

## 1. algorithm

`#include <algorithm> #include <numeric>`

Algo	Params	Funcion
sort, stable_sort	f, l	ordena el intervalo
nth_element	f, nth, l	void ordena el n-esimo, y particiona el resto
fill, fill_n	f, l / n, elem	void llena [f, l) o [f, f+n) con elem
lower_bound, upper_bound	f, l, elem	it al primer / ultimo donde se puede insertar elem para que quede ordenada
binary_search	f, l, elem	bool esta elem en [f, l)
copy	f, l, resul	hace resul+i=f+i $\forall i$
find, find_if, find_first_of	f, l, elem / pred / f2, l2	it encuentra i $\in [f, l)$ tq. i=elem, pred(i), i $\in [f2, l2)$
count, count_if	f, l, elem/pred	cuenta elem, pred(i)
search	f, l, f2, l2	busca [f2, l2) $\in [f, l)$
replace, replace_if	f, l, old / pred, new	cambia old / pred(i) por new
reverse	f, l	da vuelta
partition, stable_partition	f, l, pred	pred(i) ad, !pred(i) atras
min_element, max_element	f, l, [comp]	it min, max de [f, l)
lexicographical_compare	f1, l1, f2, l2	bool con [f1, l1] i [f2, l2]
next/prev_permutation	f, l	deja en [f, l) la perm sig, ant
set_intersection, set_difference, set_union, set_symmetric_difference,	f1, l1, f2, l2, res	[res, ...) la op. de conj
push_heap, pop_heap, make_heap	f, l, e / e /	mete/saca e en heap [f, l), hace un heap de [f, l)
is_heap	f, l	bool es [f, l) un heap
accumulate	f, l, i, [op]	$T = \sum / \text{oper de } [f, l)$
inner_product	f1, l1, f2, i	$T = i + [f1, l1) \cdot [f2, \dots)$
partial_sum	f, l, r, [op]	$r+i = \sum / \text{oper de } [f, f+i) \forall i \in [f, l)$
__builtin_ffs	unsigned int	Pos. del primer 1 desde la derecha
__builtin_clz	unsigned int	Cant. de ceros desde la izquierda.
__builtin_ctz	unsigned int	Cant. de ceros desde la derecha.
__builtin_popcount	unsigned int	Cant. de 1's en x.
__builtin_parity	unsigned int	1 si x es par, 0 si es impar.
__builtin_XXXXXXll	unsigned ll	= pero para long long's.

## 2. Estructuras

### 2.1. RMQ (static)

Dado un arreglo y una operacion asociativa *idempotente*, get(i, j) opera sobre el rango [i, j). Restriccion:  $LVL \geq \text{ceil}(\log n)$ ; Usar [ ] para llenar arreglo y luego build().

```

1 struct RMQ{
2     #define LVL 10
3     tipo vec[LVL] [1<<(LVL+1)];
4     tipo &operator[] (int p){return vec[0][p];}
5     tipo get(int i, int j) { //intervalo [i,j)
6         int p = 31-__builtin_clz(j-i);
7         return min(vec[p][i], vec[p][j-(1<<p)]);
8     }
9     void build(int n) { //O(n log n)
10        int mp = 31-__builtin_clz(n);
11        forn(p, mp) forn(x, n-(1<<p))
12            vec[p+1][x] = min(vec[p][x], vec[p][x+(1<<p)]);
13    }
};

```

### 2.2. RMQ (dynamic)

```

1 //Dado un arreglo y una operacion asociativa con neutro, get(i, j) opera
   sobre el rango [i, j).
2 #define MAXN 100000
3 #define operacion(x, y) max(x, y)
4 const int neutro=0;
5 struct RMQ{
6     int sz;
7     tipo t[4*MAXN];
8     tipo &operator[] (int p){return t[sz+p];}
9     void init(int n){ //O(n log n)
10        sz = 1 << (32-__builtin_clz(n));
11        forn(i, 2*sz) t[i]=neutro;
12    }
13     void updall(){ //O(n)
14        dforn(i, sz) t[i]=operacion(t[2*i], t[2*i+1]);}
15     tipo get(int i, int j){return get(i, j, 1, 0, sz);}
16     tipo get(int i, int j, int n, int a, int b){ //O(lgn)
17         if(j<=a || i>=b) return neutro;
18         if(i<=a && b<=j) return t[n];
19         int c=(a+b)/2;

```

```

20     return operacion(get(i, j, 2*n, a, c), get(i, j, 2*n+1, c, b));
21 }
22 void set(int p, tipo val){//0(lgn)
23     for(p+=sz; p>0 && t[p]!=val;){
24         t[p]=val;
25         p/=2;
26         val=operacion(t[p*2], t[p*2+1]);
27     }
28 }
29 }rmq;
30 //Usage:
31 cin >> n; rmq.init(n); forn(i, n) cin >> rmq[i]; rmq.updall();

```

### 2.3. RMQ (lazy)

```

1 //Dado un arreglo y una operacion asociativa con neutro, get(i, j) opera
  sobre el rango [i, j].
2 typedef int Elem; //Elem de los elementos del arreglo
3 typedef int Alt; //Elem de la alteracion
4 #define operacion(x,y) x+y
5 const Elem neutro=0; const Alt neutro2=0;
6 #define MAXN 100000
7 struct RMQ{
8     int sz;
9     Elem t[4*MAXN];
10    Alt dirty[4*MAXN]; //las alteraciones pueden ser de distinto Elem
11    Elem &operator[] (int p){return t[sz+p];}
12    void init(int n){//0(nlgn)
13        sz = 1 << (32-__builtin_clz(n));
14        forn(i, 2*sz) t[i]=neutro;
15        forn(i, 2*sz) dirty[i]=neutro2;
16    }
17    void push(int n, int a, int b){//propaga el dirty a sus hijos
18        if(dirty[n]!=0){
19            t[n]+=dirty[n]*(b-a); //altera el nodo
20            if(n<sz){
21                dirty[2*n]+=dirty[n];
22                dirty[2*n+1]+=dirty[n];
23            }
24            dirty[n]=0;
25        }
26    }
27    Elem get(int i, int j, int n, int a, int b){//0(lgn)

```

```

28     if(j<=a || i>=b) return neutro;
29     push(n, a, b); //corrige el valor antes de usarlo
30     if(i<=a && b<=j) return t[n];
31     int c=(a+b)/2;
32     return operacion(get(i, j, 2*n, a, c), get(i, j, 2*n+1, c, b));
33 }
34 Elem get(int i, int j){return get(i,j,1,0,sz);}
35 //altera los valores en [i, j] con una alteracion de val
36 void alterar(Alt val, int i, int j, int n, int a, int b){//0(lgn)
37     push(n, a, b);
38     if(j<=a || i>=b) return;
39     if(i<=a && b<=j){
40         dirty[n]+=val;
41         push(n, a, b);
42         return;
43     }
44     int c=(a+b)/2;
45     alterar(val, i, j, 2*n, a, c), alterar(val, i, j, 2*n+1, c, b);
46     t[n]=operacion(t[2*n], t[2*n+1]); //por esto es el push de arriba
47 }
48 void alterar(Alt val, int i, int j){alterar(val,i,j,1,0,sz);}
49 }rmq;

```

### 2.4. RMQ (persistente)

```

1 typedef int tipo;
2 tipo oper(const tipo &a, const tipo &b){
3     return a+b;
4 }
5 struct node{
6     tipo v; node *l,*r;
7     node(tipo v):v(v), l(NULL), r(NULL) {}
8     node(node *l, node *r) : l(l), r(r){
9         if(!l) v=r->v;
10        else if(!r) v=l->v;
11        else v=oper(l->v, r->v);
12    }
13 };
14 node *build (tipo *a, int tl, int tr) { //modificar para que tome tipo a
15     if (tl+1==tr) return new node(a[tl]);
16     int tm=(tl + tr)>>1;
17     return new node(build(a, tl, tm), build(a, tm, tr));
18 }

```

```

19 node *update(int pos, int new_val, node *t, int tl, int tr){
20     if (tl+1==tr) return new node(new_val);
21     int tm=(tl+tr)>>1;
22     if(pos < tm) return new node(update(pos, new_val, t->l, tl, tm), t->r);
23     ;
24     else return new node(t->l, update(pos, new_val, t->r, tm, tr));
25 }
26 tipo get(int l, int r, node *t, int tl, int tr){
27     if(l==tl && tr==r) return t->v;
28     int tm=(tl + tr)>>1;
29     if(r<=tm) return get(l, r, t->l, tl, tm);
30     else if(l>=tm) return get(l, r, t->r, tm, tr);
31     return oper(get(l, tm, t->l, tl, tm), get(tm, r, t->r, tm, tr));
32 }

```

## 2.5. Fenwick Tree

```

1 //For 2D threat each column as a Fenwick tree, by adding a nested for in
  //each operation
2 struct Fenwick{
3     static const int sz=1000001;
4     tipo t[sz];
5     void adjust(int p, tipo v){//valid with p in [1, sz), 0(lgn)
6         for(; p<sz; p+=(p&-p)) t[p]+=v; }
7     tipo sum(int p){//cumulative sum in [1, p], 0(lgn)
8         tipo s=0;
9         for(; p; p--=(p&-p)) s+=t[p];
10        return s;
11    }
12    tipo sum(int a, int b){return sum(b)-sum(a-1);}
13    //get largest value with cumulative sum less than or equal to x;
14    //for smallest, pass x-1 and add 1 to result
15    int getind(tipo x) {//0(lgn)
16        int idx = 0, mask = N;
17        while(mask && idx < N) {
18            int t = idx + mask;
19            if(x >= tree[t])
20                idx = t, x -= tree[t];
21            mask >>= 1;
22        }
23        return idx;
24    }
25 };

```

## 2.6. Union Find

```

1 struct UnionFind{
2     vector<int> f;//the array contains the parent of each node
3     void init(int n){f.clear(); f.insert(f.begin(), n, -1);}
4     int comp(int x){return (f[x]==-1?f[x]=comp(f[x]));};//0(1)
5     bool join(int i, int j) {
6         bool con=comp(i)==comp(j);
7         if(!con) f[comp(i)] = comp(j);
8         return con;
9     }
10 };

```

## 2.7. Disjoint Intervals

```

1 bool operator< (const ii &a, const ii &b) {return a.fst<b.fst;}
2 //Stores intervals as [first, second]
3 //in case of a collision it joins them in a single interval
4 struct disjoint_intervals {
5     set<ii> segs;
6     void insert(ii v) {//0(lgn)
7         if(v.snd-v.fst==0.) return;//0J0
8         set<ii>::iterator it,at;
9         at = it = segs.lower_bound(v);
10        if (at!=segs.begin() && (--at)->snd >= v.fst)
11            v.fst = at->fst, --it;
12        for(; it!=segs.end() && it->fst <= v.snd; segs.erase(it++))
13            v.snd=max(v.snd, it->snd);
14        segs.insert(v);
15    }
16 };

```

## 2.8. RMQ (2D)

```

1 struct RMQ2D{
2     static const int sz=1024;
3     RMQ t[sz];
4     RMQ &operator[] (int p){return t[sz/2+p];}
5     void build(int n, int m){//0(nm)
6         forr(y, sz/2, sz/2+m)
7             t[y].build(m);
8         forr(y, sz/2+m, sz)
9             forn(x, sz)
10                t[y].t[x]=0;
11        dforn(y, sz/2)

```

```

12     forn(x, sz)
13         t[y].t[x]=max(t[y*2].t[x], t[y*2+1].t[x]);
14     }
15     void set(int x, int y, tipo v){//O(lgm.lgn)
16         y+=sz/2;
17         t[y].set(x, v);
18         while(y/=2)
19             t[y].set(x, max(t[y*2][x], t[y*2+1][x]));
20     }
21     //O(lgm.lgn)
22     int get(int x1, int y1, int x2, int y2, int n=1, int a=0, int b=sz/2){
23         if(y2<=a || y1>=b) return 0;
24         if(y1<=a && b<=y2) return t[n].get(x1, x2);
25         int c=(a+b)/2;
26         return max(get(x1, y1, x2, y2, 2*n, a, c),
27                     get(x1, y1, x2, y2, 2*n+1, c, b));
28     }
29 };
30 //Example to initialize a grid of M rows and N columns:
31 RMQ2D rmq;
32 forn(i, M)
33     forn(j, N)
34         cin >> rmq[i][j];
35 rmq.build(N, M);

```

## 2.9. Big Int

```

1 #define BASEXP 6
2 #define BASE 1000000
3 #define LMAX 1000
4 struct bint{
5     int l;
6     ll n[LMAX];
7     bint(ll x=0){
8         l=1;
9         forn(i, LMAX){
10             if (x) l=i+1;
11             n[i]=x%BASE;
12             x/=BASE;
13         }
14     }
15     bint(string x){
16

```

```

17         l=(x.size()-1)/BASEXP+1;
18         fill(n, n+LMAX, 0);
19         ll r=1;
20         forn(i, sz(x)){
21             n[i / BASEXP] += r * (x[x.size()-1-i]-'0');
22             r*=10; if(r==BASE)r=1;
23         }
24     }
25     void out(){
26         cout << n[l-1];
27         dforn(i, l-1) printf("%6.6llu", n[i]);//6=BASEXP!
28     }
29     void invar(){
30         fill(n+1, n+LMAX, 0);
31         while(l>1 && !n[l-1]) l--;
32     }
33 };
34 bint operator+(const bint&a, const bint&b){
35     bint c;
36     c.l = max(a.l, b.l);
37     ll q = 0;
38     forn(i, c.l) q += a.n[i]+b.n[i], c.n[i]=q %BASE, q/=BASE;
39     if(q) c.n[c.l++] = q;
40     c.invar();
41     return c;
42 }
43 pair<bint, bool> lresta(const bint& a, const bint& b) // c = a - b
44 {
45     bint c;
46     c.l = max(a.l, b.l);
47     ll q = 0;
48     forn(i, c.l) q += a.n[i]-b.n[i], c.n[i]=(q+BASE) %BASE, q=(q+BASE)/
49         BASE-1;
50     c.invar();
51     return make_pair(c, !q);
52 }
53 bint& operator-= (bint& a, const bint& b){return a=lresta(a, b).first;}
54 bint operator- (const bint&a, const bint&b){return lresta(a, b).first;}
55 bool operator< (const bint&a, const bint&b){return !lresta(a, b).second;
56     ;}
57 bool operator<= (const bint&a, const bint&b){return lresta(b, a).second
58     ;}
59 bool operator==(const bint&a, const bint&b){return a <= b && b <= a;}

```

```

57 bint operator*(const bint&a, ll b){
58     bint c;
59     ll q = 0;
60     forn(i, a.l) q += a.n[i]*b, c.n[i] = q %BASE, q/=BASE;
61     c.l = a.l;
62     while(q) c.n[c.l++] = q %BASE, q/=BASE;
63     c.invar();
64     return c;
65 }
66 bint operator*(const bint&a, const bint&b){
67     bint c;
68     c.l = a.l+b.l;
69     fill(c.n, c.n+b.l, 0);
70     forn(i, a.l){
71         ll q = 0;
72         forn(j, b.l) q += a.n[i]*b.n[j]+c.n[i+j], c.n[i+j] = q %BASE, q
73             /=BASE;
74         c.n[i+b.l] = q;
75     }
76     c.invar();
77     return c;
78 }
79 pair<bint, ll> ldiv(const bint& a, ll b){// c = a / b ; rm = a % b
80     bint c;
81     ll rm = 0;
82     dforn(i, a.l){
83         rm = rm * BASE + a.n[i];
84         c.n[i] = rm / b;
85         rm %= b;
86     }
87     c.l = a.l;
88     c.invar();
89     return make_pair(c, rm);
90 }
91 bint operator/(const bint&a, ll b){return ldiv(a, b).first;}
92 ll operator%(const bint&a, ll b){return ldiv(a, b).second;}
93 pair<bint, bint> ldiv(const bint& a, const bint& b){
94     bint c;
95     bint rm = 0;
96     dforn(i, a.l){
97         if (rm.l==1 && !rm.n[0])
98             rm.n[0] = a.n[i];
99         else{

```

```

99             dforn(j, rm.l) rm.n[j+1] = rm.n[j];
100             rm.n[0] = a.n[i];
101             rm.l++;
102         }
103         ll q = rm.n[b.l] * BASE + rm.n[b.l-1];
104         ll u = q / (b.n[b.l-1] + 1);
105         ll v = q / b.n[b.l-1] + 1;
106         while (u < v-1){
107             ll m = (u+v)/2;
108             if (b*m <= rm) u = m;
109             else v = m;
110         }
111         c.n[i]=u;
112         rm-=b*u;
113     }
114     c.l=a.l;
115     c.invar();
116     return make_pair(c, rm);
117 }
118 bint operator/(const bint&a, const bint&b){return ldiv(a, b).first;}
119 bint operator%(const bint&a, const bint&b){return ldiv(a, b).second;}

```

## 2.10. Modnum

```

1 struct mnum{
2     static const tipo mod=12582917;
3     tipo v;
4     mnum(tipo v=0): v(v%mod) {}
5     mnum operator+(mnum b){return v+b.v;}
6     mnum operator-(mnum b){return v>=b.v? v-b.v : mod-b.v+v;}
7     mnum operator*(mnum b){return v*b.v;}
8     mnum operator^(int n){
9         if(!n) return 1;
10        return n%2? (*this)^(n/2)*(*this) : (*this)^(n/2);}
11 };

```

## 2.11. Treap

```

1 typedef int Key;
2 typedef struct node *pnode;
3 struct node{
4     Key key;
5     int prior, size;
6     pnode l,r;

```

```

7     node(Key key=0, int prior=0): key(key), prior(prior), size(1), l(0),
      r(0) {}
8 };
9 struct treap {
10     pnode root;
11     treap(): root(0) {}
12     int size(pnode p) { return p ? p->size : 0; }
13     int size() { return size(root); }
14     void push(pnode p) {
15         // modificar y propagar el dirty a los hijos aca(para lazy)
16     }
17     // Update function and size from children's values
18     void pull(pnode p) { //recalcular valor del nodo aca (para rmq)
19         p->size = 1 + size(p->l) + size(p->r);
20     }
21     pnode merge(pnode l, pnode r) {
22         if (!l || !r) return l ? l : r;
23         push(l), push(r);
24         pnode t;
25         if (l->prior < r->prior) l->r=merge(l->r, r), t = l;
26         else r->l=merge(l, r->l), t = r;
27         pull(t);
28         return t;
29     } //opcional:
30     void merge(treap t) {root = merge(root, t.root), t.root=0;}
31     //*****KEY OPERATIONS*****//
32     void splitKey(pnode t, Key key, pnode &l, pnode &r) {
33         if (!t) return void(l = r = 0);
34         push(t);
35         if (key <= t->key) splitKey(t->l, key, l, t->l), r = t;
36         else splitKey(t->r, key, t->r, r), l = t;
37         pull(t);
38     }
39     void insertKey(Key key) {
40         pnode elem = new node(key, rand());
41         pnode t1, t2; splitKey(root, key, t1, t2);
42         t1=merge(t1,elem);
43         root=merge(t1,t2);
44     }
45     void eraseKeys(Key key1, Key key2) {
46         pnode t1,t2,t3;
47         splitKey(root,key1,t1,t2);
48         splitKey(t2,key2, t2, t3);

```

```

49         root=merge(t1,t3);
50     }
51     void eraseKey(pnode &t, Key key) {
52         if (!t) return;
53         push(t);
54         if (key == t->key) t=merge(t->l, t->r);
55         else if (key < t->key) eraseKey(t->l, key);
56         else eraseKey(t->r, key);
57         pull(t);
58     }
59     void eraseKey(Key key) {eraseKey(root, key);}
60     pnode findKey(pnode t, Key key) {
61         if (!t) return 0;
62         if (key == t->key) return t;
63         if (key < t->key) return findKey(t->l, key);
64         return findKey(t->r, key);
65     }
66     pnode findKey(Key key) { return findKey(root, key); }
67     //*****POS OPERATIONS*****// No mezclar con las funciones Key
68     //(No funciona con pos:)
69     void splitSize(pnode t, int sz, pnode &l, pnode &r) {
70         if (!t) return void(l = r = 0);
71         push(t);
72         if (sz <= size(t->l)) splitSize(t->l, sz, l, t->l), r = t;
73         else splitSize(t->r, sz - 1 - size(t->l), t->r, r), l = t;
74         pull(t);
75     }
76     void insertPos(int pos, Key key) {
77         pnode elem = new node(key, rand());
78         pnode t1,t2; splitSize(root, pos, t1, t2);
79         t1=merge(t1,elem);
80         root=merge(t1,t2);
81     }
82     void erasePos(int pos1, int pos2=-1) {
83         if(pos2==-1) pos2=pos1+1;
84         pnode t1,t2,t3;
85         splitSize(root,pos1,t1,t2);
86         splitSize(t2,pos2-pos1,t2,t3);
87         root=merge(t1, t2);
88     }
89     pnode findPos(pnode t, int pos) {
90         if(!t) return 0;
91         if(pos <= size(t->l)) return findPos(t->l, pos);

```

```

92     return findPos(t->r, pos - 1 - size(t->l));
93 }
94 Key &operator[](int pos){return findPos(root, pos)->key;}//ojito
95 };

```

+

## 2.12. Convex Hull Trick

+

```

1  const ll is_query = -(1LL<<62);
2  struct Line {
3      ll m, b;
4      mutable multiset<Line>::iterator it;
5      const Line *succ(multiset<Line>::iterator it) const;
6      bool operator<(const Line& rhs) const {
7          if (rhs.b != is_query) return m < rhs.m;
8          const Line *s=succ(it);
9          if(!s) return 0;
10         ll x = rhs.m;
11         return b - s->b < (s->m - m) * x;
12     }
13 };
14 struct HullDynamic : public multiset<Line>{ // will maintain upper hull
15     for maximum
16     bool bad(iterator y) {
17         iterator z = next(y);
18         if (y == begin()) {
19             if (z == end()) return 0;
20             return y->m == z->m && y->b <= z->b;
21         }
22         iterator x = prev(y);
23         if (z == end()) return y->m == x->m && y->b <= x->b;
24         return (x->b - y->b)*(z->m - y->m) >= (y->b - z->b)*(y->m - x->m);
25     }
26     iterator next(iterator y){return ++y;}
27     iterator prev(iterator y){return --y;}
28     void insert_line(ll m, ll b) {
29         iterator y = insert((Line) { m, b });
30         y->it=y;
31         if (bad(y)) { erase(y); return; }
32         while (next(y) != end() && bad(next(y))) erase(next(y));
33         while (y != begin() && bad(prev(y))) erase(prev(y));

```

```

33     }
34     ll eval(ll x) {
35         Line l = *lower_bound((Line) { x, is_query });
36         return l.m * x + l.b;
37     }
38 }h;
39 const Line *Line::succ(multiset<Line>::iterator it) const{
40     return (++it==h.end())? NULL : &*it;};

```

## 2.13. Gain-Cost Set

```

1  //esta estructura mantiene pairs(beneficio, costo)
2  //de tal manera que en el set quedan ordenados
3  //por beneficio Y COSTO creciente. (va borrando los que no son optimos)
4  struct V{
5      int gain, cost;
6      bool operator<(const V &b)const{return gain<b.gain;}
7  };
8  set<V> s;
9  void add(V x){
10     set<V>::iterator p=s.lower_bound(x);//primer elemento mayor o igual
11     if(p!=s.end() && p->cost <= x.cost) return;//ya hay uno mejor
12     p=s.upper_bound(x);//primer elemento mayor
13     if(p!=s.begin()){//borro todos los peores (<=beneficio y >=costo)
14         --p;//ahora es ultimo elemento menor o igual
15         while(p->cost >= x.cost){
16             if(p==s.begin()){s.erase(p); break;}
17             s.erase(p--);
18         }
19     }
20     s.insert(x);
21 }
22 int get(int gain){//minimo costo de obtener tal ganancia
23     set<V>::iterator p=s.lower_bound((V){gain, 0});
24     return p==s.end()? INF : p->cost;};

```

## 3. Algos

### 3.1. Longest Increasing Subsequence

```

1  //Para non-increasing, cambiar comparaciones y revisar busq binaria

```



```

2 //Given an array, paint it in the least number of colors so that each
  color turns to a non-increasing subsequence.
3 //Solution:Min number of colors=Length of the longest increasing
  subsequence
4 int N, a[MAXN]; //secuencia y su longitud
5 ii d[MAXN+1]; //d[i]=ultimo valor de la subsecuencia de tamaño i
6 int p[MAXN]; //padres
7 vector<int> R; //respuesta
8 void rec(int i){
9     if(i== -1) return;
10    R.push_back(a[i]);
11    rec(p[i]);
12 }
13 int lis(){ //O(nlogn)
14     d[0] = ii(-INF, -1); for(i, N) d[i+1]=ii(INF, -1);
15     for(i, N){
16         int j = upper_bound(d, d+N+1, ii(a[i], INF))-d;
17         if (d[j-1].first < a[i] && a[i] < d[j].first){
18             p[i]=d[j-1].second;
19             d[j] = ii(a[i], i);
20         }
21     }
22     R.clear();
23     dforn(i, N+1) if(d[i].first!=INF){
24         rec(d[i].second); //reconstruir
25         reverse(R.begin(), R.end());
26         return i; //longitud
27     }
28     return 0;
29 }

```

### 3.2. Manacher

```

1 int d1[MAXN]; //d1[i]=long del maximo palindromo impar con centro en i
2 int d2[MAXN]; //d2[i]=analogo pero para longitud par
3 //0 1 2 3 4
4 //a a b c c <--d1[2]=3
5 //a a b b <--d2[2]=2 (están uno antes)
6 void manacher(){
7     int l=0, r=-1, n=sz(s);
8     for(i, n){
9         int k=(i>r? 1 : min(d1[l+r-i], r-i));
10        while(i+k<n && i-k>=0 && s[i+k]==s[i-k]) ++k;

```

```

11        d1[i] = k--;
12        if(i+k > r) l=i-k, r=i+k;
13    }
14    l=0, r=-1;
15    forn(i, n){
16        int k=(i>r? 0 : min(d2[l+r-i+1], r-i+1))+1;
17        while(i+k-1<n && i-k>=0 && s[i+k-1]==s[i-k]) k++;
18        d2[i] = --k;
19        if(i+k-1 > r) l=i-k, r=i+k-1;
20    }

```

### 3.3. Alpha-Beta pruning

```

1 ll alphabeta(State &s, bool player = true, int depth = 1e9, ll alpha = -
  INF, ll beta = INF) { //player = true -> Maximiza
2     if(s.isFinal()) return s.score;
3     //~ if (!depth) return s.heuristic();
4     vector<State> children;
5     s.expand(player, children);
6     int n = children.size();
7     forn(i, n) {
8         ll v = alphabeta(children[i], !player, depth-1, alpha, beta);
9         if(!player) alpha = max(alpha, v);
10        else beta = min(beta, v);
11        if(beta <= alpha) break;
12    }
13    return !player ? alpha : beta;

```

## 4. Strings

### 4.1. KMP

```

1 string T; //cadena donde buscar(where)
2 string P; //cadena a buscar(what)
3 int b[MAXLEN]; //back table
4 void kmppre(){ //by gabina with love
5     int i=0, j=-1; b[0]=-1;
6     while(i<sz(P)){
7         while(j>=0 && P[i] != P[j]) j=b[j];
8         i++, j++;
9         b[i] = j;
10    }
11 }

```



```

12
13 void kmp(){
14     int i=0, j=0;
15     while(i<sz(T)){
16         while(j>=0 && T[i]!=P[j]) j=b[j];
17         i++, j++;
18         if(j==sz(P)){
19             printf("P is found at index %d in T\n", i-j);
20             j=b[j];
21         }
22     }
23 }

```

## 4.2. Trie

```

1 struct trie{
2     map<char, trie> m;
3     void add(const string &s, int p=0){
4         if(s[p]) m[s[p]].add(s, p+1);
5     }
6     void dfs(){
7         //Do stuff
8         forall(it, m)
9             it->second.dfs();
10    }
11 };

```

## 4.3. Suffix Array (largo, nlogn)

```

1 #define MAX_N 1000
2 #define rBOUND(x) (x<n? r[x] : 0)
3 //sa will hold the suffixes in order.
4 int sa[MAX_N], r[MAX_N], n;
5 string s; //input string, n=sz(s)
6
7 int f[MAX_N], tmpsa[MAX_N];
8 void countingSort(int k){
9     zero(f);
10    forn(i, n) f[rBOUND(i+k)]++;
11    int sum=0;
12    forn(i, max(255, n)){
13        int t=f[i]; f[i]=sum; sum+=t;
14    }
15    forn(i, n)
        tmpsa[f[rBOUND(sa[i]+k)]++] = sa[i];

```

```

16    memcpy(sa, tmpsa, sizeof(sa));
17 }
18 void constructsa(){//O(n log n)
19     n=sz(s);
20     forn(i, n) sa[i]=i, r[i]=s[i];
21     for(int k=1; k<n; k<=1){
22         countingSort(k), countingSort(0);
23         int rank, tmpr[MAX_N];
24         tmpr[sa[0]]=rank=0;
25         forr(i, 1, n)
26             tmpr[sa[i]]=(r[sa[i]]==r[sa[i-1]] && r[sa[i]+k]==r[sa[i-1]+k])?
                rank : ++rank;
27         memcpy(r, tmpr, sizeof(r));
28         if(r[sa[n-1]]==n-1) break;
29     }
30 }
31 void print(){//for debug
32     forn(i, n)
33         cout << i << ' ' <<
34         s.substr(sa[i], s.find( '$', sa[i])-sa[i]) << endl;

```

## 4.4. String Matching With Suffix Array

```

1 //returns (lowerbound, upperbound) of the search
2 ii stringMatching(string P){ //O(sz(P)lgn)
3     int lo=0, hi=n-1, mid=lo;
4     while(lo<hi){
5         mid=(lo+hi)/2;
6         int res=s.compare(sa[mid], sz(P), P);
7         if(res>=0) hi=mid;
8         else lo=mid+1;
9     }
10    if(s.compare(sa[lo], sz(P), P)!=0) return ii(-1, -1);
11    ii ans; ans.fst=lo;
12    lo=0, hi=n-1, mid;
13    while(lo<hi){
14        mid=(lo+hi)/2;
15        int res=s.compare(sa[mid], sz(P), P);
16        if(res>0) hi=mid;
17        else lo=mid+1;
18    }
19    if(s.compare(sa[hi], sz(P), P)!=0) hi--;
20    ans.snd=hi;

```

```

21 | return ans;
22 | }

```

#### 4.5. LCP (Longest Common Prefix)

```

1 | //Calculates the LCP between consecutives suffixes in the Suffix Array.
2 | //LCP[i] is the length of the LCP between sa[i] and sa[i-1]
3 | int LCP[MAX_N], phi[MAX_N], PLCP[MAX_N];
4 | void computeLCP(){//O(n)
5 |     phi[sa[0]]=-1;
6 |     forr(i, 1, n) phi[sa[i]]=sa[i-1];
7 |     int L=0;
8 |     forn(i, n){
9 |         if(phi[i]==-1) {PLCP[i]=0; continue;}
10 |         while(s[i+L]==s[phi[i]+L]) L++;
11 |         PLCP[i]=L;
12 |         L=max(L-1, 0);
13 |     }
14 |     forn(i, n) LCP[i]=PLCP[sa[i]];
15 | }

```

#### 4.6. Corasick

```

1 |
2 | struct trie{
3 |     map<char, trie> next;
4 |     trie* tran[256];//transiciones del automata
5 |     int idhoja, szhoja;//id de la hoja o 0 si no lo es
6 |     //link lleva al sufijo mas largo, nxthoja lleva al mas largo pero que
7 |     //es hoja
8 |     trie *padre, *link, *nxthoja;
9 |     char pch;//caracter que conecta con padre
10 |     trie(): tran(), idhoja(), padre(), link() {}
11 |     void insert(const string &s, int id=1, int p=0){//id>0!!!
12 |         if(p<sz(s)){
13 |             trie &ch=next[s[p]];
14 |             tran[(int)s[p]]=&ch;
15 |             ch.padre=this, ch.pch=s[p];
16 |             ch.insert(s, id, p+1);
17 |         }
18 |         else idhoja=id, szhoja=sz(s);
19 |     }
20 |     trie* get_link() {
21 |         if(!link){

```

```

21 |         if(!padre) link=this;//es la raiz
22 |         else if(!padre->padre) link=padre;//hijo de la raiz
23 |         else link=padre->get_link()->get_tran(pch);
24 |     }
25 |     return link;
26 | }
27 | trie* get_tran(int c) {
28 |     if(!tran[c])
29 |         tran[c] = !padre? this : this->get_link()->get_tran(c);
30 |     return tran[c];
31 | }
32 | trie *get_nxthoja(){
33 |     if(!nxthoja) nxthoja = get_link()->idhoja? link : link->nxthoja;
34 |     return nxthoja;
35 | }
36 | void print(int p){
37 |     if(idhoja)
38 |         cout << "found_" << idhoja << "_at_position_" << p-szhoja << endl
39 |         ;
40 |     if(get_nxthoja()) get_nxthoja()->print(p);
41 | }
42 | void matching(const string &s, int p=0){
43 |     print(p);
44 |     if(p<sz(s)) get_tran(s[p])->matching(s, p+1);

```

### 5. Geometria

#### 5.1. Punto

```

1 | struct pto{
2 |     tipo x, y;
3 |     pto(tipo x=0, tipo y=0):x(x),y(y){}
4 |     pto operator+(pto a){return pto(x+a.x, y+a.y);}
5 |     pto operator-(pto a){return pto(x-a.x, y-a.y);}
6 |     pto operator+(tipo a){return pto(x+a, y+a);}
7 |     pto operator*(tipo a){return pto(x*a, y*a);}
8 |     pto operator/(tipo a){return pto(x/a, y/a);}
9 |     //dot product, producto interno:
10 |     tipo operator*(pto a){return x*a.x+y*a.y;}
11 |     //module of the cross product or vectorial product:
12 |     //if a is less than 180 clockwise from b, a^b>0
13 |     tipo operator^(pto a){return x*a.y-y*a.x;}
14 |     //returns true if this is at the left side of line qr

```

```

15 bool left(pto q, pto r){return ((q-*this)^(r-*this))>0;}
16 bool operator<(const pto &a) const{return x<a.x || (abs(x-a.x)<EPS &&
    y<a.y);}
17 bool operator==(pto a){return abs(x-a.x)<EPS && abs(y-a.y)<EPS;}
18 double norm(){return sqrt(x*x+y*y);}
19 tipo norm_sq(){return x*x+y*y;}
20 };
21 double dist(pto a, pto b){return (b-a).norm();}
22 typedef pto vec;
23
24 double angle(pto a, pto o, pto b){
25     pto oa=a-o, ob=b-o;
26     return atan2(oa^ob, oa*ob);}
27
28 //rotate p by theta rads CCW w.r.t. origin (0,0)
29 pto rotate(pto p, double theta){
30     return pto(p.x*cos(theta)-p.y*sin(theta),
31         p.x*sin(theta)+p.y*cos(theta));
32 }
33
34 //orden total de puntos alrededor de un punto r
35 struct Cmp{
36     pto r;
37     Cmp(pto _r){r = _r;}
38     int cuad(const pto &a) const{
39         if(a.x > 0 && a.y >= 0)return 0;
40         if(a.x <= 0 && a.y > 0)return 1;
41         if(a.x < 0 && a.y <= 0)return 2;
42         if(a.x >= 0 && a.y < 0)return 3;
43         assert(a.x ==0 && a.y==0);
44         return -1;
45     }
46     bool cmp(const pto&p1, const pto&p2)const{
47         int c1 = cuad(p1), c2 = cuad(p2);
48         if(c1==c2){
49             return p1.y*p2.x<p1.x*p2.y;
50         }else{
51             return c1 < c2;
52         }
53     }
54     bool operator()(const pto&p1, const pto&p2) const{
55         return cmp(pto(p1.x-r.x,p1.y-r.y),pto(p2.x-r.x,p2.y-r.y));
56     }
57 };

```

## 5.2. Line

```

1 int sgn(ll x){return x<0? -1 : !!x;}
2 struct line{
3     line() {}
4     double a,b,c;//Ax+By=C
5     //pto MUST store float coordinates!
6     line(double a, double b, double c):a(a),b(b),c(c){}
7     line(pto p, pto q): a(q.y-p.y), b(p.x-q.x), c(a*p.x+b*p.y) {}
8     int side(pto p){return sgn(ll(a) * p.x + ll(b) * p.y - c);}
9 };
10 bool parallels(line l1, line l2){return abs(l1.a*l2.b-l2.a*l1.b)<EPS;}
11 pto inter(line l1, line l2){//intersection
12     double det=l1.a*l2.b-l2.a*l1.b;
13     if(abs(det)<EPS) return pto(INF, INF);//parallels
14     return pto(l2.b*l1.c-l1.b*l2.c, l1.a*l2.c-l2.a*l1.c)/det;
15 }

```

## 5.3. Segment

```

1 struct segm{
2     pto s,f;
3     segm(pto s, pto f):s(s), f(f) {}
4     pto closest(pto p) {//use for dist to point
5         double l2 = dist_sq(s, f);
6         if(l2==0.) return s;
7         double t=((p-s)*(f-s))/l2/l2;
8         if (t<0.) return s;//not write if is a line
9         else if(t>1.)return f;//not write if is a line
10        return s+((f-s)*t);
11    }
12    bool inside(pto p){
13        return ((s-p)^(f-p))==0 && min(s, f)<=*this&&*this<max(s, f);}
14 };
15
16 bool insidebox(pto a, pto b, pto p) {
17     return (a.x-p.x)*(p.x-b.x)>-EPS && (a.y-p.y)*(p.y-b.y)>-EPS;
18 }
19 pto inter(segm s1, segm s2){
20     pto r=inter(line(s1.s, s1.f), line(s2.s, s2.f));
21     if(insidebox(s1.s,s1.f,p) && insidebox(s2.s,s2.f,p))
22         return r;
23     return pto(INF, INF);

```

24 }

## 5.4. Rectangle

```
1 struct rect{
2     //lower-left and upper-right corners
3     pto lw, up;
4 };
5 //returns if there's an intersection and stores it in r
6 bool inter(rect a, rect b, rect &r){
7     r.lw=pto(max(a.lw.x, b.lw.x), max(a.lw.y, b.lw.y));
8     r.up=pto(min(a.up.x, b.up.x), min(a.up.y, b.up.y));
9     //check case when only a edge is common
10    return r.lw.x<r.up.x && r.lw.y<r.up.y;
11 }
```

## 5.5. Polygon Area

```
1 double area(vector<pto> &p){//0(sz(p))
2     double area=0;
3     forn(i, sz(p)) area+=p[i]^p[(i+1)%sz(p)];
4     //if points are in clockwise order then area is negative
5     return abs(area)/2;
6 }
7 //Area ellipse = M_PI*a*b where a and b are the semi axis lengths
8 //Area triangle = sqrt(s*(s-a)(s-b)(s-c)) where s=(a+b+c)/2
```

## 5.6. Circle

```
1 vec perp(vec v){return vec(-v.y, v.x);}
2 line bisector(pto x, pto y){
3     line l=line(x, y); pto m=(x+y)/2;
4     return line(-l.b, l.a, -l.b*m.x+l.a*m.y);
5 }
6 struct Circle{
7     pto o;
8     double r;
9     Circle(pto x, pto y, pto z){
10        o=inter(bisector(x, y), bisector(y, z));
11        r=dist(o, x);
12    }
13    pair<pto, pto> ptosTang(pto p){
14        pto m=(p+o)/2;
15        tipo d=dist(o, m);
```

```
16        tipo a=r*r/(2*d);
17        tipo h=sqrt(r*r-a*a);
18        pto m2=o+(m-o)*a/d;
19        vec per=perp(m-o)/d;
20        return make_pair(m2-per*h, m2+per*h);
21    }
22 };
23 //finds the center of the circle containing p1 and p2 with radius r
24 //as there may be two solutions swap p1, p2 to get the other
25 bool circle2PtsRad(pto p1, pto p2, double r, pto &c){
26     double d2=(p1-p2).norm_sq(), det=r*r/d2-0.25;
27     if(det<0) return false;
28     c=(p1+p2)/2+perp(p2-p1)*sqrt(det);
29     return true;
30 }
31 #define sqr(a) ((a)*(a))
32 #define feq(a,b) (fabs((a)-(b))<EPS)
33 pair<tipo, tipo> ecCuad(tipo a, tipo b, tipo c){//a*x*x+b*x+c=0
34     tipo dx = sqrt(b*b-4.0*a*c);
35     return make_pair((-b + dx)/(2.0*a), (-b - dx)/(2.0*a));
36 }
37 pair<pto, pto> interCL(Circle c, line l){
38     bool sw=false;
39     if((sw=feq(0,l.b))){
40         swap(l.a, l.b);
41         swap(c.o.x, c.o.y);
42     }
43     pair<tipo, tipo> rc = ecCuad(
44         sqr(l.a)+sqr(l.b),
45         2.0*l.a*l.b*c.o.y-2.0*(sqr(l.b)*c.o.x+l.c*l.a),
46         sqr(l.b)*(sqr(c.o.x)+sqr(c.o.y)-sqr(c.r))+sqr(l.c)-2.0*l.c*l.b*c.o.y
47     );
48     pair<pto, pto> p( pto(rc.first, (l.c - l.a * rc.first) / l.b),
49         pto(rc.second, (l.c - l.a * rc.second) / l.b) );
50     if(sw){
51         swap(p.first.x, p.first.y);
52         swap(p.second.x, p.second.y);
53     }
54     return p;
55 }
56 pair<pto, pto> interCC(Circle c1, Circle c2){
57     line l;
58     l.a = c1.o.x-c2.o.x;
```

```

59 | 1.b = c1.o.y-c2.o.y;
60 | 1.c = (sqr(c2.r)-sqr(c1.r)+sqr(c1.o.x)-sqr(c2.o.x)+sqr(c1.o.y)
61 | -sqr(c2.o.y))/2.0;
62 | return interCL(c1, l);
63 | }

```

## 5.7. Point in Poly

```

1 | //checks if v is inside of P, using ray casting
2 | //works with convex and concave.
3 | //excludes boundaries, handle it separately using segment.inside()
4 | bool inPolygon(pto v, vector<pto>& P) {
5 |     bool c = false;
6 |     forn(i, sz(P)){
7 |         int j=(i+1)%sz(P);
8 |         if((P[j].y>v.y) != (P[i].y > v.y) &&
9 | (v.x < (P[i].x - P[j].x) * (v.y-P[j].y) / (P[i].y - P[j].y) + P[j].x))
10 |             c = !c;
11 |     }
12 |     return c;
13 | }

```

## 5.8. Convex Check CHECK

```

1 | bool isConvex(vector<int> &p){//O(N)
2 |     int N=sz(p);
3 |     if(N<3) return false;
4 |     bool isLeft=p[0].left(p[1], p[2]);
5 |     forr(i, 1, N)
6 |         if(p[i].left(p[(i+1)%N], p[(i+2)%N])!=isLeft)
7 |             return false;
8 |     return true; }

```

## 5.9. Convex Hull

```

1 | //stores convex hull of P in S, CCW order
2 | void CH(vector<pto>& P, vector<pto> &S){
3 |     S.clear();
4 |     sort(P.begin(), P.end());
5 |     forn(i, sz(P)){
6 |         while(sz(S)>= 2 && S[sz(S)-1].left(S[sz(S)-2], P[i])) S.pop_back();
7 |         S.pb(P[i]);
8 |     }
9 |     S.pop_back();

```

```

10 | int k=sz(S);
11 | dforn(i, sz(P)){
12 |     while(sz(S) >= k+2 && S[sz(S)-1].left(S[sz(S)-2], P[i])) S.pop_back
13 |         ();
14 |     S.pb(P[i]);
15 | }
16 | S.pop_back();

```

## 5.10. Cut Polygon

```

1 | //cuts polygon Q along the line ab
2 | //stores the left side (swap a, b for the right one) in P
3 | void cutPolygon(pto a, pto b, vector<pto> Q, vector<pto> &P){
4 |     P.clear();
5 |     forn(i, sz(Q)){
6 |         double left1=(b-a)^(Q[i]-a), left2=(b-a)^(Q[(i+1)%sz(Q)]-a);
7 |         if(left1>=0) P.pb(Q[i]);
8 |         if(left1*left2<0)
9 |             P.pb(inter(line(Q[i], Q[(i+1)%sz(Q)]), line(a, b)));
10 |     }
11 | }

```

## 5.11. Bresenham

```

1 | //plot a line approximation in a 2d map
2 | void bresenham(pto a, pto b){
3 |     pto d=b-a; d.x=abs(d.x), d.y=abs(d.y);
4 |     pto s(a.x<b.x? 1: -1, a.y<b.y? 1: -1);
5 |     int err=d.x-d.y;
6 |     while(1){
7 |         m[a.x][a.y]=1;//plot
8 |         if(a==b) break;
9 |         int e2=2*err;
10 |         if(e2 > -d.y){
11 |             err-=d.y, a.x+=s.x;
12 |         }
13 |         if(e2 < d.x)
14 |             err+= d.x, a.y+= s.y;
15 |     }

```

## 5.12. Rotate Matrix

```

1 | //rotates matrix t 90 degrees clockwise

```

```

2 //using auxiliary matrix t2(faster)
3 void rotate(){
4     forn(x, n) forn(y, n)
5         t2[n-y-1][x]=t[x][y];
6     memcpy(t, t2, sizeof(t));
7 }

```

### 5.13. Interseccion de Circulos en n3log(n)

```

1 struct event {
2     double x; int t;
3     event(double xx, int tt) : x(xx), t(tt) {}
4     bool operator <(const event &o) const { return x < o.x; }
5 };
6 typedef vector<Circle> VC;
7 typedef vector<event> VE;
8 int n;
9 double cuenta(VE &v, double A, double B) {
10     sort(v.begin(), v.end());
11     double res = 0.0, lx = ((v.empty())?0.0:v[0].x);
12     int contador = 0;
13     forn(i, sz(v)) {
14         // interseccion de todos (contador == n), union de todos (
15         // conjunto de puntos cubierto por exacta k Circulos (contador
16         // == k)
17         if (contador == n) res += v[i].x - lx;
18         contador += v[i].t;
19         lx = v[i].x;
20     }
21     return res;
22 }
23 // Primitiva de sqrt(r*r - x*x) como funcion double de una variable x.
24 inline double primitiva(double x, double r) {
25     if (x >= r) return r*r*M_PI/4.0;
26     if (x <= -r) return -r*r*M_PI/4.0;
27     double raiz = sqrt(r*r-x*x);
28     return 0.5 * (x * raiz + r*r*atan(x/raiz));
29 }
30 double interCircle(VC &v) {
31     vector<double> p; p.reserve(v.size() * (v.size() + 2));
32     forn(i, sz(v)) {
33         p.push_back(v[i].c.x + v[i].r);

```

```

33         p.push_back(v[i].c.x - v[i].r);
34     }
35     forn(i, sz(v)) forn(j, i) {
36         Circle &a = v[i], b = v[j];
37         double d = (a.c - b.c).norm();
38         if (fabs(a.r - b.r) < d && d < a.r + b.r) {
39             double alfa = acos((sqr(a.r) + sqr(d) - sqr(b.r)) / (2.0 * d
40             * a.r));
41             pto vec = (b.c - a.c) * (a.r / d);
42             p.pb((a.c + rotate(vec, alfa)).x);
43             p.pb((a.c + rotate(vec, -alfa)).x);
44         }
45     }
46     sort(p.begin(), p.end());
47     double res = 0.0;
48     forn(i, sz(p)-1) {
49         const double A = p[i], B = p[i+1];
50         VE ve; ve.reserve(2 * v.size());
51         forn(j, sz(v)) {
52             const Circle &c = v[j];
53             double arco = primitiva(B-c.c.x, c.r) - primitiva(A-c.c.x, c.r);
54             double base = c.c.y * (B-A);
55             ve.push_back(event(base + arco, -1));
56             ve.push_back(event(base - arco, 1));
57         }
58         res += cuenta(ve, A, B);
59     }
60     return res;

```

## 6. Math

### 6.1. Identidades

$$\begin{aligned}
 \sum_{i=0}^n \binom{n}{i} &= 2^n \\
 \sum_{i=0}^n i \binom{n}{i} &= n * 2^{n-1} \\
 \sum_{i=m}^n i &= \frac{n(n+1)}{2} - \frac{m(m-1)}{2} = \frac{(n+1-m)(n+m)}{2} \\
 \sum_{i=0}^n i &= \sum_{i=1}^n i = \frac{n(n+1)}{2} \\
 \sum_{i=0}^n i^2 &= \frac{n(n+1)(2n+1)}{6} = \frac{n^3}{3} + \frac{n^2}{2} + \frac{n}{6} \\
 \sum_{i=0}^n i(i-1) &= \frac{6}{6} \left(\frac{n}{2}\right) \left(\frac{n}{2} + 1\right) (n+1) \text{ (doubles)} \rightarrow \text{Sino ver caso impar y par} \\
 \sum_{i=0}^n i^3 &= \left(\frac{n(n+1)}{2}\right)^2 = \frac{n^4}{4} + \frac{n^3}{2} + \frac{n^2}{4} = \left[\sum_{i=1}^n i\right]^2
 \end{aligned}$$

$$\sum_{i=0}^n i^4 = \frac{n(n+1)(2n+1)(3n^2+3n-1)}{30} = \frac{n^5}{5} + \frac{n^4}{2} + \frac{n^3}{3} - \frac{n}{30}$$

$$\sum_{i=0}^n i^p = \frac{(n+1)^{p+1}}{p+1} + \sum_{k=1}^p \frac{B_k}{p-k+1} \binom{p}{k} (n+1)^{p-k+1}$$

$$r = e - v + k + 1$$

Teorema de Pick: (Area, puntos interiores y puntos en el borde)

$$A = I + \frac{B}{2} - 1$$

## 6.2. Ec. Caracteristica

$$a_0 T(n) + a_1 T(n-1) + \dots + a_k T(n-k) = 0$$

$$p(x) = a_0 x^k + a_1 x^{k-1} + \dots + a_k$$

Sean  $r_1, r_2, \dots, r_q$  las raíces distintas, de mult.  $m_1, m_2, \dots, m_q$

$$T(n) = \sum_{i=1}^q \sum_{j=0}^{m_i-1} c_{ij} n^j r_i^n$$

Las constantes  $c_{ij}$  se determinan por los casos base.

## 6.3. Combinatorio

```

1  forn(i, MAXN+1){//comb[i][k]=i tomados de a k
2      comb[i][0]=comb[i][i]=1;
3      forr(k, 1, i) comb[i][k]=(comb[i-1][k]+comb[i-1][k-1])%MOD;
4  }
5
6  ll lucas (ll n, ll k, int p){ //Calcula (n,k)%p teniendo comb[p][p]
    precalculado.
7      ll aux = 1;
8      while (n + k){
9          aux = (aux * comb[n%p][k%p]) %p;
10         n/=p, k/=p;
11     }
12     return aux;
13 }
```

## 6.4. Exp. de Numeros Mod.

```

1  ll expmod (ll b, ll e, ll m){//O(log b)
2      if(!e) return 1;
3      ll q= expmod(b,e/2,m); q=(q*q)%m;
4      return e%2? (b * q)%m : q;
5  }
```

## 6.5. Exp. de Matrices y Fibonacci en log(n)

```

1  struct M22{          // |a b|
2      tipo a,b,c,d;// |c d|
```

```

3      M22 operator*(const M22 &p) const {
4          return (M22){a*p.a+b*p.c, a*p.b+b*p.d, c*p.a+d*p.c,c*p.b+d*p.d};}
5  };
6  M22 operator^(const M22 &p, int n){
7      if(!n) return (M22){1, 0, 0, 1};//identidad
8      M22 q=p^(n/2); q=q*q;
9      return n%2? p * q : q;}
10
11 ll fibo(ll n){//calcula el fibonacci enesimo
12     M22 mat=(M22){0, 1, 1, 1}^n;
13     return mat.a*f0+mat.b*f1;//f0 y f1 son los valores iniciales
14 }
```

## 6.6. Teorema Chino del Resto

$$y = \sum_{j=1}^n (x_j * (\prod_{i=1, i \neq j}^n m_i)^{-1}_{m_j} * \prod_{i=1, i \neq j}^n m_i)$$

## 6.7. Funciones de primos

```

1  ll numPrimeFactors (ll n){
2      ll rta = 0;
3      map<ll,ll> f=fact(n);
4      forall(it, f) rta += it->second;
5      return rta;
6  }
7
8  ll numDiffPrimeFactors (ll n){
9      ll rta = 0;
10     map<ll,ll> f=fact(n);
11     forall(it, f) rta += 1;
12     return rta;
13 }
14
15 ll sumPrimeFactors (ll n){
16     ll rta = 0;
17     map<ll,ll> f=fact(n);
18     forall(it, f) rta += it->first;
19     return rta;
20 }
21
22 ll numDiv (ll n){
```



```

23 ll rta = 1;
24 map<ll,ll> f=fact(n);
25 forall(it, f) rta *= (it->second + 1);
26 return rta;
27 }
28
29 ll sumDiv (ll n){
30 ll rta = 1;
31 map<ll,ll> f=fact(n);
32 forall(it, f) rta *= ((ll)pow((double)it->first, it->second + 1.0)-1)
33 / (it->first-1);
34 return rta;
35 }
36
37 ll eulerPhi (ll n){ // con criba: O(lg n)
38 ll rta = n;
39 map<ll,ll> f=fact(n);
40 forall(it, f) rta -= rta / it->first;
41 return rta;
42 }
43
44 ll eulerPhi2 (ll n){ // O (sqrt n)
45 ll r = n;
46 forr (i,2,n+1){
47 if ((ll)i*i > n)
48 break;
49 if (n % i == 0){
50 while (n%i == 0) n/=i;
51 r -= r/i;
52 }}
53 if (n != 1)
54 r-= r/n;
55 return r;
56 }

```

## 6.8. Phollard's Rho (rolando)

```

1 ll gcd(ll a, ll b){return a?gcd(b %a, a):b;}
2
3 ll mulmod (ll a, ll b, ll c) { //returns (a*b)%c, and minimize overflow
4 ll x = 0, y = a%c;
5 while (b > 0){
6 if (b % 2 == 1) x = (x+y) % c;

```

```

7 y = (y*2) % c;
8 b /= 2;
9 }
10 return x % c;
11 }
12
13 ll expmod (ll b, ll e, ll m){//O(log b)
14 if(!e) return 1;
15 ll q= expmod(b,e/2,m); q=mulmod(q,q,m);
16 return e%2? mulmod(b,q,m) : q;
17 }
18
19 bool es_primo_prob (ll n, int a)
20 {
21 if (n == a) return true;
22 ll s = 0,d = n-1;
23 while (d % 2 == 0) s++,d/=2;
24
25 ll x = expmod(a,d,n);
26 if ((x == 1) || (x+1 == n)) return true;
27
28 forn (i, s-1){
29 x = mulmod(x, x, n);
30 if (x == 1) return false;
31 if (x+1 == n) return true;
32 }
33 return false;
34 }
35
36 bool rabin (ll n){ //devuelve true si n es primo
37 if (n == 1) return false;
38 const int ar[] = {2,3,5,7,11,13,17,19,23};
39 forn (j,9)
40 if (!es_primo_prob(n,ar[j]))
41 return false;
42 return true;
43 }
44
45 ll rho(ll n){
46 if( (n & 1) == 0 ) return 2;
47 ll x = 2 , y = 2 , d = 1;
48 ll c = rand() % n + 1;
49 while( d == 1 ){

```

```

50     x = (mulmod( x , x , n ) + c)%n;
51     y = (mulmod( y , y , n ) + c)%n;
52     y = (mulmod( y , y , n ) + c)%n;
53     if( x - y >= 0 ) d = gcd( x - y , n );
54     else d = gcd( y - x , n );
55 }
56 return d;
57 }

```

## 6.9. Criba

```

1  #define MAXP 100000 //no necesariamente primo
2  int criba[MAXP+1];
3  void crearcriba(){
4      int w[] = {4,2,4,2,4,6,2,6};
5      for(int p=25;p<=MAXP;p+=10) criba[p]=5;
6      for(int p=9;p<=MAXP;p+=6) criba[p]=3;
7      for(int p=4;p<=MAXP;p+=2) criba[p]=2;
8      for(int p=7,cur=0;p*p<=MAXP;p+=w[cur++&7]) if (!criba[p])
9          for(int j=p*p;j<=MAXP;j+=(p<<1)) if(!criba[j]) criba[j]=p;
10 }
11
12 vector<int> primos;
13 void buscarprimos(){
14     crearcriba();
15     forr (i,2,MAXP+1) if (!criba[i]) primos.push_back(i);
16 }
17
18 //~ Useful for bit trick:
19 //~ #define SET(i) ( criba[(i)>>5]|=1<<((i)&31) )
20 //~ #define INDEX(i) ( (criba[i]>>5)>>((i)&31))&1 )
21 //~ unsigned int criba[MAXP/32+1];

```

## 6.10. Factorizacion

Sea  $n = \prod p_i^{k_i}$ , fact(n) genera un map donde a cada  $p_i$  le asocia su  $k_i$

```

1  //factoriza bien numeros hasta MAXP^2
2  map<ll,ll> fact(ll n){ //0 (cant primos)
3      map<ll,ll> ret;
4      forall(p, primos){
5          while(!(n%p)){
6              ret[*p]++; //divisor found
7              n/=p;
8          }

```

```

9      }
10     if(n>1) ret[n]++;
11     return ret;
12 }
13
14 //factoriza bien numeros hasta MAXP
15 map<ll,ll> fact2(ll n){ //0 (lg n)
16     map<ll,ll> ret;
17     while (criba[n]){
18         ret[criba[n]]++;
19         n/=criba[n];
20     }
21     if(n>1) ret[n]++;
22     return ret;
23 }
24
25 map<ll,ll> f3;
26 void fact3(ll n){ //0 (lg n)^3. un solo numero
27     if (n == 1) return;
28     if (rabin(n))
29         f3[n]++;
30     else{
31         ll aux = rho(n);
32         fact3(aux); fact3(n/aux);
33     }
34     if(n>1) f3[n]++;
35     return;
36 }
37
38 //Usar asi: divisores(fac, divs, fac.begin()); NO ESTA ORDENADO
39 void divisores(const map<ll,ll> &f, vector<ll> &divs, map<ll,ll>::
40     iterator it, ll n=1){
41     if(it==f.begin()) divs.clear();
42     if(it==f.end()) {
43         if(n>1) divs.pb(n);
44         return;
45     }
46     ll p=it->fst, k=it->snd; ++it;
47     forn(_, k+1)
48         divisores(f, divs, it, n), n*=p;

```

## 6.11. GCD

```
1 | tipo gcd(tipo a, tipo b){return a?gcd(b %a, a):b;}
```

## 6.12. Extended Euclid

```
1 | void extendedEuclid (ll a, ll b){ //a * x + b * y = d
2 |   if (!b) { x = 1; y = 0; d = a; return;}
3 |   extendedEuclid (b, a%b);
4 |   ll x1 = y;
5 |   ll y1 = x - (a/b) * y;
6 |   x = x1; y = y1;
7 | }
```

## 6.13. LCM

```
1 | tipo lcm(tipo a, tipo b){return a / gcd(a,b) * b;}
```

## 6.14. Inversos

```
1 | #define MAXMOD 15485867
2 | ll inv[MAXMOD]; //inv[i]*i=1 mod MOD
3 | void calc(int p){ //0(p)
4 |   inv[1]=1;
5 |   forr(i, 2, p) inv[i]= p-((p/i)*inv[p%i])%p;
6 | }
7 | int inverso(int x){ //0(log x)
8 |   return expmod(x, eulerphi(MOD)-2); //si mod no es primo(sacar a mano)
9 |   return expmod(x, MOD-2); //si mod es primo
10 | }
```

## 6.15. Simpson

```
1 | double integral(double a, double b, int n=10000) { //0(n), n=cantdiv
2 |   double area=0, h=(b-a)/n, fa=f(a), fb;
3 |   for(i, n){
4 |     fb=f(a+h*(i+1));
5 |     area+=fa+ 4*f(a+h*(i+0.5)) +fb, fa=fb;
6 |   }
7 |   return area*h/6.;}
```

## 6.16. Fraction

```
1 | tipo mcd(tipo a, tipo b){return a?mcd(b%a, a):b;}
```

```
2 | struct frac{
```

```
3 |   tipo p,q;
4 |   frac(tipo p=0, tipo q=1):p(p),q(q) {norm();}
5 |   void norm(){
6 |     tipo a = mcd(p,q);
7 |     if(a) p/=a, q/=a;
8 |     else q=1;
9 |     if (q<0) q=-q, p=-p;}
10 |   frac operator+(const frac& o){
11 |     tipo a = mcd(q,o.q);
12 |     return frac(p*(o.q/a)+o.p*(q/a), q*(o.q/a));}
13 |   frac operator-(const frac& o){
14 |     tipo a = mcd(q,o.q);
15 |     return frac(p*(o.q/a)-o.p*(q/a), q*(o.q/a));}
16 |   frac operator*(frac o){
17 |     tipo a = mcd(q,o.p), b = mcd(o.q,p);
18 |     return frac((p/b)*(o.p/a), (q/a)*(o.q/b));}
19 |   frac operator/(frac o){
20 |     tipo a = mcd(q,o.q), b = mcd(o.p,p);
21 |     return frac((p/b)*(o.q/a), (q/a)*(o.p/b));}
22 |   bool operator<(const frac &o) const{return p*o.q < o.p*q;}
23 |   bool operator==(frac o){return p==o.p&&q==o.q;}
24 | };
```

## 6.17. Polinomio

```
1 | struct poly {
2 |   vector<tipo> c; //guarda los coeficientes del polinomio
3 |   poly(const vector<tipo> &c): c(c) {}
4 |   poly() {}
5 |   int gr(){ //calculates grade of the polynomial
6 |     return sz(c); }
7 |   bool isnull(){return c.empty();}
8 |   poly operator+(const poly &o) const {
9 |     int m = sz(c), n = sz(o.c);
10 |     vector<tipo> res(max(m,n));
11 |     for(i, m) res[i] += c[i];
12 |     for(i, n) res[i] += o.c[i];
13 |     return poly(res);
14 |   }
15 |   poly operator*(const poly &o) const {
16 |     int m = sz(c), n = sz(o.c);
17 |     vector<tipo> res(m+n-1);
18 |     for(i, m) for(j, n) res[i+j]+=c[i]*o.c[j];
```

```

19     return poly(res);
20 }
21 tipo eval(tipo v) {
22     tipo sum = 0;
23     forall(it, c) sum=sum*v + *it;
24     return sum;
25 }
26 //poly contains only a vector<int> c (the coeficients)
27 //the following function generates the roots of the polynomial
28 //it can be easily modified to return float roots
29 set<tipo> roots(){
30     set<tipo> roots;
31     tipo a0 = abs(c[0]), an = abs(c[sz(c)-1]);
32     vector<tipo> ps,qs;
33     forr(p,1,sqrt(a0)+1) if (a0%p==0) ps.pb(p),ps.pb(a0/p);
34     forr(q,1,sqrt(an)+1) if (an%q==0) qs.pb(q),qs.pb(an/q);
35     forall(pt,ps)
36         forall(qt,qs) if ( (*pt) % (*qt)==0 ) {
37             tipo root = abs((*pt) / (*qt));
38             if (eval(root)==0) roots.insert(root);
39         }
40     return roots;
41 }
42 };
43 poly interpolate(const vector<tipo> &x, const vector<tipo> &y) {
44     int n = sz(x);
45     poly p;
46     vector<tipo> aux(2);
47     forn(i, n) {
48         double a = y[i] - p.eval(x[i]);
49         forn(j, i) a /= x[i] - x[j];
50         poly add(vector<tipo>(1, a));
51         forn(j, i) aux[0]=-x[j], aux[1]=1, add = add*aux;
52         p = p + add;
53     }
54     return p;
55 }
56 //the following functions allows parsing an expression like
57 //34+150+4*45
58 //into a polynomial(el numero en funcion de la base)
59 #define LAST(s) (sz(s)? s[sz(s)-1] : 0)
60 #define POP(s) s.erase(--s.end());
61 poly D(string &s) {

```

```

62     poly d;
63     for(int i=0; isdigit(LAST(s)); i++) d.c.push_back(LAST(s)-'0'), POP(s);
64     ;
65     return d;}
66
67 poly T(string &s) {
68     poly t=D(s);
69     if (LAST(s)=='*'){POP(s); return T(s)*t;}
70     return t;
71 }
72 //main function, call this to parse
73 poly E(string &s) {
74     poly e=T(s);
75     if (LAST(s)=='+'){POP(s); return E(s)+e;}
76     return e;
77 }

```

## 6.18. Ec. Lineales

```

1 bool resolver_ev(Mat a, Vec y, Vec &x, Mat &ev){
2     int n = a.size(), m = n?a[0].size():0, rw = min(n, m);
3     vector<int> p; forn(i,m) p.push_back(i);
4     forn(i, rw) {
5         int uc=i, uf=i;
6         forr(f, i, n) forr(c, i, m) if(fabs(a[f][c])>fabs(a[uf][uc])) {uf=f;
7             uc=c;}
8         if (freq(a[uf][uc], 0)) { rw = i; break; }
9         forn(j, n) swap(a[j][i], a[j][uc]);
10        swap(a[i], a[uf]); swap(y[i], y[uf]); swap(p[i], p[uc]);
11        tipo inv = 1 / a[i][i]; //aca divide
12        forn(j, i+1, n) {
13            tipo v = a[j][i] * inv;
14            forr(k, i, m) a[j][k]-=v * a[i][k];
15            y[j] -= v*y[i];
16        }
17    } // rw = rango(a), aca la matriz esta triangulada
18    forr(i, rw, n) if (!freq(y[i],0)) return false; // chequeo de
19    compatibilidad
20    x = vector<tipo>(m, 0);
21    dforn(i, rw){
22        tipo s = y[i];
23        forn(j, i+1, rw) s -= a[i][j]*x[p[j]];
24        x[p[i]] = s / a[i][i]; //aca divide

```

```

23 }
24 ev = Mat(m-rw, Vec(m, 0)); // Esta parte va SOLO si se necesita el ev
25 forn(k, m-rw) {
26     ev[k][p[k+rw]] = 1;
27     dforn(i, rw){
28         tipo s = -a[i][k+rw];
29         forr(j, i+1, rw) s -= a[i][j]*ev[k][p[j]];
30         ev[k][p[i]] = s / a[i][i]; //aca divide
31     }
32 }
33 return true;
34 }

```

## 6.19. FFT

```

1 //~ typedef complex<double> base; //menos codigo, pero mas lento
2 //elegir si usar complejos de c (lento) o estos
3 struct base{
4     double r,i;
5     base(double r=0, double i=0):r(r), i(i){}
6     double real()const{return r;}
7     void operator/=(const int c){r/=c, i/=c;}
8 };
9 base operator*(const base &a, const base &b){
10     return base(a.r*b.r-a.i*b.i, a.r*b.i+a.i*b.r);}
11 base operator+(const base &a, const base &b){
12     return base(a.r+b.r, a.i+b.i);}
13 base operator-(const base &a, const base &b){
14     return base(a.r-b.r, a.i-b.i);}
15 vector<int> rev;
16 vector<base> wlen_pw;
17 inline static void fft(base a[], int n, bool invert) {
18     forn(i, n) if(i<rev[i]) swap(a[i], a[rev[i]]);
19     for (int len=2; len<=n; len<=1) {
20         double ang = 2*M_PI/len * (invert?-1:+1);
21         int len2 = len>>1;
22         base wlen (cos(ang), sin(ang));
23         wlen_pw[0] = base (1, 0);
24         forr(i, 1, len2) wlen_pw[i] = wlen_pw[i-1] * wlen;
25         for (int i=0; i<n; i+=len) {
26             base t,
27                 *pu = a+i,
28                 *pv = a+i+len2,

```

```

29         *pu_end = a+i+len2,
30         *pw = &wlen_pw[0];
31         for (; pu!=pu_end; ++pu, ++pv, ++pw) {
32             t = *pv * *pw;
33             *pv = *pu - t;
34             *pu = *pu + t;
35         }
36     }
37 }
38 if (invert) forn(i, n) a[i]/= n;
39 }
40 inline static void calc_rev(int n){//precalculo: llamar antes de fft!!
41     wlen_pw.resize(n);
42     rev.resize(n);
43     int lg=31-__builtin_clz(n);
44     forn(i, n){
45         rev[i] = 0;
46         forn(k, lg) if(i&(1<<k))
47             rev[i]|=1<<(lg-1-k);
48     }
49 }
50 inline static void multiply(const vector<int> &a, const vector<int> &b,
51     vector<int> &res) {
52     vector<base> fa (a.begin(), a.end()), fb (b.begin(), b.end());
53     int n=1;
54     while(n < max(sz(a), sz(b))) n <= 1;
55     n <= 1;
56     calc_rev(n);
57     fa.resize (n), fb.resize (n);
58     fft (&fa[0], n, false), fft (&fb[0], n, false);
59     forn(i, n) fa[i] = fa[i] * fb[i];
60     res.resize(n);
61     forn(i, n) res[i] = int (fa[i].real() + 0.5);
62 }
63 void toPoly(const string &s, vector<int> &P){//convierte un numero a
64     polinomio
65     P.clear();
66     dforn(i, sz(s)) P.pb(s[i]-'0');
67 }

```

## 6.20. Tablas y cotas (Primos, Divisores, Factoriales, etc)

### Factoriales

0! = 1	11! = 39.916.800
1! = 1	12! = 479.001.600 ( $\in \text{int}$ )
2! = 2	13! = 6.227.020.800
3! = 6	14! = 87.178.291.200
4! = 24	15! = 1.307.674.368.000
5! = 120	16! = 20.922.789.888.000
6! = 720	17! = 355.687.428.096.000
7! = 5.040	18! = 6.402.373.705.728.000
8! = 40.320	19! = 121.645.100.408.832.000
9! = 362.880	20! = 2.432.902.008.176.640.000 ( $\in \text{tint}$ )
10! = 3.628.800	21! = 51.090.942.171.709.400.000
max signed tint = 9.223.372.036.854.775.807	
max unsigned tint = 18.446.744.073.709.551.615	

### Primos

2 3 5 7 11 13 17 19 23 29 31 37 41 43 47 53 59 61 67 71 73 79 83 89 97 101 103 107 109  
 113 127 131 137 139 149 151 157 163 167 173 179 181 191 193 197 199 211 223 227 229  
 233 239 241 251 257 263 269 271 277 281 283 293 307 311 313 317 331 337 347 349 353  
 359 367 373 379 383 389 397 401 409 419 421 431 433 439 443 449 457 461 463 467 479  
 487 491 499 503 509 521 523 541 547 557 563 569 571 577 587 593 599 601 607 613 617  
 619 631 641 643 647 653 659 661 673 677 683 691 701 709 719 727 733 739 743 751 757  
 761 769 773 787 797 809 811 821 823 827 829 839 853 857 859 863 877 881 883 887 907  
 911 919 929 937 941 947 953 967 971 977 983 991 997 1009 1013 1019 1021 1031 1033  
 1039 1049 1051 1061 1063 1069 1087 1091 1093 1097 1103 1109 1117 1123 1129 1151  
 1153 1163 1171 1181 1187 1193 1201 1213 1217 1223 1229 1231 1237 1249 1259 1277  
 1279 1283 1289 1291 1297 1301 1303 1307 1319 1321 1327 1361 1367 1373 1381 1399  
 1409 1423 1427 1429 1433 1439 1447 1451 1453 1459 1471 1481 1483 1487 1489 1493  
 1499 1511 1523 1531 1543 1549 1553 1559 1567 1571 1579 1583 1597 1601 1607 1609  
 1613 1619 1621 1627 1637 1657 1663 1667 1669 1693 1697 1699 1709 1721 1723 1733  
 1741 1747 1753 1759 1777 1783 1787 1789 1801 1811 1823 1831 1847 1861 1867 1871  
 1873 1877 1879 1889 1901 1907 1913 1931 1933 1949 1951 1973 1979 1987 1993 1997  
 1999 2003 2011 2017 2027 2029 2039 2053 2063 2069 2081

### Primos cercanos a $10^n$

9941 9949 9967 9973 10007 10009 10037 10039 10061 10067 10069 10079  
 99961 99971 99989 99991 100003 100019 100043 100049 100057 100069  
 999959 999961 999979 999983 1000003 1000033 1000037 1000039  
 9999943 9999971 9999973 9999991 10000019 10000079 10000103 10000121  
 99999941 99999959 99999971 99999989 100000007 100000037 100000039 100000049  
 999999893 999999929 999999937 1000000007 1000000009 1000000021 1000000033

### Cantidad de primos menores que $10^n$

$\pi(10^1) = 4$  ;  $\pi(10^2) = 25$  ;  $\pi(10^3) = 168$  ;  $\pi(10^4) = 1229$  ;  $\pi(10^5) = 9592$   
 $\pi(10^6) = 78.498$  ;  $\pi(10^7) = 664.579$  ;  $\pi(10^8) = 5.761.455$  ;  $\pi(10^9) = 50.847.534$   
 $\pi(10^{10}) = 455.052,511$  ;  $\pi(10^{11}) = 4.118.054.813$  ;  $\pi(10^{12}) = 37.607.912.018$

### Divisores

Cantidad de divisores ( $\sigma_0$ ) para *algunos*  $n/\neg\exists n' < n, \sigma_0(n') \geq \sigma_0(n)$

$\sigma_0(60) = 12$  ;  $\sigma_0(120) = 16$  ;  $\sigma_0(180) = 18$  ;  $\sigma_0(240) = 20$  ;  $\sigma_0(360) = 24$   
 $\sigma_0(720) = 30$  ;  $\sigma_0(840) = 32$  ;  $\sigma_0(1260) = 36$  ;  $\sigma_0(1680) = 40$  ;  $\sigma_0(10080) = 72$   
 $\sigma_0(15120) = 80$  ;  $\sigma_0(50400) = 108$  ;  $\sigma_0(83160) = 128$  ;  $\sigma_0(110880) = 144$   
 $\sigma_0(498960) = 200$  ;  $\sigma_0(554400) = 216$  ;  $\sigma_0(1081080) = 256$  ;  $\sigma_0(1441440) = 288$   
 $\sigma_0(4324320) = 384$  ;  $\sigma_0(8648640) = 448$

Suma de divisores ( $\sigma_1$ ) para *algunos*  $n/\neg\exists n' < n, \sigma_1(n') \geq \sigma_1(n)$

$\sigma_1(96) = 252$  ;  $\sigma_1(108) = 280$  ;  $\sigma_1(120) = 360$  ;  $\sigma_1(144) = 403$  ;  $\sigma_1(168) = 480$   
 $\sigma_1(960) = 3048$  ;  $\sigma_1(1008) = 3224$  ;  $\sigma_1(1080) = 3600$  ;  $\sigma_1(1200) = 3844$   
 $\sigma_1(4620) = 16128$  ;  $\sigma_1(4680) = 16380$  ;  $\sigma_1(5040) = 19344$  ;  $\sigma_1(5760) = 19890$   
 $\sigma_1(8820) = 31122$  ;  $\sigma_1(9240) = 34560$  ;  $\sigma_1(10080) = 39312$  ;  $\sigma_1(10920) = 40320$   
 $\sigma_1(32760) = 131040$  ;  $\sigma_1(35280) = 137826$  ;  $\sigma_1(36960) = 145152$  ;  $\sigma_1(37800) = 148800$   
 $\sigma_1(60480) = 243840$  ;  $\sigma_1(64680) = 246240$  ;  $\sigma_1(65520) = 270816$  ;  $\sigma_1(70560) = 280098$   
 $\sigma_1(95760) = 386880$  ;  $\sigma_1(98280) = 403200$  ;  $\sigma_1(100800) = 409448$   
 $\sigma_1(491400) = 2083200$  ;  $\sigma_1(498960) = 2160576$  ;  $\sigma_1(514080) = 2177280$   
 $\sigma_1(982800) = 4305280$  ;  $\sigma_1(997920) = 4390848$  ;  $\sigma_1(1048320) = 4464096$   
 $\sigma_1(4979520) = 22189440$  ;  $\sigma_1(4989600) = 22686048$  ;  $\sigma_1(5045040) = 23154768$   
 $\sigma_1(9896040) = 44323200$  ;  $\sigma_1(9959040) = 44553600$  ;  $\sigma_1(9979200) = 45732192$

## 7. Grafos

### 7.1. Dijkstra

```

1  #define INF 1e9
2  int N;
3  #define MAX_V 250001
4  vector<ii> G[MAX_V];
5  //To add an edge use
6  #define add(a, b, w) G[a].pb(make_pair(w, b))
7
8  ll dijkstra(int s, int t){//O(|E| log |V|)
9      priority_queue<ii, vector<ii>, greater<ii>> > Q;
10     vector<ll> dist(N, INF); vector<int> dad(N, -1);
11     Q.push(make_pair(0, s)); dist[s] = 0;
12     while(sz(Q)){
13         ii p = Q.top(); Q.pop();

```

```

14     if(p.snd == t) break;
15     forall(it, G[p.snd])
16         if(dist[p.snd]+it->first < dist[it->snd]){
17             dist[it->snd] = dist[p.snd] + it->fst;
18             dad[it->snd] = p.snd;
19             Q.push(make_pair(dist[it->snd], it->snd));
20         }
21     }
22     return dist[t];
23     if(dist[t]<INF)//path generator
24         for(int i=t; i!=-1; i=dad[i])
25             printf("%d%c", i, (i==s?'\\n':'\\u'));
26 }

```

## 7.2. Bellman-Ford

```

1 vector<ii> G[MAX_N]; //ady. list with pairs (weight, dst)
2 int dist[MAX_N];
3 void bford(int src){ //O(VE)
4     dist[src]=0;
5     forn(i, N-1) forn(j, N) if(dist[j]!=INF) forall(it, G[j])
6         dist[it->snd]=min(dist[it->snd], dist[j]+it->fst);
7 }
8
9 bool hasNegCycle(){
10     forn(j, N) if(dist[j]!=INF) forall(it, G[j])
11         if(dist[it->snd]>dist[j]+it->fst) return true;
12     //inside if: all points reachable from it->snd will have -INF distance
13     // (do bfs)
14     return false;
15 }

```

## 7.3. Floyd-Warshall

```

1 //G[i][j] contains weight of edge (i, j) or INF
2 //G[i][i]=0
3 int G[MAX_N][MAX_N];
4 void floyd(){ //O(N^3)
5     forn(k, N) forn(i, N) if(G[i][k]!=INF) forn(j, N) if(G[k][j]!=INF)
6         G[i][j]=min(G[i][j], G[i][k]+G[k][j]);
7 }
8 bool inNegCycle(int v){
9     return G[v][v]<0;
10 }
//checks if there's a neg. cycle in path from a to b

```

```

11 bool hasNegCycle(int a, int b){
12     forn(i, N) if(G[a][i]!=INF && G[i][i]<0 && G[i][b]!=INF)
13         return true;
14     return false;
15 }

```

## 7.4. Kruskal

```

1 struct Ar{int a,b,w;};
2 bool operator<(const Ar& a, const Ar &b){return a.w<b.w;}
3 vector<Ar> E;
4 ll kruskal(){
5     ll cost=0;
6     sort(E.begin(), E.end()); //ordenar aristas de menor a mayor
7     uf.init(n);
8     forall(it, E){
9         if(uf.comp(it->a)!=uf.comp(it->b)){ //si no estan conectados
10             uf.unir(it->a, it->b); //conectar
11             cost+=it->w;
12         }
13     }
14     return cost;
15 }

```

## 7.5. Prim

```

1 bool taken[MAXN];
2 priority_queue<ii, vector<ii>, greater<ii> > pq; //min heap
3 void process(int v){
4     taken[v]=true;
5     forall(e, G[v])
6         if(!taken[e->second]) pq.push(*e);
7 }
8
9 ll prim(){
10     zero(taken);
11     process(0);
12     ll cost=0;
13     while(sz(pq)){
14         ii e=pq.top(); pq.pop();
15         if(!taken[e.second]) cost+=e.first, process(e.second);
16     }
17     return cost;
18 }

```



## 7.6. 2-SAT + Tarjan SCC

```

1 //We have a vertex representing a var and other for his negation.
2 //Every edge stored in G represents an implication. To add an equation
  of the form a||b, use addor(a, b)
3 //MAX=max cant var, n=cant var
4 #define addor(a, b) (G[neg(a)].pb(b), G[neg(b)].pb(a))
5 vector<int> G[MAX*2];
6 //idx[i]=index assigned in the dfs
7 //lw[i]=lowest index(closer from the root) reachable from i
8 int lw[MAX*2], idx[MAX*2], qidx;
9 stack<int> q;
10 int qcmp, cmp[MAX*2];
11 //verdad[cmp[i]]=valor de la variable i
12 bool verdad[MAX*2+1];
13
14 int neg(int x) { return x>=n? x-n : x+n;}
15 void tjn(int v){
16     lw[v]=idx[v]=++qidx;
17     q.push(v), cmp[v]=-2;
18     forall(it, G[v]){
19         if(!idx[*it] || cmp[*it]==-2){
20             if(!idx[*it]) tjn(*it);
21             lw[v]=min(lw[v], lw[*it]);
22         }
23     }
24     if(lw[v]==idx[v]){
25         qcmp++;
26         int x;
27         do{x=q.top(); q.pop(); cmp[x]=qcmp;}while(x!=v);
28         verdad[qcmp]=(cmp[neg(v)]<0);
29     }
30 }
31 //remember to CLEAR G!!!
32 bool satisf(){//O(n)
33     memset(idx, 0, sizeof(idx)), qidx=0;
34     memset(cmp, -1, sizeof(cmp)), qcmp=0;
35     forn(i, n){
36         if(!idx[i]) tjn(i);
37         if(!idx[neg(i)]) tjn(neg(i));
38     }
39     forn(i, n) if(cmp[i]==cmp[neg(i)]) return false;
40     return true;

```

41 | }

## 7.7. Articulation Points

```

1 int N;
2 vector<int> G[1000000];
3 //V[i]=node number(if visited), L[i]= lowest V[i] reachable from i
4 int qV, V[1000000], L[1000000], P[1000000];
5 void dfs(int v, int f){
6     L[v]=V[v]=++qV;
7     forall(it, G[v])
8         if(!V[*it]){
9             dfs(*it, v);
10            L[v] = min(L[v], L[*it]);
11            P[v] += L[*it]>=V[v];
12        }
13        else if(*it!=f)
14            L[v]=min(L[v], V[*it]);
15    }
16    int cantart(){ //O(n)
17        qV=0;
18        zero(V), zero(P);
19        dfs(1, 0); P[1]--;
20        int q=0;
21        forn(i, N) if(P[i]) q++;
22        return q;
23    }

```

## 7.8. Comp. Biconexas y Puentes

```

1 struct edge {
2     int u,v, comp;
3     bool bridge;
4 };
5 vector<edge> e;
6 void addEdge(int u, int v) {
7     G[u].pb(sz(e)), G[v].pb(sz(e));
8     e.pb((edge){u,v,-1,false});
9 }
10 //d[i]=id de la dfs
11 //b[i]=lowest id reachable from i
12 int d[MAXN], b[MAXN], t;
13 int nbc;//cant componentes
14 int comp[MAXN]; //comp[i]=cant comp biconexas a la cual pertenece i

```

```

15 void initDfs(int n) {
16     zero(G), zero(comp);
17     e.clear();
18     forn(i,n) d[i]=-1;
19     nbc = t = 0;
20 }
21 stack<int> st;
22 void dfs(int u, int pe) { // O(n + m)
23     b[u] = d[u] = t++;
24     comp[u] = (pe != -1);
25     forall(ne, G[u]) if (*ne != pe){
26         int v = e[*ne].u ^ e[*ne].v ^ u;
27         if (d[v] == -1) {
28             st.push(*ne);
29             dfs(v,*ne);
30             if (b[v] > d[u]){
31                 e[*ne].bridge = true; // bridge
32             }
33             if (b[v] >= d[u]){ // art
34                 int last;
35                 do {
36                     last = st.top(); st.pop();
37                     e[last].comp = nbc;
38                 } while (last != *ne);
39                 nbc++;
40                 comp[u]++;
41             }
42             b[u] = min(b[u], b[v]);
43         }
44         else if (d[v] < d[u]) { // back edge
45             st.push(*ne);
46             b[u] = min(b[u], d[v]);
47         }
48     }
49 }

```

## 7.9. LCA + Climb

```

1 //f[v][k] holds the 2^k father of v
2 //L[v] holds the level of v
3 int N, f[100001][20], L[100001];
4 void build(){ //f[i][0] must be filled previously, O(nlgn)
5     forn(k, 20-1) forn(i, N) f[i][k+1]=f[f[i][k]][k];}

```

```

6
7 #define lg(x) (31-__builtin_clz(x))//=floor(log2(x))
8
9 int climb(int a, int d){ // O(lgn)
10     if(!d) return a;
11     dforn(i, lg(L[a])+1)
12         if(1<<i<=d)
13             a=f[a][i], d-=1<<i;
14     return a;
15 }
16 int lca(int a, int b){ // O(lgn)
17     if(L[a]<L[b]) swap(a, b);
18     a=climb(a, L[a]-L[b]);
19     if(a==b) return a;
20     dforn(i, lg(L[a])+1)
21         if(f[a][i]!=f[b][i])
22             a=f[a][i], b=f[b][i];
23     return f[a][0];
24 }

```

## 7.10. Heavy Light Decomposition

```

1 int treesz[MAXN]; //cantidad de nodos en el subarbol del nodo v
2 int dad[MAXN]; //dad[v]=padre del nodo v
3 void dfs1(int v, int p=-1){ //pre-dfs
4     dad[v]=p;
5     treesz[v]=1;
6     forall(it, G[v]) if(*it!=p){
7         dfs1(*it, v);
8         treesz[v]+=treesz[*it];
9     }
10 }
11 int pos[MAXN], q; //pos[v]=posicion del nodo v en el recorrido de la dfs
12 //Las cadenas aparecen continuas en el recorrido!
13 int cantcad;
14 int homecad[MAXN]; //dada una cadena devuelve su nodo inicial
15 int cad[MAXN]; //cad[v]=cadena a la que pertenece el nodo
16 void heavylight(int v, int cur=-1){
17     if(cur== -1) homecad[cur=cantcad++]=v;
18     pos[v]=q++;
19     cad[v]=cur;
20     int mx=-1;
21     forn(i, sz(G[v])) if(G[v][i]!=dad[v])

```

```

22     if(mx==-1 || treesz[G[v][mx]]<treesz[G[v][i]]) mx=i;
23     if(mx!=-1) heavyLight(G[v][mx], cur);
24     forn(i, sz(G[v])) if(i!=mx && G[v][i]!=dad[v])
25         heavyLight(G[v][i], -1);
26 }
27 //ejemplo de obtener el maximo numero en el camino entre dos nodos
28 //RTA: max(query(low, u), query(low, v)), con low=lca(u, v)
29 //esta funcion va trepando por las cadenas
30 int query(int an, int v){//O(logn)
31     //si estan en la misma cadena:
32     if(cad[an]==cad[v]) return rmq.get(pos[an], pos[v]+1);
33     return max(query(an, dad[homecad[cad[v]]]),
34               rmq.get(pos[homecad[cad[v]]], pos[v]+1));
35 }

```

### 7.11. Centroid Decomposition

```

1  typedef pair<int,int> ii;
2  int n,szt[100100],letter[100100];
3  bool taken[100100];
4  vector<int> G[100100];
5
6  void calcsz(int v, int p) {
7      szt[v] = 1;
8      forall(it,G[v]) if (*it!=p && !taken[*it])
9          calcsz(*it,v), szt[v]+=szt[*it];
10 }
11
12 void centroid(int v, int lvl=0, int tam=-1) {
13     if(tam==-1) calcsz(v, -1), tam=szt[v];
14     forall(it, G[v]) if(!taken[*it] && szt[*it]>=tam/2)
15         {szt[v]=0; centroid(*it, lvl, tam); return;}
16     taken[v]=true;
17     letter[v]=lvl;
18     forall(it, G[v]) if(!taken[*it])
19         centroid(*it, lvl+1, -1);
20 }

```

### 7.12. Euler Cycle

```

1  int n,m,ars[MAXE], eq;
2  vector<int> G[MAXN]; //fill G,n,m,ars,eq
3  list<int> path;
4  int used[MAXN];

```

```

5  bool usede[MAXE];
6  queue<list<int>::iterator> q;
7  int get(int v){
8      while(used[v]<sz(G[v]) && usede[ G[v][used[v]] ]) used[v]++;
9      return used[v];
10 }
11 void explore(int v, int r, list<int>::iterator it){
12     int ar=G[v][get(v)]; int u=v^ars[ar];
13     usede[ar]=true;
14     list<int>::iterator it2=path.insert(it, u);
15     if(u!=r) explore(u, r, it2);
16     if(get(v)<sz(G[v])) q.push(it);
17 }
18 void euler(){
19     zero(used), zero(usede);
20     path.clear();
21     q=queue<list<int>::iterator>();
22     path.push_back(0); q.push(path.begin());
23     while(sz(q)){
24         list<int>::iterator it=q.front(); q.pop();
25         if(used[*it]<sz(G[*it])) explore(*it, *it, it);
26     }
27     reverse(path.begin(), path.end());
28 }
29 void addEdge(int u, int v){
30     G[u].pb(eq), G[v].pb(eq);
31     ars[eq++]=u^v;
32 }

```

### 7.13. Chu-liu

```

1  void visit(graph &h, int v, int s, int r,
2      vector<int> &no, vector< vector<int> > &comp,
3      vector<int> &prev, vector< vector<int> > &next, vector<weight> &mcost,
4      vector<int> &mark, weight &cost, bool &found) {
5      if (mark[v]) {
6          vector<int> temp = no;
7          found = true;
8          do {
9              cost += mcost[v];
10             v = prev[v];
11             if (v != s) {
12                 while (comp[v].size() > 0) {

```

```

13     no[comp[v].back()] = s;
14     comp[s].push_back(comp[v].back());
15     comp[v].pop_back();
16 }
17 }
18 } while (v != s);
19 forall(j, comp[s]) if (*j != r) forall(e, h[*j])
20     if (no[e->src] != s) e->w -= mcost[ temp[*j] ];
21 }
22 mark[v] = true;
23 forall(i, next[v]) if (no[*i] != no[v] && prev[no[*i]] == v)
24     if (!mark[no[*i]] || *i == s)
25         visit(h, *i, s, r, no, comp, prev, next, mcost, mark, cost, found)
26         ;
27 }
28 weight minimumSpanningArborescence(const graph &g, int r) {
29     const int n=sz(g);
30     graph h(n);
31     forn(u,n) forall(e,g[u]) h[e->dst].pb(*e);
32     vector<int> no(n);
33     vector<vector<int>> > comp(n);
34     forn(u, n) comp[u].pb(no[u] = u);
35     for (weight cost = 0; ;) {
36         vector<int> prev(n, -1);
37         vector<weight> mcost(n, INF);
38         forn(j,n) if (j != r) forall(e,h[j])
39             if (no[e->src] != no[j])
40                 if (e->w < mcost[ no[j] ])
41                     mcost[ no[j] ] = e->w, prev[ no[j] ] = no[e->src];
42         vector< vector<int> > next(n);
43         forn(u,n) if (prev[u] >= 0)
44             next[ prev[u] ].push_back(u);
45         bool stop = true;
46         vector<int> mark(n);
47         forn(u,n) if (u != r && !mark[u] && !comp[u].empty()) {
48             bool found = false;
49             visit(h, u, u, r, no, comp, prev, next, mcost, mark, cost, found);
50             if (found) stop = false;
51         }
52         if (stop) {
53             forn(u,n) if (prev[u] >= 0) cost += mcost[u];
54             return cost;
55         }
56     }

```

```

55     }
56 }

```

## 7.14. Hungarian

```

1  #define MAXN 256
2  #define INFTO 0x7f7f7f7f
3  int n;
4  int mt[MAXN][MAXN]; // Matriz de costos (X * Y)
5  int xy[MAXN], yx[MAXN]; // Matching resultante (X->Y, Y->X)
6  int lx[MAXN], ly[MAXN], slk[MAXN], slkx[MAXN], prv[MAXN];
7  char S[MAXN], T[MAXN];
8  void updtree(int x) {
9      forn(y, n) if (lx[x] + ly[y] - mt[x][y] < slk[y]) {
10         slk[y] = lx[x] + ly[y] - mt[x][y];
11         slkx[y] = x;
12     } }
13 int hungar() { // Matching maximo de mayor costo en grafos dirigidos (N^3)
14     forn(i, n) {
15         ly[i] = 0;
16         lx[i] = *max_element(mt[i], mt[i]+n); }
17     memset(xy, -1, sizeof(xy));
18     memset(yx, -1, sizeof(yx));
19     forn(m, n) {
20         memset(S, 0, sizeof(S));
21         memset(T, 0, sizeof(T));
22         memset(prv, -1, sizeof(prv));
23         memset(slk, 0x7f, sizeof(slk));
24         queue<int> q;
25         #define bpone(e, p) { q.push(e); prv[e] = p; S[e] = 1; updtree(e); }
26         forn(i, n) if (xy[i] == -1) { bpone(i, -2); break; }
27         int x=0, y=-1;
28         while (y!=-1) {
29             while (!q.empty() && y!=-1) {
30                 x = q.front(); q.pop();
31                 forn(j, n) if (mt[x][j] == lx[x] + ly[j] && !T[j]) {
32                     if (yx[j] == -1) { y = j; break; }
33                     T[j] = 1;
34                     bpone(yx[j], x);
35                 }
36             }
37             if (y!=-1) break;
38             int dlt = INFTO;

```

```

39     forn(j, n) if (!T[j]) dlt = min(dlt, slk[j]);
40     forn(k, n) {
41         if (S[k]) lx[k] -= dlt;
42         if (T[k]) ly[k] += dlt;
43         if (!T[k]) slk[k] -= dlt;
44     }
45     forn(j, n) if (!T[j] && !slk[j]) {
46         if (yx[j] == -1) {
47             x = slkx[j]; y = j; break;
48         } else {
49             T[j] = 1;
50             if (!S[yx[j]]) bpone(yx[j], slkx[j]);
51         }
52     }
53 }
54 if (y!=-1) {
55     for(int p = x; p != -2; p = prv[p]) {
56         yx[y] = p;
57         int ty = xy[p]; xy[p] = y; y = ty;
58     }
59 } else break;
60 }
61 int res = 0;
62 forn(i, n) res += mt[i][xy[i]];
63 return res;
64 }

```

## 8. Network Flow

### 8.1. Dinic

```

1  int nodes, src, dest;
2  int dist[MAX], q[MAX], work[MAX];
3
4  struct Edge {
5      int to, rev;
6      ll f, cap;
7      Edge(int to, int rev, ll f, ll cap) : to(to), rev(rev), f(f), cap(cap)
8      {}
9  };
10 vector<Edge> G[MAX];
11

```

```

12 // Adds bidirectional edge
13 void addEdge(int s, int t, ll cap){
14     G[s].push_back(Edge(t, G[t].size(), 0, cap));
15     G[t].push_back(Edge(s, G[s].size()-1, 0, 0));
16 }
17
18 bool dinic_bfs() {
19     fill(dist, dist + nodes, -1);
20     dist[src] = 0;
21     int qt = 0;
22     q[qt++] = src;
23     for (int qh = 0; qh < qt; qh++) {
24         int u = q[qh];
25         forall(e, G[u]){
26             int v = e->to;
27             if(dist[v]<0 && e->f < e->cap){
28                 dist[v]=dist[u]+1;
29                 q[qt++]=v;
30             }
31         }
32     }
33     return dist[dest] >= 0;
34 }
35
36 ll dinic_dfs(int u, ll f) {
37     if (u == dest) return f;
38     for (int &i = work[u]; i < (int) G[u].size(); i++) {
39         Edge &e = G[u][i];
40         if (e.cap <= e.f) continue;
41         int v = e.to;
42         if (dist[v] == dist[u] + 1) {
43             ll df = dinic_dfs(v, min(f, e.cap - e.f));
44             if (df > 0) {
45                 e.f += df;
46                 G[v][e.rev].f -= df;
47                 return df;
48             }
49         }
50     }
51     return 0;
52 }
53
54 ll maxFlow(int _src, int _dest) { //O(V^2 E)<

```

```

55 src = _src;
56 dest = _dest;
57 ll result = 0;
58 while (dinic_bfs()) {
59     fill(work, work + nodes, 0);
60     while(ll delta = dinic_dfs(src, INF))
61         result += delta;
62 }
63
64 // todos los nodos con dist[v]!=-1 vs los que tienen dist[v]==-1
65 // forman el min cut
66
67 return result;
68 }

```

## 8.2. Konig

```

1 // asume que el dinic YA ESTA tirado
2 // asume que nodes-1 y nodes-2 son la fuente y destino
3 int match[maxnodes]; // match[v]=u si u-v esta en el matching, -1 si v
4 // no esta matcheado
5 int s[maxnodes]; // numero de la bfs del koning
6 queue<int> kq;
7 // s[e]%2==1 o si e esta en V1 y s[e]==-1-> lo agarras
8 void koning() { // O(n)
9     forn(v,nodes-2) s[v] = match[v] = -1;
10    forn(v,nodes-2) forall(it,g[v]) if (it->to < nodes-2 && it->f>0)
11        { match[v]=it->to; match[it->to]=v;}
12    forn(v,nodes-2) if (match[v]==-1) {s[v]=0;kq.push(v);}
13    while(!kq.empty()) {
14        int e = kq.front(); kq.pop();
15        if (s[e]%2==1) {
16            s[match[e]] = s[e]+1;
17            kq.push(match[e]);
18        } else {
19            forall(it,g[e]) if (it->to < nodes-2 && s[it->to]==-1) {
20                s[it->to] = s[e]+1;
21                kq.push(it->to);
22            }
23        }
24    }
25 }

```

## 8.3. Edmonds Karp's

```

1 #define MAX_V 1000
2 #define INF 1e9
3 //special nodes
4 #define SRC 0
5 #define SNK 1
6 map<int, int> G[MAX_V]; //limpiar esto
7 //To add an edge use
8 #define add(a, b, w) G[a][b]=w
9 int f, p[MAX_V];
10 void augment(int v, int minE){
11     if(v==SRC) f=minE;
12     else if(p[v]!=-1){
13         augment(p[v], min(minE, G[p[v]][v]));
14         G[p[v]][v]-=f, G[v][p[v]]+=f;
15     }
16 }
17 ll maxflow(){ //O(VE^2)
18     ll Mf=0;
19     do{
20         f=0;
21         char used[MAX_V]; queue<int> q; q.push(SRC);
22         zero(used), memset(p, -1, sizeof(p));
23         while(sz(q)){
24             int u=q.front(); q.pop();
25             if(u==SNK) break;
26             forall(it, G[u])
27                 if(it->snd>0 && !used[it->fst])
28                     used[it->fst]=true, q.push(it->fst), p[it->fst]=u;
29         }
30         augment(SNK, INF);
31         Mf+=f;
32     }while(f);
33     return Mf;
34 }

```

## 8.4. Push-Relabel O(N<sup>3</sup>)

```

1 #define MAX_V 1000
2 int N; //valid nodes are [0...N-1]
3 #define INF 1e9
4 //special nodes

```

```

5  #define SRC 0
6  #define SNK 1
7  map<int, int> G[MAX_V];
8  //To add an edge use
9  #define add(a, b, w) G[a][b]=w
10 ll excess[MAX_V];
11 int height[MAX_V], active[MAX_V], count[2*MAX_V+1];
12 queue<int> Q;
13 void enqueue(int v) {
14     if (!active[v] && excess[v] > 0) active[v]=true, Q.push(v); }
15 void push(int a, int b) {
16     int amt = min(excess[a], ll(G[a][b]));
17     if(height[a] <= height[b] || amt == 0) return;
18     G[a][b]-=amt, G[b][a]+=amt;
19     excess[b] += amt, excess[a] -= amt;
20     enqueue(b);
21 }
22 void gap(int k) {
23     forn(v, N){
24         if (height[v] < k) continue;
25         count[height[v]]--;
26         height[v] = max(height[v], N+1);
27         count[height[v]]++;
28         enqueue(v);
29     }
30 }
31 void relabel(int v) {
32     count[height[v]]--;
33     height[v] = 2*N;
34     forall(it, G[v])
35         if(it->snd)
36             height[v] = min(height[v], height[it->fst] + 1);
37     count[height[v]]++;
38     enqueue(v);
39 }
40 ll maxflow() { //O(V^3)
41     zero(height), zero(active), zero(count), zero(excess);
42     count[0] = N-1;
43     count[N] = 1;
44     height[SRC] = N;
45     active[SRC] = active[SNK] = true;
46     forall(it, G[SRC]){
47         excess[SRC] += it->snd;

```

```

48     push(SRC, it->fst);
49 }
50 while(sz(Q)) {
51     int v = Q.front(); Q.pop();
52     active[v]=false;
53     forall(it, G[v]) push(v, it->fst);
54     if(excess[v] > 0)
55         count[height[v]] == 1? gap(height[v]):relabel(v);
56 }
57 ll mf=0;
58 forall(it, G[SRC]) mf+=G[it->fst][SRC];
59 return mf;
60 }

```

## 8.5. Min-cost Max-flow

```

1  struct edge {
2      int u, v;
3      ll cap, cost, flow;
4      ll rem() { return cap - flow; }
5  };
6  int n; //numero de nodos
7  vector<int> G[MAXN];
8  vector<edge> e;
9  void addEdge(int u, int v, ll cap, ll cost) {
10     G[u].pb(si(e)); e.pb((edge){u,v,cap,cost,0});
11     G[v].pb(si(e)); e.pb((edge){v,u,0,-cost,0});
12 }
13 ll pot[MAXN], dist[MAXN], pre[MAXN], cap[MAXN];
14 ll mxFlow, mnCost;
15 void flow(int s, int t) {
16     fill(pot, pot+n, 0);
17     mxFlow=mnCost=0;
18     while(1){
19         fill(dist, dist+n, INF); dist[s] = 0;
20         fill(pre, pre+n, -1); pre[s]=0;
21         fill(cap, cap+n, 0); cap[s] = INF;
22         priority_queue<pair<ll,int> > q; q.push(make_pair(0,s));
23         while (!q.empty()) {
24             pair<ll,int> top = q.top(); q.pop();
25             int u = top.second, d = -top.first;
26             if (u == t) break;
27             if (d > dist[u]) continue;

```



```

28     forn(i, si(G[u])) {
29         edge E = e[G[u][i]];
30         int c = E.cost + pot[u] - pot[E.v];
31         if (E.rem() && dist[E.v] > dist[u] + c) {
32             dist[E.v] = dist[u] + c;
33             pre[E.v] = G[u][i];
34             cap[E.v] = min(cap[u], E.rem());
35             q.push(make_pair(-dist[E.v], E.v));
36         }
37     }
38 }
39 if (pre[t] == -1) break;
40 forn(u, n)
41     if (dist[u] == INF) pot[u] = INF;
42     else pot[u] += dist[u];
43 mxFlow += cap[t];
44 mnCost += cap[t] * pot[t];
45 for (int v = t; v != s; v = e[pre[v]].u) {
46     e[pre[v]].flow += cap[t];
47     e[pre[v]^1].flow -= cap[t];
48 }
49 }
50 }

```

## 9. Template

```

1  #include <bits/stdc++.h>
2  using namespace std;
3  #define dprint(v) cerr << #v "=" << v << endl //;)
4  #define forr(i,a,b) for(int i=(a); i<(b); i++)
5  #define forn(i,n) forr(i,0,n)
6  #define dforn(i,n) for(int i=n-1; i>=0; i--)
7  #define forall(it,v) for(typeof(v.begin()) it=v.begin(); it!=v.end(); ++it)
8
9  #define sz(c) ((int)c.size())
10 #define zero(v) memset(v, 0, sizeof(v))
11 #define pb push_back
12 #define fst first
13 #define snd second
14 typedef long long ll;
15 typedef pair<int,int> ii;
16
17 int main() {

```

```

17     freopen("input.in", "r", stdin);
18     ios::sync_with_stdio(0);
19     while(){
20
21     }
22     return 0;
23 }

```

## 10. Ayudamemoria

### Cant. decimales

```

1  #include <iomanip>
2  cout << setprecision(2) << fixed;

```

### Rellenar con espacios(para justificar)

```

1  #include <iomanip>
2  cout << setfill(' ') << setw(3) << 2 << endl;

```

### Leer hasta fin de linea

```

1  #include <sstream>
2  //hacer cin.ignore() antes de getline()
3  while(getline(cin, line)){
4      istringstream is(line);
5      while(is >> X)
6          cout << X << " ";
7      cout << endl;
8  }

```

### Aleatorios

```

1  #define RAND(a, b) (rand()%(b-a+1)+a)
2  srand(time(NULL));

```

### Doubles Comp.

```

1  const double EPS = 1e-9;
2  x == y <=> fabs(x-y) < EPS
3  x > y <=> x > y + EPS
4  x >= y <=> x > y - EPS

```

### Limites

```
1 | #include <limits>
2 | numeric_limits<T>
3 |     ::max()
4 |     ::min()
5 |     ::epsilon()
```

## Muahaha

```
1 | #include <signal.h>
2 | void divzero(int p){
3 |     while(true);}
4 | void segm(int p){
5 |     exit(0);}
6 | //in main
7 | signal(SIGFPE, divzero);
8 | signal(SIGSEGV, segm);
```

## Mejorar velocidad

```
1 | ios::sync_with_stdio(false);
```

## Mejorar velocidad 2

```
1 | //Solo para enteros positivos
2 | inline void Scanf(int& a){
3 |     char c = 0;
4 |     while(c<33) c = getc(stdin);
5 |     a = 0;
6 |     while(c>33) a = a*10 + c - '0', c = getc(stdin);
7 | }
```

## Leer del teclado

```
1 | freopen("/dev/tty", "a", stdin);
```

## Iterar subconjunto

```
1 | for(int sbm=bm; sbm; sbm=(sbm-1)&bm)
```

## File setup

```
1 | //tambien se pueden usar comas: {a, x, m, l}
2 | touch {a..l}.in; tee {a..l}.cpp < template.cpp
```