# Índice

## 1. algorithm

#include <algorithm> #include <numeric>

Algo	Params	Funcion
sort, stable_sort	f, 1	ordena el intervalo
nth_element	f, nth, l	void ordena el n-esimo, y
		particiona el resto
fill, fill_n	f, l / n, elem	void llena [f, l) o [f,
		f+n) con elem
lower_bound, upper_bound	f, l, elem	it al primer / ultimo donde se
		puede insertar elem para que
		quede ordenada
binary_search	f, l, elem	bool esta elem en [f, l)
copy	f, l, resul	hace resul $+i=f+i \ \forall i$
find, find_if, find_first_of	f, l, elem	$it$ encuentra i $\in$ [f,l) tq. i=elem,
	/ pred / f2, l2	$\operatorname{pred}(i), i \in [f2,l2)$
count, count_if	f, l, elem/pred	cuenta elem, pred(i)
search	f, l, f2, l2	busca $[f2,l2) \in [f,l)$
replace, replace_if	f, l, old	cambia old / pred(i) por new
	/ pred, new	
reverse	f, l	da vuelta
partition, stable_partition	f, l, pred	pred(i) ad, !pred(i) atras
min_element, max_element	f, l, [comp]	it min, max de [f,l]
lexicographical_compare	f1,l1,f2,l2	bool con [f1,l1];[f2,l2]
next/prev_permutation	f,l	deja en [f,l) la perm sig, ant
set_intersection,	f1, l1, f2, l2, res	[res,) la op. de conj
set_difference, set_union,		
set_symmetric_difference,		
push_heap, pop_heap,	f, l, e / e /	mete/saca e en heap [f,l),
make_heap		hace un heap de [f,l)
is_heap	f,l	bool es [f,l) un heap
accumulate	f,l,i,[op]	$T = \sum / \text{oper de [f,l)}$
inner_product	f1, l1, f2, i	$T = i + [f1, 11) \cdot [f2, \dots)$
partial_sum	f, l, r, [op]	$r+i = \sum /oper de [f,f+i] \forall i \in [f,l)$
builtin_ffs	unsigned int	Pos. del primer 1 desde la derecha
_builtin_clz	unsigned int	Cant. de ceros desde la izquierda.
_builtin_ctz	unsigned int	Cant. de ceros desde la derecha.
_builtin_popcount	unsigned int	Cant. de 1's en x.
1:14::4	unsigned int	1 si x es par, 0 si es impar.
builtin_paritybuiltin_XXXXXXIl	unsigned ll	= pero para long long's.

## 2. Estructuras

## 2.1. RMQ (static)

Dado un arreglo y una operacion asociativa *idempotente*, get(i, j) opera sobre el rango [i, j). Restriccion: LVL  $\geq$  ceil(logn); Usar [] para llenar arreglo y luego build().

```
1 struct RMQ{
     #define LVL 10
2
     tipo vec[LVL] [1<<(LVL+1)];
     tipo &operator[](int p){return vec[0][p];}
     tipo get(int i, int j) {//intervalo [i,j)
5
       int p = 31-_builtin_clz(j-i);
6
       return min(vec[p][i],vec[p][j-(1<<p)]);</pre>
7
     }
8
     void build(int n) {//O(nlogn)
9
       int mp = 31-__builtin_clz(n);
10
       forn(p, mp) forn(x, n-(1<<p))
11
         vec[p+1][x] = min(vec[p][x], vec[p][x+(1<<p)]);
12
     }
13
14 };
```

## 2.2. RMQ (dynamic)

```
1 //Dado un arreglo y una operacion asociativa con neutro, get(i, j) opera
        sobre el rango [i, j).
  #define MAXN 100000
   #define operacion(x, y) max(x, y)
   const int neutro=0;
   struct RMQ{
5
     int sz;
6
     tipo t[4*MAXN];
7
     tipo &operator[](int p){return t[sz+p];}
8
     void init(int n){//O(nlgn)
9
       sz = 1 \ll (32-\_builtin\_clz(n));
10
       forn(i, 2*sz) t[i]=neutro;
11
12
     void updall(){//0(n)}
13
       dforn(i, sz) t[i]=operacion(t[2*i], t[2*i+1]);}
14
     tipo get(int i, int j){return get(i,j,1,0,sz);}
15
     tipo get(int i, int j, int n, int a, int b){\frac{1}{0}}
16
       if(j<=a || i>=b) return neutro;
17
       if(i<=a && b<=j) return t[n];
18
```

```
int c=(a+b)/2;
19
       return operacion(get(i, j, 2*n, a, c), get(i, j, 2*n+1, c, b));
20
21
     void set(int p, tipo val){//O(lgn)
22
       for(p+=sz; p>0 && t[p]!=val;){
23
         t[p]=val;
24
         p/=2;
25
         val=operacion(t[p*2], t[p*2+1]);
26
    }
   }rmq;
29
   //Usage:
31 | cin >> n; rmg.init(n); forn(i, n) cin >> rmg[i]; rmg.updall();
2.3. RMQ (lazy)
1 //Dado un arreglo y una operacion asociativa con neutro, get(i, j) opera
        sobre el rango [i, j).
typedef int Elem;//Elem de los elementos del arreglo
   typedef int Alt;//Elem de la alteracion
   #define operacion(x,y) x+y
   const Elem neutro=0; const Alt neutro2=0;
   #define MAXN 100000
   struct RMO{
     int sz:
     Elem t[4*MAXN]:
     Alt dirty[4*MAXN];//las alteraciones pueden ser de distinto Elem
10
     Elem &operator[](int p){return t[sz+p];}
11
     void init(int n){//O(nlgn)
12
       sz = 1 \ll (32-\_builtin\_clz(n));
13
       forn(i, 2*sz) t[i]=neutro;
14
       forn(i, 2*sz) dirty[i]=neutro2;
15
16
     void push(int n, int a, int b){//propaga el dirty a sus hijos
17
       if(dirty[n]!=0){
18
         t[n]+=dirty[n]*(b-a);//altera el nodo
19
         if(n<sz){
20
           dirty[2*n]+=dirty[n];
21
           dirty[2*n+1]+=dirty[n];
22
         }
23
         dirty[n]=0;
24
25
26
```

```
Elem get(int i, int j, int n, int a, int b){\frac{1}{0}}
27
       if(j<=a || i>=b) return neutro;
28
       push(n, a, b);//corrige el valor antes de usarlo
29
       if(i<=a && b<=j) return t[n];</pre>
30
       int c=(a+b)/2;
31
       return operacion(get(i, j, 2*n, a, c), get(i, j, 2*n+1, c, b));
32
33
     Elem get(int i, int j){return get(i,j,1,0,sz);}
34
     //altera los valores en [i, j) con una alteracion de val
35
     void alterar(Alt val, int i, int j, int n, int a, int b)\{//0(lgn)\}
36
       push(n, a, b);
37
       if(j<=a || i>=b) return;
38
       if(i<=a && b<=j){
39
         dirty[n]+=val;
         push(n, a, b);
41
         return;
42
       }
43
       int c=(a+b)/2;
44
       alterar(val, i, j, 2*n, a, c), alterar(val, i, j, 2*n+1, c, b);
45
       t[n]=operacion(t[2*n], t[2*n+1]);//por esto es el push de arriba
46
     }
47
     void alterar(Alt val, int i, int j){alterar(val,i,j,1,0,sz);}
  |}rmq;
```

#### 2.4. Fenwick Tree

```
1 //For 2D threat each column as a Fenwick tree, by adding a nested for in
        each operation
  struct Fenwick{
     static const int sz=1000001;
     tipo t[sz];
4
     void adjust(int p, tipo v){//valid with p in [1, sz), O(lgn)
5
      for(; p<sz; p+=(p&-p)) t[p]+=v; }</pre>
6
     tipo sum(int p){//cumulative sum in [1, p], O(lgn)
       tipo s=0;
8
      for(; p; p-=(p&-p)) s+=t[p];
9
       return s;
10
11
     tipo sum(int a, int b){return sum(b)-sum(a-1);}
12
     //get largest value with cumulative sum less than or equal to x;
13
     //for smallest, pass x-1 and add 1 to result
14
     int getind(tipo x) {//0(lgn)
15
         int idx = 0, mask = N;
16
```

```
while(mask && idx < N) {
    int t = idx + mask;
    if(x >= tree[t])
        idx = t, x -= tree[t];
    mask >>= 1;
    }
    return idx;
}
```

#### 2.5. Union Find

```
struct UnionFind{
vector<int> f;//the array contains the parent of each node
void init(int n){f.clear(); f.insert(f.begin(), n, -1);}
int comp(int x){return (f[x]==-1?x:f[x]=comp(f[x]));}//O(1)
bool join(int i, int j) {
   bool con=comp(i)==comp(j);
   if(!con) f[comp(i)] = comp(j);
   return con;
}
return con;
}
```

## 2.6. Disjoint Intervals

```
bool operator (const ii &a, const ii &b) {return a.fst <b.fst;}
   //Stores intervals as [first, second]
   //in case of a collision it joins them in a single interval
   struct disjoint_intervals {
     set<ii>> segs;
     void insert(ii v) {//O(lgn)
6
       if(v.snd-v.fst==0.) return;//0J0
7
       set<ii>>::iterator it.at:
8
       at = it = segs.lower_bound(v);
       if (at!=segs.begin() && (--at)->snd >= v.fst)
         v.fst = at->fst, --it;
11
       for(; it!=segs.end() && it->fst <= v.snd; segs.erase(it++))</pre>
         v.snd=max(v.snd, it->snd);
13
       segs.insert(v):
14
15
<sub>16</sub> | };
```

## 2.7. RMQ (2D)

int 1;

```
1 | struct RMQ2D{
                                                                                          11 n[LMAX];
     static const int sz=1024;
                                                                                   7
                                                                                          bint(11 x=0){
2
     RMQ t[sz];
                                                                                   8
                                                                                              1=1;
3
     RMQ &operator[](int p){return t[sz/2+p];}
                                                                                              forn(i, LMAX){
4
     void build(int n, int m){\frac{}{0(nm)}}
                                                                                                   if (x) l=i+1;
       forr(v, sz/2, sz/2+m)
                                                                                                  n[i]=x BASE;
                                                                                   11
         t[v].build(m);
                                                                                                   x/=BASE;
                                                                                   12
       forr(y, sz/2+m, sz)
                                                                                   13
8
         forn(x, sz)
                                                                                              }
9
                                                                                   14
                                                                                          }
           t[v].t[x]=0;
                                                                                   15
10
       dforn(y, sz/2)
                                                                                          bint(string x){
                                                                                   16
11
         forn(x, sz)
                                                                                          l=(x.size()-1)/BASEXP+1;
12
           t[y].t[x]=max(t[y*2].t[x], t[y*2+1].t[x]);
                                                                                              fill(n, n+LMAX, 0);
13
                                                                                   18
     }
                                                                                              ll r=1;
                                                                                   19
14
     void set(int x, int y, tipo v){//O(lgm.lgn)
                                                                                              forn(i, sz(x)){
15
       v + = sz/2:
                                                                                                   n[i / BASEXP] += r * (x[x.size()-1-i]-'0');
                                                                                  21
16
       t[y].set(x, v);
                                                                                                  r*=10; if(r==BASE)r=1;
17
       while(y/=2)
                                                                                              }
18
         t[y].set(x, max(t[y*2][x], t[y*2+1][x]));
                                                                                          }
                                                                                   24
19
     }
                                                                                          void out(){
20
     //0(lgm.lgn)
                                                                                          cout << n[l-1];
21
     int get(int x1, int y1, int x2, int y2, int n=1, int a=0, int b=sz/2){
                                                                                          dforn(i, 1-1) printf("%6.611u", n[i]);//6=BASEXP!
22
       if(y2<=a || y1>=b) return 0;
                                                                                        }
                                                                                   28
23
       if(y1<=a && b<=y2) return t[n].get(x1, x2);
                                                                                        void invar(){
                                                                                  29
24
       int c=(a+b)/2;
                                                                                          fill(n+1, n+LMAX, 0);
                                                                                  30
25
                                                                                          while(1>1 && !n[1-1]) 1--;
       return max(get(x1, y1, x2, y2, 2*n, a, c),
                                                                                  31
26
            get(x1, y1, x2, y2, 2*n+1, c, b));
                                                                                       }
                                                                                   32
27
     }
                                                                                   33
28
                                                                                      bint operator+(const bint&a, const bint&b){
29
   //Example to initialize a grid of M rows and N columns:
                                                                                        bint c;
   RMQ2D rmq;
                                                                                          c.1 = max(a.1, b.1);
                                                                                   36
  forn(i, M)
                                                                                          11 q = 0;
    forn(j, N)
                                                                                          forn(i, c.1) q += a.n[i]+b.n[i], c.n[i]=q %BASE, q/=BASE;
33
       cin >> rmq[i][j];
                                                                                          if(q) c.n[c.l++] = q;
35 rmq.build(N, M);
                                                                                          c.invar();
                                                                                          return c;
                                                                                   41
2.8. Big Int
                                                                                      pair<br/>bint, bool> lresta(const bint& a, const bint& b) // c = a - b
                                                                                      {
1 #define BASEXP 6
                                                                                   44
                                                                                   45
                                                                                        bint c;
2 #define BASE 1000000
                                                                                          c.1 = max(a.1, b.1);
  #define LMAX 1000
                                                                                          11 q = 0;
                                                                                   47
  struct bint{
```

```
forn(i, c.1) q += a.n[i]-b.n[i], c.n[i]=(q+BASE) %ASE, q=(q+BASE)/
48
           BASE-1:
       c.invar():
49
       return make_pair(c, !q);
50
51
   bint& operator-= (bint& a, const bint& b){return a=lresta(a, b).first;}
   bint operator- (const bint&a, const bint&b){return lresta(a, b).first;}
   bool operator< (const bint&a, const bint&b){return !lresta(a, b).second
       ;}
   bool operator <= (const bint&a, const bint&b) {return lresta(b, a).second
   bool operator==(const bint&a, const bint&b){return a <= b && b <= a;}
   bint operator*(const bint&a, 11 b){
       bint c:
58
       11 q = 0;
59
       forn(i, a.l) q += a.n[i]*b, c.n[i] = q \text{BASE}, q/=BASE};
60
       c.1 = a.1:
61
       while(q) c.n[c.l++] = q %BASE, q/=BASE;
62
       c.invar():
63
       return c;
64
65
   bint operator*(const bint&a, const bint&b){
       bint c;
67
       c.1 = a.1+b.1;
68
       fill(c.n, c.n+b.1, 0);
69
       forn(i, a.1){
70
           11 q = 0;
71
           forn(j, b.1) q += a.n[i]*b.n[j]+c.n[i+j], c.n[i+j] = q BASE, q
72
           c.n[i+b.1] = q;
73
       }
74
       c.invar();
75
       return c:
76
77
   pair<br/>bint, 11> ldiv(const bint& a, 11 b)\{// c = a / b : rm = a \% b\}
78
     bint c:
79
     11 \text{ rm} = 0:
80
     dforn(i, a.1){
81
               rm = rm * BASE + a.n[i];
82
               c.n[i] = rm / b;
83
               rm %= b;
84
       }
85
       c.l = a.l;
86
```

```
c.invar():
       return make_pair(c, rm);
88
   }
89
   bint operator/(const bint&a, ll b){return ldiv(a, b).first;}
   11 operator %(const bint&a, ll b){return ldiv(a, b).second;}
   pair<bint, bint> ldiv(const bint& a, const bint& b){
     bint c;
93
       bint rm = 0;
94
       dform(i, a.1){
95
            if (rm.l==1 && !rm.n[0])
                rm.n[0] = a.n[i];
97
98
                dforn(j, rm.l) rm.n[j+1] = rm.n[j];
99
                rm.n[0] = a.n[i]:
100
                rm.l++:
101
           }
102
            ll q = rm.n[b.1] * BASE + rm.n[b.1-1];
103
           ll u = q / (b.n[b.l-1] + 1);
           ll v = a / b.n[b.l-1] + 1:
            while (u < v-1){
                11 m = (u+v)/2;
                if (b*m \le rm) u = m;
                else v = m;
109
            c.n[i]=u;
111
            rm-=b*u;
112
       }
113
     c.l=a.l:
114
       c.invar();
115
116
       return make_pair(c, rm);
117
   bint operator/(const bint&a, const bint&b){return ldiv(a, b).first;}
bint operator %(const bint&a, const bint&b) {return ldiv(a, b).second;}
2.9. Modnum
 1 | struct mnum{
      static const tipo mod=12582917:
     tipo v:
3
     mnum(tipo v=0): v(v mod) {}
     mnum operator+(mnum b){return v+b.v;}
     mnum operator-(mnum b){return v>=b.v? v-b.v : mod-b.v+v;}
     mnum operator*(mnum b){return v*b.v;}
```

```
mnum operator^(int n){
                                                                                               if (\text{key} \le \text{t->key}) splitKey(t->1, key, 1, t->1), r = t;
                                                                                   36
                                                                                               else splitKey(t->r, key, t->r, r), l = t;
       if(!n) return 1;
9
                                                                                   37
       return n%2? (*this)^(n/2)*(this) : (*this)^(n/2);}
                                                                                               pull(t);
10
                                                                                   38
11 };
                                                                                          }
                                                                                   39
                                                                                           void insertKey(Key key) {
                                                                                   40
2.10. Treap
                                                                                               pnode elem = new node(key, rand());
                                                                                   41
                                                                                               pnode t1, t2; splitKey(root, key, t1, t2);
                                                                                   42
                                                                                               t1=merge(t1,elem);
                                                                                   43
   typedef int Key;
                                                                                               root=merge(t1,t2);
                                                                                   44
   typedef struct node *pnode;
                                                                                   45
                                                                                           void eraseKeys(Key key1, Key key2) {
                                                                                   46
   struct node{
                                                                                               pnode t1,t2,t3;
       Key key;
                                                                                   47
                                                                                               splitKey(root,key1,t1,t2);
       int prior, size;
                                                                                   48
                                                                                               splitKey(t2,key2, t2, t3);
       pnode 1,r;
                                                                                               root=merge(t1,t3);
       node(Key key=0, int prior=0): key(key), prior(prior), size(1), 1(0),
                                                                                   50
                                                                                   51
            r(0) {}
                                                                                           void eraseKey(pnode &t, Key key) {
  };
                                                                                   52
9
                                                                                               if (!t) return;
                                                                                   53
   struct treap {
10
                                                                                               push(t):
       pnode root;
                                                                                   54
                                                                                               if (key == t->key) t=merge(t->1, t->r);
       treap(): root(0) {}
                                                                                               else if (key < t->key) eraseKey(t->1, key);
       int size(pnode p) { return p ? p->size : 0; }
                                                                                   56
13
                                                                                               else eraseKey(t->r, key);
       int size() { return size(root); }
                                                                                   57
14
                                                                                               pull(t);
       void push(pnode p) {
                                                                                   58
15
                                                                                           }
           // modificar y propagar el dirty a los hijos aca(para lazy)
                                                                                   59
16
                                                                                          void eraseKey(Key key) {eraseKey(root, key);}
       }
                                                                                   60
17
                                                                                          pnode findKey(pnode t, Key key) {
       // Update function and size from children's values
                                                                                   61
18
                                                                                               if (!t) return 0;
       void pull(pnode p) {//recalcular valor del nodo aca (para rmq)
                                                                                   62
19
                                                                                               if (key == t->key) return t;
           p->size = 1 + size(p->1) + size(p->r);
                                                                                   63
20
                                                                                               if (key < t->key) return findKey(t->1, key);
                                                                                   64
^{21}
                                                                                               return findKey(t->r, key);
       pnode merge(pnode 1, pnode r) {
                                                                                   65
22
                                                                                           }
           if (!1 || !r) return 1 ? 1 : r;
                                                                                   66
23
                                                                                           pnode findKey(Key key) { return findKey(root, key); }
           push(1), push(r);
                                                                                   67
^{24}
                                                                                          //****POS OPERATIONS****// No mezclar con las funciones Key
                                                                                   68
           pnode t;
25
                                                                                          //(No funciona con pos:)
           if (1->prior < r->prior) 1->r=merge(1->r, r), t = 1;
                                                                                   69
26
                                                                                          void splitSize(pnode t, int sz, pnode &1, pnode &r) {
           else r\rightarrow l=merge(1, r\rightarrow 1), t = r;
                                                                                   70
27
                                                                                               if (!t) return void(1 = r = 0);
                                                                                   71
           pull(t);
28
                                                                                               push(t);
                                                                                   72
           return t;
29
                                                                                               if (sz \le size(t->1)) splitSize(t->1, sz, 1, t->1), r = t;
       }//opcional:
                                                                                   73
30
                                                                                               else splitSize(t->r, sz - 1 - size(t->l), t->r, r), l = t;
       void merge(treap t) {root = merge(root, t.root), t.root=0;}
                                                                                   74
31
                                                                                               pull(t);
       //*****KEY OPERATIONS****//
                                                                                   75
32
                                                                                          }
                                                                                   76
       void splitKey(pnode t, Key key, pnode &1, pnode &r) {
33
                                                                                           void insertPos(int pos, Key key) {
           if (!t) return void(1 = r = 0);
                                                                                   77
34
                                                                                               pnode elem = new node(key, rand());
                                                                                   78
           push(t);
35
```

```
pnode t1,t2; splitSize(root, pos, t1, t2);
79
           t1=merge(t1,elem);
80
           root=merge(t1,t2);
81
       }
82
       void erasePos(int pos1, int pos2=-1) {
83
       if(pos2==-1) pos2=pos1+1;
84
           pnode t1, t2, t3;
85
           splitSize(root,pos1,t1,t2);
86
           splitSize(t2,pos2-pos1,t2,t3);
87
           root=merge(t1, t2);
88
89
       pnode findPos(pnode t, int pos) {
90
           if(!t) return 0:
91
           if(pos <= size(t->1)) return findPos(t->1, pos);
92
           return findPos(t->r, pos - 1 - size(t->l));
93
94
       Key &operator[](int pos){return findPos(root, pos)->key;}//ojito
95
96
```

## 3. Algos

## 3.1. Longest Increasing Subsecuence

```
//Para non-increasing, cambiar comparaciones y revisar busq binaria
   //Given an array, paint it in the least number of colors so that each
       color turns to a non-increasing subsequence.
  //Solution:Min number of colors=Length of the longest increasing
       subsequence
  int N, a[MAXN];//secuencia y su longitud
  ii d[MAXN+1];//d[i]=ultimo valor de la subsecuencia de tamanio i
   int p[MAXN];//padres
  vector<int> R;//respuesta
   void rec(int i){
     if(i==-1) return;
9
     R.push_back(a[i]);
10
     rec(p[i]);
11
12
  int lis(){//O(nlogn)
13
     d[0] = ii(-INF, -1); forn(i, N) d[i+1]=ii(INF, -1);
14
     forn(i, N){
15
       int j = upper_bound(d, d+N+1, ii(a[i], INF))-d;
16
       if (d[j-1].first < a[i]&&a[i] < d[j].first){</pre>
17
         p[i]=d[j-1].second;
18
```

```
d[j] = ii(a[i], i);
19
20
     }
21
     R.clear();
22
     dforn(i, N+1) if(d[i].first!=INF){
23
       rec(d[i].second);//reconstruir
24
       reverse(R.begin(), R.end());
       return i;//longitud
    }
27
     return 0;
28
29 }
```

#### 3.2. Manacher

```
1 int d1[MAXN];//d1[i]=long del maximo palindromo impar con centro en i
  int d2[MAXN];//d2[i]=analogo pero para longitud par
3 //0 1 2 3 4
   //a a b c c <--d1[2]=3
   //a a b b <--d2[2]=2 (estan uno antes)
   void manacher(){
     int l=0, r=-1, n=sz(s);
     forn(i, n){
       int k=(i>r? 1 : min(d1[l+r-i], r-i));
9
       while(i+k<n && i-k>=0 && s[i+k]==s[i-k]) ++k;
10
       d1[i] = k--:
11
       if(i+k > r) l=i-k, r=i+k;
12
13
     l=0, r=-1;
14
     forn(i, n){
15
       int k=(i>r? 0 : min(d2[1+r-i+1], r-i+1))+1;
       while(i+k-1 \le k = 0 \ k \le [i+k-1] == s[i-k]) k++;
17
       d2[i] = --k;
       if(i+k-1 > r) l=i-k, r=i+k-1;
19
20
```

## 4. Strings

#### 4.1. KMP

```
string T;//cadena donde buscar(where)
string P;//cadena a buscar(what)
int b[MAXLEN];//back table
void kmppre(){//by gabina with love
```

```
int i = 0, j=-1; b[0]=-1;
                                                                                         zero(f);
 5
                                                                                        forn(i, n) f[rBOUND(i+k)]++;
       while(i<sz(P)){</pre>
 6
                                                                                   10
           while(j>=0 && P[i] != P[j]) j=b[j];
                                                                                         int sum=0;
                                                                                   11
           i++, j++;
                                                                                        forn(i, max(255, n)){
                                                                                   12
 8
                                                                                           int t=f[i]; f[i]=sum; sum+=t;}
           b[i] = j;
                                                                                   13
 9
       }
                                                                                        forn(i, n)
10
                                                                                   14
                                                                                           tmpsa[f[rBOUND(sa[i]+k)]++]=sa[i];
11
                                                                                   15
                                                                                        memcpy(sa, tmpsa, sizeof(sa));
                                                                                   16
^{12}
   void kmp(){
                                                                                   17
13
                                                                                       void constructsa(){\frac{}{0} (n \log n)}
       int i=0, j=0;
14
                                                                                        n=sz(s);
       while(i<sz(T)){</pre>
15
                                                                                   19
                                                                                        forn(i, n) sa[i]=i, r[i]=s[i];
           while(j>=0 && T[i]!=P[j]) j=b[j];
                                                                                   20
16
                                                                                        for(int k=1: k<n: k<<=1){
           i++, j++;
17
                                                                                   21
           if(j==sz(P)){
                                                                                           countingSort(k), countingSort(0);
                                                                                   22
18
                printf("P_is_found_at_index_i/d_in_T\n", i-j);
                                                                                          int rank, tmpr[MAX_N];
                                                                                   23
19
                                                                                           tmpr[sa[0]]=rank=0;
                i=b[i];
20
                                                                                   24
           }
                                                                                           forr(i, 1, n)
21
                                                                                   25
       }
                                                                                             tmpr[sa[i]] = r[sa[i-1]] \&\& r[sa[i]+k] = r[sa[i-1]+k])?
22
23 |}
                                                                                                 rank: ++rank:
                                                                                           memcpy(r, tmpr, sizeof(r));
       Trie
                                                                                           if(r[sa[n-1]]==n-1) break;
                                                                                   28
                                                                                        }
                                                                                   29
   struct trie{
                                                                                   30
     map<char, trie> m;
 2
                                                                                      void print(){//for debug
     void add(const string &s, int p=0){
                                                                                        forn(i, n)
                                                                                   32
       if(s[p]) m[s[p]].add(s, p+1);
 4
                                                                                           cout << i << ',,' <<
                                                                                   33
     }
 5
                                                                                           s.substr(sa[i], s.find( '$', sa[i])-sa[i]) << endl;}
                                                                                   34
     void dfs(){
       //Do stuff
                                                                                          String Matching With Suffix Array
       forall(it, m)
 8
         it->second.dfs();
9
                                                                                    1 //returns (lowerbound, upperbound) of the search
     }
10
                                                                                      ii stringMatching(string P){ //O(sz(P)lgn)
11 | };
                                                                                         int lo=0, hi=n-1, mid=lo;
       Suffix Array (largo, nlogn)
                                                                                         while(lo<hi){</pre>
                                                                                           mid=(lo+hi)/2;
                                                                                    5
                                                                                          int res=s.compare(sa[mid], sz(P), P);
  #define MAX_N 1000
                                                                                    6
   #define rBOUND(x) (x<n? r[x] : 0)
                                                                                           if(res>=0) hi=mid;
   //sa will hold the suffixes in order.
                                                                                           else lo=mid+1:
                                                                                    8
   int sa[MAX_N], r[MAX_N], n;
                                                                                        }
                                                                                    9
                                                                                        if(s.compare(sa[lo], sz(P), P)!=0) return ii(-1, -1);
   string s; //input string, n=sz(s)
                                                                                   10
 5
                                                                                        ii ans; ans.fst=lo;
                                                                                   11
  int f[MAX_N], tmpsa[MAX_N];
                                                                                        lo=0, hi=n-1, mid;
                                                                                   12
  void countingSort(int k){
                                                                                         while(lo<hi){</pre>
                                                                                   13
```

char pch;//caracter que conecta con padre trie(): tran(), idhoja(), padre(), link() {}

10

11

12

13

 $if(p \le z(s))$ {

trie &ch=next[s[p]];

tran[(int)s[p]]=&ch;

void insert(const string &s, int id=1, int p=0){//id>0!!!

```
mid=(lo+hi)/2:
                                                                                           ch.padre=this, ch.pch=s[p];
                                                                                  14
14
       int res=s.compare(sa[mid], sz(P), P);
                                                                                           ch.insert(s, id, p+1);
                                                                                  15
15
       if(res>0) hi=mid;
16
                                                                                  16
       else lo=mid+1;
                                                                                         else idhoja=id, szhoja=sz(s);
17
                                                                                  17
                                                                                  18
18
     if(s.compare(sa[hi], sz(P), P)!=0) hi--;
                                                                                       trie* get_link() {
19
                                                                                  19
     ans.snd=hi;
                                                                                         if(!link){
                                                                                 20
     return ans;
                                                                                           if(!padre) link=this;//es la raiz
21
                                                                                 21
22
     LCP (Longest Common Prefix)
                                                                                 24
                                                                                         return link;
                                                                                 25
   //Calculates the LCP between consecutives suffixes in the Suffix Array.
                                                                                      }
                                                                                 26
   //LCP[i] is the length of the LCP between sa[i] and sa[i-1]
                                                                                       trie* get_tran(int c) {
                                                                                 27
  int LCP[MAX_N], phi[MAX_N], PLCP[MAX_N];
                                                                                         if(!tran[c])
                                                                                 28
   void computeLCP(){//0(n)}
                                                                                  29
     phi[sa[0]]=-1;
                                                                                         return tran[c];
                                                                                 30
    forr(i, 1, n) phi[sa[i]]=sa[i-1];
                                                                                      }
                                                                                 31
     int L=0;
                                                                                       trie *get_nxthoja(){
                                                                                 32
     forn(i, n){
       if(phi[i]==-1) {PLCP[i]=0; continue;}
9
                                                                                         return nxthoja;
                                                                                 34
       while(s[i+L] == s[phi[i]+L]) L++;
10
                                                                                      }
                                                                                 35
       PLCP[i]=L:
11
                                                                                      void print(int p){
                                                                                 36
       L=max(L-1, 0);
12
                                                                                         if(idhoja)
                                                                                 37
13
                                                                                 38
     forn(i, n) LCP[i]=PLCP[sa[i]];
14
15
                                                                                 39
      Corasick
                                                                                  40
                                                                                 41
                                                                                         print(p);
                                                                                  42
1
                                                                                  43
  struct trie{
2
     map<char, trie> next;
3
                                                                                        Geometria
     trie* tran[256];//transiciones del automata
4
     int idhoja, szhoja;//id de la hoja o 0 si no lo es
                                                                                  #define EPS 1e-9
     //link lleva al sufijo mas largo, nxthoja lleva al mas largo pero que
6
         es hoja
                                                                                  5.1. Punto
     trie *padre, *link, *nxthoja;
```

```
else if(!padre->padre) link=padre;//hijo de la raiz
        else link=padre->get_link()->get_tran(pch);
        tran[c] = !padre? this : this->get_link()->get_tran(c);
      if(!nxthoja) nxthoja = get_link()->idhoja? link : link->nxthoja;
        cout << "found" << idhoja << "LUatuposition" << p-szhoja << endl
      if(get_nxthoja()) get_nxthoja()->print(p);
    void matching(const string &s, int p=0){
      if(p<sz(s)) get_tran(s[p])->matching(s, p+1);
1 struct pto{
    tipo x, y;
```

pto(tipo x=0, tipo y=0):x(x),y(y){}

pto operator+(pto a){return pto(x+a.x, y+a.y);}

pto operator-(pto a){return pto(x-a.x, y-a.y);}

```
pto operator+(tipo a){return pto(x+a, y+a);}
                                                                                         if(c1==c2){}
     pto operator*(tipo a){return pto(x*a, y*a);}
                                                                                           return p1.y*p2.x<p1.x*p2.y;
                                                                                  49
     pto operator/(tipo a){return pto(x/a, y/a);}
     //dot product, producto interno:
                                                                                           return c1 < c2;
9
     tipo operator*(pto a){return x*a.x+y*a.y;}
                                                                                      }}
                                                                                  52
     //module of the cross product or vectorial product:
                                                                                     bool operator()(const pto&p1, const pto&p2) const{
                                                                                     return cmp(pto(p1.x-r.x,p1.y-r.y),pto(p2.x-r.x,p2.y-r.y));
     //if a is less than 180 clockwise from b, a^b>0
     tipo operator^(pto a){return x*a.y-y*a.x;}
                                                                                     }
                                                                                  55
13
     //returns true if this is at the left side of line qr
                                                                                  56 };
14
     bool left(pto q, pto r){return ((q-*this)^(r-*this))>0;}
15
                                                                                  5.2. Line
     bool operator<(const pto &a) const{return x<a.x || (abs(x-a.x)<EPS &&
16
         y<a.y);}
                                                                                  1 | struct line{
   bool operator == (pto a) {return abs(x-a.x) < EPS && abs(y-a.y) < EPS;}
                                                                                       line() {}
     double norm(){return sqrt(x*x+y*y);}
                                                                                       double a,b,c;//Ax+By=C
     tipo norm_sq(){return x*x+y*y;}
19
                                                                                     //pto MUST store float coordinates!
20
                                                                                      line(double a, double b, double c):a(a),b(b),c(c){}
   double dist(pto a, pto b){return (b-a).norm();}
                                                                                       line(pto p, pto q): a(q.y-p.y), b(p.x-q.x), c(a*p.x+b*p.y) {}
                                                                                  6
   typedef pto vec;
                                                                                     }:
                                                                                  7
23
                                                                                     bool parallels(line 11, line 12){return abs(11.a*12.b-12.a*11.b)<EPS;}
   double angle(pto a, pto o, pto b){
                                                                                     pto inter(line 11, line 12){//intersection
     pto oa=a-o, ob=b-o;
25
                                                                                       double det=11.a*12.b-12.a*11.b:
     return atan2(oa^ob, oa*ob);}
26
                                                                                       if(abs(det) < EPS) return pto(INF, INF); //parallels
                                                                                 11
27
                                                                                       return pto(12.b*11.c-11.b*12.c, 11.a*12.c-12.a*11.c)/det;
                                                                                  12
    //rotate p by theta rads CCW w.r.t. origin (0,0)
                                                                                  13 }
   pto rotate(pto p, double theta){
29
     return pto(p.x*cos(theta)-p.y*sin(theta),
                                                                                  5.3. Segment
30
        p.x*sin(theta)+p.y*cos(theta));
31
                                                                                  1 | struct segm{
32
                                                                                       pto s,f;
33
   //orden total de puntos alrededor de un punto r
                                                                                       segm(pto s, pto f):s(s), f(f) {}
   struct Cmp{
                                                                                      pto closest(pto p) {//use for dist to point
35
     pto r;
36
                                                                                          double 12 = dist_sq(s, f);
                                                                                  5
     Cmp(pto _r)\{r = _r;\}
                                                                                          if(12==0.) return s;
37
                                                                                  6
     int cuad(const pto &a) const{
                                                                                          double t = ((p-s)*(f-s))/12;
38
       if(a.x > 0 \&\& a.y >= 0)return 0;
                                                                                          if (t<0.) return s;//not write if is a line
39
       if(a.x <= 0 && a.y > 0)return 1;
                                                                                          else if(t>1.)return f;//not write if is a line
40
       if(a.x < 0 && a.y <= 0)return 2;
41
                                                                                          return s+((f-s)*t);
                                                                                  10
       if(a.x >= 0 \&\& a.y < 0)return 3;
                                                                                      }
42
                                                                                 11
       assert(a.x ==0 && a.v==0);
                                                                                       bool inside(pto p){
43
       return -1;
                                                                                     return ((s-p)^(f-p))==0 \&\& min(s, f)<*this&&*this<max(s, f);}
44
                                                                                     };
45
                                                                                  14
     bool cmp(const pto&p1, const pto&p2)const{
46
                                                                                  15
       int c1 = cuad(p1), c2 = cuad(p2);
47
                                                                                 bool insidebox(pto a, pto b, pto p) {
```

```
return (a.x-p.x)*(p.x-b.x)>-EPS && (a.y-p.y)*(p.y-b.y)>-EPS;
                                                                                  23 //finds the center of the circle containing p1 and p2 with radius r
                                                                                     //as there may be two solutions swap p1, p2 to get the other
18
                                                                                     bool circle2PtsRad(pto p1, pto p2, double r, pto &c){
   pto inter(segm s1, segm s2){
19
     pto r=inter(line(s1.s, s1.f), line(s2.s, s2.f));
                                                                                              double d2=(p1-p2).norm_sq(), det=r*r/d2-0.25;
     if(insidebox(s1.s,s1.f,p) && insidebox(s2.s,s2.f,p))
                                                                                              if(det<0) return false;</pre>
                                                                                  27
21
                                                                                              c=(p1+p2)/2+perp(p2-p1)*sqrt(det);
         return r;
22
     return pto(INF, INF);
                                                                                              return true;
23
                                                                                  29
24 }
                                                                                  30
                                                                                     #define sqr(a) ((a)*(a))
     Polygon Area
                                                                                     #define feq(a,b) (fabs((a)-(b))<EPS)</pre>
                                                                                     pair<tipo, tipo > ecCuad(tipo a, tipo b, tipo c){//a*x*x+b*x+c=0
  |double area(vector<pto> &p){//0(sz(p))
                                                                                       tipo dx = sqrt(b*b-4.0*a*c);
     double area=0;
2
                                                                                       return make_pair((-b + dx)/(2.0*a), (-b - dx)/(2.0*a));
                                                                                  35
    forn(i, sz(p)) area+=p[i]^p[(i+1) %z(p)];
                                                                                  36
    //if points are in clockwise order then area is negative
                                                                                     pair<pto, pto> interCL(Circle c, line 1){
                                                                                  37
    return abs(area)/2:
5
                                                                                       bool sw=false;
  | }
6
                                                                                       if((sw=feq(0,1.b))){
  //Area ellipse = M_PI*a*b where a and b are the semi axis lengths
                                                                                       swap(1.a, 1.b);
\frac{1}{4} //Area triangle = sqrt(s*(s-a)(s-b)(s-c)) where s=(a+b+c)/2
                                                                                        swap(c.o.x, c.o.y);
                                                                                  41
5.5. Circle
                                                                                       pair<tipo, tipo> rc = ecCuad(
                                                                                  43
                                                                                       sqr(l.a)+sqr(l.b),
  vec perp(vec v){return vec(-v.y, v.x);}
                                                                                       2.0*1.a*1.b*c.o.y-2.0*(sqr(1.b)*c.o.x+1.c*1.a),
  line bisector(pto x, pto y){
                                                                                  45
                                                                                       sqr(1.b)*(sqr(c.o.x)+sqr(c.o.y)-sqr(c.r))+sqr(1.c)-2.0*1.c*1.b*c.o.y
     line l=line(x, y); pto m=(x+y)/2;
                                                                                  46
     return line(-1.b, 1.a, -1.b*m.x+1.a*m.y);
                                                                                  47
                                                                                       pair<pto, pto> p( pto(rc.first, (1.c - 1.a * rc.first) / 1.b),
                                                                                  48
5
                                                                                                  pto(rc.second, (1.c - 1.a * rc.second) / 1.b) );
   struct Circle{
                                                                                  49
                                                                                       if(sw){
                                                                                  50
     pto o;
7
                                                                                       swap(p.first.x, p.first.y);
     double r;
                                                                                  51
8
     Circle(pto x, pto y, pto z){
                                                                                       swap(p.second.x, p.second.y);
9
       o=inter(bisector(x, y), bisector(y, z));
                                                                                  53
10
                                                                                       return p;
       r=dist(o, x);
                                                                                  54
11
                                                                                  55
12
                                                                                     pair<pto, pto> interCC(Circle c1, Circle c2){
     pair<pto, pto> ptosTang(pto p){
13
                                                                                       line 1:
       pto m=(p+o)/2;
                                                                                  57
14
                                                                                       1.a = c1.o.x-c2.o.x;
       tipo d=dist(o, m);
15
                                                                                       1.b = c1.o.y-c2.o.y;
       tipo a=r*r/(2*d);
16
                                                                                       1.c = (sqr(c2.r) - sqr(c1.r) + sqr(c1.o.x) - sqr(c2.o.x) + sqr(c1.o.y)
       tipo h=sqrt(r*r-a*a);
17
                                                                                       -sqr(c2.o.y))/2.0;
       pto m2=o+(m-o)*a/d;
18
                                                                                       return interCL(c1, 1);
                                                                                  62
       vec per=perp(m-o)/d;
19
                                                                                  63 }
       return mkp(m2-per*h, m2+per*h);
20
21
22 | };
```

## 5.6. Point in Poly

```
1 //checks if v is inside of P, using ray casting
   //works with convex and concave.
   //excludes boundaries, handle it separately using segment.inside()
  bool inPolygon(pto v, vector<pto>& P) {
     bool c = false;
    forn(i, sz(P)){
6
       int j=(i+1) \%z(P);
      if((P[j].y>v.y) != (P[i].y > v.y) &&
8
     (v.x < (P[i].x - P[i].x) * (v.y-P[i].y) / (P[i].y - P[i].y) + P[i].x))
         c = !c;
    }
11
    return c;
12
13 }
```

#### 5.7. Convex Check CHECK

```
bool isConvex(vector<int> &p){//O(N)
int N=sz(p);
if(N<3) return false;
bool isLeft=p[0].left(p[1], p[2]);
forr(i, 1, N)
if(p[i].left(p[(i+1) %], p[(i+2) %])!=isLeft)
return false;
return true; }</pre>
```

#### 5.8. Convex Hull

```
//stores convex hull of P in S, CCW order
   void CH(vector<pto>& P, vector<pto> &S){
     S.clear();
3
     sort(P.begin(), P.end());
4
     forn(i, sz(P)){
5
       while(sz(S) \ge 2 \&\& S[sz(S)-1].left(S[sz(S)-2], P[i])) S.pop_back();
6
       S.pb(P[i]);
7
8
     S.pop_back();
9
     int k=sz(S):
10
     dforn(i, sz(P)){
11
       \label{eq:while(sz(S) >= k+2 && S[sz(S)-1].left(S[sz(S)-2], P[i])) S.pop_back} \\
12
            ();
       S.pb(P[i]);
13
     }
14
```

```
S.pop_back();
16 }
5.9. Cut Polygon
1 //cuts polygon Q along the line ab
2 //stores the left side (swap a, b for the right one) in P
   void cutPolygon(pto a, pto b, vector<pto> Q, vector<pto> &P){
    P.clear():
    forn(i, sz(Q)){
      double left1=(b-a)^(Q[i]-a), left2=(b-a)^(Q[(i+1) \sz(Q)]-a);
      if(left1>=0) P.pb(Q[i]);
7
      if(left1*left2<0)
8
        P.pb(inter(line(Q[i], Q[(i+1) \slashz(Q)]), line(a, b)));
9
    }
10
11 }
5.10. Bresenham
1 //plot a line approximation in a 2d map
  void bresenham(pto a, pto b){
     pto d=b-a; d.x=abs(d.x), d.y=abs(d.y);
    pto s(a.x<b.x? 1: -1, a.y<b.y? 1: -1);
    int err=d.x-d.v;
    while(1){
      m[a.x][a.y]=1;//plot
      if(a==b) break;
      int e2=2*err;
      if(e2 > -d.y){
         err-=d.y, a.x+=s.x;
      if(e2 < d.x)
         err+= d.x, a.y+= s.y;
13
    }
14
15 }
5.11. Interseccion de Circulos en n3log(n)
1 struct event {
       double x: int t:
       event(double xx, int tt) : x(xx), t(tt) {}
       bool operator <(const event &o) const { return x < o.x; }</pre>
4
  };
5
  typedef vector<Circle> VC;
```

7 typedef vector<event> VE;

```
8 | int n;
   double cuenta(VE &v, double A,double B) {
       sort(v.begin(), v.end());
10
       double res = 0.0, lx = ((v.empty())?0.0:v[0].x);
11
       int contador = 0;
12
       forn(i,sz(v)) {
13
           // interseccion de todos (contador == n), union de todos (
14
                contador > 0),
           // conjunto de puntos cubierto por exacta k Circulos (contador
15
               == k
           if (contador == n) res += v[i].x - lx;
16
           contador += v[i].t;
17
           lx = v[i].x:
18
       }
19
       return res;
20
21
    / Primitiva de sqrt(r*r - x*x) como funcion double de una variable x.
22
   inline double primitiva(double x,double r) {
23
       if (x \ge r) return r*r*M PI/4.0:
24
       if (x \le -r) return -r*r*M_PI/4.0;
25
       double raiz = sqrt(r*r-x*x);
26
       return 0.5 * (x * raiz + r*r*atan(x/raiz));
27
28
   double interCircle(VC &v) {
29
       vector<double> p; p.reserve(v.size() * (v.size() + 2));
30
       forn(i,sz(v)) {
31
           p.push_back(v[i].c.x + v[i].r);
32
           p.push_back(v[i].c.x - v[i].r);
33
       }
34
       forn(i,sz(v)) forn(j,i) {
35
           Circle &a = v[i], b = v[j];
36
           double d = (a.c - b.c).norm();
37
           if (fabs(a.r - b.r) < d \&\& d < a.r + b.r) {
38
               double alfa = acos((sqr(a.r) + sqr(d) - sqr(b.r)) / (2.0 * d)
39
                     * a.r)):
               pto vec = (b.c - a.c) * (a.r / d);
40
               p.pb((a.c + rotate(vec, alfa)).x);
41
               p.pb((a.c + rotate(vec, -alfa)).x);
42
           }
43
44
       sort(p.begin(), p.end());
45
       double res = 0.0;
46
       forn(i,sz(p)-1) {
47
```

```
const double A = p[i], B = p[i+1];
48
           VE ve; ve.reserve(2 * v.size());
49
           forn(j,sz(v)) {
50
                const Circle &c = v[j];
51
                double arco = primitiva(B-c.c.x,c.r) - primitiva(A-c.c.x,c.r
52
                    );
                double base = c.c.v * (B-A);
53
                ve.push_back(event(base + arco,-1));
                ve.push_back(event(base - arco, 1));
55
           res += cuenta(ve,A,B);
57
58
       return res;
59
60 }
```

## 6. Math

#### 6.1. Identidades

```
\sum_{i=0}^{n} \binom{n}{i} = 2^{n}
\sum_{i=0}^{n} i \binom{n}{i} = n * 2^{n-1}
\sum_{i=m}^{n} i = \frac{n(n+1)}{2} - \frac{m(m-1)}{2} = \frac{(n+1-m)(n+m)}{2}
\sum_{i=0}^{n} i = \sum_{i=1}^{n} i = \frac{n(n+1)}{2}
\sum_{i=0}^{n} i^{2} = \frac{n(n+1)(2n+1)}{6} = \frac{n^{3}}{3} + \frac{n^{2}}{2} + \frac{n}{6}
\sum_{i=0}^{n} i(i-1) = \frac{8}{6} (\frac{n}{2})(\frac{n}{2}+1)(n+1) \text{ (doubles)} \rightarrow \text{Sino ver caso impar y par}
\sum_{i=0}^{n} i^{3} = \left(\frac{n(n+1)}{2}\right)^{2} = \frac{n^{4}}{4} + \frac{n^{3}}{2} + \frac{n^{2}}{4} = \left[\sum_{i=1}^{n} i\right]^{2}
\sum_{i=0}^{n} i^{4} = \frac{n(n+1)(2n+1)(3n^{2}+3n-1)}{30} = \frac{n^{5}}{5} + \frac{n^{4}}{2} + \frac{n^{3}}{3} - \frac{n}{30}
\sum_{i=0}^{n} i^{p} = \frac{(n+1)^{p+1}}{p+1} + \sum_{k=1}^{p} \frac{B_{k}}{p-k+1} \binom{p}{k}(n+1)^{p-k+1}
r = e - v + k + 1
Teorema de Pick: (Area, puntos interiores y puntos en el borde)
A = I + \frac{B}{2} - 1
```

#### 6.2. Ec. Caracteristica

```
a_0T(n) + a_1T(n-1) + \dots + a_kT(n-k) = 0
p(x) = a_0x^k + a_1x^{k-1} + \dots + a_k
Sean r_1, r_2, \dots, r_q las raíces distintas, de mult. m_1, m_2, \dots, m_q
T(n) = \sum_{i=1}^q \sum_{j=0}^{m_i-1} c_{ij} n^j r_i^n
Las constantes c_{ij} se determinan por los casos base.
```

#### 6.3. Combinatorio

```
forn(i, MAXN+1){//comb[i][k]=i tomados de a k
    comb[i][0]=comb[i][i]=1;
    forr(k, 1, i) comb[i][k]=(comb[i-1][k]+comb[i-1][k-1]) MOD;
}

ll lucas (ll n, ll k, int p){ //Calcula (n,k) %p teniendo comb[p][p]
    precalculado.

ll aux = 1;
    while (n + k){
    aux = (aux * comb[n %p][k %p]) %p;
    n/=p, k/=p;
}
return aux;
}
```

## 6.4. Exp. de Numeros Mod.

```
1  | ll expmod (ll b, ll e, ll m){//0(log b)
2  | if(!e) return 1;
3  | ll q= expmod(b,e/2,m); q=(q*q) %m;
4  | return e%2? (b * q) %m : q;
5  |}
```

## 6.5. Exp. de Matrices y Fibonacci en log(n)

```
struct M22{
                    // |a b|
     tipo a,b,c,d;// |c d|
     M22 operator*(const M22 &p) const {
       return (M22){a*p.a+b*p.c, a*p.b+b*p.d, c*p.a+d*p.c,c*p.b+d*p.d};}
4
5
   M22 operator (const M22 &p, int n){
     if(!n) return (M22){1, 0, 0, 1};//identidad
     M22 q=p^(n/2); q=q*q;
     return n%2? p * q : q;}
10
   ll fibo(ll n){//calcula el fibonacci enesimo
    M22 \text{ mat}=(M22)\{0, 1, 1, 1\}^n;
    return mat.a*f0+mat.b*f1;//f0 y f1 son los valores iniciales
13
14 }
```

#### 6.6. Teorema Chino del Resto

$$y = \sum_{j=1}^{n} (x_j * (\prod_{i=1, i \neq j}^{n} m_i)_{m_j}^{-1} * \prod_{i=1, i \neq j}^{n} m_i)$$

## 6.7. Funciones de primos

```
1 | 11 numPrimeFactors (11 n){
     ll rta = 0;
    map<ll,11> f=fact(n);
     forall(it, f) rta += it->second;
    return rta;
   11 numDiffPrimeFactors (11 n){
    ll rta = 0;
    map<11,11> f=fact(n);
    forall(it, f) rta += 1;
     return rta:
13
   11 sumPrimeFactors (11 n){
    ll rta = 0:
     map<11,11> f=fact(n);
     forall(it, f) rta += it->first;
     return rta;
19
20
21
   11 numDiv (ll n){
    ll rta = 1;
     map<ll,ll> f=fact(n);
     forall(it, f) rta *= (it->second + 1);
25
     return rta;
26
27
28
   ll sumDiv (ll n){
    ll rta = 1:
     map<11,11> f=fact(n);
31
     forall(it, f) rta *= ((ll)pow((double)it->first, it->second + 1.0)-1)
         / (it->first-1);
33
    return rta;
34 }
```

19

```
forn (i, s-1){
35
   ll eulerPhi (ll n){ // con criba: O(lg n)
                                                                                          x = (x*x) n;
                                                                                   21
36
     11 \text{ rta} = n;
                                                                                          if (x == 1) return false;
37
     map<ll,ll> f=fact(n);
                                                                                          if (x+1 == n) return true;
38
     forall(it, f) rta -= rta / it->first;
                                                                                        }
                                                                                   24
     return rta;
                                                                                        return false;
40
                                                                                   25
                                                                                   26
41
42
                                                                                   27
   11 eulerPhi2 (11 n){ // 0 (sqrt n)
                                                                                      bool miller_rabin (ll n){ //devuelve true si n es primo
                                                                                        if (n == 1) return false;
     11 r = n;
44
                                                                                        const int ar[] = \{2,3,5,7,11,13,17,19,23\};
     forr (i,2,n+1){
45
       if ((ll)i*i > n)
                                                                                        forn (j,9)
                                                                                          if (!es_primo_prob(n,ar[j]))
         break:
47
       if (n \% i == 0){
                                                                                            return false:
                                                                                   33
         while (n\%i == 0) n/=i;
                                                                                        return true;
                                                                                   34
49
         r = r/i;
50
                                                                                   35
       }}
51
                                                                                      ll pollard_rho (ll n, ll c=1){
     if (n != 1)
                                                                                        int i = 0, k = 2;
       r=r/n:
53
                                                                                        11 x = 3, y = 3;
     return r;
54
55 }
                                                                                        if(c>=n) return -1;//FAILURE
                                                                                        //~ if(c!=1) dprint(c);
6.8. Phollard's Rho (rolando)
                                                                                        while (1){
                                                                                         i++;
                                                                                          x = (mulmod (x,x,n) + c) %n;
1 | 11 mulmod (11 a, 11 b, 11 c) { //returns (a*b) %, and minimize overfloor
                                                                                   44
                                                                                          ll d = gcd (abs(y-x), n);
     11 x = 0, y = a\%;
                                                                                          if(d==n) return pollard_rho(n, c+1);
     while (b > 0){
                                                                                   46
                                                                                          if (d != 1) return d;
      if (b \% 2 == 1) x = (x+y) \% c;
                                                                                          if (i == k) y = x, k*=2;
       y = (y*2) \% c;
                                                                                   48
                                                                                   49
       b /= 2;
                                                                                   50
7
                                                                                   51
     return x % c;
8
                                                                                   52 | 11 brent(11 n){
9
                                                                                          srand(time(NULL));
10
                                                                                              if (n \%2 == 0)
   bool es_primo_prob (ll n, int a)
                                                                                   54
11
                                                                                   55
12
                                                                                              ll y = rand() \%(n-1)+1, c = rand() \%(n-1)+1, m = rand() \%(n-1)+1;
     if (n == a) return true;
13
                                                                                              ll g,r,q,x,k,ys;
     ll s = 0.d = n-1:
                                                                                   57
14
                                                                                               g = r = q = 1;
     while (d \% 2 == 0) s++, d/=2;
15
                                                                                              while (g==1){
16
                                                                                                       x = y;
     11 x = expmod(a,d,n);
17
                                                                                                       forn (i,r)
     if ((x == 1) \mid | (x+1 == n)) return true;
                                                                                   61
18
```

62

y = ((y\*y) /n+c) /n;

```
k = 0:
63
                     while (k < r \&\& g == 1){
64
                               ys = y;
65
                               forn (i,min(m,r-k)){
66
                                        y = ((y*y) /n+c) /n;
67
                                        q = q*(x-y) n;
68
                   }
69
                               g = gcd(q,n);
70
                               k = k + m;
71
72
                     r = r*2;
73
74
            if (g==n){
75
                      while (true){
76
                              ys = ((ys*ys) %n+c) %n;
77
                              g = gcd(abs(x-ys),n);
78
                               if (g>1)
79
                                      break;
80
              }
81
82
83
84
            return g;
85 | }
```

### 6.9. Criba

```
#define MAXP 80000 //no necesariamente primo
  int criba[MAXP+1];
   vector<int> primos;
   void buscarprimos(){
     int sq=sqrt(MAXP)+1;
5
     forr(p, 2, MAXP+1) if(!criba[p]){
6
       primos.push_back(p);
7
       if(p \le sq)
8
         for(int m=p*p; m<=MAXP; m+=p)//borro los multiplos de p</pre>
           if(!criba[m])criba[m]=p;
     }
11
12
```

## 6.10. Factorizacion

```
Sea n = \prod p_i^{k_i}, fact(n) genera un map donde a cada p_i le asocia su k_i

//factoriza bien numeros hasta MAXP^2
map<11,11> fact(11 n){ //0 (cant primos)
```

```
map<ll,ll> ret;
    forall(p, primos){
4
      while(!(n %*p)){
        ret[*p]++;//divisor found
        n/=*p;
8
     if(n>1) ret[n]++;
     return ret;
11
12
13
   //factoriza bien numeros hasta MAXP
   map<11,11> fact2(11 n) { //0 (lg n)
     map<ll,ll> ret;
     while (criba[n]){
      ret[criba[n]]++;
      n/=criba[n];
19
    }
20
    if(n>1) ret[n]++;
21
     return ret;
23 }
6.11. GCD
tipo gcd(tipo a, tipo b){return a?gcd(b %a, a):b;}
6.12. Extended Euclid
void extendedEuclid (ll a, ll b) \{ //a * x + b * y = d \}
    if (!b) { x = 1; y = 0; d = a; return;}
    extendedEuclid (b, a%);
    11 x1 = y;
    11 y1 = x - (a/b) * y;
     x = x1; y = y1;
6
7 }
6.13. LCM
tipo lcm(tipo a, tipo b){return a / gcd(a,b) * b;}
6.14. Inversos
```

1 #define MAXMOD 15485867

```
6 MATH - 6.17 Polinomio
2 | ll inv[MAXMOD];//inv[i]*i=1 mod MOD
  void calc(int p){\frac{1}{0}}
     inv[1]=1;
4
    forr(i, 2, p) inv[i] = p-((p/i)*inv[p\%i])\%;
6
   int inverso(int x){\frac{1}{0}} \log x
     return expmod(x, eulerphi(MOD)-2);//si mod no es primo(sacar a mano)
     return expmod(x, MOD-2);//si mod es primo
10 }
6.15. Simpson
  double integral(double a, double b, int n=10000) {//O(n), n=cantdiv
     double area=0, h=(b-a)/n, fa=f(a), fb;
2
     forn(i, n){
3
       fb=f(a+h*(i+1)):
       area+=fa+ 4*f(a+h*(i+0.5)) +fb. fa=fb:
5
6
```

#### 6.16. Fraction

return area\*h/6.;}

```
tipo mcd(tipo a, tipo b){return a?mcd(b%a, a):b;}
  struct frac{
     tipo p,q;
     frac(tipo p=0, tipo q=1):p(p),q(q) {norm();}
     void norm(){
       tipo a = mcd(p,q);
      if(a) p/=a, q/=a;
       else q=1;
8
       if (q<0) q=-q, p=-p;}
9
     frac operator+(const frac& o){
10
       tipo a = mcd(q, o.q);
11
       return frac(p*(o.q/a)+o.p*(q/a), q*(o.q/a));}
12
     frac operator-(const frac& o){
13
       tipo a = mcd(q, o.q);
14
       return frac(p*(o.q/a)-o.p*(q/a), q*(o.q/a));}
15
     frac operator*(frac o){
16
       tipo a = mcd(q,o.p), b = mcd(o.q,p);
17
       return frac((p/b)*(o.p/a), (q/a)*(o.q/b));}
18
     frac operator/(frac o){
19
       tipo a = mcd(q,o.q), b = mcd(o.p,p);
20
       return frac((p/b)*(o.q/a),(q/a)*(o.p/b));}
21
     bool operator<(const frac &o) const{return p*o.q < o.p*q;}</pre>
22
```

```
bool operator==(frac o){return p==o.p&kq==o.q;}
24 };
6.17. Polinomio
struct poly {
       vector<tipo> c;//guarda los coeficientes del polinomio
2
       poly(const vector<tipo> &c): c(c) {}
3
       poly() {}
4
     int gr(){//calculates grade of the polynomial
       return sz(c); }
6
     bool isnull() {return c.empty();}
7
       poly operator+(const poly &o) const {
           int m = sz(c), n = sz(o.c);
9
           vector<tipo> res(max(m,n));
           forn(i, m) res[i] += c[i];
11
           forn(i, n) res[i] += o.c[i];
           return poly(res);
13
14
       poly operator*(const poly &o) const {
15
           int m = sz(c), n = sz(o.c);
           vector<tipo> res(m+n-1);
17
           forn(i, m) forn(j, n) res[i+j]+=c[i]*o.c[j];
           return poly(res);
19
20
     tipo eval(tipo v) {
21
       tipo sum = 0;
22
       forall(it, c) sum=sum*v + *it;
       return sum;
24
25
     //the following function generates the roots of the polynomial
26
   //it can be easily modified to return float roots
27
     set<tipo> roots(){
28
       set<tipo> roots;
29
       tipo a0 = abs(c[0]), an = abs(c[sz(c)-1]);
30
       vector<tipo> ps,qs;
31
       forr(p,1,sqrt(a0)+1) if (a0\%==0) ps.pb(p),ps.pb(a0/p);
32
       forr(q,1,sqrt(an)+1) if (an)(q=0) qs.pb(q),qs.pb(an/q);
33
       forall(pt,ps)
34
         forall(qt,qs) if ( (*pt) % (*qt)==0 ) {
35
           tipo root = abs((*pt) / (*qt));
36
           if (eval(root)==0) roots.insert(root);
37
         }
38
```

```
return roots;
39
    }
40
41
   poly interpolate(const vector<tipo> &x, const vector<tipo> &y) {
       int n = sz(x);
43
       poly p;
44
       vector<tipo> aux(2);
45
       forn(i, n) {
46
          double a = y[i] - p.eval(x[i]);
47
          forn(j, i) a /= x[i] - x[j];
48
          poly add(vector<tipo>(1, a));
49
          forn(j, i) aux[0]=-x[j], aux[1]=1, add = add*aux;
50
          p = p + add;
51
       }
52
       return p;
53
54
    //the following functions allows parsing an expression like
   //34+150+4*45
   //into a polynomial(el numero en funcion de la base)
   #define LAST(s) (sz(s)? s[sz(s)-1]:0)
   #define POP(s) s.erase(--s.end());
   poly D(string &s) {
     poly d;
61
     for(int i=0; isdigit(LAST(s)); i++) d.c.push_back(LAST(s)-'0'), POP(s)
62
     return d;}
63
64
   poly T(string &s) {
65
     poly t=D(s);
66
     if (LAST(s)=='*'){POP(s); return T(s)*t;}
67
     return t;
68
69
   //main function, call this to parse
   poly E(string &s) {
71
     poly e=T(s);
72
     if (LAST(s)=='+')\{POP(s); return E(s)+e;\}
73
     return e;
74
75 | }
```

## 7. Grafos

## 7.1. Dijkstra

```
1 #define INF 1e9
  int N;
2
   #define MAX_V 250001
   vector<ii> G[MAX_V];
   //To add an edge use
   #define add(a, b, w) G[a].pb(mkp(w, b))
   ll dijkstra(int s, int t){\frac{}{|0(|E| \log |V|)}}
     priority_queue<ii, vector<ii>, greater<ii> > Q;
     vector<ll> dist(N, INF); vector<int> dad(N, -1);
     Q.push(mkp(0, s)); dist[s] = 0;
11
     while(sz(Q)){
12
       ii p = Q.top(); Q.pop();
13
       if(p.snd == t) break;
       forall(it, G[p.snd])
15
         if(dist[p.snd]+it->first < dist[it->snd]){
           dist[it->snd] = dist[p.snd] + it->fst;
17
           dad[it->snd] = p.snd;
18
           Q.push(mkp(dist[it->snd], it->snd));
19
         }
20
     }
21
     return dist[t];
     if(dist[t]<INF)//path generator</pre>
23
       for(int i=t; i!=-1; i=dad[i])
         printf("%d%c", i, (i==s?'\n':'\_'));
25
26 }
7.2. Bellman-Ford
vector<ii> G[MAX_N];//ady. list with pairs (weight, dst)
  int dist[MAX_N];
   void bford(int src){//O(VE)
     dist[src]=0;
     forn(i, N-1) forn(j, N) if(dist[j]!=INF) forall(it, G[j])
       dist[it->snd]=min(dist[it->snd], dist[j]+it->fst);
6
   }
7
   bool hasNegCycle(){
     forn(j, N) if(dist[j]!=INF) forall(it, G[j])
       if(dist[it->snd]>dist[j]+it->fst) return true;
     //inside if: all points reachable from it->snd will have -INF distance
12
         (do bfs)
     return false;
13
```

qcmp++;

25

```
14 }
7.3. Floyd-Warshall
1 //G[i][j] contains weight of edge (i, j) or INF
   //G[i][i]=0
   int G[MAX_N] [MAX_N];
   void floyd(){//0(N^3)}
  forn(k, N) forn(i, N) if(G[i][k]!=INF) forn(j, N) if(G[k][j]!=INF)
     G[i][j]=min(G[i][j], G[i][k]+G[k][j]);
6
7
  bool inNegCycle(int v){
8
    return G[v][v]<0;}
   //checks if there's a neg. cycle in path from a to b
   bool hasNegCycle(int a, int b){
    forn(i, N) if(G[a][i]!=INF && G[i][i]<0 && G[i][b]!=INF)
12
       return true:
13
     return false;
14
15 }
7.4. Kruskal
struct Ar{int a,b,w;};
  |bool operator<(const Ar& a, const Ar &b){return a.w<b.w;}
   vector<Ar> E;
   11 kruskal(){
       11 cost=0;
5
       sort(E.begin(), E.end());//ordenar aristas de menor a mayor
6
       uf.init(n);
7
       forall(it, E){
8
           if(uf.comp(it->a)!=uf.comp(it->b)){//si no estan conectados
               uf.unir(it->a, it->b);//conectar
10
               cost+=it->w;
11
           }
12
       }
13
       return cost;
14
15 }
     \mathbf{Prim}
bool taken[MAXN];
  priority_queue<ii, vector<ii>, greater<ii> > pq;//min heap
  void process(int v){
3
       taken[v]=true;
```

```
forall(e, G[v])
5
           if(!taken[e->second]) pq.push(*e);
6
   }
7
   ll prim(){
       zero(taken);
       process(0);
       ll cost=0;
       while(sz(pq)){
13
           ii e=pq.top(); pq.pop();
           if(!taken[e.second]) cost+=e.first, process(e.second);
15
       }
16
       return cost;
17
18 }
7.6. 2-SAT + Tarjan SCC
1 //We have a vertex representing a var and other for his negation.
2 //Every edge stored in G represents an implication. To add an equation
       of the form a | |b, use addor(a, b)
3 //MAX=max cant var, n=cant var
   #define addor(a, b) (G[neg(a)].pb(b), G[neg(b)].pb(a))
   vector<int> G[MAX*2];
   //idx[i]=index assigned in the dfs
  //lw[i]=lowest index(closer from the root) reachable from i
   int lw[MAX*2], idx[MAX*2], qidx;
   stack<int> q;
   int qcmp, cmp[MAX*2];
   //verdad[cmp[i]]=valor de la variable i
   bool verdad[MAX*2+1];
13
   int neg(int x) { return x>=n? x-n : x+n;}
   void tjn(int v){
    lw[v]=idx[v]=++qidx;
16
    q.push(v), cmp[v]=-2;
17
    forall(it, G[v]){
18
       if(!idx[*it] || cmp[*it]==-2){
19
         if(!idx[*it]) tjn(*it);
20
         lw[v]=min(lw[v], lw[*it]);
21
       }
22
    }
23
     if(lw[v]==idx[v]){
24
```

```
int x:
26
       do{x=q.top(); q.pop(); cmp[x]=qcmp;}while(x!=v);
27
       verdad[qcmp] = (cmp[neg(v)] < 0);</pre>
28
29
30
    //remember to CLEAR G!!!
    bool satisf(){\frac{}{0(n)}}
     memset(idx, 0, sizeof(idx)), qidx=0;
33
     memset(cmp, -1, sizeof(cmp)), qcmp=0;
34
     forn(i, n){
35
       if(!idx[i]) tjn(i);
36
       if(!idx[neg(i)]) tjn(neg(i));
37
     }
38
     forn(i, n) if(cmp[i] == cmp[neg(i)]) return false;
     return true;
41 | }
```

#### 7.7. Articulation Points

```
1 | int N;
   vector<int> G[1000000];
   //V[i]=node number(if visited), L[i]= lowest V[i] reachable from i
   int qV, V[1000000], L[1000000], P[1000000];
   void dfs(int v, int f){
     L[v]=V[v]=++qV;
6
     forall(it, G[v])
7
       if(!V[*it]){
8
         dfs(*it, v);
9
         L[v] = min(L[v], L[*it]);
10
         P[v] += L[*it] >= V[v];
11
12
       else if(*it!=f)
13
         L[v]=min(L[v], V[*it]);
14
15
   int cantart() { //0(n)
16
17
     zero(V), zero(P);
18
     dfs(1, 0); P[1]--;
19
     int q=0;
     forn(i, N) if(P[i]) q++;
  return q;
22
23 }
```

## 7.8. Comp. Biconexas y Puentas

```
struct edge {
     int u,v, comp;
     bool bridge;
   };
4
   vector<edge> e;
   void addEdge(int u, int v) {
     G[u].pb(sz(e)), G[v].pb(sz(e));
     e.pb((edge){u,v,-1,false});
9
   //d[i]=id de la dfs
   //b[i]=lowest id reachable from i
   int d[MAXN], b[MAXN], t;
   int nbc;//cant componentes
   int comp[MAXN];//comp[i]=cant comp biconexas a la cual pertenece i
   void initDfs(int n) {
     zero(G), zero(comp);
    e.clear();
     forn(i,n) d[i]=-1;
     nbc = t = 0;
20
   stack<int> st;
   void dfs(int u, int pe) \{//0(n + m)\}
     b[u] = d[u] = t++;
     comp[u] = (pe != -1);
     forall(ne, G[u]) if (*ne != pe){
       int v = e[*ne].u ^ e[*ne].v ^ u;
26
       if (d[v] == -1) {
27
         st.push(*ne);
28
         dfs(v,*ne);
29
         if (b[v] > d[u]){
30
           e[*ne].bridge = true; // bridge
31
32
         if (b[v] >= d[u]) \{ // art \}
33
           int last;
34
           do {
35
             last = st.top(); st.pop();
36
             e[last].comp = nbc;
37
           } while (last != *ne);
38
           nbc++;
39
           comp[u]++;
40
41
```

```
b[u] = min(b[u], b[v]);
42
       }
43
       else if (d[v] < d[u]) \{ // back edge
44
         st.push(*ne);
45
         b[u] = min(b[u], d[v]);
46
47
48
49
     LCA + Climb
   //f[v][k] holds the 2^k father of v
   //L[v] holds the level of v
   int N, f[100001][20], L[100001];
   void build(){//f[i][0] must be filled previously, O(nlgn)
     forn(k, 20-1) forn(i, N) f[i][k+1]=f[f[i][k]][k];}
   #define lg(x) (31-_builtin_clz(x))//=floor(log2(x))
   int climb(int a, int d){\frac{1}{0}}
9
     if(!d) return a;
10
     dforn(i, lg(L[a])+1)
11
       if(1<<i<=d)
12
         a=f[a][i], d-=1<<i;
13
       return a;
14
15
   int lca(int a, int b){\frac{1}{0(lgn)}}
     if(L[a]<L[b]) swap(a, b);
     a=climb(a, L[a]-L[b]);
18
     if(a==b) return a;
19
     dforn(i, lg(L[a])+1)
20
       if(f[a][i]!=f[b][i])
21
         a=f[a][i], b=f[b][i];
22
     return f[a][0];
23
24 | }
        Heavy Light Decomposition
  int treesz[MAXN]://cantidad de nodos en el subarbol del nodo v
   int dad[MAXN];//dad[v]=padre del nodo v
   void dfs1(int v, int p=-1){//pre-dfs
     dad[v]=p;
4
     treesz[v]=1;
5
     forall(it, G[v]) if(*it!=p){
```

```
dfs1(*it, v);
       treesz[v]+=treesz[*it];
8
9
10
   int pos[MAXN], q;//pos[v]=posicion del nodo v en el recorrido de la dfs
   //Las cadenas aparecen continuas en el recorrido!
   int cantcad;
   int homecad[MAXN];//dada una cadena devuelve su nodo inicial
   int cad[MAXN];//cad[v]=cadena a la que pertenece el nodo
   void heavylight(int v, int cur=-1){
     if(cur==-1) homecad[cur=cantcad++]=v;
17
     pos[v]=q++;
     cad[v]=cur:
19
     int mx=-1:
20
     forn(i, sz(G[v])) if(G[v][i]!=dad[v])
21
       if(mx==-1 || treesz[G[v][mx]]<treesz[G[v][i]]) mx=i;</pre>
     if(mx!=-1) heavylight(G[v][mx], cur);
     forn(i, sz(G[v])) if(i!=mx && G[v][i]!=dad[v])
       heavylight(G[v][i], -1);
25
26
   //ejemplo de obtener el maximo numero en el camino entre dos nodos
   //RTA: max(query(low, u), query(low, v)), con low=lca(u, v)
   //esta funcion va trepando por las cadenas
   int query(int an, int v){//0(logn)
    //si estan en la misma cadena:
    if(cad[an] == cad[v]) return rmq.get(pos[an], pos[v]+1);
     return max(query(an, dad[homecad[cad[v]]]),
33
            rmq.get(pos[homecad[cad[v]]], pos[v]+1));
34
35 }
7.11. Euler Cycle
int n,m,ars[MAXE], eq;
  vector<int> G[MAXN];//fill G,n,m,ars,eq
  list<int> path;
   int used[MAXN];
   bool usede[MAXE];
   queue<list<int>::iterator> q;
   int get(int v){
     while(used[v]\leq z(G[v]) && usede[G[v][used[v]]]) used[v]++;
     return used[v];
9
   }
10
void explore(int v, int r, list<int>::iterator it){
```

while (comp[v].size() > 0) {

19

```
int ar=G[v][get(v)]; int u=v^ars[ar];
                                                                                                            no[comp[v].back()] = s;
                                                                                   20
                                                                                                            comp[s].push_back(comp[v].back());
     usede[ar]=true;
                                                                                   21
13
     list<int>::iterator it2=path.insert(it, u);
                                                                                                            comp[v].pop_back();
14
                                                                                   22
     if(u!=r) explore(u, r, it2);
                                                                                                       }
15
                                                                                   23
     if(get(v)<sz(G[v])) q.push(it);</pre>
                                                                                                   }
16
                                                                                   24
                                                                                               } while (v != s);
17
                                                                                   25
   void euler(){
                                                                                   26
18
     zero(used), zero(usede);
                                                                                               for (int k = 0; k < sz(comp[s]); ++k) {
19
                                                                                   27
     path.clear();
                                                                                                   int j = comp[s][k];
20
     q=queue<list<int>::iterator>();
                                                                                                   if (j != r)
21
     path.push_back(0); q.push(path.begin());
                                                                                                   for (int l = 0; l < sz(gg[j]); ++1)
22
                                                                                   30
     while(sz(q)){
                                                                                                   if (no[ gg[j][1].src ] != s)
23
                                                                                   31
       list<int>::iterator it=q.front(); q.pop();
                                                                                                   gg[j][l].w -= mcost[ temp[j] ];
                                                                                   32
       if(used[*it] < sz(G[*it])) explore(*it, *it, it);</pre>
                                                                                               }
25
                                                                                   33
                                                                                           }
                                                                                   34
26
     reverse(path.begin(), path.end());
                                                                                           mark[v] = true;
27
                                                                                           for (int k = 0; k < sz(next[v]); ++k) {
28
                                                                                   36
   void addEdge(int u, int v){
                                                                                               int i = next[v][k];
                                                                                   37
     G[u].pb(eq), G[v].pb(eq);
                                                                                               if (no[i] != no[v] && prev[ no[i] ] == v)
                                                                                   38
30
     ars[eq++]=u^v;
                                                                                               if (!mark[ no[i] ] || i == s)
31
                                                                                               backward_traverse(i, s, r, gg,
32 }
                                                                                   40
                                                                                               no, comp, prev, next, mcost, mark, cost, found);
                                                                                   41
7.12. Chu-liu
                                                                                           }
                                                                                   42
                                                                                   43
                                                                                       weight minimum_spanning_arborescence(int r, graph &g) {
1 struct Arf
                                                                                           const int n = g.size();
       int src,dst,w;
2
                                                                                           graph gg(n);
   };
                                                                                   46
3
                                                                                           for (int i = 0; i < sz(g); ++i)
   typedef int weight;
                                                                                   47
                                                                                           for (int j = 0; j < sz(g[i]); ++j)
   const weight inf=1e9;
                                                                                   48
                                                                                             gg[ g[i][j].dst ].push_back( g[i][j] );
   typedef vector< vector< Ar > > graph;
                                                                                   49
                                                                                           vector<int> no(n);
   void backward_traverse(int v, int s, int r,
                                                                                   50
                                                                                           vector< vector<int> > comp(n);
   graph &gg,
                                                                                   51
                                                                                           for (int i = 0; i < n; ++i) {
   vector<int> &no, vector< vector<int> > &comp,
                                                                                   52
                                                                                               no[i] = i:
                                                                                   53
   vector<int> &prev, vector< vector<int> > &next, vector<weight> &mcost,
                                                                                               comp[i].push_back(i);
   vector<int> &mark, weight &cost, bool &found) {
                                                                                   54
11
                                                                                   55
       if (mark[v]) {
12
                                                                                           weight cost = 0;
           vector<int> temp = no;
                                                                                   56
13
                                                                                           while (1) {
           found = true:
                                                                                   57
14
                                                                                               vector<int> prev(n, -1);
           do {
                                                                                   58
15
                                                                                               vector<weight> mcost(n, inf);
               cost += mcost[v]:
                                                                                   59
16
                                                                                               for (int j = 0; j < n; ++j) {
               v = prev[v];
                                                                                   60
17
                                                                                                   if (j == r) continue;
               if (v != s) {
                                                                                   61
18
                                                                                                   for (int k = 0; k < sz(gg[j]); ++k) {
```

62

void updtree(int x) {

slkx[y] = x;

10

11

forn(y, n) if (lx[x] + ly[y] - mt[x][y] < slk[y]) {

slk[y] = lx[x] + ly[y] - mt[x][y];

```
int i = gg[j][k].src;
                                                                                   12 }
63
                                                                                     int hungar(){//Matching maximo de mayor costo en grafos dirigidos (N^3)
                   if (no[i] != no[j]) {
64
                       if (gg[j][k].w < mcost[ no[j] ]) {</pre>
                                                                                        forn(i, n) {
65
                                                                                  14
                            mcost[no[j]] = gg[j][k].w;
                                                                                          lv[i] = 0;
                                                                                   15
66
                           prev[ no[j] ] = no[i];
                                                                                          lx[i] = *max_element(mt[i], mt[i]+n); }
                                                                                  16
67
                       }
                                                                                        memset(xy, -1, sizeof(xy));
                                                                                  17
                                                                                       memset(yx, -1, sizeof(yx));
                   }
                                                                                  18
69
               }
                                                                                        forn(m, n) {
                                                                                  19
70
           }
                                                                                          memset(S, 0, sizeof(S));
71
           vector< vector<int> > next(n);
                                                                                          memset(T, 0, sizeof(T));
72
           for (int i = 0; i < n; ++i)
                                                                                          memset(prv, -1, sizeof(prv));
73
                                                                                  22
           if (prev[i] >= 0)
                                                                                          memset(slk, 0x7f, sizeof(slk));
74
                                                                                  23
           next[ prev[i] ].push_back( i );
                                                                                          queue<int> q;
75
                                                                                  24
                                                                                      #define bpone(e, p) { q.push(e); prv[e] = p; S[e] = 1; updtree(e); }
76
                                                                                          forn(i, n) if (xy[i] == -1) { bpone(i, -2); break; }
           bool stop = true;
77
                                                                                  26
           vector<int> mark(n);
                                                                                          int x=0, y=-1;
78
                                                                                  27
           for (int i = 0; i < n; ++i) {
                                                                                          while (y==-1) {
79
                                                                                  28
               if (i == r || mark[i] || comp[i].size() == 0) continue;
                                                                                            while (!q.empty() && y==-1) {
80
                                                                                  29
               bool found = false:
                                                                                              x = q.front(); q.pop();
                                                                                  30
81
                                                                                              forn(j, n) if (mt[x][j] == lx[x] + ly[j] && !T[j]) {
               backward_traverse(i, i, r, gg,
                                                                                  31
82
               no, comp, prev, next, mcost, mark, cost, found);
                                                                                                if (yx[j] == -1) \{ y = j; break; \}
                                                                                  32
83
                                                                                                T[i] = 1;
               if (found) stop = false;
84
                                                                                  33
           }
                                                                                                bpone(yx[j], x);
85
                                                                                  34
           if (stop) {
86
                                                                                   35
               for (int i = 0; i < n; ++i)
                                                                                            }
87
                                                                                   36
                 if (prev[i] >= 0)
                                                                                            if (y!=-1) break;
                                                                                  37
88
                   cost += mcost[i];
                                                                                            int dlt = INFTO;
                                                                                  38
89
                                                                                            forn(j, n) if (!T[j]) dlt = min(dlt, slk[j]);
               return cost;
                                                                                  39
90
                                                                                            forn(k, n) {
91 | } }}
                                                                                              if (S[k]) lx[k] = dlt;
       Hungarian
7.13.
                                                                                              if (T[k]) ly[k] += dlt;
                                                                                              if (!T[k]) slk[k] -= dlt;
                                                                                  43
                                                                                            }
  #define MAXN 256
                                                                                  44
                                                                                            forn(j, n) if (!T[j] && !slk[j]) {
   #define INFTO Ox7f7f7f7f
                                                                                  45
                                                                                              if (yx[j] == -1) {
                                                                                  46
   int n;
3
                                                                                                x = slkx[j]; y = j; break;
   int mt[MAXN] [MAXN]; // Matriz de costos (X * Y)
                                                                                  47
                                                                                              } else {
   int xy[MAXN], yx[MAXN]; // Matching resultante (X->Y, Y->X)
                                                                                  48
                                                                                                T[i] = 1:
   int lx[MAXN], ly[MAXN], slk[MAXN], slkx[MAXN], prv[MAXN];
                                                                                  49
                                                                                                if (!S[yx[j]]) bpone(yx[j], slkx[j]);
   char S[MAXN], T[MAXN];
                                                                                              }
```

51

52

53

54

}

if (y!=-1) {

```
for(int p = x; p != -2; p = prv[p]) {
55
           yx[y] = p;
56
           int ty = xy[p]; xy[p] = y; y = ty;
57
58
       } else break;
59
60
     int res = 0;
61
     forn(i, n) res += mt[i][xy[i]];
62
     return res;
63
64 }
```

## 8. Network Flow

#### 8.1. Dinic

```
int nodes, src, dest;
   int dist[MAX], q[MAX], work[MAX];
3
   struct Edge {
     int to, rev;
     ll f, cap;
     Edge(int to, int rev, ll f, ll cap): to(to), rev(rev), f(f), cap(cap)
          {}
   };
8
   vector<Edge> G[MAX];
10
11
   // Adds bidirectional edge
12
   void addEdge(int s, int t, ll cap){
13
     G[s].push_back(Edge(t, G[t].size(), 0, cap));
14
     G[t].push_back(Edge(s, G[s].size()-1, 0, 0));
15
16
17
   bool dinic_bfs() {
18
     fill(dist, dist + nodes, -1);
19
     dist[src] = 0;
20
     int qt = 0;
^{21}
     q[qt++] = src;
22
     for (int qh = 0; qh < qt; qh++) {</pre>
23
       int u = q[qh];
24
       forall(e, G[u]){
25
         int v = e \rightarrow to;
26
         if(dist[v]<0 \&\& e->f < e->cap){
27
```

```
dist[v]=dist[u]+1;
28
           q[qt++]=v;
29
         }
30
       }
31
     }
32
     return dist[dest] >= 0;
33
34
   ll dinic_dfs(int u, ll f) {
     if (u == dest) return f;
    for (int &i = work[u]; i < (int) G[u].size(); i++) {</pre>
       Edge &e = G[u][i];
39
       if (e.cap <= e.f) continue;</pre>
40
       int v = e.to;
       if (dist[v] == dist[u] + 1) {
         11 df = dinic_dfs(v, min(f, e.cap - e.f));
         if (df > 0) {
44
           e.f += df;
           G[v][e.rev].f -= df;
46
           return df;
47
         }
48
       }
50
     return 0;
51
52
53
   11 maxFlow(int _src, int _dest) {//O(V^2 E)
     src = _src;
55
     dest = _dest;
     11 result = 0;
57
     while (dinic_bfs()) {
58
       fill(work, work + nodes, 0);
59
       while(ll delta = dinic_dfs(src, INF))
60
         result += delta;
61
    }
62
63
     // todos los nodos con dist[v]!=-1 vs los que tienen dist[v]==-1
64
         forman el min cut
65
     return result;
66
67 }
```

## **8.2.** Konig

```
1 // asume que el dinic YA ESTA tirado
   // asume que nodes-1 y nodes-2 son la fuente y destino
  int match[maxnodes]; // match[v]=u si u-v esta en el matching, -1 si v
       no esta matcheado
  int s[maxnodes]; // numero de la bfs del koning
   queue<int> kq;
   // s[e] %2==1 o si e esta en V1 y s[e]==-1-> lo agarras
   void koning() \{//0(n)
     forn(v,nodes-2) s[v] = match[v] = -1;
     forn(v,nodes-2) forall(it,g[v]) if (it->to < nodes-2 && it->f>0)
       { match[v]=it->to; match[it->to]=v;}
     form(v,nodes-2) if (match[v]==-1) {s[v]=0;kq.push(v);}
11
     while(!kq.empty()) {
12
       int e = kq.front(); kq.pop();
13
       if (s[e] %2==1) {
14
         s[match[e]] = s[e]+1;
15
         kq.push(match[e]);
16
       } else {
17
18
         forall(it,g[e]) if (it->to < nodes-2 && s[it->to]==-1) {
19
           s[it->to] = s[e]+1;
20
           kq.push(it->to);
21
22
23
24
25
```

## 8.3. Edmonds Karp's

```
#define MAX_V 1000
#define INF 1e9
//special nodes
#define SRC 0
#define SNK 1
map<int, int> G[MAX_V];//limpiar esto
//To add an edge use
#define add(a, b, w) G[a][b]=w
int f, p[MAX_V];
void augment(int v, int minE){
if(v==SRC) f=minE;
else if(p[v]!=-1){
```

```
augment(p[v], min(minE, G[p[v]][v]));
       G[p[v]][v]=f, G[v][p[v]]+=f;
14
     }
15
   }
16
   ll maxflow(){//O(VE^2)
     ll Mf=0;
     do{
19
       f=0;
20
       char used[MAX_V]; queue<int> q; q.push(SRC);
21
       zero(used), memset(p, -1, sizeof(p));
22
       while(sz(q)){
23
         int u=q.front(); q.pop();
24
         if(u==SNK) break:
25
         forall(it, G[u])
           if(it->snd>0 && !used[it->fst])
27
             used[it->fst]=true, q.push(it->fst), p[it->fst]=u;
28
       }
29
       augment(SNK, INF);
       Mf+=f:
31
     }while(f);
     return Mf;
34 }
```

#### 8.4. Push-Relabel

```
1 #define MAX V 1000
int N://valid nodes are [0...N-1]
   #define INF 1e9
   //special nodes
   #define SRC 0
   #define SNK 1
  map<int, int> G[MAX_V];
  //To add an edge use
   #define add(a, b, w) G[a][b]=w
   11 excess[MAX_V];
  int height[MAX_V], active[MAX_V], count[2*MAX_V+1];
   queue<int> Q;
   void enqueue(int v) {
    if (!active[v] && excess[v] > 0) active[v]=true, Q.push(v); }
   void push(int a, int b) {
    int amt = min(excess[a], ll(G[a][b]));
    if(height[a] <= height[b] || amt == 0) return;</pre>
17
    G[a][b]-=amt, G[b][a]+=amt;
```

```
excess[b] += amt, excess[a] -= amt;
     enqueue(b);
20
^{21}
   void gap(int k) {
22
     forn(v, N){
23
       if (height[v] < k) continue;
24
       count[height[v]]--;
25
       height[v] = max(height[v], N+1);
26
       count[height[v]]++;
27
       enqueue(v);
28
     }
29
30
   void relabel(int v) {
31
     count[height[v]]--;
32
     height[v] = 2*N;
33
     forall(it, G[v])
34
       if(it->snd)
35
         height[v] = min(height[v], height[it->fst] + 1);
36
     count[height[v]]++;
37
     enqueue(v);
38
39
   ll maxflow() \{//0(V^3)
     zero(height), zero(active), zero(count), zero(excess);
41
     count[0] = N-1;
42
     count[N] = 1;
43
     height[SRC] = N;
44
     active[SRC] = active[SNK] = true;
45
     forall(it, G[SRC]){
46
       excess[SRC] += it->snd;
47
       push(SRC, it->fst);
48
     }
49
     while(sz(Q)) {
50
       int v = Q.front(); Q.pop();
51
       active[v]=false:
52
     forall(it, G[v]) push(v, it->fst);
53
     if(excess[v] > 0)
54
       count[height[v]] == 1? gap(height[v]):relabel(v);
55
     }
56
     ll mf=0;
57
     forall(it, G[SRC]) mf+=G[it->fst][SRC];
     return mf:
59
60 }
```

#### 8.5. Min-cost Max-flow

```
struct edge {
     int u, v;
     11 cap, cost, flow;
     ll rem() { return cap - flow; }
   };
5
   int n;//numero de nodos
   vector<int> G[MAXN];
   vector<edge> e;
   void addEdge(int u, int v, ll cap, ll cost) {
     G[u].pb(si(e)); e.pb((edge){u,v,cap,cost,0});
     G[v].pb(si(e)); e.pb((edge){v,u,0,-cost,0});
11
12
   11 pot[MAXN], dist[MAXN], pre[MAXN], cap[MAXN];
   11 mxFlow, mnCost;
   void flow(int s, int t) {
     fill(pot, pot+n, 0);
     mxFlow=mnCost=0;
     while(1){
       fill(dist, dist+n, INF); dist[s] = 0;
       fill(pre, pre+n, -1); pre[s]=0;
20
       fill(cap, cap+n, 0); cap[s] = INF;
21
       priority_queue<pair<ll,int> > q; q.push(mkp(0,s));
       while (!q.empty()) {
23
         pair<ll,int> top = q.top(); q.pop();
24
         int u = top.second, d = -top.first;
         if (u == t) break;
26
         if (d > dist[u]) continue;
27
         forn(i,si(G[u])) {
28
           edge E = e[G[u][i]];
29
           int c = E.cost + pot[u] - pot[E.v];
30
           if (E.rem() && dist[E.v] > dist[u] + c) {
31
             dist[E.v] = dist[u] + c;
32
             pre[E.v] = G[u][i];
33
             cap[E.v] = min(cap[u], E.rem());
34
             q.push(mkp(-dist[E.v], E.v));
35
36
         }
37
38
       if (pre[t] == -1) break;
39
       forn(u,n)
40
         if (dist[u] == INF) pot[u] = INF;
41
```

```
else pot[u] += dist[u];
mxFlow +=cap[t];
mnCost +=cap[t]*pot[t];
for (int v = t; v != s; v = e[pre[v]].u) {
    e[pre[v]].flow += cap[t];
    e[pre[v]^1].flow -= cap[t];
}
```

## 9. Ayudamemoria

#### Cant. decimales

```
#include <iomanip>
cout << setprecision(2) << fixed;</pre>
```

## Rellenar con espacios(para justificar)

```
#include <iomanip>
cout << setfill(''') << setw(3) << 2 << endl;</pre>
```

#### Leer hasta fin de linea

```
#include <sstream>
//hacer cin.ignore() antes de getline()

while(getline(cin, line)){
   istringstream is(line);
   while(is >> X)
   cout << X << "";
   cout << endl;
}</pre>
```

#### Aleatorios

```
#define RAND(a, b) (rand() %(b-a+1)+a)
rand(time(NULL));
```

## Doubles Comp.

```
const double EPS = 1e-9;
    x == y <=> fabs(x-y) < EPS
    x > y <=> x > y + EPS
    x >= y <=> x > y - EPS
```

#### Limites

#### Muahaha

```
#include <signal.h>
void divzero(int p){
while(true);}
void segm(int p){
exit(0);}
//in main
signal(SIGFPE, divzero);
signal(SIGSEGV, segm);
```

## Mejorar velocidad

```
ios::sync_with_stdio(false);
```

## Mejorar velocidad 2

```
//Solo para enteros positivos
inline void Scanf(int& a){
    char c = 0;
    while(c<33) c = getc(stdin);
    a = 0;
    while(c>33) a = a*10 + c - '0', c = getc(stdin);
}
```

#### Leer del teclado

```
freopen("/dev/tty", "a", stdin);
```

## Iterar subconjunto

```
for(int sbm=bm; sbm; sbm=(sbm-1)&bm)
```

## File setup

```
//tambien se pueden usar comas: {a, x, m, 1}
touch {a..l}.in; tee {a..l}.cpp < template.cpp
```