));
24     memset(prev2, -1, sizeof(prev2));
25     for (x, n) if (xy[x] == -1){
26         q[wr++] = root = x, prev2[x] = -2;
27         S[x] = true; break; }
28     for (y, n) slack[y] = lx[root] + ly[y] - cost[
        root][y], slackx[y] = root;
29     while (true){
30         while (rd < wr){
31             x = q[rd++];
32             for (y = 0; y < n; y++) if (cost[x][y] ==
                lx[x] + ly[y] && !T[y]){
33                 if (yx[y] == -1) break; T[y] = true;

```

```

34     q[wr++] = yx[y], add_to_tree(yx[y], x); }
35     if (y < n) break; }
36     if (y < n) break;
37     update_labels(), wr = rd = 0;
38     for (y = 0; y < n; y++) if (!T[y] && slack[y]
39         == 0){
40         if (yx[y] == -1){x = slackx[y]; break;}
41         else{
42             T[y] = true;
43             if (!S[yx[y]]) q[wr++] = yx[y],
44                 add_to_tree(yx[y], slackx[y]);
45         }
46         if (y < n) break; }
47     if (y < n){
48         max_match++;
49         for (int cx = x, cy = y, ty; cx != -2; cx =
50             prev2[cx], cy = ty)
51             ty = xy[cx], yx[cy] = cx, xy[cx] = cy;
52         augment(); }
53 }
54 tipo hungarian(){
55     tipo ret = 0; max_match = 0, memset(xy, -1,
56         sizeof(xy));
57     memset(yx, -1, sizeof(yx)), init_labels(),
58         augment(); //steps 1-3
59     forn (x,n) ret += cost[x][xy[x]]; return ret;
60 }

```

## 7.10. Dynamic Connectivity

```

1 struct UnionFind {
2     int n, comp;
3     vector<int> pre, si, c;
4     UnionFind(int n=0):n(n), comp(n), pre(n), si(
5         n, 1) {
6         forn(i,n) pre[i] = i; }
7     int find(int u){return u==pre[u]?u:find(pre[u]
8         );}
9     bool merge(int u, int v) {
10         if((u=find(u))==v) return false;
11         ;
12         if(si[u]<si[v]) swap(u, v);
13         si[u]+=si[v], pre[v]=u, comp--, c.pb(v);
14         return true;
15     }
16     int snap(){return sz(c);}
17     void rollback(int snap){
18         while(sz(c)>snap){
19             int v = c.back(); c.pop_back();
20             si[pre[v]] -= si[v], pre[v] = v, comp
21                 ++;
22         }
23     }
24 };
25 enum {ADD,DEL,QUERY};
26 struct Query {int type,u,v;};
27 struct DynCon {
28     vector<Query> q;
29     UnionFind dsu;
30     vector<int> match,res;

```

```

27     map<ii,int> last;//se puede no usar cuando
28         hay identificador para cada arista (
29         mejora poco)
30     DynCon(int n=0):dsu(n){}
31     void add(int u, int v) {
32         if(u>v) swap(u,v);
33         q.pb((Query){ADD, u, v}), match.pb(-1);
34         last[ii(u,v)] = sz(q)-1;
35     }
36     void remove(int u, int v) {
37         if(u>v) swap(u,v);
38         q.pb((Query){DEL, u, v});
39         int prev = last[ii(u,v)];
40         match[prev] = sz(q)-1;
41         match.pb(prev);
42     }
43     void query() {//podria pasarle un puntero
44         donde guardar la respuesta
45         q.pb((Query){QUERY, -1, -1}), match.pb
46             (-1);}
47     void process() {
48         forn(i,sz(q)) if (q[i].type == ADD &&
49             match[i] == -1) match[i] = sz(q);
50         go(0,sz(q));
51     }
52     void go(int l, int r) {
53         if(l+1==r){
54             if (q[l].type == QUERY)//Aqui
55                 responder la query usando el dsu!
56                 res.pb(dsu.comp);//aqui query=
57                     cantidad de componentes
58                     conexas
59             return;
60         }
61         int s=dsu.snap(), m = (l+r) / 2;
62         forr(i,m,r) if(match[i]!=-1 && match[i]<l

```

## 8. Network Flow

### 8.1. Dinic

```

1
2 const int MAX = 300;
3 // Corte minimo: vertices con dist[v]>=0 (del
4     lado de src) VS. dist[v]==-1 (del lado del
5     dst)
6 // Para el caso de la red de Bipartite Matching (
7     Sean V1 y V2 los conjuntos mas proximos a src
8     y dst respectivamente):
9 // Reconstruir matching: para todo v1 en V1 ver

```



```

    las aristas a vertices de V2 con it->f>0, es
    arista del Matching
6 // Min Vertex Cover: vertices de V1 con dist[v
  ]==1 + vertices de V2 con dist[v]>0
7 // Max Independent Set: tomar los vertices NO
  tomados por el Min Vertex Cover
8 // Max Clique: construir la red de G complemento
  (debe ser bipartito!) y encontrar un Max
  Independet Set
9 // Min Edge Cover: tomar las aristas del matching
  + para todo vertices no cubierto hasta el
  momento, tomar cualquier arista de el
10 int nodes, src, dst;
11 int dist[MAX], q[MAX], work[MAX];
12 struct Edge {
13     int to, rev;
14     ll f, cap;
15     Edge(int to, int rev, ll f, ll cap) : to(to),
        rev(rev), f(f), cap(cap) {}
16 };
17 vector<Edge> G[MAX];
18 void addEdge(int s, int t, ll cap){
19     G[s].pb(Edge(t, sz(G[t]), 0, cap)), G[t].pb(
        Edge(s, sz(G[s])-1, 0, 0));}
20 bool dinic_bfs(){
21     fill(dist, dist+nodes, -1), dist[src]=0;
22     int qt=0; q[qt++]=src;
23     for(int qh=0; qh<qt; qh++){
24         int u =q[qh];
25         forall(e, G[u]){
26             int v=e->to;
27             if(dist[v]<0 && e->f < e->cap)
28                 dist[v]=dist[u]+1, q[qt++]=v;
29         }
30     }
31     return dist[dst]>=0;
32 }
33 ll dinic_dfs(int u, ll f){
34     if(u==dst) return f;
35     for(int &i=work[u]; i<sz(G[u]); i++){
36         Edge &e = G[u][i];
37         if(e.cap<=e.f) continue;
38         int v=e.to;
39         if(dist[v]==dist[u]+1){
40             ll df=dinic_dfs(v, min(f, e.cap-e
                .f));
41             if(df>0){
42                 e.f+=df, G[v][e.rev].f-=
                    df;
43                 return df; }
44         }
45     }
46     return 0;
47 }
48 ll maxFlow(int _src, int _dst){
49     src=_src, dst=_dst;
50     ll result=0;
51     while(dinic_bfs()){
52         fill(work, work+nodes, 0);
53         while(ll delta=dinic_dfs(src,INF))

```

```

54         result+=delta;
55     }
56     // todos los nodos con dist[v]!=-1 vs los que
        tienen dist[v]==-1 forman el min-cut
57     return result; }

```

## 8.2. Edmonds Karp's

```

1 #define MAX_V 1000
2 #define INF 1e9
3 //special nodes
4 #define SRC 0
5 #define SNK 1
6 map<int, int> G[MAX_V]; //limpiar esto
7 //To add an edge use
8 #define add(a, b, w) G[a][b]=w
9 int f, p[MAX_V];
10 void augment(int v, int minE){
11     if(v==SRC) f=minE;
12     else if(p[v]!=-1){
13         augment(p[v], min(minE, G[p[v]][v]));
14         G[p[v]][v]-=f, G[v][p[v]]+=f;
15     }
16 }
17 ll maxflow(){//O(VE^2)
18     ll Mf=0;
19     do{
20         f=0;
21         char used[MAX_V]; queue<int> q; q.push(SRC);
22         zero(used), memset(p, -1, sizeof(p));
23         while(sz(q)){
24             int u=q.front(); q.pop();
25             if(u==SNK) break;
26             forall(it, G[u])
27                 if(it->snd>0 && !used[it->fst])
28                     used[it->fst]=true, q.push(it->fst), p[
                        it->fst]=u;
29         }
30         augment(SNK, INF);
31         Mf+=f;
32     }while(f);
33     return Mf;
34 }

```

## 8.3. Max Matching

```

1 int LEFT, r[MAXV]; bool seen[MAXV]; VI AdjList[
    MAXV];
2 bool can_match(int u) {
3     for (auto & v : AdjList[u]) {
4         if (!seen[v]) {
5             seen[v] = true;
6             if (r[v] < 0 || can_match(r[v])) {
7                 r[v] = u; return true;
8             }
9         }
10    } return false;
11 }
12 int max_matching() {
13     memset(r, -1, sizeof r);
14     int ans = 0;

```

```

15     for (int u=0 ; u<LEFT ; u++) {
16         memset(seen, 0, sizeof seen);
17         if (can_match(u)) ans++;
18     } return ans;
19 }

```

## 8.4. Min-cost Max-flow

```

1  const int MAXN=10000;
2  typedef ll tf;
3  typedef ll tc;
4  const tf INFFLUJO = 1e14;
5  const tc INFCOSTO = 1e14;
6  struct edge {
7      int u, v;
8      tf cap, flow;
9      tc cost;
10     tf rem() { return cap - flow; }
11 };
12 int nodes; //numero de nodos
13 vector<int> G[MAXN]; // limpiar!
14 vector<edge> e; // limpiar!
15 void addEdge(int u, int v, tf cap, tc cost) {
16     G[u].pb(sz(e)); e.pb((edge){u,v,cap,0,cost});
17     G[v].pb(sz(e)); e.pb((edge){v,u,0,0,-cost});
18 }
19 tc dist[MAXN], mnCost;
20 int pre[MAXN];
21 tf cap[MAXN], mxFlow;
22 bool in_queue[MAXN];
23 void flow(int s, int t) {
24     zero(in_queue);
25     mxFlow=mnCost=0;
26     while(1){
27         fill(dist, dist+nodes, INFCOSTO); dist[s] =
28             0;
29         memset(pre, -1, sizeof(pre)); pre[s]=0;
30         zero(cap); cap[s] = INFFLUJO;
31         queue<int> q; q.push(s); in_queue[s]=1;
32         while(sz(q)){
33             int u=q.front(); q.pop(); in_queue[u]=0;
34             for(auto it:G[u]) {
35                 edge &E = e[it];
36                 if(E.rem() && dist[E.v] > dist[u] + E.
37                     cost + 1e-9){ // ojo EPS
38                     dist[E.v]=dist[u]+E.cost;
39                     pre[E.v] = it;
40                     cap[E.v] = min(cap[u], E.rem());
41                     if(!in_queue[E.v]) q.push(E.v),
42                         in_queue[E.v]=1;
43                 }
44             }
45         }
46         if (pre[t] == -1) break;
47         mxFlow +=cap[t];
48         mnCost +=cap[t]*dist[t];
49         for (int v = t; v != s; v = e[pre[v]].u) {
50             e[pre[v]].flow += cap[t];
51             e[pre[v]^1].flow -= cap[t];
52         }
53     }
54 }

```

```

50 }
51 }

```

## 9. Template y Otros

### Template

```

1  //touch {a..m}.in; tee {a..m}.cpp < template.cpp
2  #include <bits/stdc++.h>
3  using namespace std;
4  #define forr(i,a,b) for(int i=(a); i<(b); i++)
5  #define forn(i,n) forr(i,0,n)
6  #define sz(c) ((int)c.size())
7  #define zero(v) memset(v, 0, sizeof(v))
8  #define forall(it,v) for(auto it=v.begin();it!=v.
9      end();++it)
10 #define pb push_back
11 #define fst first
12 #define snd second
13 typedef long long ll;
14 typedef pair<int,int> ii;
15 #define dform(i,n) for(int i=n-1; i>=0; i--)
16 #define dprint(v) cout << #v"<=" << v << endl //;
17
18 const int MAXN=100100;
19 int n;
20
21 int main() {
22     freopen("input.in", "r", stdin);
23     ios::sync_with_stdio(0);
24     while(cin >> n){
25
26     }
27     return 0;
28 }

```

### Rellenar con espacios(para justificar)

```

1  #include <iomanip>
2  cout << setfill(' ') << setw(3) << 2 << endl;

```

### Aleatorios

```

1  #define RAND(a, b) (rand()%(b-a+1)+a)
2  srand(time(NULL));

```

### Doubles Comp.

```

1  const double EPS = 1e-9;
2  x == y <=> fabs(x-y) < EPS, x > y <=> x > y +
3      EPS
4  x >= y <=> x > y - EPS

```

### Expandir pila

```

1  #include <sys/resource.h>
2  rlimit rl;
3  getrlimit(RLIMIT_STACK, &rl);
4  rl.rlim_cur=1024L*1024L*256L;//256mb
5  setrlimit(RLIMIT_STACK, &rl);

```

### Iterar subconjunto

```
1 | for(int sbm=bm; sbm; sbm=(sbm-1)&bm)
```

## Split

```
1 | vector<string> split(string str,string sep){  
2 |     char* cstr=const_cast<char*>(str.c_str());  
3 |     char* current;  
4 |     vector<string> arr;  
5 |     current=strtok(cstr,sep.c_str());  
6 |     while(current!=NULL){  
7 |         arr.push_back(current);  
8 |         current=strtok(NULL,sep.c_str());  
9 |     }  
10 |     return arr;  
11 | }
```