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1. algorithm

#include <algorithm> #include <numeric>

Algo	Params	Funcion
sort, stable_sort	f, l	ordena el intervalo
nth_element	f, nth, l	<i>void</i> ordena el n-esimo, y particiona el resto
fill, fill_n	f, l / n, elem	<i>void</i> llena [f, l) o [f, f+n) con elem
lower_bound, upper_bound	f, l, elem	<i>it</i> al primer / ultimo donde se puede insertar elem para que quede ordenada
binary_search	f, l, elem	<i>bool</i> esta elem en [f, l)
copy	f, l, resul	hace resul+i=f+i $\forall i$
find, find_if, find_first_of	f, l, elem / pred / f2, l2	<i>it</i> encuentra i $\in [f, l)$ tq. i=elem, pred(i), i $\in [f2, l2)$
count, count_if	f, l, elem/pred	cuenta elem, pred(i)
search	f, l, f2, l2	busca [f2,l2) $\in [f, l)$
replace, replace_if	f, l, old / pred, new	cambia old / pred(i) por new
reverse	f, l	da vuelta
partition, stable_partition	f, l, pred	pred(i) ad, !pred(i) atras
min_element, max_element	f, l, [comp]	<i>it</i> min, max de [f,l]
lexicographical_compare	f1,l1,f2,l2	<i>bool</i> con [f1,l1]i[f2,l2]
next/prev_permutation	f,l	deja en [f,l) la perm sig, ant
set_intersection, set_difference, set_union, set_symmetric_difference,	f1, l1, f2, l2, res	[res, ...) la op. de conj
push_heap, pop_heap, make_heap	f, l, e / e /	mete/saca e en heap [f,l), hace un heap de [f,l)
is_heap	f,l	<i>bool</i> es [f,l) un heap
accumulate	f,l,i,[op]	$T = \sum$ /oper de [f,l)
inner_product	f1, l1, f2, i	$T = i + [f1, l1) \cdot [f2, \dots)$
partial_sum	f, l, r, [op]	r+i = \sum /oper de [f,f+i] $\forall i \in [f, l)$
__builtin_ffs	unsigned int	Pos. del primer 1 desde la derecha
__builtin_clz	unsigned int	Cant. de ceros desde la izquierda.
__builtin_ctz	unsigned int	Cant. de ceros desde la derecha.
__builtin_popcount	unsigned int	Cant. de 1's en x.
__builtin_parity	unsigned int	1 si x es par, 0 si es impar.
__builtin_XXXXXXll	unsigned ll	= pero para long long's.

2. Estructuras

2.1. RMQ (static)

Dado un arreglo y una operacion asociativa *idempotente*, `get(i, j)` opera sobre el rango `[i, j]`. Restriccion: $LVL \geq \text{ceil}(\log n)$; Usar `[]` para llenar arreglo y luego `build()`.

```

1 struct RMQ{
2     #define LVL 10
3     tipo vec[LVL][1<<(LVL+1)];
4     tipo &operator[] (int p){return vec[0][p];}
5     tipo get(int i, int j) { //intervalo [i,j]
6         int p = 31-__builtin_clz(j-i);
7         return min(vec[p][i], vec[p][j-(1<<p)]);
8     }
9     void build(int n) { //O(nlogn)
10        int mp = 31-__builtin_clz(n);
11        forn(p, mp) forn(x, n-(1<<p))
12            vec[p+1][x] = min(vec[p][x], vec[p][x+(1<<p)]);
13    };

```

2.2. RMQ (dynamic)

```

1 //Dado un arreglo y una operacion asociativa con neutro, get(i, j) opera
   sobre el rango [i, j].
2 #define MAXN 100000
3 #define operacion(x, y) max(x, y)
4 const int neutro=0;
5 struct RMQ{
6     int sz;
7     tipo t[4*MAXN];
8     tipo &operator[] (int p){return t[sz+p];}
9     void init(int n){ //O(nlgn)
10        sz = 1 << (32-__builtin_clz(n));
11        forn(i, 2*sz) t[i]=neutro;
12    }
13     void updall(){ //O(n)
14        dforn(i, sz) t[i]=operacion(t[2*i], t[2*i+1]);}
15     tipo get(int i, int j){return get(i,j,1,0,sz);}
16     tipo get(int i, int j, int n, int a, int b){ //O(lgn)
17         if(j<=a || i>=b) return neutro;
18         if(i<=a && b<=j) return t[n];
19         int c=(a+b)/2;

```

```

20         return operacion(get(i, j, 2*n, a, c), get(i, j, 2*n+1, c, b));
21     }
22     void set(int p, tipo val){ //O(lgn)
23         for(p+=sz; p>0 && t[p]!=val;){
24             t[p]=val;
25             p/=2;
26             val=operacion(t[p*2], t[p*2+1]);
27         }
28     }
29 }rmq;
30 //Usage:
31 cin >> n; rmq.init(n); forn(i, n) cin >> rmq[i]; rmq.updall();

```

2.3. RMQ (lazy)

```

1 //Dado un arreglo y una operacion asociativa con neutro, get(i, j) opera
   sobre el rango [i, j].
2 typedef int Elem; //Elem de los elementos del arreglo
3 typedef int Alt; //Elem de la alteracion
4 #define operacion(x,y) x+y
5 const Elem neutro=0; const Alt neutro2=0;
6 #define MAXN 100000
7 struct RMQ{
8     int sz;
9     Elem t[4*MAXN];
10    Alt dirty[4*MAXN]; //las alteraciones pueden ser de distinto Elem
11    Elem &operator[] (int p){return t[sz+p];}
12    void init(int n){ //O(nlgn)
13        sz = 1 << (32-__builtin_clz(n));
14        forn(i, 2*sz) t[i]=neutro;
15        forn(i, 2*sz) dirty[i]=neutro2;
16    }
17    void push(int n, int a, int b){ //propaga el dirty a sus hijos
18        if(dirty[n]!=0){
19            t[n]+=dirty[n]*(b-a); //altera el nodo
20            if(n<sz){
21                dirty[2*n]+=dirty[n];
22                dirty[2*n+1]+=dirty[n];
23            }
24            dirty[n]=0;
25        }
26    }
27    Elem get(int i, int j, int n, int a, int b){ //O(lgn)

```

```

28     if(j<=a || i>=b) return neutro;
29     push(n, a, b); //corrige el valor antes de usarlo
30     if(i<=a && b<=j) return t[n];
31     int c=(a+b)/2;
32     return operacion(get(i, j, 2*n, a, c), get(i, j, 2*n+1, c, b));
33 }
34 Elem get(int i, int j){return get(i,j,1,0,sz);}
35 //altera los valores en [i, j) con una alteracion de val
36 void alterar(Alt val, int i, int j, int n, int a, int b){//O(lgn)
37     push(n, a, b);
38     if(j<=a || i>=b) return;
39     if(i<=a && b<=j){
40         dirty[n]+=val;
41         push(n, a, b);
42         return;
43     }
44     int c=(a+b)/2;
45     alterar(val, i, j, 2*n, a, c), alterar(val, i, j, 2*n+1, c, b);
46     t[n]=operacion(t[2*n], t[2*n+1]); //por esto es el push de arriba
47 }
48 void alterar(Alt val, int i, int j){alterar(val,i,j,1,0,sz);}
49 }rmq;

```

2.4. RMQ (persistente)

```

1  typedef int tipo;
2  tipo oper(const tipo &a, const tipo &b){
3      return a+b;
4  }
5  struct node{
6      tipo v; node *l,*r;
7      node(tipo v):v(v), l(NULL), r(NULL) {}
8      node(node *l, node *r) : l(l), r(r){
9          if(!l) v=r->v;
10         else if(!r) v=l->v;
11         else v=oper(l->v, r->v);
12     }
13 };
14 node *build (tipo *a, int tl, int tr) { //modificar para que tome tipo a
15     if (tl+1==tr) return new node(a[tl]);
16     int tm=(tl + tr)>>1;
17     return new node(build(a, tl, tm), build(a, tm, tr));
18 }

```

```

19 node *update(int pos, int new_val, node *t, int tl, int tr){
20     if (tl+1==tr) return new node(new_val);
21     int tm=(tl+tr)>>1;
22     if(pos < tm) return new node(update(pos, new_val, t->l, tl, tm), t->r);
23     ;
24     else return new node(t->l, update(pos, new_val, t->r, tm, tr));
25 }
26 tipo get(int l, int r, node *t, int tl, int tr){
27     if(l==tl && tr==r) return t->v;
28     int tm=(tl + tr)>>1;
29     if(r<=tm) return get(l, r, t->l, tl, tm);
30     else if(l>=tm) return get(l, r, t->r, tm, tr);
31     return oper(get(l, tm, t->l, tl, tm), get(tm, r, t->r, tm, tr));
32 }

```

2.5. Union Find

```

1  struct UnionFind{
2      vector<int> f; //the array contains the parent of each node
3      void init(int n){f.clear(); f.insert(f.begin(), n, -1);}
4      int comp(int x){return (f[x]==-1?f[x]=comp(f[x]));} //O(1)
5      bool join(int i, int j) {
6          bool con=comp(i)==comp(j);
7          if(!con) f[comp(i)] = comp(j);
8          return con;
9      }
10 };

```

2.6. Disjoint Intervals

```

1  bool operator< (const ii &a, const ii &b) {return a.fst<b.fst;}
2  //Stores intervals as [first, second]
3  //in case of a collision it joins them in a single interval
4  struct disjoint_intervals {
5      set<ii> segs;
6      void insert(ii v) { //O(lgn)
7          if(v.snd-v.fst==0.) return; //OJO
8          set<ii>::iterator it,at;
9          at = it = segs.lower_bound(v);
10         if (at!=segs.begin() && (--at)->snd >= v.fst)
11             v.fst = at->fst, --it;
12         for(; it!=segs.end() && it->fst <= v.snd; segs.erase(it++))
13             v.snd=max(v.snd, it->snd);
14         segs.insert(v);
15     }
16 }

```

```
16 };
```

2.7. RMQ (2D)

```
1 struct RMQ2D{//n filas x m columnas
2     int sz;
3     RMQ t[4*MAXN];
4     RMQ &operator[](int p){return t[sz/2+p];};//t[i][j]=i fila, j col
5     void init(int n, int m){//O(n*m)
6         sz = 1 << (32-__builtin_clz(n));
7         for(i, 2*sz) t[i].init(m); }
8     void set(int i, int j, tipo val){//O(lgm.lgn)
9         for(i+=sz; i>0;){
10             t[i].set(j, val);
11             i/=2;
12             val=operacion(t[i*2][j], t[i*2+1][j]);
13         } }
14     tipo get(int i1, int j1, int i2, int j2){return get(i1,j1,i2,j2,1,0,
15         sz);}
16     //O(lgm.lgn), rangos cerrado abierto
17     int get(int i1, int j1, int i2, int j2, int n, int a, int b){
18         if(i2<=a || i1>=b) return 0;
19         if(i1<=a && b<=i2) return t[n].get(j1, j2);
20         int c=(a+b)/2;
21         return operacion(get(i1, j1, i2, j2, 2*n, a, c),
22             get(i1, j1, i2, j2, 2*n+1, c, b));
23     }
24 } rmq;
25 //Example to initialize a grid of M rows and N columns:
26 RMQ2D rmq; rmq.init(n,m);
27 for(i, n) for(j, m){
28     int v; cin >> v; rmq.set(i, j, v);}
```

2.8. HashTables

```
1 //Compilar: g++ --std=c++11
2 struct Hash{
3     size_t operator()(const ii &a)const{
4         size_t s=hash<int>()(a.fst);
5         return hash<int>()(a.snd)+0x9e3779b9+(s<<6)+(s>>2);
6     }
7     size_t operator()(const vector<int> &v)const{
8         size_t s=0;
9         for(auto &e : v)
```

```
10         s ^= hash<int>()(e)+0x9e3779b9+(s<<6)+(s>>2);
11         return s;
12     }
13 };
14 unordered_set<ii, Hash> s;
15 unordered_map<ii, int, Hash> m;//map<key, value, hasher>
```

2.9. Treap para set

```
1 typedef int Key;
2 typedef struct node *pnode;
3 struct node{
4     Key key;
5     int prior, size;
6     pnode l,r;
7     node(Key key=0): key(key), prior(rand()), size(1), l(0), r(0) {}
8 };
9 static int size(pnode p) { return p ? p->size : 0; }
10 void push(pnode p) {
11     // modificar y propagar el dirty a los hijos aca(para lazy)
12 }
13 // Update function and size from children's Value
14 void pull(pnode p) { //recalcular valor del nodo aca (para rmq)
15     p->size = 1 + size(p->l) + size(p->r);
16 }
17 //junta dos arreglos
18 pnode merge(pnode l, pnode r) {
19     if (!l || !r) return l ? l : r;
20     push(l), push(r);
21     pnode t;
22     if (l->prior < r->prior) l->r=merge(l->r, r), t = l;
23     else r->l=merge(l, r->l), t = r;
24     pull(t);
25     return t;
26 }
27 //parte el arreglo en dos, l<key<=r
28 void split(pnode t, Key key, pnode &l, pnode &r) {
29     if (!t) return void(l = r = 0);
30     push(t);
31     if (key <= t->key) split(t->l, key, l, t->l), r = t;
32     else split(t->r, key, t->r, r), l = t;
33     pull(t);
34 }
```

```

35
36 void erase(pnode &t, Key key) {
37     if (!t) return;
38     push(t);
39     if (key == t->key) t=merge(t->l, t->r);
40     else if (key < t->key) erase(t->l, key);
41     else erase(t->r, key);
42     if(t) pull(t);
43 }
44
45 ostream& operator<<(ostream &out, const pnode &t) {
46     if(!t) return out;
47     return out << t->l << t->key << ' ' << t->r;
48 }
49 pnode find(pnode t, Key key) {
50     if (!t) return 0;
51     if (key == t->key) return t;
52     if (key < t->key) return find(t->l, key);
53     return find(t->r, key);
54 }
55 struct treap {
56     pnode root;
57     treap(pnode root=0): root(root) {}
58     int size() { return ::size(root); }
59     void insert(Key key) {
60         pnode t1, t2; split(root, key, t1, t2);
61         t1::merge(t1, new node(key));
62         root=:merge(t1,t2);
63     }
64     void erase(Key key1, Key key2) {
65         pnode t1,t2,t3;
66         split(root,key1,t1,t2);
67         split(t2,key2, t2, t3);
68         root=merge(t1,t3);
69     }
70     void erase(Key key) {::erase(root, key);}
71     pnode find(Key key) { return ::find(root, key); }
72     Key &operator[](int pos){return find(pos)->key;}//ojito
73 };
74 treap merge(treap a, treap b) {return treap(merge(a.root, b.root));}

```

2.10. Treap para arreglo

```

1  typedef struct node *pnode;
2  struct node{
3      Value val, mini;
4      int dirty;
5      int prior, size;
6      pnode l,r,parent;
7      node(Value val): val(val), mini(val), dirty(0), prior(rand()), size
          (1), l(0), r(0), parent(0) {}
8  };
9  static int size(pnode p) { return p ? p->size : 0; }
10 void push(pnode p) { //propagar dirty a los hijos(aca para lazy)
11     p->val.fst+=p->dirty;
12     p->mini.fst+=p->dirty;
13     if(p->l) p->l->dirty+=p->dirty;
14     if(p->r) p->r->dirty+=p->dirty;
15     p->dirty=0;
16 }
17 static Value mini(pnode p) { return p ? push(p), p->mini : ii(1e9, -1);
    }
18 // Update function and size from children's Value
19 void pull(pnode p) { //recalcular valor del nodo aca (para rmq)
20     p->size = 1 + size(p->l) + size(p->r);
21     p->mini = min(min(p->val, mini(p->l)), mini(p->r)); //operacion del rmq
22     !
23     p->parent=0;
24     if(p->l) p->l->parent=p;
25     if(p->r) p->r->parent=p;
26 }
27 //junta dos arreglos
28 pnode merge(pnode l, pnode r) {
29     if (!l || !r) return l ? l : r;
30     push(l), push(r);
31     pnode t;
32     if (l->prior < r->prior) l->r=merge(l->r, r), t = l;
33     else r->l=merge(l, r->l), t = r;
34     pull(t);
35     return t;
36 }
37 //parte el arreglo en dos, sz(l)==tam
38 void split(pnode t, int tam, pnode &l, pnode &r) {
39     if (!t) return void(l = r = 0);
40     push(t);
41     if (tam <= size(t->l)) split(t->l, tam, l, t->l), r = t;

```

```

41 else split(t->r, tam - 1 - size(t->l), t->r, r), l = t;
42 pull(t);
43 }
44 pnode at(pnode t, int pos) {
45     if(!t) exit(1);
46     push(t);
47     if(pos == size(t->l)) return t;
48     if(pos < size(t->l)) return at(t->l, pos);
49     return at(t->r, pos - 1 - size(t->l));
50 }
51 int getpos(pnode t){//inversa de at
52     if(!t->parent) return size(t->l);
53     if(t==t->parent->l) return getpos(t->parent)-size(t->r)-1;
54     return getpos(t->parent)+size(t->l)+1;
55 }
56 void split(pnode t, int i, int j, pnode &l, pnode &m, pnode &r) {
57     split(t, i, l, t), split(t, j-i, m, r);}
58 Value get(pnode &p, int i, int j){//like rmq
59     pnode l,m,r;
60     split(p, i, j, l, m, r);
61     Value ret=mini(m);
62     p=merge(l, merge(m, r));
63     return ret;
64 }
65 void print(const pnode &t) {//for debugging
66     if(!t) return;
67     push(t);
68     print(t->l);
69     cout << t->val.fst << '␣';
70     print(t->r);
71 }

```

2.11. Convex Hull Trick

```

1 struct Line{tipo m,h;};
2 tipo inter(Line a, Line b){
3     tipo x=b.h-a.h, y=a.m-b.m;
4     return x/y+(x%y?!((x>0)^(y>0)):0);//==ceil(x/y)
5 }
6 struct CHT {
7     vector<Line> c;
8     bool mx;
9     int pos;

```

```

10 CHT(bool mx=0):mx(mx),pos(0){} //mx=1 si las query devuelven el max
11 inline Line acc(int i){return c[c[0].m>c.back().m? i : sz(c)-1-i];}
12 inline bool irre(Line x, Line y, Line z){
13     return c[0].m>z.m? inter(y, z) <= inter(x, y)
14         : inter(y, z) >= inter(x, y);
15 }
16 void add(tipo m, tipo h) { //O(1), los m tienen que entrar ordenados
17     if(mx) m*=-1, h*=-1;
18     Line l=(Line){m, h};
19     if(sz(c) && m==c.back().m) { l.h=min(h, c.back().h), c.pop_back
20         (); if(pos) pos--; }
21     while(sz(c)>=2 && irre(c[sz(c)-2], c[sz(c)-1], l)) { c.pop_back
22         (); if(pos) pos--; }
23     c.pb(l);
24 }
25 inline bool fbin(tipo x, int m) {return inter(acc(m), acc(m+1))>x;}
26 tipo eval(tipo x){
27     int n = sz(c);
28     //query con x no ordenados O(lgn)
29     int a=-1, b=n-1;
30     while(b-a>1) { int m = (a+b)/2;
31         if(fbin(x, m)) b=m;
32         else a=m;
33     }
34     return (acc(b).m*x+acc(b).h)*(mx?-1:1);
35     //query O(1)
36     while(pos>0 && fbin(x, pos-1)) pos--;
37     while(pos<n-1 && !fbin(x, pos)) pos++;
38     return (acc(pos).m*x+acc(pos).h)*(mx?-1:1);
39 }
40 } ch;

```

2.12. Convex Hull Trick (Dynamic)

```

1 const ll is_query = -(1LL<<62);
2 struct Line {
3     ll m, b;
4     mutable multiset<Line>::iterator it;
5     const Line *succ(multiset<Line>::iterator it) const;
6     bool operator<(const Line& rhs) const {
7         if (rhs.b != is_query) return m < rhs.m;
8         const Line *s=succ(it);
9         if(!s) return 0;

```

```

10     ll x = rhs.m;
11     return b - s->b < (s->m - m) * x;
12 }
13 };
14 struct HullDynamic : public multiset<Line>{ // will maintain upper hull
15     for maximum
16     bool bad(iterator y) {
17         iterator z = next(y);
18         if (y == begin()) {
19             if (z == end()) return 0;
20             return y->m == z->m && y->b <= z->b;
21         }
22         iterator x = prev(y);
23         if (z == end()) return y->m == x->m && y->b <= x->b;
24         return (x->b - y->b)*(z->m - y->m) >= (y->b - z->b)*(y->m - x->m);
25     }
26     iterator next(iterator y){return ++y;}
27     iterator prev(iterator y){return --y;}
28     void insert_line(ll m, ll b) {
29         iterator y = insert((Line) { m, b });
30         y->it=y;
31         if (bad(y)) { erase(y); return; }
32         while (next(y) != end() && bad(next(y))) erase(next(y));
33         while (y != begin() && bad(prev(y))) erase(prev(y));
34     }
35     ll eval(ll x) {
36         Line l = *lower_bound((Line) { x, is_query });
37         return l.m * x + l.b;
38     }
39 }h;
40 const Line *Line::succ(multiset<Line>::iterator it) const{
41     return (++it==h.end())? NULL : &*it;};

```

2.13. Gain-Cost Set

```

1 //esta estructura mantiene pairs(beneficio, costo)
2 //de tal manera que en el set quedan ordenados
3 //por beneficio Y COSTO creciente. (va borrando los que no son optimos)
4 struct V{
5     int gain, cost;
6     bool operator<(const V &b)const{return gain<b.gain;}
7 };

```

```

8 set<V> s;
9 void add(V x){
10     set<V>::iterator p=s.lower_bound(x); //primer elemento mayor o igual
11     if(p!=s.end() && p->cost <= x.cost) return; //ya hay uno mejor
12     p=s.upper_bound(x); //primer elemento mayor
13     if(p!=s.begin()){ //borro todos los peores (<=beneficio y >=costo)
14         --p; //ahora es ultimo elemento menor o igual
15         while(p->cost >= x.cost){
16             if(p==s.begin()){s.erase(p); break;}
17             s.erase(p--);
18         }
19     }
20     s.insert(x);
21 }
22 int get(int gain){ //minimo costo de obtener tal ganancia
23     set<V>::iterator p=s.lower_bound((V){gain, 0});
24     return p==s.end()? INF : p->cost;}

```

2.14. Set con busq binaria

```

1 #include <ext/pb_ds/assoc_container.hpp>
2 #include <ext/pb_ds/tree_policy.hpp>
3 using namespace __gnu_pbds;
4 typedef tree<int,null_type,less<int>, //key,mapped type, comparator
5         rb_tree_tag,tree_order_statistics_node_update> set_t;
6 //find_by_order(i) devuelve iterador al i-esimo elemento
7 //order_of_key(k): devuelve la pos del lower bound de k
8 //Ej: 12, 100, 505, 1000, 10000.
9 //order_of_key(10) == 0, order_of_key(100) == 1,
10 //order_of_key(707) == 3, order_of_key(9999999) == 5

```

2.15. Wavelet tree/matrix

```

1 ==> bitmap.hpp <==
2 #ifndef BITMAP_HPP
3 #define BITMAP_HPP
4 #include <vector>
5 #include "utils.hpp"
6 using namespace std;
7
8 // Indices start from 0
9 struct BitmapRank {
10     const int bits = sizeof(int)*8;
11     vector<int> vec;

```



```

12 vector<int> count;
13
14 BitmapRank() {}
15
16 void resize(int n) {
17     vec.resize((n+bits-1)/bits);
18     count.resize(vec.size());
19 }
20
21 void set(int i, bool b) {
22     set_bit(vec[i/bits], i %bits, b);
23 }
24
25 void build_rank() {
26     for (int i = 1; i < (int)vec.size(); ++i)
27         count[i] = count[i-1] + popcnt(vec[i-1]);
28 }
29
30 int rank1(int i) const {
31     return i < 0 ? 0 : count[i/bits] + popcnt(vec[i/bits] << (bits - i%
32         bits - 1));
33 }
34
35 int rank1(int i, int j) const {
36     return rank1(j) - rank1(i-1);
37 }
38
39 int rank0(int i) const {
40     return i < 0 ? 0 : i - rank1(i) + 1;
41 }
42
43 int rank0(int i, int j) const {
44     return rank0(j) - rank0(i-1);
45 }
46 };
47 #endif
48
49 ==> utils.hpp <==
50 #ifndef UTILS_HPP
51 #define UTILS_HPP
52
53 #define log2(x) (sizeof(uint)*8 - __builtin_clz(x))

```

```

54
55 #define popcnt(x) __builtin_popcount(x)
56
57 #define set_bit(v, i, b) v |= ((b) << (i))
58 #define get_bit(v, i) ((v) & (1 << (i)))
59
60 #endif
61
62 ==> wavelet-matrix.cpp <==
63 /*
64  *
65  * -----
66  * "THE BEER-WARE LICENSE" (Revision 42):
67  * <nlehmann@dcc.uchile.cl> wrote this file. As long as you retain this
68  * notice
69  * you can do whatever you want with this stuff. If we meet some day,
70  * and you
71  * think this stuff is worth it, you can buy me a beer in return Nicol'
72  * as Lehmann
73  *
74  * -----
75  */
76 #include <vector>
77 #include <cstdio>
78 #include <algorithm>
79 #include "utils.hpp"
80 #include "bitmap.hpp"
81 using namespace std;
82
83 typedef unsigned int uint;
84
85 // Wavelet Matrix with succinct representation of bitmaps
86 struct WaveMatrixSucc {
87     uint height;
88     vector<BitmapRank> B;
89     vector<int> z;
90
91     WaveMatrixSucc(vector<int> &A) :
92         WaveMatrixSucc(A, *max_element(A.begin(), A.end()) + 1) {}
93
94     // sigma = size of the alphabet, ie., one more than the maximum

```

```

    element
// in A.
WaveMatrixSucc(vector<int> &A, int sigma)
: height(log2(sigma - 1)),
  B(height), z(height) {
for (uint l = 0; l < height; ++l) {
    B[l].resize(A.size());
    for (uint i = 0; i < A.size(); ++i)
        B[l].set(i, get_bit(A[i], height - l - 1));
    B[l].build_rank();

    auto it = stable_partition(A.begin(), A.end(), [=] (int c) {
        return not get_bit(c, height - l - 1);
    });
    z[l] = distance(A.begin(), it);
}
}

// Count occurrences of number c until position i.
// ie, occurrences of c in positions [i,j]
int rank(int c, int i) const {
    int p = -1;
    for (uint l = 0; l < height; ++l) {
        if (get_bit(c, height - l - 1)) {
            p = z[l] + B[l].rank1(p) - 1;
            i = z[l] + B[l].rank1(i) - 1;
        } else {
            p = B[l].rank0(p) - 1;
            i = B[l].rank0(i) - 1;
        }
    }
    return i - p;
}

// Find the k-th smallest element in positions [i,j].
// The smallest element is k=1
int quantile(int k, int i, int j) const {
    int element = 0;
    for (uint l = 0; l < height; ++l) {
        int r = B[l].rank0(i, j);
        if (r >= k) {
            i = B[l].rank0(i-1);
            j = B[l].rank0(j) - 1;

```

```

    } else {
        i = z[l] + B[l].rank1(i-1);
        j = z[l] + B[l].rank1(j) - 1;
        k -= r;
        set_bit(element, height - l - 1, 1);
    }
}
return element;
}

// Count number of occurrences of numbers in the range [a, b]
// present in the sequence in positions [i, j], ie, if representing a
// grid it
// counts number of points in the specified rectangle.
int range(int i, int j, int a, int b) const {
    return range(i, j, a, b, 0, (1 << height)-1, 0);
}

int range(int i, int j, int a, int b, int L, int U, int l) const {
    if (b < L || U < a)
        return 0;

    int M = L + (U-L)/2;
    if (a <= L && U <= b)
        return j - i + 1;
    else {
        int left = range(B[l].rank0(i-1), B[l].rank0(j) - 1,
                        a, b, L, M, l + 1);
        int right = range(z[l] + B[l].rank1(i-1), z[l] + B[l].rank1(j) -
                        1,
                        a, b, M+1, U, l+1);
        return left + right;
    }
}

};

==> wavelet-tree.cpp <==
#include<vector>
#include<algorithm>
#include "bitmap.hpp"
using namespace std;
typedef vector<int>::iterator iter;

```

```

173 //Wavelet tree with succinct representation of bitmaps
174 struct WaveTreeSucc {
175     vector<vector<int>> > C; int s;
176
177     // sigma = size of the alphabet, ie., one more than the maximum
178     // element
179     // in S.
180     WaveTreeSucc(vector<int> &A, int sigma) : C(sigma*2), s(sigma) {
181         build(A.begin(), A.end(), 0, s-1, 1);
182     }
183
184     void build(iter b, iter e, int L, int U, int u) {
185         if (L == U)
186             return;
187         int M = (L+U)/2;
188
189         // C[u][i] contains number of zeros until position i-1: [0,i)
190         C[u].reserve(e-b+1); C[u].push_back(0);
191         for (iter it = b; it != e; ++it)
192             C[u].push_back(C[u].back() + (*it<=M));
193
194         iter p = stable_partition(b, e, [=](int i){return i<=M;});
195
196         build(b, p, L, M, u*2);
197         build(p, e, M+1, U, u*2+1);
198     }
199
200     // Count occurrences of number c until position i.
201     // ie, occurrences of c in positions [i,j]
202     int rank(int c, int i) const {
203         // Internally we consider an interval open on the left: [0, i)
204         i++;
205         int L = 0, U = s-1, u = 1, M, r;
206         while (L != U) {
207             M = (L+U)/2;
208             r = C[u][i]; u*=2;
209             if (c <= M)
210                 i = r, U = M;
211             else
212                 i -= r, L = M+1, ++u;
213         }
214         return i;
215     }

```

```

215
216     // Find the k-th smallest element in positions [i,j].
217     // The smallest element is k=1
218     int quantile(int k, int i, int j) const {
219         // internally we consider an interval open on the left: [i, j)
220         j++;
221         int L = 0, U = s-1, u = 1, M, ri, rj;
222         while (L != U) {
223             M = (L+U)/2;
224             ri = C[u][i]; rj = C[u][j]; u*=2;
225             if (k <= rj-ri)
226                 i = ri, j = rj, U = M;
227             else
228                 k -= rj-ri, i -= ri, j -= rj,
229                 L = M+1, ++u;
230         }
231         return U;
232     }
233
234     // Count number of occurrences of numbers in the range [a, b]
235     // present in the sequence in positions [i, j], ie, if representing a
236     // grid it
237     // counts number of points in the specified rectangle.
238     mutable int L, U;
239     int range(int i, int j, int a, int b) const {
240         if (b < a or j < i)
241             return 0;
242         L = a; U = b;
243         return range(i, j+1, 0, s-1, 1);
244     }
245
246     int range(int i, int j, int a, int b, int u) const {
247         if (b < L or U < a)
248             return 0;
249         if (L <= a and b <= U)
250             return j-i;
251         int M = (a+b)/2, ri = C[u][i], rj = C[u][j];
252         return range(ri, rj, a, M, u*2) +
253             range(i-ri, j-rj, M+1, b, u*2+1);
254     }

```

3. Algos

3.1. Longest Increasing Subsequence

```

1 //Para non-increasing, cambiar comparaciones y revisar busq binaria
2 //Given an array, paint it in the least number of colors so that each
   color turns to a non-increasing subsequence.
3 //Solution:Min number of colors=Length of the longest increasing
   subsequence
4 int N, a[MAXN]; //secuencia y su longitud
5 ii d[MAXN+1]; //d[i]=ultimo valor de la subsecuencia de tamaño i
6 int p[MAXN]; //padres
7 vector<int> R; //respuesta
8 void rec(int i){
9     if(i== -1) return;
10    R.push_back(a[i]);
11    rec(p[i]);
12 }
13 int lis(){ //O(nlogn)
14     d[0] = ii(-INF, -1); for(i, N) d[i+1]=ii(INF, -1);
15     for(i, N){
16         int j = upper_bound(d, d+N+1, ii(a[i], INF))-d;
17         if (d[j-1].first < a[i] && a[i] < d[j].first){
18             p[i]=d[j-1].second;
19             d[j] = ii(a[i], i);
20         }
21     }
22     R.clear();
23     dforn(i, N+1) if(d[i].first!=INF){
24         rec(d[i].second); //reconstruir
25         reverse(R.begin(), R.end());
26         return i; //longitud
27     }
28     return 0;
29 }
```

3.2. Alpha-Beta pruning

```

1 ll alphabeta(State &s, bool player = true, int depth = 1e9, ll alpha = -
   INF, ll beta = INF) { //player = true -> Maximiza
2     if(s.isFinal()) return s.score;
3     //~ if (!depth) return s.heuristic();
4     vector<State> children;
```

```

5     s.expand(player, children);
6     int n = children.size();
7     forn(i, n) {
8         ll v = alphabeta(children[i], !player, depth-1, alpha, beta);
9         if(!player) alpha = max(alpha, v);
10        else beta = min(beta, v);
11        if(beta <= alpha) break;
12    }
13    return !player ? alpha : beta;}
```

3.3. Mo's algorithm

```

1 int n, sq;
2 struct Qu{ //queries [l, r]
3     //intervalos cerrado abiertos !!! importante!!
4     int l, r, id;
5 }qs[MAXN];
6 int ans[MAXN], curans; //ans[i]=ans to ith query
7 bool bymos(const Qu &a, const Qu &b){
8     if(a.l/sq != b.l/sq) return a.l < b.l;
9     return (a.l/sq & 1 ? a.r < b.r : a.r > b.r);
10 }
11 void mos(){
12     forn(i, t) qs[i].id=i;
13     sort(qs, qs+t, bymos);
14     int cl=0, cr=0;
15     sq=sqrt(n);
16     curans=0;
17     forn(i, t){ //intervalos cerrado abiertos !!! importante!!
18         Qu &q=qs[i];
19         while(cl>q.l) add(--cl);
20         while(cr<q.r) add(cr++);
21         while(cl<q.l) remove(cl++);
22         while(cr>q.r) remove(--cr);
23         ans[q.id]=curans;
24     }
25 }
```

3.4. Ternary search

```

1 #include <functional>
2 //Retorna argmax de una funcion unimodal 'f' en el rango [left, right]
3 double ternarySearch(double l, double r, function<double(double)> f){
4     for(int i = 0; i < 300; i++){
```

```

5     double m1 = l+(r-l)/3, m2 = r-(r-l)/3;
6     if (f(m1) < f(m2)) l = m1; else r = m2;
7 }
8 return (left + right)/2;
9 }

```

4. Strings

4.1. Manacher

```

1 int d1[MAXN]; //d1[i]=long del maximo palindromo impar con centro en i
2 int d2[MAXN]; //d2[i]=analogo pero para longitud par
3 //0 1 2 3 4
4 //a a b c c <--d1[2]=3
5 //a a b b <--d2[2]=2 (estan uno antes)
6 void manacher(){
7     int l=0, r=-1, n=sz(s);
8     forn(i, n){
9         int k=(i>r? 1 : min(d1[l+r-i], r-i));
10        while(i+k<n && i-k>=0 && s[i+k]==s[i-k]) ++k;
11        d1[i] = k--;
12        if(i+k > r) l=i-k, r=i+k;
13    }
14    l=0, r=-1;
15    forn(i, n){
16        int k=(i>r? 0 : min(d2[l+r-i+1], r-i+1))+1;
17        while(i+k-1<n && i-k>=0 && s[i+k-1]==s[i-k]) k++;
18        d2[i] = --k;
19        if(i+k-1 > r) l=i-k, r=i+k-1;
20    }

```

4.2. KMP

```

1 string T; //cadena donde buscar(what)
2 string P; //cadena a buscar(what)
3 int b[MAXLEN]; //back table b[i] maximo borde de [0..i)
4 void kmppre(){ //by gabina with love
5     int i =0, j=-1; b[0]=-1;
6     while(i<sz(P)){
7         while(j>=0 && P[i] != P[j]) j=b[j];
8         i++, j++, b[i] = j;
9     }
10 }

```

```

11 void kmp(){
12     int i=0, j=0;
13     while(i<sz(T)){
14         while(j>=0 && T[i]!=P[j]) j=b[j];
15         i++, j++;
16         if(j==sz(P)) printf("P is found at index %d in T\n", i-j), j=b[j];
17     }
18 }
19
20 int main(){
21     cout << "T=";
22     cin >> T;
23     cout << "P=";

```

4.3. Trie

```

1 struct trie{
2     map<char, trie> m;
3     void add(const string &s, int p=0){
4         if(s[p]) m[s[p]].add(s, p+1);
5     }
6     void dfs(){
7         //Do stuff
8         forall(it, m)
9             it->second.dfs();
10    }
11 };

```

4.4. Suffix Array (largo, nlogn)

```

1 #define MAX_N 1000
2 #define rBOUND(x) (x<n? r[x] : 0)
3 //sa will hold the suffixes in order.
4 int sa[MAX_N], r[MAX_N], n;
5 string s; //input string, n=sz(s)
6
7 int f[MAX_N], tmpsa[MAX_N];
8 void countingSort(int k){
9     zero(f);
10    forn(i, n) f[rBOUND(i+k)]++;
11    int sum=0;
12    forn(i, max(255, n)){
13        int t=f[i]; f[i]=sum; sum+=t; }

```

```

14  forn(i, n)
15      tmpsa[f[rBOUND(sa[i]+k)]++] = sa[i];
16  memcpy(sa, tmpsa, sizeof(sa));
17  }
18  void constructsa(){//O(n log n)
19      n=sz(s);
20      forn(i, n) sa[i]=i, r[i]=s[i];
21      for(int k=1; k<n; k<=1){
22          countingSort(k), countingSort(0);
23          int rank, tmpr[MAX_N];
24          tmpr[sa[0]]=rank=0;
25          forr(i, 1, n)
26              tmpr[sa[i]]=(r[sa[i]]==r[sa[i-1]] && r[sa[i]+k]==r[sa[i-1]+k])?
                  rank : ++rank;
27          memcpy(r, tmpr, sizeof(r));
28          if(r[sa[n-1]]==n-1) break;
29      }
30  }
31  void print(){//for debug
32      forn(i, n)
33          cout << i << ' ' <<
34          s.substr(sa[i], s.find( '$', sa[i])-sa[i]) << endl;

```

4.5. String Matching With Suffix Array

```

1  //returns (lowerbound, upperbound) of the search
2  ii stringMatching(string P){ //O(sz(P)lgn)
3      int lo=0, hi=n-1, mid=lo;
4      while(lo<hi){
5          mid=(lo+hi)/2;
6          int res=s.compare(sa[mid], sz(P), P);
7          if(res>=0) hi=mid;
8          else lo=mid+1;
9      }
10     if(s.compare(sa[lo], sz(P), P)!=0) return ii(-1, -1);
11     ii ans; ans.fst=lo;
12     lo=0, hi=n-1, mid;
13     while(lo<hi){
14         mid=(lo+hi)/2;
15         int res=s.compare(sa[mid], sz(P), P);
16         if(res>0) hi=mid;
17         else lo=mid+1;
18     }

```

```

19     if(s.compare(sa[hi], sz(P), P)!=0) hi--;
20     ans.snd=hi;
21     return ans;
22 }

```

4.6. LCP (Longest Common Prefix)

```

1  //Calculates the LCP between consecutives suffixes in the Suffix Array.
2  //LCP[i] is the length of the LCP between sa[i] and sa[i-1]
3  int LCP[MAX_N], phi[MAX_N], PLCP[MAX_N];
4  void computeLCP(){//O(n)
5      phi[sa[0]]=-1;
6      forr(i, 1, n) phi[sa[i]]=sa[i-1];
7      int L=0;
8      forn(i, n){
9          if(phi[i]==-1) {PLCP[i]=0; continue;}
10         while(s[i+L]==s[phi[i]+L]) L++;
11         PLCP[i]=L;
12         L=max(L-1, 0);
13     }
14     forn(i, n) LCP[i]=PLCP[sa[i]];
15 }

```

4.7. Corasick

```

1
2  struct trie{
3      map<char, trie> next;
4      trie* tran[256]; //transiciones del automata
5      int idhoja, szhoja; //id de la hoja o 0 si no lo es
6      //link lleva al sufijo mas largo, nxthoja lleva al mas largo pero que
7      //es hoja
8      trie *padre, *link, *nxthoja;
9      char pch; //caracter que conecta con padre
10     trie(): tran(), idhoja(), padre(), link() {}
11     void insert(const string &s, int id=1, int p=0){//id>0!!!
12         if(p<sz(s)){
13             trie &ch=next[s[p]];
14             tran[(int)s[p]]=&ch;
15             ch.padre=this, ch.pch=s[p];
16             ch.insert(s, id, p+1);
17         }
18         else idhoja=id, szhoja=sz(s);
19     }

```

```

19  trie* get_link() {
20      if(!link){
21          if(!padre) link=this; //es la raiz
22          else if(!padre->padre) link=padre; //hijo de la raiz
23          else link=padre->get_link()->get_tran(pch);
24      }
25      return link; }
26  trie* get_tran(int c) {
27      if(!tran[c]) tran[c] = !padre? this : this->get_link()->get_tran(c);
28      return tran[c]; }
29  trie *get_nxthoja(){
30      if(!nxthoja) nxthoja = get_link()->idhoja? link : link->nxthoja;
31      return nxthoja; }
32  void print(int p){
33      if(idhoja) cout << "found_" << idhoja << "_at_position_" << p-
34          szhoja << endl;
35      if(get_nxthoja()) get_nxthoja()->print(p); }
36  void matching(const string &s, int p=0){
37      print(p); if(p<sz(s)) get_tran(s[p])->matching(s, p+1); }
38  }tri;
39
40  int main(){
41      tri=trie(); //clear
42      tri.insert("ho", 1);
43      tri.insert("hoho", 2);

```

4.8. Suffix Automaton

```

1  struct state {
2      int len, link;
3      map<char,int> next;
4      state() { }
5  };
6  const int MAXLEN = 10010;
7  state st[MAXLEN*2];
8  int sz, last;
9  void sa_init() {
10     forn(i,sz) st[i].next.clear();
11     sz = last = 0;
12     st[0].len = 0;
13     st[0].link = -1;
14     ++sz;

```

```

15 }
16 // Es un DAG de una sola fuente y una sola hoja
17 // cantidad de endpos = cantidad de apariciones = cantidad de caminos de
18 // la clase al nodo terminal
19 // cantidad de miembros de la clase = st[v].len-st[st[v].link].len (v>0)
20 // = caminos del inicio a la clase
21 // El arbol de los suffix links es el suffix tree de la cadena invertida
22 // La string de la arista link(v)->v son los caracteres que difieren
23 void sa_extend (char c) {
24     int cur = sz++;
25     st[cur].len = st[last].len + 1;
26     // en cur agregamos la posicion que estamos extendiendo
27     //podria agregar tambien un identificador de las cadenas a las cuales
28     //pertenece (si hay varias)
29     int p;
30     for (p=last; p!=-1 && !st[p].next.count(c); p=st[p].link) // modificar
31         esta linea para hacer separadores unicos entre varias cadenas (c
32         =='$')
33     st[p].next[c] = cur;
34     if (p == -1)
35         st[cur].link = 0;
36     else {
37         int q = st[p].next[c];
38         if (st[p].len + 1 == st[q].len)
39             st[cur].link = q;
40         else {
41             int clone = sz++;
42             // no le ponemos la posicion actual a clone sino indirectamente
43             // por el link de cur
44             st[clone].len = st[p].len + 1;
45             st[clone].next = st[q].next;
46             st[clone].link = st[q].link;
47             for (; p!=-1 && st[p].next.count(c) && st[p].next[c]==q; p=st[p].
48                 link)
49                 st[p].next[c] = clone;
50             st[q].link = st[cur].link = clone;
51         }
52     }
53     last = cur;
54 }

```

4.9. Z Function

```

1 char s[MAXN];
2 int z[MAXN]; // z[i] = i==0 ? 0 : max k tq s[0,k) match with s[i,i+k)
3 void z_function(char s[],int z[]) {
4     int n = strlen(s);
5     forn(i, n) z[i]=0;
6     for (int i = 1, l = 0, r = 0; i < n; ++i) {
7         if (i <= r) z[i] = min (r - i + 1, z[i - l]);
8         while (i + z[i] < n && s[z[i]] == s[i + z[i]]) ++z[i];
9         if (i + z[i] - 1 > r) l = i, r = i + z[i] - 1;
10    }
11 }
12
13 int main() {
14     ios::sync_with_stdio(0);

```

4.10. Palindromic tree

```

1 using namespace std;
2
3 const int maxn = 10100100;
4
5 int len[maxn];
6 int suffLink[maxn];
7 int to[maxn][2];
8 int cnt[maxn];
9 int numV;
10 char str[maxn];
11
12 int v;
13
14 void addLetter(int n)
15 {
16     while (str[n - len[v] - 1] != str[n] )
17         v = suffLink[v];
18     int u = suffLink[v];
19     while (str[n - len[u] - 1] != str[n] )
20         u = suffLink[u];
21     int u_ = to[u][str[n] - 'a'];
22     int v_ = to[v][str[n] - 'a'];
23     if (v_ == -1)
24     {
25         v_ = to[v][str[n] - 'a'] = numV;
26         len[numV++] = len[v] + 2;

```

```

27         suffLink[v_] = u_;
28     }
29     v = v_;
30     cnt[v]++;
31 }
32
33 void init()
34 {
35     memset(to, -1, sizeof to);
36     str[0] = '#';
37     len[0] = -1;
38     len[1] = 0;
39     len[2] = len[3] = 1;
40     suffLink[1] = 0;
41     suffLink[0] = 0;
42     suffLink[2] = 1;
43     suffLink[3] = 1;
44     to[0][0] = 2;
45     to[0][1] = 3;
46     numV = 4;
47 }
48
49 int main()
50 {
51     init();
52     scanf("%s", str + 1);
53     int n = strlen(str);
54     for (int i = 1; i < n; i++)
55         addLetter(i);
56
57     long long ans = 0;
58     for (int i = numV - 1; i > 0; i--)
59     {
60         cnt[suffLink[i]] += cnt[i];
61         ans = max(ans, cnt[i] * 1LL * len[i] );
62         // fprintf(stderr, "i = %d, cnt = %d, len = %d\n", i, cnt[i]
63         // ], len[i] );
64     }
65     printf("%lld\n", ans);
66
67     return 0;

```


4.11. Rabin Karp Fixed Length

```

1  #include <bits/stdc++.h>
2  #include <functional>
3  using namespace std;
4  #define MAXN 100005
5
6  typedef long long ll;
7  typedef function<char(int)> f_getter;
8  typedef function<void(ll)> f_matcher;
9
10
11
12 struct RobinKarpMatchSetting {
13     int p_length; //Largo pattern a buscar
14     int t_length; //Largo texto en el que buscar
15     f_getter t_getter; //Funcion que devuelve el iesimo elemento del texto
16     f_matcher matcher; //Funcion que se activa cada vez que hay match
17 };
18
19 ll rk_pot[MAXN];
20 ll rk_p = 257, rk_M = 1000000007, rk_p_inv = 70038911; //pow
    (257,10**9+7-2,10**9+7)
21 void initRK(){
22     ll p = 1;
23     for (int i = 0; i < MAX_LENGTH; i++, p=(p*rk_p)%rk_M){
24         rk_pot[i]=p;
25     }
26 }
27
28 ll calcHashRK(int start, int offset, f_getter getter){
29     ll r = 0;
30     for (int i = start; i < start+offset; i++) r=(r*rk_pot[i-start]*getter
        (i))%rk_M;
31     return r;
32 }
33
34 void RKSearch(RobinKarpMatchSetting &ms){
35     ll h = calcHashRK(0,ms.p_length,ms.t_getter);
36     ms.matcher(h);
37     for (int i = ms.p_length; i < ms.t_length; i++){
38         h = ((h-ms.t_getter(i-ms.p_length))%rk_M+rk_M)%rk_M;
39         h = ( h * rk_p_inv ) %rk_M;

```

```

40     h = (h + ms.t_getter(i)*rk_pot[ms.p_length-1]) % rk_M;
41     ms.matcher(h);
42 }
43 }
44
45 string text[35];
46 int N;
47
48 //Return 2 if not shared, 1 if shared
49 int evalLength(int length){
50     set<ll> shared;
51     RobinKarpMatchSetting ms;
52     ms.t_length = text[0].size();
53     ms.t_getter = [](int j)->char{return text[0][j]};
54     ms.p_length = length;
55     ms.matcher = [&shared](ll h){shared.insert(h)};
56     RKSearch(ms);
57     for (int i = 1; i < N; i++){
58         set<ll> newShared;
59         ms.matcher = [&shared,&newShared](ll h){if (shared.count(h))
            newShared.insert(h)};
60         ms.t_getter = [i](int j)->char{return text[i][j]};
61         ms.t_length = text[i].size();
62         RKSearch(ms);
63         if (newShared.size() == 0) return 2;
64         shared = newShared;
65     }
66     return 1;
67 }
68
69 int main() {
70     ios_base::sync_with_stdio(false);
71     cin.tie(0);
72     initRK();
73     while (cin >> N){
74         int minLength = 100005;
75         for (int i = 0; i < N; i++) {
76             cin >> text[i];
77             minLength=min(minLength,(int)text[i].size());
78         }
79         cout << (lowerBound(1,minLength,evalLength,2) - 1) << "\n";
80     }
81 }

```

5. Geometria

5.1. Punto

```

1 struct pto{
2     double x, y;
3     pto(double x=0, double y=0):x(x),y(y){}
4     pto operator+(pto a){return pto(x+a.x, y+a.y);}
5     pto operator-(pto a){return pto(x-a.x, y-a.y);}
6     pto operator+(double a){return pto(x+a, y+a);}
7     pto operator*(double a){return pto(x*a, y*a);}
8     pto operator/(double a){return pto(x/a, y/a);}
9     //dot product, producto interno:
10    double operator*(pto a){return x*a.x+y*a.y;}
11    //module of the cross product or vectorial product:
12    //if a is less than 180 clockwise from b, a^b>0
13    double operator^(pto a){return x*a.y-y*a.x;}
14    //returns true if this is at the left side of line qr
15    bool left(pto q, pto r){return ((q-*this)^(r-*this))>0;}
16    bool operator<(const pto &a) const{return x<a.x-EPS || (abs(x-a.x)<EPS
17        && y<a.y-EPS);}
18    bool operator==(pto a){return abs(x-a.x)<EPS && abs(y-a.y)<EPS;}
19    double norm(){return sqrt(x*x+y*y);}
20    double norm_sq(){return x*x+y*y;}
21 };
22 typedef pto vec;
23
24 double angle(pto a, pto o, pto b){
25     pto oa=a-o, ob=b-o;
26     return atan2(oa^ob, oa*ob);}
27
28 //rotate p by theta rads CCW w.r.t. origin (0,0)
29 pto rotate(pto p, double theta){
30     return pto(p.x*cos(theta)-p.y*sin(theta),
31         p.x*sin(theta)+p.y*cos(theta));
32 }

```

5.2. Orden radial de puntos

```

1 struct Cmp{//orden total de puntos alrededor de un punto r
2     pto r;
3     Cmp(pto r):r(r) {}

```

```

4     int cuad(const pto &a) const{
5         if(a.x > 0 && a.y >= 0)return 0;
6         if(a.x <= 0 && a.y > 0)return 1;
7         if(a.x < 0 && a.y <= 0)return 2;
8         if(a.x >= 0 && a.y < 0)return 3;
9         assert(a.x ==0 && a.y==0);
10        return -1;
11    }
12    bool cmp(const pto&p1, const pto&p2)const{
13        int c1 = cuad(p1), c2 = cuad(p2);
14        if(c1==c2) return p1.y*p2.x<p1.x*p2.y;
15        else return c1 < c2;
16    }
17    bool operator()(const pto&p1, const pto&p2) const{
18        return cmp(pto(p1.x-r.x,p1.y-r.y),pto(p2.x-r.x,p2.y-r.y));
19    }
20 };

```

5.3. Line

```

1 int sgn(ll x){return x<0? -1 : !!x;}
2 struct line{
3     line() {}
4     double a,b,c;//Ax+By=C
5     //pto MUST store float coordinates!
6     line(double a, double b, double c):a(a),b(b),c(c){}
7     line(pto p, pto q): a(q.y-p.y), b(p.x-q.x), c(a*p.x+b*p.y) {}
8     int side(pto p){return sgn(11(a) * p.x + 11(b) * p.y - c);}
9 };
10 bool parallels(line l1, line l2){return abs(11.a*12.b-12.a*11.b)<EPS;}
11 pto inter(line l1, line l2){//intersection
12     double det=11.a*12.b-12.a*11.b;
13     if(abs(det)<EPS) return pto(INF, INF);//parallels
14     return pto(12.b*11.c-11.b*12.c, 11.a*12.c-12.a*11.c)/det;
15 }

```

5.4. Segment

```

1 struct segm{
2     pto s,f;
3     segm(pto s, pto f):s(s), f(f) {}
4     pto closest(pto p) {//use for dist to point
5         double l2 = dist_sq(s, f);
6         if(l2==0.) return s;

```

```

7     double t=((p-s)*(f-s))/l2;
8     if (t<0.) return s;//not write if is a line
9     else if(t>1.)return f;//not write if is a line
10    return s+((f-s)*t);
11 }
12 bool inside(pto p){return abs(dist(s, p)+dist(p, f)-dist(s, f))<EPS
    ;}
13 };
14
15 pto inter(seg m s1, seg m s2){
16     pto r=inter(line(s1.s, s1.f), line(s2.s, s2.f));
17     if(s1.inside(r) && s2.inside(r)) return r;
18     return pto(INF, INF);
19 }

```

5.5. Rectangle

```

1 struct rect{
2     //lower-left and upper-right corners
3     pto lw, up;
4 };
5 //returns if there's an intersection and stores it in r
6 bool inter(rect a, rect b, rect &r){
7     r.lw=pto(max(a.lw.x, b.lw.x), max(a.lw.y, b.lw.y));
8     r.up=pto(min(a.up.x, b.up.x), min(a.up.y, b.up.y));
9     //check case when only a edge is common
10    return r.lw.x<r.up.x && r.lw.y<r.up.y;
11 }

```

5.6. Polygon Area

```

1 double area(vector<pto> &p){//0(sz(p))
2     double area=0;
3     forn(i, sz(p)) area+=p[i]^p[(i+1)%sz(p)];
4     //if points are in clockwise order then area is negative
5     return abs(area)/2;
6 }
7 //Area ellipse = M_PI*a*b where a and b are the semi axis lengths
8 //Area triangle = sqrt(s*(s-a)(s-b)(s-c)) where s=(a+b+c)/2

```

5.7. Circle

```

1 vec perp(vec v){return vec(-v.y, v.x);}
2 line bisector(pto x, pto y){

```

```

3     line l=line(x, y); pto m=(x+y)/2;
4     return line(-l.b, l.a, -l.b*m.x+l.a*m.y);
5 }
6 struct Circle{
7     pto o;
8     double r;
9     Circle(pto x, pto y, pto z){
10         o=inter(bisector(x, y), bisector(y, z));
11         r=dist(o, x);
12     }
13     pair<pto, pto> ptosTang(pto p){
14         pto m=(p+o)/2;
15         tipo d=dist(o, m);
16         tipo a=r*r/(2*d);
17         tipo h=sqrt(r*r-a*a);
18         pto m2=o+(m-o)*a/d;
19         vec per=perp(m-o)/d;
20         return make_pair(m2-per*h, m2+per*h);
21     }
22 };
23 //finds the center of the circle containing p1 and p2 with radius r
24 //as there may be two solutions swap p1, p2 to get the other
25 bool circle2PtsRad(pto p1, pto p2, double r, pto &c){
26     double d2=(p1-p2).norm_sq(), det=r*r/d2-0.25;
27     if(det<0) return false;
28     c=(p1+p2)/2+perp(p2-p1)*sqrt(det);
29     return true;
30 }
31 #define sqr(a) ((a)*(a))
32 #define feq(a,b) (fabs((a)-(b))<EPS)
33 pair<tipo, tipo> ecCuad(tipo a, tipo b, tipo c){//a*x*x+b*x+c=0
34     tipo dx = sqrt(b*b-4.0*a*c);
35     return make_pair((-b + dx)/(2.0*a), (-b - dx)/(2.0*a));
36 }
37 pair<pto, pto> interCL(Circle c, line l){
38     bool sw=false;
39     if((sw=feq(0,l.b))){
40         swap(l.a, l.b);
41         swap(c.o.x, c.o.y);
42     }
43     pair<tipo, tipo> rc = ecCuad(
44         sqr(l.a)+sqr(l.b),
45         2.0*l.a*l.b*c.o.y-2.0*(sqr(l.b)*c.o.x+l.c*l.a),

```

```

46   sqr(1.b)*(sqr(c.o.x)+sqr(c.o.y)-sqr(c.r))+sqr(1.c)-2.0*1.c*1.b*c.o.y
47   );
48   pair<pto, pto> p( pto(rc.first, (1.c - 1.a * rc.first) / 1.b),
49                   pto(rc.second, (1.c - 1.a * rc.second) / 1.b) );
50   if(sw){
51       swap(p.first.x, p.first.y);
52       swap(p.second.x, p.second.y);
53   }
54   return p;
55 }
56 pair<pto, pto> interCC(Circle c1, Circle c2){
57     line l;
58     l.a = c1.o.x-c2.o.x;
59     l.b = c1.o.y-c2.o.y;
60     l.c = (sqr(c2.r)-sqr(c1.r)+sqr(c1.o.x)-sqr(c2.o.x)+sqr(c1.o.y)
61            -sqr(c2.o.y))/2.0;
62     return interCL(c1, l);
63 }

```

5.8. Point in Poly

```

1 //checks if v is inside of P, using ray casting
2 //works with convex and concave.
3 //excludes boundaries, handle it separately using segment.inside()
4 bool inPolygon(pto v, vector<pto>& P) {
5     bool c = false;
6     forn(i, sz(P)){
7         int j=(i+1)%sz(P);
8         if((P[j].y>v.y) != (P[i].y > v.y) &&
9            (v.x < (P[i].x - P[j].x) * (v.y-P[j].y) / (P[i].y - P[j].y) + P[j].x))
10            c = !c;
11     }
12     return c;
13 }

```

5.9. Point in Convex Poly log(n)

```

1 void normalize(vector<pto> &pt){//delete collinear points first!
2     //this makes it clockwise:
3     if(pt[2].left(pt[0], pt[1])) reverse(pt.begin(), pt.end());
4     int n=sz(pt), pi=0;
5     forn(i, n)
6         if(pt[i].x<pt[pi].x || (pt[i].x==pt[pi].x && pt[i].y<pt[pi].y))
7             pi=i;

```

```

8     vector<pto> shift(n);//puts pi as first point
9     forn(i, n) shift[i]=pt[(pi+i)%n];
10    pt.swap(shift);
11 }
12 bool inPolygon(pto p, const vector<pto> &pt){
13     //call normalize first!
14     if(p.left(pt[0], pt[1]) || p.left(pt[sz(pt)-1], pt[0])) return false;
15     int a=1, b=sz(pt)-1;
16     while(b-a>1){
17         int c=(a+b)/2;
18         if(!p.left(pt[0], pt[c])) a=c;
19         else b=c;
20     }
21     return !p.left(pt[a], pt[a+1]);
22 }

```

5.10. Convex Check CHECK

```

1 bool isConvex(vector<int> &p){//O(N), delete collinear points!
2     int N=sz(p);
3     if(N<3) return false;
4     bool isLeft=p[0].left(p[1], p[2]);
5     forr(i, 1, N)
6         if(p[i].left(p[(i+1)%N], p[(i+2)%N])!=isLeft)
7             return false;
8     return true; }

```

5.11. Convex Hull

```

1 //stores convex hull of P in S, CCW order
2 //left must return >=0 to delete collinear points!
3 void CH(vector<pto>& P, vector<pto> &S){
4     S.clear();
5     sort(P.begin(), P.end());//first x, then y
6     forn(i, sz(P)){//lower hull
7         while(sz(S)>= 2 && S[sz(S)-1].left(S[sz(S)-2], P[i])) S.pop_back();
8         S.pb(P[i]);
9     }
10    S.pop_back();
11    int k=sz(S);
12    dforn(i, sz(P)){//upper hull
13        while(sz(S) >= k+2 && S[sz(S)-1].left(S[sz(S)-2], P[i])) S.pop_back();
14        S.pb(P[i]);

```

```

15 }
16 S.pop_back();
17 }

```

5.12. Cut Polygon

```

1 //cuts polygon Q along the line ab
2 //stores the left side (swap a, b for the right one) in P
3 void cutPolygon(pto a, pto b, vector<pto> Q, vector<pto> &P){
4     P.clear();
5     forn(i, sz(Q)){
6         double left1=(b-a)^(Q[i]-a), left2=(b-a)^(Q[(i+1)%sz(Q)]-a);
7         if(left1>=0) P.pb(Q[i]);
8         if(left1*left2<0)
9             P.pb(inter(line(Q[i], Q[(i+1)%sz(Q)]), line(a, b)));
10    }
11 }

```

5.13. Bresenham

```

1 //plot a line approximation in a 2d map
2 void bresenham(pto a, pto b){
3     pto d=b-a; d.x=abs(d.x), d.y=abs(d.y);
4     pto s(a.x<b.x? 1: -1, a.y<b.y? 1: -1);
5     int err=d.x-d.y;
6     while(1){
7         m[a.x][a.y]=1;//plot
8         if(a==b) break;
9         int e2=err;
10        if(e2 >= 0) err-=2*d.y, a.x+=s.x;
11        if(e2 <= 0) err+= 2*d.x, a.y+= s.y;
12    }
13 }

```

5.14. Rotate Matrix

```

1 //rotates matrix t 90 degrees clockwise
2 //using auxiliary matrix t2(faster)
3 void rotate(){
4     forn(x, n) forn(y, n)
5         t2[n-y-1][x]=t[x][y];
6     memcpy(t, t2, sizeof(t));
7 }

```

5.15. Interseccion de Circulos en n3log(n)

```

1 struct event {
2     double x; int t;
3     event(double xx, int tt) : x(xx), t(tt) {}
4     bool operator <(const event &o) const { return x < o.x; }
5 };
6 typedef vector<Circle> VC;
7 typedef vector<event> VE;
8 int n;
9 double cuenta(VE &v, double A, double B) {
10    sort(v.begin(), v.end());
11    double res = 0.0, lx = ((v.empty())?0.0:v[0].x);
12    int contador = 0;
13    forn(i, sz(v)) {
14        //interseccion de todos (contador == n), union de todos (
15            contador > 0)
16        //conjunto de puntos cubierto por exacta k Circulos (contador ==
17            k)
18        if (contador == n) res += v[i].x - lx;
19        contador += v[i].t, lx = v[i].x;
20    }
21    return res;
22 }
23 // Primitiva de sqrt(r*r - x*x) como funcion double de una variable x.
24 inline double primitiva(double x, double r) {
25     if (x >= r) return r*r*M_PI/4.0;
26     if (x <= -r) return -r*r*M_PI/4.0;
27     double raiz = sqrt(r*r-x*x);
28     return 0.5 * (x * raiz + r*r*atan(x/raiz));
29 }
30 double interCircle(VC &v) {
31     vector<double> p; p.reserve(v.size() * (v.size() + 2));
32     forn(i, sz(v)) p.push_back(v[i].c.x + v[i].r), p.push_back(v[i].c.x
33         - v[i].r);
34     forn(i, sz(v)) forn(j, i) {
35         Circle &a = v[i], b = v[j];
36         double d = (a.c - b.c).norm();
37         if (fabs(a.r - b.r) < d && d < a.r + b.r) {
38             double alfa = acos((sqr(a.r) + sqr(d) - sqr(b.r)) / (2.0 * d
39                 * a.r));
40             pto vec = (b.c - a.c) * (a.r / d);
41             p.pb((a.c + rotate(vec, alfa)).x), p.pb((a.c + rotate(vec, -

```

```

38         alfa)).x);
39     }
40     sort(p.begin(), p.end());
41     double res = 0.0;
42     forn(i,sz(p)-1) {
43         const double A = p[i], B = p[i+1];
44         VE ve; ve.reserve(2 * v.size());
45         forn(j,sz(v)) {
46             const Circle &c = v[j];
47             double arco = primitiva(B-c.c.x,c.r) - primitiva(A-c.c.x,c.r);
48             double base = c.c.y * (B-A);
49             ve.push_back(event(base + arco,-1));
50             ve.push_back(event(base - arco, 1));
51         }
52         res += cuenta(ve,A,B);
53     }
54     return res;
55 }

```

6. Math

6.1. Identidades

$$\sum_{i=0}^n i \binom{n}{i} = n * 2^{n-1}$$

$$\sum_{i=m}^n i = \frac{n(n+1)}{2} - \frac{m(m-1)}{2} = \frac{(n+1-m)(n+m)}{2}$$

$$\sum_{i=0}^n i = \sum_{i=1}^n i = \frac{n(n+1)}{2}$$

$$\sum_{i=0}^n i^2 = \frac{n(n+1)(2n+1)}{6} = \frac{n^3}{3} + \frac{n^2}{2} + \frac{n}{6}$$

$$\sum_{i=0}^n i(i-1) = \frac{8}{6} \left(\frac{n}{2}\right) \left(\frac{n}{2} + 1\right) (n+1) \text{ (doubles)} \rightarrow \text{Sino ver caso impar y par}$$

$$\sum_{i=0}^n i^3 = \left(\frac{n(n+1)}{2}\right)^2 = \frac{n^4}{4} + \frac{n^3}{2} + \frac{n^2}{4} = \left[\sum_{i=1}^n i\right]^2$$

$$\sum_{i=0}^n i^4 = \frac{n(n+1)(2n+1)(3n^2+3n-1)}{30} = \frac{n^5}{5} + \frac{n^4}{2} + \frac{n^3}{3} - \frac{n}{30}$$

$$\sum_{i=0}^n i^p = \frac{(n+1)^{p+1}}{p+1} + \sum_{k=1}^p \frac{B_k}{p-k+1} \binom{p}{k} (n+1)^{p-k+1}$$

$$r = e - v + k + 1$$

Teorema de Pick: (Area, puntos interiores y puntos en el borde)

$$A = I + \frac{B}{2} - 1$$

6.2. Ec. Característica

$$a_0 T(n) + a_1 T(n-1) + \dots + a_k T(n-k) = 0$$

$$p(x) = a_0 x^k + a_1 x^{k-1} + \dots + a_k$$

Sean r_1, r_2, \dots, r_q las raíces distintas, de mult. m_1, m_2, \dots, m_q

$$T(n) = \sum_{i=1}^q \sum_{j=0}^{m_i-1} c_{ij} n^j r_i^n$$

Las constantes c_{ij} se determinan por los casos base.

6.3. Combinatorio

```

1  forn(i, MAXN+1){ //comb[i][k]=i tomados de a k
2      comb[i][0]=comb[i][i]=1;
3      forr(k, 1, i) comb[i][k]=(comb[i-1][k]+comb[i-1][k-1])%MOD;
4  }
5  ll lucas (ll n, ll k, int p){ //Calcula (n,k)%p teniendo comb[p][p]
6      //precalculado.
7      ll aux = 1;
8      while (n + k) aux = (aux * comb[n%p][k%p]) %p, n/=p, k/=p;
9      return aux;
10 }

```

6.4. Gauss Jordan, Determinante $O(n^3)$

```

1  #include <iostream>
2  #include <vector>
3  #include <cmath>
4
5  using namespace std;
6
7  const double EPS = 1e-10;
8
9  typedef vector<int> VI;
10 typedef double T;
11 typedef vector<T> VT;
12 typedef vector<VT> VVT;
13
14 T GaussJordan(VVT &a, VVT &b) {
15     const int n = a.size();
16     const int m = b[0].size();
17     VI irow(n), icol(n), ipiv(n);
18     T det = 1;
19
20     for (int i = 0; i < n; i++) {
21         int pj = -1, pk = -1;
22         for (int j = 0; j < n; j++) if (!ipiv[j])
23             for (int k = 0; k < n; k++) if (!ipiv[k])
24                 if (pj == -1 || fabs(a[j][k]) > fabs(a[pj][pk])) { pj = j; pk = k; }

```

```

25     if (fabs(a[pj][pk]) < EPS) { cerr << "Matrix_is_singular." << endl;
        exit(0); }
26     ipiv[pk]++;
27     swap(a[pj], a[pk]);
28     swap(b[pj], b[pk]);
29     if (pj != pk) det *= -1;
30     irow[i] = pj;
31     icol[i] = pk;
32
33     T c = 1.0 / a[pk][pk];
34     det *= a[pk][pk];
35     a[pk][pk] = 1.0;
36     for (int p = 0; p < n; p++) a[pk][p] *= c;
37     for (int p = 0; p < m; p++) b[pk][p] *= c;
38     for (int p = 0; p < n; p++) if (p != pk) {
39         c = a[p][pk];
40         a[p][pk] = 0;
41         for (int q = 0; q < n; q++) a[p][q] -= a[pk][q] * c;
42         for (int q = 0; q < m; q++) b[p][q] -= b[pk][q] * c;
43     }
44 }
45
46 for (int p = n-1; p >= 0; p--) if (irow[p] != icol[p]) {
47     for (int k = 0; k < n; k++) swap(a[k][irow[p]], a[k][icol[p]]);
48 }
49
50 return det;
51 }
52
53 int main() {
54     const int n = 4;
55     const int m = 2;
56     double A[n][n] = { {1,2,3,4},{1,0,1,0},{5,3,2,4},{6,1,4,6} };
57     double B[n][m] = { {1,2},{4,3},{5,6},{8,7} };
58     VVT a(n), b(n);
59     for (int i = 0; i < n; i++) {
60         a[i] = VT(A[i], A[i] + n);
61         b[i] = VT(B[i], B[i] + m);

```

6.5. Teorema Chino del Resto

$$y = \sum_{j=1}^n (x_j * (\prod_{i=1, i \neq j}^n m_i)^{-1}_{m_j} * \prod_{i=1, i \neq j}^n m_i)$$

```

1 // Chinese remainder theorem (special case): find z such that
2 // z %m1 = r1, z %m2 = r2. Here, z is unique modulo M = lcm(m1, m2).
3 // Return (z, M). On failure, M = -1.
4 PII chinese_remainder_theorem(int m1, int r1, int m2, int r2) {
5     int s, t;
6     int g = extended_euclid(m1, m2, s, t);
7     if (r1%g != r2%g) return make_pair(0, -1);
8     return make_pair(mod(s*r2*m1 + t*r1*m2, m1*m2) / g, m1*m2 / g);
9 }
10
11 // Chinese remainder theorem: find z such that
12 // z %m[i] = r[i] for all i. Note that the solution is
13 // unique modulo M = lcm_i (m[i]). Return (z, M). On
14 // failure, M = -1. Note that we do not require the a[i]'s
15 // to be relatively prime.
16 PII chinese_remainder_theorem(const VI &m, const VI &r) {
17     PII ret = make_pair(r[0], m[0]);
18     for (int i = 1; i < m.size(); i++) {
19         ret = chinese_remainder_theorem(ret.second, ret.first, m[i], r[i]);
20         if (ret.second == -1) break;
21     }
22     return ret;
23 }

```

6.6. Funciones de primos

Iterar mientras el $p^2 \leq N$. Revisar que $N! = 1$, en este caso N es primo. **NumDiv**: Producto (exponentes+1). **SumDiv**: Product suma geom. factores. **EulerPhi** (coprimos): Inicia ans=N. Para cada primo divisor: ans=ans/primo (una vez) y dividir luego N todo lo posible por p .

6.7. Phollard's Rho (rolando)

```

1 ll gcd(ll a, ll b){return a?gcd(b %a, a):b;}
2
3 ll mulmod (ll a, ll b, ll c) { //returns (a*b)%c, and minimize overflow

```



```

4  ll x = 0, y = a%c;
5  while (b > 0){
6      if (b % 2 == 1) x = (x+y) % c;
7      y = (y*2) % c;
8      b /= 2;
9  }
10 return x % c;
11 }

12
13 ll expmod (ll b, ll e, ll m){//O(log b)
14     if(!e) return 1;
15     ll q= expmod(b,e/2,m); q=mulmod(q,q,m);
16     return e%2? mulmod(b,q,m) : q;
17 }

18
19 bool es_primo_prob (ll n, int a)
20 {
21     if (n == a) return true;
22     ll s = 0,d = n-1;
23     while (d % 2 == 0) s++,d/=2;
24
25     ll x = expmod(a,d,n);
26     if ((x == 1) || (x+1 == n)) return true;
27
28     forn (i, s-1){
29         x = mulmod(x, x, n);
30         if (x == 1) return false;
31         if (x+1 == n) return true;
32     }
33     return false;
34 }

35
36 bool rabin (ll n){ //devuelve true si n es primo
37     if (n == 1) return false;
38     const int ar[] = {2,3,5,7,11,13,17,19,23};
39     forn (j,9)
40         if (!es_primo_prob(n,ar[j]))
41             return false;
42     return true;
43 }

44
45 ll rho(ll n){
46     if( (n & 1) == 0 ) return 2;

```

```

47     ll x = 2 , y = 2 , d = 1;
48     ll c = rand() % n + 1;
49     while( d == 1 ){
50         x = (mulmod( x , x , n ) + c)%n;
51         y = (mulmod( y , y , n ) + c)%n;
52         y = (mulmod( y , y , n ) + c)%n;
53         if( x - y >= 0 ) d = gcd( x - y , n );
54         else d = gcd( y - x , n );
55     }
56     return d==n? rho(n):d;
57 }

58
59 map<ll,ll> prim;
60 void factRho (ll n){ //O (lg n)^3. un solo numero
61     if (n == 1) return;
62     if (rabin(n)){
63         prim[n]++;
64         return;
65     }
66     ll factor = rho(n);
67     factRho(factor);
68     factRho(n/factor);
69 }

```

6.8. GCD

```
1 | tipo gcd(tipo a, tipo b){return a?gcd(b %a, a):b;}
```

6.9. Extended Euclid

```

1 | void extendedEuclid (ll a, ll b){ //a * x + b * y = d
2 |     if (!b) { x = 1; y = 0; d = a; return;}
3 |     extendedEuclid (b, a%b);
4 |     ll x1 = y;
5 |     ll y1 = x - (a/b) * y;
6 |     x = x1; y = y1;
7 | }

```

6.10. Inversos

```

1 | #define MAXMOD 15485867
2 | ll inv[MAXMOD]; //inv[i]*i=1 mod MOD
3 | void calc(int p){//O(p)
4 |     inv[1]=1;

```



```

5   forr(i, 2, p) inv[i]= p-((p/i)*inv[p%i])%p;
6   }
7   int inverso(int x){//0(log x)
8       return expmod(x, eulerphi(MOD)-2);//si mod no es primo(sacar a mano)
9       return expmod(x, MOD-2);//si mod es primo
10  }

```

6.11. Simpson

```

1   double integral(double a, double b, int n=10000) {//0(n), n=cantdiv
2       double area=0, h=(b-a)/n, fa=f(a), fb;
3       forn(i, n){
4           fb=f(a+h*(i+1));
5           area+=fa+ 4*f(a+h*(i+0.5)) +fb, fa=fb;
6       }
7       return area*h/6.;}

```

6.12. Polinomio

```

1       int m = sz(c), n = sz(o.c);
2       vector<tipo> res(max(m,n));
3       forn(i, m) res[i] += c[i];
4       forn(i, n) res[i] += o.c[i];
5       return poly(res);    }
6   poly operator*(const tipo cons) const {
7       vector<tipo> res(sz(c));
8       forn(i, sz(c)) res[i]=c[i]*cons;
9       return poly(res);    }
10  poly operator*(const poly &o) const {
11      int m = sz(c), n = sz(o.c);
12      vector<tipo> res(m+n-1);
13      forn(i, m) forn(j, n) res[i+j]+=c[i]*o.c[j];
14      return poly(res);    }
15  tipo eval(tipo v) {
16      tipo sum = 0;
17      dforn(i, sz(c)) sum=sum*v + c[i];
18      return sum; }
19      //poly contains only a vector<int> c (the coefficients)
20      //the following function generates the roots of the polynomial
21      //it can be easily modified to return float roots
22      set<tipo> roots(){
23          set<tipo> roots;
24          tipo a0 = abs(c[0]), an = abs(c[sz(c)-1]);
25          vector<tipo> ps,qs;

```

```

26      forr(p,1,sqrt(a0)+1) if (a0%p==0) ps.pb(p),ps.pb(a0/p);
27      forr(q,1,sqrt(an)+1) if (an%q==0) qs.pb(q),qs.pb(an/q);
28      forall(pt,ps)
29          forall(qt,qs) if ( (*pt) % (*qt)==0 ) {
30              tipo root = abs((*pt) / (*qt));
31              if (eval(root)==0) roots.insert(root);
32          }
33      return roots; }
34  };
35  pair<poly,tipo> ruffini(const poly p, tipo r) {
36      int n = sz(p.c) - 1 ;
37      vector<tipo> b(n);
38      b[n-1] = p.c[n];
39      dforn(k,n-1) b[k] = p.c[k+1] + r*b[k+1];
40      tipo resto = p.c[0] + r*b[0];
41      poly result(b);
42      return make_pair(result,resto);
43  }
44  poly interpolate(const vector<tipo>& x,const vector<tipo>& y) {
45      poly A; A.c.pb(1);
46      forn(i,sz(x)) { poly aux; aux.c.pb(-x[i]), aux.c.pb(1), A = A * aux;
47          }
48      poly S; S.c.pb(0);
49      forn(i,sz(x)) { poly Li;
50          Li = ruffini(A,x[i]).fst;
51          Li = Li * (1.0 / Li.eval(x[i])); // here put a multiple of the
52              coefficients instead of 1.0 to avoid using double
53          S = S + Li * y[i]; }
54      return S;
55  }
56  int main(){
57      return 0;
58  }

```

6.13. FFT

```

1   //~ typedef complex<double> base; //menos codigo, pero mas lento
2   //elegir si usar complejos de c (lento) o estos
3   struct base{
4       double r,i;
5       base(double r=0, double i=0):r(r), i(i){}
6       double real()const{return r;}

```

```

7   void operator/=(const int c){r/=c, i/=c;}
8   };
9   base operator*(const base &a, const base &b){
10      return base(a.r*b.r-a.i*b.i, a.r*b.i+a.i*b.r);}
11   base operator+(const base &a, const base &b){
12      return base(a.r+b.r, a.i+b.i);}
13   base operator-(const base &a, const base &b){
14      return base(a.r-b.r, a.i-b.i);}
15   vector<int> rev; vector<base> wlen_pw;
16   inline static void fft(base a[], int n, bool invert) {
17       forn(i, n) if(i<rev[i]) swap(a[i], a[rev[i]]);
18       for (int len=2; len<=n; len<=<=1) {
19           double ang = 2*M_PI/len * (invert?-1:+1);
20           int len2 = len>>1;
21           base wlen (cos(ang), sin(ang));
22           wlen_pw[0] = base (1, 0);
23           forr(i, 1, len2) wlen_pw[i] = wlen_pw[i-1] * wlen;
24           for (int i=0; i<n; i+=len) {
25               base t, *pu = a+i, *pv = a+i+len2, *pu_end = a+i+len2, *pw = &
                wlen_pw[0];
26               for (; pu!=pu_end; ++pu, ++pv, ++pw)
27                   t = *pv * *pw, *pv = *pu - t, *pu = *pu + t;
28           }
29       }
30       if (invert) forn(i, n) a[i]/= n;}
31   inline static void calc_rev(int n){//precalculo: llamar antes de fft!!
32       wlen_pw.resize(n), rev.resize(n);
33       int lg=31-__builtin_clz(n);
34       forn(i, n){
35           rev[i] = 0;
36           forn(k, lg) if(i&(1<<k)) rev[i]|=1<<(lg-1-k);
37       }}
38   inline static void multiply(const vector<int> &a, const vector<int> &b,
        vector<int> &res) {
39       vector<base> fa (a.begin(), a.end()), fb (b.begin(), b.end());
40       int n=1; while(n < max(sz(a), sz(b))) n <=<= 1; n <=<= 1;
41       calc_rev(n);
42       fa.resize (n), fb.resize (n);
43       fft (&fa[0], n, false), fft (&fb[0], n, false);
44       forn(i, n) fa[i] = fa[i] * fb[i];
45       fft (&fa[0], n, true);
46       res.resize(n);
47       forn(i, n) res[i] = int (fa[i].real() + 0.5); }

```

```

48   void toPoly(const string &s, vector<int> &P){//convierte un numero a
        polinomio
49       P.clear();
50       dforn(i, sz(s)) P.pb(s[i]-'0');}

```

6.14. Tablas y cotas (Primos, Divisores, Factoriales, etc)

max signed tint = 9.223.372.036.854.775.807

max unsigned tint = 18.446.744.073.709.551.615

7. Grafos

7.1. Dijkstra

```

1   #define INF 1e9
2   int N;
3   #define MAX_V 250001
4   vector<ii> G[MAX_V];
5   //To add an edge use
6   #define add(a, b, w) G[a].pb(make_pair(w, b))
7   ll dijkstra(int s, int t){//O(|E| log |V|)
8       priority_queue<ii, vector<ii>, greater<ii> > Q;
9       vector<ll> dist(N, INF); vector<int> dad(N, -1);
10      Q.push(make_pair(0, s)); dist[s] = 0;
11      while(sz(Q)){
12          ii p = Q.top(); Q.pop();
13          if(p.snd == t) break;
14          forall(it, G[p.snd])
15              if(dist[p.snd]+it->first < dist[it->snd]){
16                  dist[it->snd] = dist[p.snd] + it->fst;
17                  dad[it->snd] = p.snd;
18                  Q.push(make_pair(dist[it->snd], it->snd)); }
19      }
20      return dist[t];
21      if(dist[t]<INF)//path generator
22          for(int i=t; i!=-1; i=dad[i])
23              printf("%d%c", i, (i==s?'\\n':' '));}

```

7.2. Bellman-Ford

```

1   vector<ii> G[MAX_N]; //ady. list with pairs (weight, dst)
2   int dist[MAX_N];
3   void bford(int src){//O(VE)

```

```

4   dist[src]=0;
5   forn(i, N-1) forn(j, N) if(dist[j]!=INF) forall(it, G[j])
6       dist[it->snd]=min(dist[it->snd], dist[j]+it->fst);
7   }
8
9   bool hasNegCycle(){
10      forn(j, N) if(dist[j]!=INF) forall(it, G[j])
11          if(dist[it->snd]>dist[j]+it->fst) return true;
12      //inside if: all points reachable from it->snd will have -INF distance
13      (do bfs)
14      return false;
15  }

```

7.3. Floyd-Warshall

```

1  //G[i][j] contains weight of edge (i, j) or INF
2  //G[i][i]=0
3  int G[MAX_N][MAX_N];
4  void floyd(){//O(N^3)
5      forn(k, N) forn(i, N) if(G[i][k]!=INF) forn(j, N) if(G[k][j]!=INF)
6          G[i][j]=min(G[i][j], G[i][k]+G[k][j]);
7  }
8  bool inNegCycle(int v){
9      return G[v][v]<0;
10 }
11 //checks if there's a neg. cycle in path from a to b
12 bool hasNegCycle(int a, int b){
13     forn(i, N) if(G[a][i]!=INF && G[i][i]<0 && G[i][b]!=INF)
14         return true;
15     return false;
16 }

```

7.4. 2-SAT + Tarjan SCC

```

1  //We have a vertex representing a var and other for his negation.
2  //Every edge stored in G represents an implication. To add an equation
3  //of the form a||b, use addor(a, b)
4  //MAX=max cant var, n=cant var
5  #define addor(a, b) (G[neg(a)].pb(b), G[neg(b)].pb(a))
6  vector<int> G[MAX*2];
7  //idx[i]=index assigned in the dfs
8  //lw[i]=lowest index (closer from the root) reachable from i
9  int lw[MAX*2], idx[MAX*2], qidx;
10 stack<int> q;
11 int qcmp, cmp[MAX*2];

```

```

11 //verdad[cmp[i]]=valor de la variable i
12 bool verdad[MAX*2+1];
13
14 int neg(int x) { return x>n? x-n : x+n;}
15 void tjn(int v){
16     lw[v]=idx[v]=++qidx;
17     q.push(v), cmp[v]=-2;
18     forall(it, G[v]){
19         if(!idx[*it] || cmp[*it]==-2){
20             if(!idx[*it]) tjn(*it);
21             lw[v]=min(lw[v], lw[*it]);
22         }
23     }
24     if(lw[v]==idx[v]){
25         int x;
26         do{x=q.top(); q.pop(); cmp[x]=qcmp;}while(x!=v);
27         verdad[qcmp]=(cmp[neg(v)]<0);
28         qcmp++;
29     }
30 }
31 //remember to CLEAR G!!!
32 bool satisf(){//O(n)
33     memset(idx, 0, sizeof(idx)), qidx=0;
34     memset(cmp, -1, sizeof(cmp)), qcmp=0;
35     forn(i, n){
36         if(!idx[i]) tjn(i);
37         if(!idx[neg(i)]) tjn(neg(i));
38     }
39     forn(i, n) if(cmp[i]==cmp[neg(i)]) return false;
40     return true;
41 }

```

7.5. Articulation Points

```

1  int N;
2  vector<int> G[1000000];
3  //V[i]=node number(if visited), L[i]= lowest V[i] reachable from i
4  int qV, V[1000000], L[1000000], P[1000000];
5  void dfs(int v, int f){
6      L[v]=V[v]=++qV;
7      forall(it, G[v])
8          if(!V[*it]){
9              dfs(*it, v);

```

```

10     L[v] = min(L[v], L[*it]);
11     P[v] += L[*it] >= V[v];
12 }
13 else if (*it != f)
14     L[v] = min(L[v], V[*it]);
15 }
16 int cantart(){ //O(n)
17     qV=0;
18     zero(V), zero(P);
19     dfs(1, 0); P[1]--;
20     int q=0;
21     forn(i, N) if(P[i]) q++;
22 return q;
23 }

```

7.6. Comp. Biconexas y Puentes

```

1 struct edge {
2     int u,v, comp;
3     bool bridge;
4 };
5 vector<edge> e;
6 void addEdge(int u, int v) {
7     G[u].pb(sz(e)), G[v].pb(sz(e));
8     e.pb((edge){u,v,-1,false});
9 }
10 //d[i]=id de la dfs
11 //b[i]=lowest id reachable from i
12 int d[MAXN], b[MAXN], t;
13 int nbc;//cant componentes
14 int comp[MAXN]; //comp[i]=cant comp biconexas a la cual pertenece i
15 void initDfs(int n) {
16     zero(G), zero(comp);
17     e.clear();
18     forn(i,n) d[i]=-1;
19     nbc = t = 0;
20 }
21 stack<int> st;
22 void dfs(int u, int pe) { //O(n + m)
23     b[u] = d[u] = t++;
24     comp[u] = (pe != -1);
25     forall(ne, G[u]) if (*ne != pe){
26         int v = e[*ne].u ^ e[*ne].v ^ u;

```

```

27     if (d[v] == -1) {
28         st.push(*ne);
29         dfs(v,*ne);
30         if (b[v] > d[u]){
31             e[*ne].bridge = true; // bridge
32         }
33         if (b[v] >= d[u]){ // art
34             int last;
35             do {
36                 last = st.top(); st.pop();
37                 e[last].comp = nbc;
38             } while (last != *ne);
39             nbc++;
40             comp[u]++;
41         }
42         b[u] = min(b[u], b[v]);
43     }
44     else if (d[v] < d[u]) { // back edge
45         st.push(*ne);
46         b[u] = min(b[u], d[v]);
47     }
48 }
49 }

```

7.7. LCA + Climb

```

1 const int MAXN=100001;
2 const int LOGN=20;
3 //f[v][k] holds the 2^k father of v
4 //L[v] holds the level of v
5 int N, f[MAXN][LOGN], L[MAXN];
6 //call before build:
7 void dfs(int v, int fa=-1, int lvl=0){ //generate required data
8     f[v][0]=fa, L[v]=lvl;
9     forall(it, G[v]) if (*it != fa) dfs(*it, v, lvl+1); }
10 void build(){ //f[i][0] must be filled previously, 0<nlgn
11     forn(k, LOGN-1) forn(i, N) f[i][k+1]=f[f[i][k]][k]; }
12 #define lg(x) (31-__builtin_clz(x)) // = floor(log2(x))
13 int climb(int a, int d){ //O(lgn)
14     if(!d) return a;
15     dforn(i, lg(L[a])+1) if (1<=i<=d) a=f[a][i], d-=1<=i;
16     return a; }
17 int lca(int a, int b){ //O(lgn)

```

```

18 if(L[a]<L[b]) swap(a, b);
19 a=climb(a, L[a]-L[b]);
20 if(a==b) return a;
21 dform(i, lg(L[a])+1) if(f[a][i]!=f[b][i]) a=f[a][i], b=f[b][i];
22 return f[a][0]; }
23 int dist(int a, int b) { //returns distance between nodes
24 return L[a]+L[b]-2*L[lca(a, b)];}

```

7.8. Heavy Light Decomposition

```

1 int treesz[MAXN]; //cantidad de nodos en el subarbol del nodo v
2 int dad[MAXN]; //dad[v]=padre del nodo v
3 void dfs1(int v, int p=-1) { //pre-dfs
4     dad[v]=p;
5     treesz[v]=1;
6     forall(it, G[v]) if(*it!=p){
7         dfs1(*it, v);
8         treesz[v]+=treesz[*it];
9     }
10 }
11 //PONER Q EN 0 !!!!
12 int pos[MAXN], q; //pos[v]=posicion del nodo v en el recorrido de la dfs
13 //Las cadenas aparecen continuas en el recorrido!
14 int cantcad;
15 int homecad[MAXN]; //dada una cadena devuelve su nodo inicial
16 int cad[MAXN]; //cad[v]=cadena a la que pertenece el nodo
17 void heavylight(int v, int cur=-1){
18     if(cur==--1) homecad[cur=cantcad++]=v;
19     pos[v]=q++;
20     cad[v]=cur;
21     int mx=-1;
22     forn(i, sz(G[v])) if(G[v][i]!=dad[v]){
23         if(mx==--1 || treesz[G[v][mx]]<treesz[G[v][i]]) mx=i;
24         if(mx!=-1) heavylight(G[v][mx], cur);
25         forn(i, sz(G[v])) if(i!=mx && G[v][i]!=dad[v]){
26             heavylight(G[v][i], -1);
27         }
28     }
29     //ejemplo de obtener el maximo numero en el camino entre dos nodos
30     //RTA: max(query(low, u), query(low, v)), con low=lca(u, v)
31     //esta funcion va trepando por las cadenas
32     int query(int an, int v) { //O(logn)
33         //si estan en la misma cadena:
34         if(cad[an]==cad[v]) return rmq.get(pos[an], pos[v]+1);

```

```

34 return max(query(an, dad[homecad[cad[v]]]),
35             rmq.get(pos[homecad[cad[v]]], pos[v]+1));
36 }

```

7.9. Centroid Decomposition

```

1 int n;
2 vector<int> G[MAXN];
3 bool taken[MAXN]; //poner todos en FALSE al principio!!
4 int padre[MAXN]; //padre de cada nodo en el centroid tree
5
6 int szt[MAXN];
7 void calcsz(int v, int p) {
8     szt[v] = 1;
9     forall(it, G[v]) if (*it!=p && !taken[*it])
10         calcsz(*it, v), szt[v]+=szt[*it];
11 }
12 void centroid(int v=0, int f=-1, int lvl=0, int tam=-1) { //O(nlogn)
13     if(tam==--1) calcsz(v, -1), tam=szt[v];
14     forall(it, G[v]) if(!taken[*it] && szt[*it]>=tam/2)
15         {szt[v]=0; centroid(*it, f, lvl, tam); return;}
16     taken[v]=true;
17     padre[v]=f;
18     forall(it, G[v]) if(!taken[*it])
19         centroid(*it, v, lvl+1, -1);
20 }

```

7.10. Euler Cycle

```

1 int n,m,ars[MAXE], eq;
2 vector<int> G[MAXN]; //fill G,n,m,ars,eq
3 list<int> path;
4 int used[MAXN];
5 bool usede[MAXE];
6 queue<list<int>::iterator> q;
7 int get(int v){
8     while(used[v]<sz(G[v]) && usede[ G[v][used[v]] ]) used[v]++;
9     return used[v];
10 }
11 void explore(int v, int r, list<int>::iterator it){
12     int ar=G[v][get(v)]; int u=v^ars[ar];
13     usede[ar]=true;
14     list<int>::iterator it2=path.insert(it, u);
15     if(u!=r) explore(u, r, it2);

```

```

16   if(get(v)<sz(G[v])) q.push(it);
17 }
18 void euler(){
19     zero(used), zero(usede);
20     path.clear();
21     q=queue<list<int>::iterator>();
22     path.push_back(0); q.push(path.begin());
23     while(sz(q)){
24         list<int>::iterator it=q.front(); q.pop();
25         if(used[*it]<sz(G[*it])) explore(*it, *it, it);
26     }
27     reverse(path.begin(), path.end());
28 }
29 void addEdge(int u, int v){
30     G[u].pb(eq), G[v].pb(eq);
31     ars[eq++]=u^v;
32 }

```

7.11. Diametro árbol

```

1  vector<int> G[MAXN]; int n,m,p[MAXN],d[MAXN],d2[MAXN];
2  int bfs(int r, int *d) {
3      queue<int> q;
4      d[r]=0; q.push(r);
5      int v;
6      while(sz(q)) { v=q.front(); q.pop();
7          forall(it,G[v]) if (d[*it]==-1)
8              d[*it]=d[v]+1, p[*it]=v, q.push(*it);
9      }
10     return v;//ultimo nodo visitado
11 }
12 vector<int> diams; vector<ii> centros;
13 void diametros(){
14     memset(d,-1,sizeof(d));
15     memset(d2,-1,sizeof(d2));
16     diams.clear(), centros.clear();
17     forn(i, n) if(d[i]==-1){
18         int v,c;
19         c=bfs(bfs(i, d2), d);
20         forn(_,d[v]/2) c=p[c];
21         diams.pb(d[v]);
22         if(d[v]&1) centros.pb(ii(c, p[c]));
23         else centros.pb(ii(c, c));

```

```

24     }
25 }
26
27 int main() {
28     freopen("in", "r", stdin);
29     while(cin >> n >> m){
30         forn(i,m) { int a,b; cin >> a >> b; a--, b--;
31             G[a].pb(b);
32             G[b].pb(a);

```

7.12. Chu-liu

```

1  void visit(graph &h, int v, int s, int r,
2      vector<int> &no, vector< vector<int> > &comp,
3      vector<int> &prev, vector< vector<int> > &next, vector<weight> &mcost,
4      vector<int> &mark, weight &cost, bool &found) {
5      if (mark[v]) {
6          vector<int> temp = no;
7          found = true;
8          do {
9              cost += mcost[v];
10             v = prev[v];
11             if (v != s) {
12                 while (comp[v].size() > 0) {
13                     no[comp[v].back()] = s;
14                     comp[s].push_back(comp[v].back());
15                     comp[v].pop_back();
16                 }
17             }
18             } while (v != s);
19             forall(j,comp[s]) if (*j != r) forall(e,h[*j])
20                 if (no[e->src] != s) e->w -= mcost[ temp[*j] ];
21         }
22         mark[v] = true;
23         forall(i,next[v]) if (no[*i] != no[v] && prev[no[*i]] == v)
24             if (!mark[no[*i]] || *i == s)
25                 visit(h, *i, s, r, no, comp, prev, next, mcost, mark, cost, found)
26             ;
27     }
28     weight minimumSpanningArborescence(const graph &g, int r) {
29         const int n=sz(g);
30         graph h(n);
31         forn(u,n) forall(e,g[u]) h[e->dst].pb(*e);

```

```

31 vector<int> no(n);
32 vector<vector<int> > comp(n);
33 for(u, n) comp[u].pb(no[u] = u);
34 for (weight cost = 0; ;) {
35     vector<int> prev(n, -1);
36     vector<weight> mcost(n, INF);
37     for(j,n) if (j != r) forall(e,h[j])
38         if (no[e->src] != no[j])
39             if (e->w < mcost[ no[j] ])
40                 mcost[ no[j] ] = e->w, prev[ no[j] ] = no[e->src];
41     vector< vector<int> > next(n);
42     for(u,n) if (prev[u] >= 0)
43         next[ prev[u] ].push_back(u);
44     bool stop = true;
45     vector<int> mark(n);
46     for(u,n) if (u != r && !mark[u] && !comp[u].empty()) {
47         bool found = false;
48         visit(h, u, u, r, no, comp, prev, next, mcost, mark, cost, found);
49         if (found) stop = false;
50     }
51     if (stop) {
52         for(u,n) if (prev[u] >= 0) cost += mcost[u];
53         return cost;
54     }
55 }
56 }

```

7.13. Hungarian

```

1 //Dado un grafo bipartito completo con costos no negativos, encuentra el
  matching perfecto de minimo costo.
2 tipo cost[N][N], lx[N], ly[N], slack[N]; //llenar: cost=matriz de
  adyacencia
3 int n, max_match, xy[N], yx[N], slackx[N], prev2[N]; //n=cantidad de nodos
4 bool S[N], T[N]; //sets S and T in algorithm
5 void add_to_tree(int x, int prevx) {
6     S[x] = true, prev2[x] = prevx;
7     for(y, n) if (lx[x] + ly[y] - cost[x][y] < slack[y] - EPS)
8         slack[y] = lx[x] + ly[y] - cost[x][y], slackx[y] = x;
9 }
10 void update_labels(){
11     tipo delta = INF;
12     for (y, n) if (!T[y]) delta = min(delta, slack[y]);

```

```

13     for (x, n) if (S[x]) lx[x] -= delta;
14     for (y, n) if (T[y]) ly[y] += delta; else slack[y] -= delta;
15 }
16 void init_labels(){
17     zero(lx), zero(ly);
18     for (x,n) for(y,n) lx[x] = max(lx[x], cost[x][y]);
19 }
20 void augment() {
21     if (max_match == n) return;
22     int x, y, root, q[N], wr = 0, rd = 0;
23     memset(S, false, sizeof(S)), memset(T, false, sizeof(T));
24     memset(prev2, -1, sizeof(prev2));
25     for (x, n) if (xy[x] == -1){
26         q[wr++] = root = x, prev2[x] = -2;
27         S[x] = true; break; }
28     for (y, n) slack[y] = lx[root] + ly[y] - cost[root][y], slackx[y] =
        root;
29     while (true){
30         while (rd < wr){
31             x = q[rd++];
32             for (y = 0; y < n; y++) if (cost[x][y] == lx[x] + ly[y] && !T[y]){
33                 if (yx[y] == -1) break; T[y] = true;
34                 q[wr++] = yx[y], add_to_tree(yx[y], x); }
35             if (y < n) break; }
36         if (y < n) break;
37         update_labels(), wr = rd = 0;
38         for (y = 0; y < n; y++) if (!T[y] && slack[y] == 0){
39             if (yx[y] == -1){x = slackx[y]; break;}
40             else{
41                 T[y] = true;
42                 if (!S[yx[y]]) q[wr++] = yx[y], add_to_tree(yx[y], slackx[y]);
43             }
44             if (y < n) break; }
45         if (y < n){
46             max_match++;
47             for (int cx = x, cy = y, ty; cx != -2; cx = prev2[cx], cy = ty)
48                 ty = xy[cx], yx[cy] = cx, xy[cx] = cy;
49             augment(); }
50     }
51     tipo hungarian(){
52         tipo ret = 0; max_match = 0, memset(xy, -1, sizeof(xy));
53         memset(yx, -1, sizeof(yx)), init_labels(), augment(); //steps 1-3
54         for (x,n) ret += cost[x][xy[x]]; return ret;

```


55 | }

7.14. Dynamic Connectivity

```

1 struct UnionFind {
2     int n, comp;
3     vector<int> pre, si, c;
4     UnionFind(int n=0):n(n), comp(n), pre(n), si(n, 1) {
5         forn(i,n) pre[i] = i; }
6     int find(int u){return u==pre[u]?u:find(pre[u]);}
7     bool merge(int u, int v) {
8         if((u=find(u))==v) return false;
9         if(si[u]<si[v]) swap(u, v);
10        si[u]+=si[v], pre[v]=u, comp--, c.pb(v);
11        return true;
12    }
13    int snap(){return sz(c);}
14    void rollback(int snap){
15        while(sz(c)>snap){
16            int v = c.back(); c.pop_back();
17            si[pre[v]] -= si[v], pre[v] = v, comp++;
18        }
19    }
20 };
21 enum {ADD,DEL,QUERY};
22 struct Query {int type,u,v};
23 struct DynCon {
24     vector<Query> q;
25     UnionFind dsu;
26     vector<int> match,res;
27     map<ii,int> last; //se puede no usar cuando hay identificador para
                        //cada arista (mejora poco)
28     DynCon(int n=0):dsu(n){}
29     void add(int u, int v) {
30         if(u>v) swap(u,v);
31         q.pb((Query){ADD, u, v}), match.pb(-1);
32         last[ii(u,v)] = sz(q)-1;
33     }
34     void remove(int u, int v) {
35         if(u>v) swap(u,v);
36         q.pb((Query){DEL, u, v});
37         int prev = last[ii(u,v)];
38         match[prev] = sz(q)-1;

```

```

39         match.pb(prev);
40     }
41     void query() { //podria pasarle un puntero donde guardar la respuesta
42         q.pb((Query){QUERY, -1, -1}), match.pb(-1);}
43     void process() {
44         forn(i,sz(q)) if (q[i].type == ADD && match[i] == -1) match[i] =
45             sz(q);
46         go(0,sz(q));
47     }
48     void go(int l, int r) {
49         if(l+1==r){
50             if (q[l].type == QUERY) //Aqui responder la query usando el
51                 dsu!
52                 res.pb(dsu.comp); //aqui query=cantidad de componentes
53                 conexas
54             return;
55         }
56         int s=dsu.snap(), m = (l+r) / 2;
57         forr(i,m,r) if(match[i]!=-1 && match[i]<l) dsu.merge(q[i].u, q[i]
58             ].v);
59         go(l,m);
60         dsu.rollback(s);
61         s = dsu.snap();
62         forr(i,l,m) if(match[i]!=-1 && match[i]>=r) dsu.merge(q[i].u, q[
63             i].v);
64         go(m,r);
65         dsu.rollback(s);
66     }
67 }dc;

```

8. Network Flow

8.1. Dinic

```

1
2 const int MAX = 300;
3 // Corte minimo: vertices con dist[v]>=0 (del lado de src) VS. dist[v]
4 // ==-1 (del lado del dst)
5 // Para el caso de la red de Bipartite Matching (Sean V1 y V2 los
6 // conjuntos mas proximos a src y dst respectivamente):
7 // Reconstruir matching: para todo v1 en V1 ver las aristas a vertices
8 // de V2 con it->f>0, es arista del Matching
9 // Min Vertex Cover: vertices de V1 con dist[v]==-1 + vertices de V2 con

```



```

    dist[v]>0
7 // Max Independent Set: tomar los vertices NO tomados por el Min Vertex
  Cover
8 // Max Clique: construir la red de G complemento (debe ser bipartito!) y
  encontrar un Max Independet Set
9 // Min Edge Cover: tomar las aristas del matching + para todo vertices
  no cubierto hasta el momento, tomar cualquier arista de el
10 int nodes, src, dst;
11 int dist[MAX], q[MAX], work[MAX];
12 struct Edge {
13     int to, rev;
14     ll f, cap;
15     Edge(int to, int rev, ll f, ll cap) : to(to), rev(rev), f(f), cap(
        cap) {}
16 };
17 vector<Edge> G[MAX];
18 void addEdge(int s, int t, ll cap){
19     G[s].pb(Edge(t, sz(G[t]), 0, cap)), G[t].pb(Edge(s, sz(G[s])-1, 0,
        0));}
20 bool dinic_bfs(){
21     fill(dist, dist+nodes, -1), dist[src]=0;
22     int qt=0; q[qt++]=src;
23     for(int qh=0; qh<qt; qh++){
24         int u = q[qh];
25         forall(e, G[u]){
26             int v=e->to;
27             if(dist[v]<0 && e->f < e->cap)
28                 dist[v]=dist[u]+1, q[qt++]=v;
29         }
30     }
31     return dist[dst]>=0;
32 }
33 ll dinic_dfs(int u, ll f){
34     if(u==dst) return f;
35     for(int &i=work[u]; i<sz(G[u]); i++){
36         Edge &e = G[u][i];
37         if(e.cap<=e.f) continue;
38         int v=e.to;
39         if(dist[v]==dist[u]+1){
40             ll df=dinic_dfs(v, min(f, e.cap-e.f));
41             if(df>0){
42                 e.f+=df, G[v][e.rev].f-= df;
43                 return df; }

```

```

44     }
45 }
46 return 0;
47 }
48 ll maxFlow(int _src, int _dst){
49     src=_src, dst=_dst;
50     ll result=0;
51     while(dinic_bfs()){
52         fill(work, work+nodes, 0);
53         while(ll delta=dinic_dfs(src,INF))
54             result+=delta;
55     }
56     // todos los nodos con dist[v]!=-1 vs los que tienen dist[v]==-1
    forman el min-cut
57     return result; }

```

8.2. Konig

```

1 // asume que el dinic YA ESTA tirado
2 // asume que nodes-1 y nodes-2 son la fuente y destino
3 int match[maxnodes]; // match[v]=u si u-v esta en el matching, -1 si v
    no esta matcheado
4 int s[maxnodes]; // numero de la bfs del koning
5 queue<int> kq;
6 // s[e] %2==1 o si e esta en V1 y s[e]==-1-> lo agarras
7 void koning() { // O(n)
8     forn(v,nodes-2) s[v] = match[v] = -1;
9     forn(v,nodes-2) forall(it,g[v]) if (it->to < nodes-2 && it->f>0)
10         { match[v]=it->to; match[it->to]=v;}
11     forn(v,nodes-2) if (match[v]==-1) {s[v]=0;kq.push(v);}
12     while(!kq.empty()) {
13         int e = kq.front(); kq.pop();
14         if (s[e] %2==1) {
15             s[match[e]] = s[e]+1;
16             kq.push(match[e]);
17         } else {
18
19             forall(it,g[e]) if (it->to < nodes-2 && s[it->to]==-1) {
20                 s[it->to] = s[e]+1;
21                 kq.push(it->to);
22             }
23         }
24     }

```

25 | }

8.3. Edmonds Karp's

```

1 #define MAX_V 1000
2 #define INF 1e9
3 //special nodes
4 #define SRC 0
5 #define SNK 1
6 map<int, int> G[MAX_V]; //limpiar esto
7 //To add an edge use
8 #define add(a, b, w) G[a][b]=w
9 int f, p[MAX_V];
10 void augment(int v, int minE){
11     if(v==SRC) f=minE;
12     else if(p[v]!=-1){
13         augment(p[v], min(minE, G[p[v]][v]));
14         G[p[v]][v]-=f, G[v][p[v]]+=f;
15     }
16 }
17 ll maxflow(){//O(VE^2)
18     ll Mf=0;
19     do{
20         f=0;
21         char used[MAX_V]; queue<int> q; q.push(SRC);
22         zero(used), memset(p, -1, sizeof(p));
23         while(sz(q)){
24             int u=q.front(); q.pop();
25             if(u==SNK) break;
26             forall(it, G[u])
27                 if(it->snd>0 && !used[it->fst])
28                     used[it->fst]=true, q.push(it->fst), p[it->fst]=u;
29         }
30         augment(SNK, INF);
31         Mf+=f;
32     }while(f);
33     return Mf;
34 }
```

8.4. Push-Relabel O(N³)

```

1 #define MAX_V 1000
2 int N;//valid nodes are [0...N-1]
3 #define INF 1e9
```

```

4 //special nodes
5 #define SRC 0
6 #define SNK 1
7 map<int, int> G[MAX_V];
8 //To add an edge use
9 #define add(a, b, w) G[a][b]=w
10 ll excess[MAX_V];
11 int height[MAX_V], active[MAX_V], count[2*MAX_V+1];
12 queue<int> Q;
13 void enqueue(int v) {
14     if (!active[v] && excess[v] > 0) active[v]=true, Q.push(v); }
15 void push(int a, int b) {
16     int amt = min(excess[a], ll(G[a][b]));
17     if(height[a] <= height[b] || amt == 0) return;
18     G[a][b]-=amt, G[b][a]+=amt;
19     excess[b] += amt, excess[a] -= amt;
20     enqueue(b);
21 }
22 void gap(int k) {
23     forn(v, N){
24         if (height[v] < k) continue;
25         count[height[v]]--;
26         height[v] = max(height[v], N+1);
27         count[height[v]]++;
28         enqueue(v);
29     }
30 }
31 void relabel(int v) {
32     count[height[v]]--;
33     height[v] = 2*N;
34     forall(it, G[v])
35         if(it->snd)
36             height[v] = min(height[v], height[it->fst] + 1);
37     count[height[v]]++;
38     enqueue(v);
39 }
40 ll maxflow() { //O(V^3)
41     zero(height), zero(active), zero(count), zero(excess);
42     count[0] = N-1;
43     count[N] = 1;
44     height[SRC] = N;
45     active[SRC] = active[SNK] = true;
46     forall(it, G[SRC]){
```

```

47     excess[SRC] += it->snd;
48     push(SRC, it->fst);
49 }
50 while(sz(Q)) {
51     int v = Q.front(); Q.pop();
52     active[v]=false;
53     forall(it, G[v]) push(v, it->fst);
54     if(excess[v] > 0)
55         count[height[v]] == 1? gap(height[v]):relabel(v);
56 }
57 ll mf=0;
58 forall(it, G[SRC]) mf+=G[it->fst][SRC];
59 return mf;
60 }

```

8.5. Min-cost Max-flow

```

1  const int MAXN=10000;
2  typedef ll tf;
3  typedef ll tc;
4  const tf INFFLUJO = 1e14;
5  const tc INFCOSTO = 1e14;
6  struct edge {
7      int u, v;
8      tf cap, flow;
9      tc cost;
10     tf rem() { return cap - flow; }
11 };
12 int nodes; //numero de nodos
13 vector<int> G[MAXN]; // limpiar!
14 vector<edge> e; // limpiar!
15 void addEdge(int u, int v, tf cap, tc cost) {
16     G[u].pb(sz(e)); e.pb((edge){u,v,cap,0,cost});
17     G[v].pb(sz(e)); e.pb((edge){v,u,0,0,-cost});
18 }
19 tc dist[MAXN], mnCost;
20 int pre[MAXN];
21 tf cap[MAXN], mxFlow;
22 bool in_queue[MAXN];
23 void flow(int s, int t) {
24     zero(in_queue);
25     mxFlow=mnCost=0;
26     while(1){

```

```

27     fill(dist, dist+nodes, INFCOSTO); dist[s] = 0;
28     memset(pre, -1, sizeof(pre)); pre[s]=0;
29     zero(cap); cap[s] = INFFLUJO;
30     queue<int> q; q.push(s); in_queue[s]=1;
31     while(sz(q)){
32         int u=q.front(); q.pop(); in_queue[u]=0;
33         for(auto it:G[u]) {
34             edge &E = e[it];
35             if(E.rem() && dist[E.v] > dist[u] + E.cost + 1e-9){ // ojo EPS
36                 dist[E.v]=dist[u]+E.cost;
37                 pre[E.v] = it;
38                 cap[E.v] = min(cap[u], E.rem());
39                 if(!in_queue[E.v]) q.push(E.v), in_queue[E.v]=1;
40             }
41         }
42     }
43     if (pre[t] == -1) break;
44     mxFlow +=cap[t];
45     mnCost +=cap[t]*dist[t];
46     for (int v = t; v != s; v = e[pre[v]].u) {
47         e[pre[v]].flow += cap[t];
48         e[pre[v]^1].flow -= cap[t];
49     }
50 }
51 }

```

9. Template

```

1  //touch {a..m}.in; tee {a..m}.cpp < template.cpp
2  #include <bits/stdc++.h>
3  using namespace std;
4  #define forr(i,a,b) for(int i=(a); i<(b); i++)
5  #define forn(i,n) forr(i,0,n)
6  #define sz(c) ((int)c.size())
7  #define zero(v) memset(v, 0, sizeof(v))
8  #define forall(it,v) for(auto it=v.begin();it!=v.end();++it)
9  #define pb push_back
10 #define fst first
11 #define snd second
12 typedef long long ll;
13 typedef pair<int,int> ii;
14 #define dform(i,n) for(int i=n-1; i>=0; i--)
15 #define dprint(v) cout << #v"=" << v << endl //;)

```

```

16
17 const int MAXN=100100;
18 int n;
19
20 int main() {
21     freopen("input.in", "r", stdin);
22     ios::sync_with_stdio(0);
23     while(cin >> n){
24
25     }
26     return 0;
27 }

```

10. Ayudamemoria

Rellenar con espacios(para justificar)

```

1 #include <iomanip>
2 cout << setfill('␣') << setw(3) << 2 << endl;

```

Leer hasta fin de linea

```

1 #include <sstream>
2 //hacer cin.ignore() antes de getline()
3 while(getline(cin, line)){
4     istringstream is(line);
5     while(is >> X)
6         cout << X << "␣";
7     cout << endl;
8 }

```

Aleatorios

```

1 #define RAND(a, b) (rand()%(b-a+1)+a)
2 srand(time(NULL));

```

Doubles Comp.

```

1 const double EPS = 1e-9;
2 x == y <=> fabs(x-y) < EPS
3 x > y <=> x > y + EPS
4 x >= y <=> x > y - EPS

```

Limites

```

1 #include <limits>
2 numeric_limits<T>
3     ::max()
4     ::min()
5     ::epsilon()

```

Expandir pila

```

1 #include <sys/resource.h>
2 rlimit rl;
3 getrlimit(RLIMIT_STACK, &rl);
4 rl.rlim_cur=1024L*1024L*256L;//256mb
5 setrlimit(RLIMIT_STACK, &rl);

```

C++11

```

1 g++ --std=c++11

```

Iterar subconjunto

```

1 for(int sbm=bm; sbm; sbm=(sbm-1)&bm)

```