

# Índice

<b>1. algorithm</b>	<b>2</b>	5.9. Convex Check CHECK . . . . .	13
<b>2. Estructuras</b>	<b>2</b>	5.10. Convex Hull . . . . .	13
2.1. Easy segment . . . . .	2	5.11. Cut Polygon . . . . .	13
2.2. RMQ (static) . . . . .	2	5.12. Bresenham . . . . .	13
2.3. RMQ (dynamic) . . . . .	3	5.13. Interseccion de Circulos en $n^3 \log(n)$ . . . . .	13
2.4. RMQ (lazy) . . . . .	3	<b>6. Math</b>	<b>14</b>
2.5. Union Find . . . . .	4	6.1. Identidades . . . . .	14
2.6. Disjoint Intervals . . . . .	4	6.2. Ec. Caracteristica . . . . .	14
2.7. RMQ (2D) . . . . .	4	6.3. Combinatorio . . . . .	14
2.8. Treap para set . . . . .	5	6.4. Gauss Jordan, Determinante $O(n^3)$ . . . . .	14
2.9. Treap para arreglo . . . . .	5	6.5. Teorema Chino del Resto . . . . .	16
2.10. Set con busq binaria . . . . .	6	6.6. Funciones de primos . . . . .	16
<b>3. Algos</b>	<b>6</b>	6.7. Phollard's Rho (rolando) . . . . .	16
3.1. Longest Increasing Subsequence . . . . .	6	6.8. GCD . . . . .	17
3.2. Alpha-Beta pruning . . . . .	6	6.9. Extended Euclid . . . . .	17
3.3. Mo's algorithm . . . . .	6	6.10. Polinomio . . . . .	17
3.4. Ternary search . . . . .	7	6.11. FFT . . . . .	18
<b>4. Strings</b>	<b>7</b>	<b>7. Grafos</b>	<b>19</b>
4.1. Manacher . . . . .	7	7.1. Bellman-Ford . . . . .	19
4.2. KMP . . . . .	7	7.2. 2-SAT + Tarjan SCC . . . . .	19
4.3. Trie . . . . .	7	7.3. Puentes y Articulation Points . . . . .	19
4.4. Suffix Array (largo, $n \log n$ ) . . . . .	7	7.4. LCA + Climb . . . . .	20
4.5. String Matching With Suffix Array . . . . .	8	7.5. Heavy Light Decomposition . . . . .	20
4.6. LCP (Longest Common Prefix) . . . . .	8	7.6. Centroid Decomposition . . . . .	20
4.7. Corasick . . . . .	8	7.7. Euler Cycle . . . . .	21
4.8. Suffix Automaton . . . . .	9	7.8. Chu-liu . . . . .	21
4.9. Z Function . . . . .	10	7.9. Hungarian . . . . .	22
4.10. Rabin Karp - Distinct Substrings . . . . .	10	7.10. Dynamic Conectivity . . . . .	23
<b>5. Geometria</b>	<b>10</b>	<b>8. Network Flow</b>	<b>23</b>
5.1. Punto . . . . .	10	8.1. Dinic . . . . .	23
5.2. Orden radial de puntos . . . . .	10	8.2. Edmonds Karp's . . . . .	24
5.3. Line . . . . .	11	8.3. Max Matching . . . . .	25
5.4. Segment . . . . .	11	8.4. Min-cost Max-flow . . . . .	25
5.5. Polygon Area . . . . .	11	<b>9. Template y Otros</b>	<b>26</b>
5.6. Circle . . . . .	11		
5.7. Point in Poly . . . . .	12		
5.8. Point in Convex Poly $\log(n)$ . . . . .	12		

## 1. algorithm

```
#include <algorithm> #include <numeric>
```

Algo	Params	Funcion
sort, stable_sort	f, l	ordena el intervalo
nth_element	f, nth, l	void ordena el n-esimo, y particiona el resto
fill, fill_n	f, l / n, elem	void llena [f, l) o [f, f+n) con elem
lower_bound, upper_bound	f, l, elem	it al primer / ultimo donde se puede insertar elem para que quede ordenada
binary_search	f, l, elem	bool esta elem en [f, l)
copy	f, l, resul	hace resul+i=f+i $\forall i$
find, find_if, find_first_of	f, l, elem / pred / f2, l2	it encuentra i $\in [f, l)$ tq. i=elem, pred(i), $i \in [f2, l2)$
count, count_if	f, l, elem/pred	cuenta elem, pred(i)
search	f, l, f2, l2	busca [f2, l2) $\in [f, l)$
replace, replace_if	f, l, old / pred, new	cambia old / pred(i) por new
reverse	f, l	da vuelta
partition, stable_partition	f, l, pred	pred(i) ad, !pred(i) atras
min_element, max_element	f, l, [comp]	it min, max de [f, l)
lexicographical_compare	f1, l1, f2, l2	bool con [f1, l1] i [f2, l2]
next/prev_permutation	f, l	deja en [f, l) la perm sig, ant
set_intersection, set_difference, set_union, set_symmetric_difference,	f1, l1, f2, l2, res	[res, ...) la op. de conj
push_heap, pop_heap, make_heap	f, l, e / e /	mete/saca e en heap [f, l), hace un heap de [f, l)
is_heap	f, l	bool es [f, l) un heap
accumulate	f, l, i, [op]	$T = \sum / \text{oper de } [f, l)$
inner_product	f1, l1, f2, i	$T = i + [f1, l1) \cdot [f2, \dots)$
partial_sum	f, l, r, [op]	$r+i = \sum / \text{oper de } [f, f+i) \forall i \in [f, l)$
__builtin_ffs	unsigned int	Pos. del primer 1 desde la derecha
__builtin_clz	unsigned int	Cant. de ceros desde la izquierda.
__builtin_ctz	unsigned int	Cant. de ceros desde la derecha.
__builtin_popcount	unsigned int	Cant. de 1's en x.
__builtin_parity	unsigned int	1 si x es par, 0 si es impar.
__builtin_XXXXXXll	unsigned ll	= pero para long long's.

## 2. Estructuras

### 2.1. Easy segment

```
1  const int N = 1e5; // limit for array size
2  int n; // array size
3  int t[2 * N];
4
5  void build() { // build the tree
6      for (int i = n - 1; i > 0; --i) t[i] = t[i<<1] + t[i<<1|1];
7  }
8
9  void modify(int p, int value) { // set value at position p
10     for (t[p += n] = value; p > 1; p >>= 1) t[p>>1] = t[p] + t[p^1];
11 }
12
13 int query(int l, int r) { // sum on interval [l, r)
14     int res = 0;
15     for (l += n, r += n; l < r; l >>= 1, r >>= 1) {
16         if (l&1) res += t[l++];
17         if (r&1) res += t[--r];
18     }
19     return res;
20 }
21
22 int main() {
23     scanf("%d", &n);
24     for (int i = 0; i < n; ++i) scanf("%d", t + n + i);
25     build();
26     modify(0, 1);
27     printf("%d\n", query(3, 11));
28     return 0;
29 }
```

### 2.2. RMQ (static)

Dado un arreglo y una operacion asociativa *idempotente*, get(i, j) opera sobre el rango [i, j). Restriccion:  $LVL \geq \text{ceil}(\log n)$ ; Usar [ ] para llenar arreglo y luego build().

```
1  struct RMQ{
2      #define LVL 10
3      tipo vec[LVL][1<<(LVL+1)];
4      tipo &operator[] (int p){return vec[0][p];}
5      tipo get(int i, int j) { // intervalo [i, j)
```

```

6   int p = 31-__builtin_clz(j-i);
7   return min(vec[p][i],vec[p][j-(1<<p)]);
8   }
9   void build(int n) { //O(nlogn)
10      int mp = 31-__builtin_clz(n);
11      forn(p, mp) forn(x, n-(1<<p))
12          vec[p+1][x] = min(vec[p][x], vec[p][x+(1<<p)]);
13  };

```

## 2.3. RMQ (dynamic)

```

1  //Dado un arreglo y una operacion asociativa con neutro, get(i, j) opera
   sobre el rango [i, j].
2  #define MAXN 100000
3  #define operacion(x, y) max(x, y)
4  const int neutro=0;
5  struct RMQ{
6      int sz;
7      tipo t[4*MAXN];
8      tipo &operator[](int p){return t[sz+p];}
9      void init(int n){ //O(nlgn)
10         sz = 1 << (32-__builtin_clz(n));
11         forn(i, 2*sz) t[i]=neutro;
12     }
13     void updall(){ //O(n)
14         dforn(i, sz) t[i]=operacion(t[2*i], t[2*i+1]);}
15     tipo get(int i, int j){return get(i,j,1,0,sz);}
16     tipo get(int i, int j, int n, int a, int b){ //O(lgn)
17         if(j<=a || i>=b) return neutro;
18         if(i<=a && b<=j) return t[n];
19         int c=(a+b)/2;
20         return operacion(get(i, j, 2*n, a, c), get(i, j, 2*n+1, c, b));
21     }
22     void set(int p, tipo val){ //O(lgn)
23         for(p+=sz; p>0 && t[p]!=val;){
24             t[p]=val;
25             p/=2;
26             val=operacion(t[p*2], t[p*2+1]);
27         }
28     }
29 }rmq;
30 //Usage:

```

```

31 cin >> n; rmq.init(n); forn(i, n) cin >> rmq[i]; rmq.updall();

```

## 2.4. RMQ (lazy)

```

1  //Dado un arreglo y una operacion asociativa con neutro, get(i, j) opera
   sobre el rango [i, j].
2  typedef int Elem; //Elem de los elementos del arreglo
3  typedef int Alt; //Elem de la alteracion
4  #define operacion(x,y) x+y
5  const Elem neutro=0; const Alt neutro2=0;
6  #define MAXN 100000
7  struct RMQ{
8      int sz;
9      Elem t[4*MAXN];
10     Alt dirty[4*MAXN]; //las alteraciones pueden ser de distinto Elem
11     Elem &operator[](int p){return t[sz+p];}
12     void init(int n){ //O(nlgn)
13         sz = 1 << (32-__builtin_clz(n));
14         forn(i, 2*sz) t[i]=neutro;
15         forn(i, 2*sz) dirty[i]=neutro2;
16     }
17     void push(int n, int a, int b){ //propaga el dirty a sus hijos
18         if(dirty[n]!=0){
19             t[n]+=dirty[n]*(b-a); //altera el nodo
20             if(n<sz){
21                 dirty[2*n]+=dirty[n];
22                 dirty[2*n+1]+=dirty[n];
23             }
24             dirty[n]=0;
25         }
26     }
27     Elem get(int i, int j, int n, int a, int b){ //O(lgn)
28         if(j<=a || i>=b) return neutro;
29         push(n, a, b); //corrige el valor antes de usarlo
30         if(i<=a && b<=j) return t[n];
31         int c=(a+b)/2;
32         return operacion(get(i, j, 2*n, a, c), get(i, j, 2*n+1, c, b));
33     }
34     Elem get(int i, int j){return get(i,j,1,0,sz);}
35     //altera los valores en [i, j] con una alteracion de val
36     void alterar(Alt val, int i, int j, int n, int a, int b){ //O(lgn)
37         push(n, a, b);
38         if(j<=a || i>=b) return;

```

```

39     if(i<=a && b<=j){
40         dirty[n]+=val;
41         push(n, a, b);
42         return;
43     }
44     int c=(a+b)/2;
45     alterar(val, i, j, 2*n, a, c), alterar(val, i, j, 2*n+1, c, b);
46     t[n]=operacion(t[2*n], t[2*n+1]);//por esto es el push de arriba
47 }
48 void alterar(Alt val, int i, int j){alterar(val,i,j,1,0,sz);}
49 }rmq;

```

## 2.5. Union Find

```

1 class UnionFind {
2 private:
3     vi p, rank, setSize;
4     int numSets;
5 public:
6     UnionFind(int N) {
7         setSize.assign(N, 1); numSets = N; rank.assign(N, 0);
8         p.assign(N, 0); for (int i = 0; i < N; i++) p[i] = i; }
9     int findSet(int i) { return (p[i] == i) ? i : (p[i] = findSet(p[i])); }
10
11     bool isSameSet(int i, int j) { return findSet(i) == findSet(j); }
12     void unionSet(int i, int j) {
13         if (!isSameSet(i, j)) { numSets--;
14             int x = findSet(i), y = findSet(j);
15             // rank is used to keep the tree short
16             if (rank[x] > rank[y]) { p[y] = x; setSize[x] += setSize[y]; }
17             else { p[x] = y; setSize[y] += setSize[x];
18                 if (rank[x] == rank[y]) rank[y]++; } } }
19     int numDisjointSets() { return numSets; }
20     int sizeOfSet(int i) { return setSize[findSet(i)]; }
21 };

```

## 2.6. Disjoint Intervals

```

1 bool operator< (const ii &a, const ii &b) {return a.fst<b.fst;}
2 //Stores intervals as [first, second]
3 //in case of a collision it joins them in a single interval
4 struct disjoint_intervals {
5     set<ii> segs;
6     void insert(ii v) {//O(lgn)

```

```

7     if(v.snd-v.fst==0.) return;//OJO
8     set<ii>::iterator it,at;
9     at = it = segs.lower_bound(v);
10    if (at!=segs.begin() && (--at)->snd >= v.fst)
11        v.fst = at->fst, --it;
12    for(; it!=segs.end() && it->fst <= v.snd; segs.erase(it++))
13        v.snd=max(v.snd, it->snd);
14    segs.insert(v);
15 }
16 };

```

## 2.7. RMQ (2D)

```

1 struct RMQ2D{//n filas x m columnas
2     int sz;
3     RMQ t[4*MAXN];
4     RMQ &operator[] (int p){return t[sz/2+p];}//t[i][j]=i fila, j col
5     void init(int n, int m){//O(n*m)
6         sz = 1 << (32-__builtin_clz(n));
7         forn(i, 2*sz) t[i].init(m); }
8     void set(int i, int j, tipo val){//O(lgm.lgn)
9         for(i+=sz; i>0;){
10             t[i].set(j, val);
11             i/=2;
12             val=operacion(t[i*2][j], t[i*2+1][j]);
13         } }
14     tipo get(int i1, int j1, int i2, int j2){return get(i1,j1,i2,j2,1,0,
15         sz);}
16     //O(lgm.lgn), rangos cerrado abierto
17     int get(int i1, int j1, int i2, int j2, int n, int a, int b){
18         if(i2<=a || i1>=b) return 0;
19         if(i1<=a && b<=i2) return t[n].get(j1, j2);
20         int c=(a+b)/2;
21         return operacion(get(i1, j1, i2, j2, 2*n, a, c),
22             get(i1, j1, i2, j2, 2*n+1, c, b));
23     }
24 } rmq;
25 //Example to initialize a grid of M rows and N columns:
26 RMQ2D rmq; rmq.init(n,m);
27 forn(i, n) forn(j, m){
28     int v; cin >> v; rmq.set(i, j, v);}

```

## 2.8. Treap para set

Treap para set tiene un Key unico por nodo. En el split if (key <= t->key). En at, if(key == t->key) return t; en lugar de pos.

```
1 void erase(pnode &t, Key key) {
2     if (!t) return; push(t);
3     if (key == t->key) t=merge(t->l, t->r);
4     else if (key < t->key) erase(t->l, key);
5     else erase(t->r, key);
6     if(t) pull(t);}
```

## 2.9. Treap para arreglo

```
1 typedef struct node *pnode;
2 struct node{
3     Value val, mini;
4     int dirty;
5     int prior, size;
6     pnode l,r,parent;
7     node(Value val): val(val), mini(val), dirty(0), prior(rand()), size
8         (1), l(0), r(0), parent(0) {}
9 };
10 static int size(pnode p) { return p ? p->size : 0; }
11 void push(pnode p) { //propagar dirty a los hijos(aca para lazy)
12     p->val.fst+=p->dirty;
13     p->mini.fst+=p->dirty;
14     if(p->l) p->l->dirty+=p->dirty;
15     if(p->r) p->r->dirty+=p->dirty;
16     p->dirty=0;
17 }
18 static Value mini(pnode p) { return p ? push(p), p->mini : ii(1e9, -1);
19     }
20 // Update function and size from children's Value
21 void pull(pnode p) { //recalcular valor del nodo aca (para rmq)
22     p->size = 1 + size(p->l) + size(p->r);
23     p->mini = min(min(p->val, mini(p->l)), mini(p->r)); //operacion del rmq
24     !
25     p->parent=0;
26     if(p->l) p->l->parent=p;
27     if(p->r) p->r->parent=p;
28 }
29 //junta dos arreglos
```

```
27 pnode merge(pnode l, pnode r) {
28     if (!l || !r) return l ? l : r;
29     push(l), push(r);
30     pnode t;
31     if (l->prior < r->prior) l->r=merge(l->r, r), t = l;
32     else r->l=merge(l, r->l), t = r;
33     pull(t);
34     return t;
35 }
36 //parte el arreglo en dos, sz(l)==tam
37 void split(pnode t, int tam, pnode &l, pnode &r) {
38     if (!t) return void(l = r = 0);
39     push(t);
40     if (tam <= size(t->l)) split(t->l, tam, l, t->l), r = t;
41     else split(t->r, tam - 1 - size(t->l), t->r, r), l = t;
42     pull(t);
43 }
44 pnode at(pnode t, int pos) {
45     if(!t) exit(1);
46     push(t);
47     if(pos == size(t->l)) return t;
48     if(pos < size(t->l)) return at(t->l, pos);
49     return at(t->r, pos - 1 - size(t->l));
50 }
51 int getpos(pnode t){ //inversa de at
52     if(!t->parent) return size(t->l);
53     if(t==t->parent->l) return getpos(t->parent)-size(t->r)-1;
54     return getpos(t->parent)+size(t->l)+1;
55 }
56 void split(pnode t, int i, int j, pnode &l, pnode &m, pnode &r) {
57     split(t, i, l, t), split(t, j-i, m, r);}
58 Value get(pnode &p, int i, int j){ //like rmq
59     pnode l,m,r;
60     split(p, i, j, l, m, r);
61     Value ret=mini(m);
62     p=merge(l, merge(m, r));
63     return ret;
64 }
65
66 //Sample program: C. LCA Online from Petrozavodsk Summer-2012.
67 //Petrozavodsk SU Contest
68 //Available at http://opentrains.snarknews.info/~ejudge
69 const int MAXN=300100;
```

```

69 int n;
70 pnode beg[MAXN], fin[MAXN];
71 pnode lista;

```

## 2.10. Set con busq binaria

```

1 #include <ext/pb_ds/assoc_container.hpp>
2 #include <ext/pb_ds/tree_policy.hpp>
3 using namespace __gnu_pbds;
4 typedef tree<int,null_type,less<int>,//key,mapped type, comparator
5     rb_tree_tag,tree_order_statistics_node_update> set_t;
6 //find_by_order(i) devuelve iterador al i-esimo elemento
7 //order_of_key(k): devuelve la pos del lower bound de k

```

## 3. Algos

### 3.1. Longest Increasing Subsequence

```

1
2 typedef vector<int> VI;
3 typedef pair<int,int> PII;
4 typedef vector<PII> VPII;
5
6 #define STRICTLY_INCREASNG
7
8 VI LongestIncreasingSubsequence(VI v) {
9     VPII best;
10    VI dad(v.size(), -1);
11
12    for (int i = 0; i < v.size(); i++) {
13        #ifdef STRICTLY_INCREASNG
14            PII item = make_pair(v[i], 0);
15            VPII::iterator it = lower_bound(best.begin(), best.end(), item);
16            item.second = i;
17        #else
18            PII item = make_pair(v[i], i);
19            VPII::iterator it = upper_bound(best.begin(), best.end(), item);
20        #endif
21        if (it == best.end()) {
22            dad[i] = (best.size() == 0 ? -1 : best.back().second);
23            best.push_back(item);
24        } else {
25            dad[i] = it == best.begin() ? -1 : prev(it)->second;

```

```

26     *it = item;
27 }
28 }

```

### 3.2. Alpha-Beta pruning

```

1 ll alphabeta(State &s, bool player = true, int depth = 1e9, ll alpha = -
    INF, ll beta = INF) { //player = true -> Maximiza
2     if(s.isFinal()) return s.score;
3     //~ if (!depth) return s.heuristic();
4     vector<State> children;
5     s.expand(player, children);
6     int n = children.size();
7     forn(i, n) {
8         ll v = alphabeta(children[i], !player, depth-1, alpha, beta);
9         if(!player) alpha = max(alpha, v);
10        else beta = min(beta, v);
11        if(beta <= alpha) break;
12    }
13    return !player ? alpha : beta;}

```

### 3.3. Mo's algorithm

```

1 int n,sq;
2 struct Qu{//queries [l, r]
3     //intervalos cerrado abiertos !!! importante!!
4     int l, r, id;
5 }qs[MAXN];
6 int ans[MAXN], curans;//ans[i]=ans to ith query
7 bool bymos(const Qu &a, const Qu &b){
8     if(a.l/sq!=b.l/sq) return a.l<b.l;
9     return (a.l/sq)&1? a.r<b.r : a.r>b.r;
10 }
11 void mos(){
12     forn(i, t) qs[i].id=i;
13     sort(qs, qs+t, bymos);
14     int cl=0, cr=0;
15     sq=sqrt(n);
16     curans=0;
17     forn(i, t){ //intervalos cerrado abiertos !!! importante!!
18         Qu &q=qs[i];
19         while(cl>q.l) add(--cl);
20         while(cr<q.r) add(cr++);
21         while(cl<q.l) remove(cl++);

```

```

22     while(cr>q.r) remove(--cr);
23     ans[q.id]=curans;
24 }
25 }

```

### 3.4. Ternary search

```

1 #include <functional>
2 //Retorna argmax de una funcion unimodal 'f' en el rango [left,right]
3 double ternarySearch(double l, double r, function<double(double)> f){
4     for(int i = 0; i < 300; i++){
5         double m1 = l+(r-l)/3, m2 = r-(r-l)/3;
6         if (f(m1) < f(m2)) l = m1; else r = m2;
7     }
8     return (left + right)/2;
9 }

```

## 4. Strings

### 4.1. Manacher

```

1 int d1[MAXN]; //d1[i]=long del maximo palindromo impar con centro en i
2 int d2[MAXN]; //d2[i]=analogo pero para longitud par
3 //0 1 2 3 4
4 //a a b c c <--d1[2]=3
5 //a a b b <--d2[2]=2 (estan uno antes)
6 void manacher(){
7     int l=0, r=-1, n=sz(s);
8     forn(i, n){
9         int k=(i>r? 1 : min(d1[l+r-i], r-i));
10        while(i+k<n && i-k>=0 && s[i+k]==s[i-k]) ++k;
11        d1[i] = k--;
12        if(i+k > r) l=i-k, r=i+k;
13    }
14    l=0, r=-1;
15    forn(i, n){
16        int k=(i>r? 0 : min(d2[l+r-i+1], r-i+1))+1;
17        while(i+k-1<n && i-k>=0 && s[i+k-1]==s[i-k]) k++;
18        d2[i] = --k;
19        if(i+k-1 > r) l=i-k, r=i+k-1;
20    }

```

### 4.2. KMP

```

1 string T;//cadena donde buscar(what)
2 string P;//cadena a buscar(what)
3 int b[MAXLEN]; //back table b[i] maximo borde de [0..i)
4 void kmppre(){ //by gabina with love
5     int i =0, j=-1; b[0]=-1;
6     while(i<sz(P)){
7         while(j>=0 && P[i] != P[j]) j=b[j];
8         i++, j++, b[i] = j;
9     }
10 }
11 void kmp(){
12     int i=0, j=0;
13     while(i<sz(T)){
14         while(j>=0 && T[i]!=P[j]) j=b[j];
15         i++, j++;
16         if(j==sz(P)) printf("P is found at index %d in T\n", i-j), j=b[j];
17     }
18 }
19
20 int main(){
21     cout << "T=";
22     cin >> T;
23     cout << "P=";

```

### 4.3. Trie

```

1 struct trie{ map<char, trie> m;
2     void add(const string &s, int p=0){ if(s[p]) m[s[p]].add(s, p+1);}
3     void dfs(){/*Do stuff*/ forall(it, m) it->second.dfs();}};

```

### 4.4. Suffix Array (largo, nlogn)

```

1 #define MAX_N 1000
2 #define rBOUND(x) (x<n? r[x] : 0)
3 //sa will hold the suffixes in order.
4 int sa[MAX_N], r[MAX_N], n;
5 string s; //input string, n=sz(s)
6
7 int f[MAX_N], tmpsa[MAX_N];
8 void countingSort(int k){
9     zero(f);
10    forn(i, n) f[rBOUND(i+k)]++;
11    int sum=0;

```

```

12  forn(i, max(255, n)){
13      int t=f[i]; f[i]=sum; sum+=t;}
14  forn(i, n)
15      tmpsa[f[rBOUND(sa[i]+k)]++] = sa[i];
16  memcpy(sa, tmpsa, sizeof(sa));
17  }
18  void constructsa(){//O(n log n)
19      n=sz(s);
20      forn(i, n) sa[i]=i, r[i]=s[i];
21      for(int k=1; k<n; k<=1){
22          countingSort(k), countingSort(0);
23          int rank, tmpr[MAX_N];
24          tmpr[sa[0]]=rank=0;
25          forr(i, 1, n)
26              tmpr[sa[i]]=(r[sa[i]]==r[sa[i-1]] && r[sa[i]+k]==r[sa[i-1]+k])?
                  rank : ++rank;
27          memcpy(r, tmpr, sizeof(r));
28          if(r[sa[n-1]]==n-1) break;
29      }
30  }
31  //returns (lowerbound, upperbound) of the search
32  ii stringMatching(string P){ //O(sz(P)lgn)
33      int lo=0, hi=n-1, mid=lo;
34      while(lo<hi){

```

#### 4.5. String Matching With Suffix Array

```

1  //returns (lowerbound, upperbound) of the search
2  ii stringMatching(string P){ //O(sz(P)lgn)
3      int lo=0, hi=n-1, mid=lo;
4      while(lo<hi){
5          mid=(lo+hi)/2;
6          int res=s.compare(sa[mid], sz(P), P);
7          if(res>=0) hi=mid;
8          else lo=mid+1;
9      }
10     if(s.compare(sa[lo], sz(P), P)!=0) return ii(-1, -1);
11     ii ans; ans.fst=lo;
12     lo=0, hi=n-1, mid;
13     while(lo<hi){
14         mid=(lo+hi)/2;
15         int res=s.compare(sa[mid], sz(P), P);
16         if(res>0) hi=mid;

```

```

17         else lo=mid+1;
18     }
19     if(s.compare(sa[hi], sz(P), P)!=0) hi--;
20     ans.snd=hi;
21     return ans;
22 }

```

#### 4.6. LCP (Longest Common Prefix)

```

1  //Calculates the LCP between consecutives suffixes in the Suffix Array.
2  //LCP[i] is the length of the LCP between sa[i] and sa[i-1]
3  int LCP[MAX_N], phi[MAX_N], PLCP[MAX_N];
4  void computeLCP(){//O(n)
5      phi[sa[0]]=-1;
6      forr(i, 1, n) phi[sa[i]]=sa[i-1];
7      int L=0;
8      forn(i, n){
9          if(phi[i]==-1) {PLCP[i]=0; continue;}
10         while(s[i+L]==s[phi[i]+L]) L++;
11         PLCP[i]=L;
12         L=max(L-1, 0);
13     }
14     forn(i, n) LCP[i]=PLCP[sa[i]];
15 }

```

#### 4.7. Corasick

```

1
2  struct trie{
3      map<char, trie> next;
4      trie* tran[256]; //transiciones del automata
5      int idhoja, szhoja; //id de la hoja o 0 si no lo es
6      //link lleva al sufijo mas largo, nxthoja lleva al mas largo pero que
        es hoja
7      trie *padre, *link, *nxthoja;
8      char pch; //caracter que conecta con padre
9      trie(): tran(), idhoja(), padre(), link() {}
10     void insert(const string &s, int id=1, int p=0){//id>0!!!
11         if(p<sz(s)){
12             trie &ch=next[s[p]];
13             tran[(int)s[p]]=&ch;
14             ch.padre=this, ch.pch=s[p];
15             ch.insert(s, id, p+1);
16         }

```



```

17     else idhoja=id, szhoja=sz(s);
18 }
19 trie* get_link() {
20     if(!link){
21         if(!padre) link=this;//es la raiz
22         else if(!padre->padre) link=padre;//hijo de la raiz
23         else link=padre->get_link()->get_tran(pch);
24     }
25     return link; }
26 trie* get_tran(int c) {
27     if(!tran[c]) tran[c] = !padre? this : this->get_link()->get_tran(c);
28     return tran[c]; }
29 trie *get_nxthoja(){
30     if(!nxthoja) nxthoja = get_link()->idhoja? link : link->nxthoja;
31     return nxthoja; }
32 void print(int p){
33     if(idhoja) cout << "found_" << idhoja << "_at_position_" << p-
34         szhoja << endl;
35     if(get_nxthoja()) get_nxthoja()->print(p); }
36 void matching(const string &s, int p=0){
37     print(p); if(p<sz(s)) get_tran(s[p])->matching(s, p+1); }
38 }tri;
39
40 int main(){
41     tri=trie();//clear
42     tri.insert("ho", 1);
43     tri.insert("hoho", 2);

```

#### 4.8. Suffix Automaton

```

1 struct state {
2     int len, link;
3     map<char,int> next;
4     state() { }
5 };
6 const int MAXLEN = 10010;
7 state st[MAXLEN*2];
8 int sz, last;
9 void sa_init() {
10     forn(i,sz) st[i].next.clear();
11     sz = last = 0;
12     st[0].len = 0;

```

```

13     st[0].link = -1;
14     ++sz;
15 }
16 // Es un DAG de una sola fuente y una sola hoja
17 // cantidad de endpos = cantidad de apariciones = cantidad de caminos de
18 // la clase al nodo terminal
19 // cantidad de miembros de la clase = st[v].len-st[st[v].link].len (v>0)
20 // = caminos del inicio a la clase
21 // El arbol de los suffix links es el suffix tree de la cadena invertida
22 // La string de la arista link(v)->v son los caracteres que difieren
23 void sa_extend (char c) {
24     int cur = sz++;
25     st[cur].len = st[last].len + 1;
26     // en cur agregamos la posicion que estamos extendiendo
27     //podria agregar tambien un identificador de las cadenas a las cuales
28     //pertenece (si hay varias)
29     int p;
30     for (p=last; p!=-1 && !st[p].next.count(c); p=st[p].link) // modificar
31         // esta linea para hacer separadores unicos entre varias cadenas (c
32         // == '$')
33     st[p].next[c] = cur;
34     if (p == -1)
35         st[cur].link = 0;
36     else {
37         int q = st[p].next[c];
38         if (st[p].len + 1 == st[q].len)
39             st[cur].link = q;
40         else {
41             int clone = sz++;
42             // no le ponemos la posicion actual a clone sino indirectamente
43             // por el link de cur
44             st[clone].len = st[p].len + 1;
45             st[clone].next = st[q].next;
46             st[clone].link = st[q].link;
47             for (; p!=-1 && st[p].next.count(c) && st[p].next[c]==q; p=st[p].
48                 link)
49                 st[p].next[c] = clone;
50             st[q].link = st[cur].link = clone;
51         }
52     }
53     last = cur;
54 }

```

## 4.9. Z Function

```

1 char s[MAXN];
2 int z[MAXN]; // z[i] = i==0 ? 0 : max k tq s[0,k) match with s[i,i+k)
3 void z_function(char s[],int z[]) {
4     int n = strlen(s);
5     forn(i, n) z[i]=0;
6     for (int i = 1, l = 0, r = 0; i < n; ++i) {
7         if (i <= r) z[i] = min (r - i + 1, z[i - l]);
8         while (i + z[i] < n && s[z[i]] == s[i + z[i]]) ++z[i];
9         if (i + z[i] - 1 > r) l = i, r = i + z[i] - 1;
10    }
11 }
12
13 int main() {
14     ios::sync_with_stdio(0);

```

## 4.10. Rabin Karp - Distinct Substrings

```

1 int count_unique_substrings(string const& s) {
2     int n = s.size();
3
4     const int p = 31;
5     const int m = 1e9 + 9;
6     vector<long long> p_pow(n);
7     p_pow[0] = 1;
8     for (int i = 1; i < n; i++)
9         p_pow[i] = (p_pow[i-1] * p) % m;
10
11     vector<long long> h(n + 1, 0);
12     for (int i = 0; i < n; i++)
13         h[i+1] = (h[i] + (s[i] - 'a' + 1) * p_pow[i]) % m;
14
15     int cnt = 0;
16     for (int l = 1; l <= n; l++) {
17         set<long long> hs;
18         for (int i = 0; i <= n - l; i++) {
19             long long cur_h = (h[i + l] + m - h[i]) % m;
20             cur_h = (cur_h * p_pow[n-i-1]) % m;
21             hs.insert(cur_h);
22         }
23         cnt += hs.size();
24     }

```

```

25     return cnt;
26 }

```

## 5. Geometria

### 5.1. Punto

```

1 struct pto{
2     double x, y;
3     pto(double x=0, double y=0):x(x),y(y){}
4     pto operator+(pto a){return pto(x+a.x, y+a.y);}
5     pto operator-(pto a){return pto(x-a.x, y-a.y);}
6     pto operator+(double a){return pto(x+a, y+a);}
7     pto operator*(double a){return pto(x*a, y*a);}
8     pto operator/(double a){return pto(x/a, y/a);}
9     //dot product, producto interno:
10    double operator*(pto a){return x*a.x+y*a.y;}
11    //module of the cross product or vectorial product:
12    //if a is less than 180 clockwise from b, a^b>0
13    double operator^(pto a){return x*a.y-y*a.x;}
14    //returns true if this is at the left side of line qr
15    bool left(pto q, pto r){return ((q-*this)^(r-*this))>0;}
16    bool operator<(const pto &a) const{return x<a.x-EPS || (abs(x-a.x)<EPS
17        && y<a.y-EPS);}
18    bool operator==(pto a){return abs(x-a.x)<EPS && abs(y-a.y)<EPS;}
19    double norm(){return sqrt(x*x+y*y);}
20    double norm_sq(){return x*x+y*y;}
21 };
22 double dist(pto a, pto b){return (b-a).norm();}
23 typedef pto vec;
24
25 double angle(pto a, pto o, pto b){
26     pto oa=a-o, ob=b-o;
27     return atan2(oa^ob, oa*ob);}
28
29 //rotate p by theta rads CCW w.r.t. origin (0,0)
30 pto rotate(pto p, double theta){
31     return pto(p.x*cos(theta)-p.y*sin(theta),
32         p.x*sin(theta)+p.y*cos(theta));
33 }

```

### 5.2. Orden radial de puntos

```

1 struct Cmp{//orden total de puntos alrededor de un punto r
2   pto r;
3   Cmp(pto r):r(r) {}
4   int cuad(const pto &a) const{
5     if(a.x > 0 && a.y >= 0)return 0;
6     if(a.x <= 0 && a.y > 0)return 1;
7     if(a.x < 0 && a.y <= 0)return 2;
8     if(a.x >= 0 && a.y < 0)return 3;
9     assert(a.x ==0 && a.y==0);
10    return -1;
11  }
12  bool cmp(const pto&p1, const pto&p2)const{
13    int c1 = cuad(p1), c2 = cuad(p2);
14    if(c1==c2) return p1.y*p2.x<p1.x*p2.y;
15    else return c1 < c2;
16  }
17  bool operator()(const pto&p1, const pto&p2) const{
18    return cmp(pto(p1.x-r.x,p1.y-r.y),pto(p2.x-r.x,p2.y-r.y));
19  }
20 };

```

### 5.3. Line

```

1 int sgn(ll x){return x<0? -1 : !!x;}
2 struct line{
3   line() {}
4   double a,b,c;//Ax+By=C
5   //pto MUST store float coordinates!
6   line(double a, double b, double c):a(a),b(b),c(c){}
7   line(pto p, pto q): a(q.y-p.y), b(p.x-q.x), c(a*p.x+b*p.y) {}
8   int side(pto p){return sgn(11(a) * p.x + 11(b) * p.y - c);}
9 };
10 bool parallels(line l1, line l2){return abs(11.a*12.b-12.a*11.b)<EPS;}
11 pto inter(line l1, line l2){//intersection
12   double det=11.a*12.b-12.a*11.b;
13   if(abs(det)<EPS) return pto(INF, INF);//parallels
14   return pto(12.b*11.c-11.b*12.c, 11.a*12.c-12.a*11.c)/det;
15 }

```

### 5.4. Segment

```

1 struct segm{
2   pto s,f;
3   segm(pto s, pto f):s(s), f(f) {}

```

```

4   pto closest(pto p) {//use for dist to point
5     double l2 = dist_sq(s, f);
6     if(l2==0.) return s;
7     double t =((p-s)*(f-s))/l2;
8     if (t<0.) return s;//not write if is a line
9     else if(t>1.)return f;//not write if is a line
10    return s+((f-s)*t);
11  }
12  bool inside(pto p){return abs(dist(s, p)+dist(p, f)-dist(s, f))<EPS;
13    };
14 };
15 pto inter(segm s1, segm s2){
16   pto r=inter(line(s1.s, s1.f), line(s2.s, s2.f));
17   if(s1.inside(r) && s2.inside(r)) return r;
18   return pto(INF, INF);
19 }

```

### 5.5. Polygon Area

```

1 double area(vector<pto> &p){//0(sz(p))
2   double area=0;
3   forn(i, sz(p)) area+=p[i]^p[(i+1)%sz(p)];
4   //if points are in clockwise order then area is negative
5   return abs(area)/2;
6 }
7 //Area ellipse = M_PI*a*b where a and b are the semi axis lengths
8 //Area triangle = sqrt(s*(s-a)(s-b)(s-c)) where s=(a+b+c)/2

```

### 5.6. Circle

```

1 vec perp(vec v){return vec(-v.y, v.x);}
2 line bisector(pto x, pto y){
3   line l=line(x, y); pto m=(x+y)/2;
4   return line(-1.b, 1.a, -1.b*m.x+1.a*m.y);
5 }
6 struct Circle{
7   pto o;
8   double r;
9   Circle(pto x, pto y, pto z){
10     o=inter(bisector(x, y), bisector(y, z));
11     r=dist(o, x);
12   }
13   pair<pto, pto> ptosTang(pto p){

```

```

14   pto m=(p+o)/2;
15   tipo d=dist(o, m);
16   tipo a=r*r/(2*d);
17   tipo h=sqrt(r*r-a*a);
18   pto m2=o+(m-o)*a/d;
19   vec per=perp(m-o)/d;
20   return make_pair(m2-per*h, m2+per*h);
21 }
22 };
23 //finds the center of the circle containing p1 and p2 with radius r
24 //as there may be two solutions swap p1, p2 to get the other
25 bool circle2PtsRad(pto p1, pto p2, double r, pto &c){
26     double d2=(p1-p2).norm_sq(), det=r*r/d2-0.25;
27     if(det<0) return false;
28     c=(p1+p2)/2+perp(p2-p1)*sqrt(det);
29     return true;
30 }
31 #define sqr(a) ((a)*(a))
32 #define feq(a,b) (fabs((a)-(b))<EPS)
33 pair<tipo, tipo> ecCuad(tipo a, tipo b, tipo c){//a*x*x+b*x+c=0
34     tipo dx = sqrt(b*b-4.0*a*c);
35     return make_pair((-b + dx)/(2.0*a), (-b - dx)/(2.0*a));
36 }
37 pair<pto, pto> interCL(Circle c, line l){
38     bool sw=false;
39     if((sw=feq(0,l.b))){
40         swap(l.a, l.b);
41         swap(c.o.x, c.o.y);
42     }
43     pair<tipo, tipo> rc = ecCuad(
44         sqr(l.a)+sqr(l.b),
45         2.0*l.a*l.b*c.o.y-2.0*(sqr(l.b)*c.o.x+l.c*l.a),
46         sqr(l.b)*(sqr(c.o.x)+sqr(c.o.y)-sqr(c.r))+sqr(l.c)-2.0*l.c*l.b*c.o.y
47     );
48     pair<pto, pto> p( pto(rc.first, (l.c - l.a * rc.first) / l.b),
49         pto(rc.second, (l.c - l.a * rc.second) / l.b) );
50     if(sw){
51         swap(p.first.x, p.first.y);
52         swap(p.second.x, p.second.y);
53     }
54     return p;
55 }
56 pair<pto, pto> interCC(Circle c1, Circle c2){

```

```

57     line l;
58     l.a = c1.o.x-c2.o.x;
59     l.b = c1.o.y-c2.o.y;
60     l.c = (sqr(c2.r)-sqr(c1.r)+sqr(c1.o.x)-sqr(c2.o.x)+sqr(c1.o.y)
61         -sqr(c2.o.y))/2.0;
62     return interCL(c1, l);
63 }

```

## 5.7. Point in Poly

```

1 //checks if v is inside of P, using ray casting
2 //works with convex and concave.
3 //excludes boundaries, handle it separately using segment.inside()
4 bool inPolygon(pto v, vector<pto>& P) {
5     bool c = false;
6     for(i, sz(P)){
7         int j=(i+1)%sz(P);
8         if((P[j].y>v.y) != (P[i].y > v.y) &&
9             (v.x < (P[i].x - P[j].x) * (v.y-P[j].y) / (P[i].y - P[j].y) + P[j].x))
10             c = !c;
11     }
12     return c;
13 }

```

## 5.8. Point in Convex Poly log(n)

```

1 void normalize(vector<pto> &pt){//delete collinear points first!
2     //this makes it clockwise:
3     if(pt[2].left(pt[0], pt[1])) reverse(pt.begin(), pt.end());
4     int n=sz(pt), pi=0;
5     for(i, n)
6         if(pt[i].x<pt[pi].x || (pt[i].x==pt[pi].x && pt[i].y<pt[pi].y))
7             pi=i;
8     vector<pto> shift(n);//puts pi as first point
9     for(i, n) shift[i]=pt[(pi+i)%n];
10    pt.swap(shift);
11 }
12 bool inPolygon(pto p, const vector<pto> &pt){
13     //call normalize first!
14     if(p.left(pt[0], pt[1]) || p.left(pt[sz(pt)-1], pt[0])) return false;
15     int a=1, b=sz(pt)-1;
16     while(b-a>1){
17         int c=(a+b)/2;
18         if(!p.left(pt[0], pt[c])) a=c;

```

```

19     else b=c;
20 }
21 return !p.left(pt[a], pt[a+1]);
22 }

```

## 5.9. Convex Check CHECK

```

1 bool isConvex(vector<int> &p){//O(N), delete collinear points!
2     int N=sz(p);
3     if(N<3) return false;
4     bool isLeft=p[0].left(p[1], p[2]);
5     forr(i, 1, N)
6         if(p[i].left(p[(i+1)%N], p[(i+2)%N])!=isLeft)
7             return false;
8     return true; }

```

## 5.10. Convex Hull

```

1 //stores convex hull of P in S, CCW order
2 //left must return >=0 to delete collinear points!
3 void CH(vector<pto>& P, vector<pto> &S){
4     S.clear();
5     sort(P.begin(), P.end());//first x, then y
6     forn(i, sz(P)){//lower hull
7         while(sz(S)>= 2 && S[sz(S)-1].left(S[sz(S)-2], P[i])) S.pop_back();
8         S.pb(P[i]);
9     }
10    S.pop_back();
11    int k=sz(S);
12    dforn(i, sz(P)){//upper hull
13        while(sz(S) >= k+2 && S[sz(S)-1].left(S[sz(S)-2], P[i])) S.pop_back
14            ();
15        S.pb(P[i]);
16    }
17    S.pop_back();
18 }

```

## 5.11. Cut Polygon

```

1 //cuts polygon Q along the line ab
2 //stores the left side (swap a, b for the right one) in P
3 void cutPolygon(pto a, pto b, vector<pto> Q, vector<pto> &P){
4     P.clear();
5     forn(i, sz(Q)){

```

```

6         double left1=(b-a)^(Q[i]-a), left2=(b-a)^(Q[(i+1)%sz(Q)]-a);
7         if(left1>=0) P.pb(Q[i]);
8         if(left1*left2<0)
9             P.pb(inter(line(Q[i], Q[(i+1)%sz(Q)]), line(a, b)));
10    }
11 }

```

## 5.12. Bresenham

```

1 //plot a line approximation in a 2d map
2 void bresenham(pto a, pto b){
3     pto d=b-a; d.x=abs(d.x), d.y=abs(d.y);
4     pto s(a.x<b.x? 1: -1, a.y<b.y? 1: -1);
5     int err=d.x-d.y;
6     while(1){
7         m[a.x][a.y]=1;//plot
8         if(a==b) break;
9         int e2=err;
10        if(e2 >= 0) err-=2*d.y, a.x+=s.x;
11        if(e2 <= 0) err+= 2*d.x, a.y+= s.y;
12    }
13 }

```

## 5.13. Interseccion de Circulos en $n^3 \log(n)$

```

1 struct event {
2     double x; int t;
3     event(double xx, int tt) : x(xx), t(tt) {}
4     bool operator <(const event &o) const { return x < o.x; }
5 };
6 typedef vector<Circle> VC;
7 typedef vector<event> VE;
8 int n;
9 double cuenta(VE &v, double A,double B) {
10    sort(v.begin(), v.end());
11    double res = 0.0, lx = ((v.empty())?0.0:v[0].x);
12    int contador = 0;
13    forn(i,sz(v)) {
14        //interseccion de todos (contador == n), union de todos (
15            contador > 0)
16        //conjunto de puntos cubierto por exacta k Circulos (contador ==
17            k)
18        if (contador == n) res += v[i].x - lx;
19        contador += v[i].t, lx = v[i].x;

```

```

18     }
19     return res;
20 }
21 // Primitiva de sqrt(r*r - x*x) como funcion double de una variable x.
22 inline double primitiva(double x,double r) {
23     if (x >= r) return r*r*M_PI/4.0;
24     if (x <= -r) return -r*r*M_PI/4.0;
25     double raiz = sqrt(r*r-x*x);
26     return 0.5 * (x * raiz + r*r*atan(x/raiz));
27 }
28 double interCircle(VC &v) {
29     vector<double> p; p.reserve(v.size() * (v.size() + 2));
30     forn(i,sz(v)) p.push_back(v[i].c.x + v[i].r), p.push_back(v[i].c.x
31         - v[i].r);
32     forn(i,sz(v)) forn(j,i) {
33         Circle &a = v[i], b = v[j];
34         double d = (a.c - b.c).norm();
35         if (fabs(a.r - b.r) < d && d < a.r + b.r) {
36             double alfa = acos((sqr(a.r) + sqr(d) - sqr(b.r)) / (2.0 * d
37                 * a.r));
38             pto vec = (b.c - a.c) * (a.r / d);
39             p.pb((a.c + rotate(vec, alfa)).x), p.pb((a.c + rotate(vec, -
40                 alfa)).x);
41         }
42     }
43     sort(p.begin(), p.end());
44     double res = 0.0;
45     forn(i,sz(p)-1) {
46         const double A = p[i], B = p[i+1];
47         VE ve; ve.reserve(2 * v.size());
48         forn(j,sz(v)) {
49             const Circle &c = v[j];
50             double arco = primitiva(B-c.c.x,c.r) - primitiva(A-c.c.x,c.r
51                 );
52             double base = c.c.y * (B-A);
53             ve.push_back(event(base + arco,-1));
54             ve.push_back(event(base - arco, 1));
55         }
56         res += cuenta(ve,A,B);
57     }
58     return res;
59 }

```

## 6. Math

### 6.1. Identidades

$$\sum_{i=0}^n i \binom{n}{i} = n * 2^{n-1}$$

$$\sum_{i=0}^n i(i-1) = \frac{8}{6} \left(\frac{n}{2}\right) \left(\frac{n}{2} + 1\right) (n+1) \text{ (doubles)} \rightarrow \text{Sino ver caso impar y par}$$

$$\sum_{i=0}^n i^4 = \frac{n(n+1)(2n+1)(3n^2+3n-1)}{30} = \frac{n^5}{5} + \frac{n^4}{2} + \frac{n^3}{3} - \frac{n}{30}$$

$$\sum_{i=0}^n i^p = \frac{(n+1)^{p+1}}{p+1} + \sum_{k=1}^p \frac{B_k}{p-k+1} \binom{p}{k} (n+1)^{p-k+1}$$

$$r = e - v + k + 1$$

Teorema de Pick: (Area, puntos interiores y puntos en el borde)

$$A = I + \frac{B}{2} - 1$$

$$\left\{ \begin{matrix} n+1 \\ k \end{matrix} \right\} = k \left\{ \begin{matrix} n \\ k \end{matrix} \right\} + \left\{ \begin{matrix} n \\ k-1 \end{matrix} \right\} \text{ for } k > 0 \text{ with initial conditions } \left\{ \begin{matrix} 0 \\ 0 \end{matrix} \right\} = 1 \quad \text{and} \quad \left\{ \begin{matrix} n \\ 0 \end{matrix} \right\} =$$

$$\left\{ \begin{matrix} 0 \\ n \end{matrix} \right\} = 0 \text{ for } n > 0. \text{ Same as } \frac{1}{k!} \sum_{j=0}^k (-1)^{k-j} \binom{k}{j} j^n$$

$$\left[ \begin{matrix} n+1 \\ k \end{matrix} \right] = n \left[ \begin{matrix} n \\ k \end{matrix} \right] + \left[ \begin{matrix} n \\ k-1 \end{matrix} \right] \text{ for } k > 0, \text{ with the initial conditions } \left[ \begin{matrix} 0 \\ 0 \end{matrix} \right] = 1 \quad \text{and} \quad \left[ \begin{matrix} n \\ 0 \end{matrix} \right] =$$

$$\left[ \begin{matrix} 0 \\ n \end{matrix} \right] = 0 \text{ for } n > 0.$$

### 6.2. Ec. Caracteristica

$$a_0 T(n) + a_1 T(n-1) + \dots + a_k T(n-k) = 0$$

$$p(x) = a_0 x^k + a_1 x^{k-1} + \dots + a_k$$

Sean  $r_1, r_2, \dots, r_q$  las raíces distintas, de mult.  $m_1, m_2, \dots, m_q$

$$T(n) = \sum_{i=1}^q \sum_{j=0}^{m_i-1} c_{ij} n^j r_i^n$$

Las constantes  $c_{ij}$  se determinan por los casos base.

### 6.3. Combinatorio

```

1  forn(i, MAXN+1){ //comb[i][k]=i tomados de a k
2      comb[i][0]=comb[i][i]=1;
3      forr(k, 1, i) comb[i][k]=(comb[i-1][k]+comb[i-1][k-1])%MOD;
4  }
5  ll lucas (ll n, ll k, int p){ //Calcula (n,k)%p teniendo comb[p][p]
6      //precalculado.
7      ll aux = 1;
8      while (n + k) aux = (aux * comb[n%p][k%p]) %p, n/=p, k/=p;
9      return aux;
10 }

```

### 6.4. Gauss Jordan, Determinante $O(n^3)$

```

1  // Gauss-Jordan elimination with full pivoting.
2  //

```

```

3 // Uses:
4 // (1) solving systems of linear equations (AX=B)
5 // (2) inverting matrices (AX=I)
6 // (3) computing determinants of square matrices
7 //
8 // Running time:  $O(n^3)$ 
9 //
10 // INPUT:   a[] [] = an nxn matrix
11 //          b[] [] = an nxm matrix
12 //
13 // OUTPUT:  X      = an nxm matrix (stored in b[] [])
14 //          A^{-1} = an nxn matrix (stored in a[] [])
15 //          returns determinant of a[] []
16
17 #include <iostream>
18 #include <vector>
19 #include <cmath>
20
21 using namespace std;
22
23 const double EPS = 1e-10;
24
25 typedef vector<int> VI;
26 typedef double T;
27 typedef vector<T> VT;
28 typedef vector<VT> VVT;
29
30 T GaussJordan(VVT &a, VVT &b) {
31     const int n = a.size();
32     const int m = b[0].size();
33     VI irow(n), icol(n), ipiv(n);
34     T det = 1;
35
36     for (int i = 0; i < n; i++) {
37         int pj = -1, pk = -1;
38         for (int j = 0; j < n; j++) if (!ipiv[j])
39             for (int k = 0; k < n; k++) if (!ipiv[k])
40                 if (fabs(a[j][k]) > fabs(a[pj][pk])) { pj = j; pk = k; }
41         if (fabs(a[pj][pk]) < EPS) { cerr << "Matrix is singular." << endl;
42             exit(0); }
43         ipiv[pj]++;
44         swap(a[pj], a[pk]);
45         swap(b[pj], b[pk]);

```

```

45         if (pj != pk) det *= -1;
46         irow[i] = pj;
47         icol[i] = pk;
48
49         T c = 1.0 / a[pk][pk];
50         det *= a[pk][pk];
51         a[pk][pk] = 1.0;
52         for (int p = 0; p < n; p++) a[pk][p] *= c;
53         for (int p = 0; p < m; p++) b[pk][p] *= c;
54         for (int p = 0; p < n; p++) if (p != pk) {
55             c = a[p][pk];
56             a[p][pk] = 0;
57             for (int q = 0; q < n; q++) a[p][q] -= a[pk][q] * c;
58             for (int q = 0; q < m; q++) b[p][q] -= b[pk][q] * c;
59         }
60     }
61
62     for (int p = n-1; p >= 0; p--) if (irow[p] != icol[p]) {
63         for (int k = 0; k < n; k++) swap(a[k][irow[p]], a[k][icol[p]]);
64     }
65
66     return det;
67 }
68
69 int main() {
70     const int n = 4;
71     const int m = 2;
72     double A[n][n] = { {1,2,3,4},{1,0,1,0},{5,3,2,4},{6,1,4,6} };
73     double B[n][m] = { {1,2},{4,3},{5,6},{8,7} };
74     VVT a(n), b(n);
75     for (int i = 0; i < n; i++) {
76         a[i] = VT(A[i], A[i] + n);
77         b[i] = VT(B[i], B[i] + m);
78     }
79
80     double det = GaussJordan(a, b);
81
82     // expected: 60
83     cout << "Determinant: " << det << endl;
84
85     // expected: -0.233333 0.166667 0.133333 0.066667
86     //              0.166667 0.166667 0.333333 -0.333333
87     //              0.233333 0.833333 -0.133333 -0.066667

```



```

88 //          0.05 -0.75 -0.1 0.2
89 cout << "Inverse:␣" << endl;
90 for (int i = 0; i < n; i++) {
91     for (int j = 0; j < n; j++)
92         cout << a[i][j] << '␣';
93     cout << endl;
94 }
95
96 // expected: 1.63333 1.3
97 //          -0.166667 0.5
98 //          2.36667 1.7
99 //          -1.85 -1.35
100 cout << "Solution:␣" << endl;
101 for (int i = 0; i < n; i++) {
102     for (int j = 0; j < m; j++)
103         cout << b[i][j] << '␣';
104     cout << endl;
105 }
106 }

```

## 6.5. Teorema Chino del Resto

```

1 // Chinese remainder theorem (special case): find z such that
2 // z % m1 = r1, z % m2 = r2. Here, z is unique modulo M = lcm(m1, m2).
3 // Return (z, M). On failure, M = -1.
4 PII chinese_remainder_theorem(int m1, int r1, int m2, int r2) {
5     int s, t;
6     int g = extended_euclid(m1, m2, s, t);
7     if (r1 % g != r2 % g) return make_pair(0, -1);
8     return make_pair(mod(s*r2*m1 + t*r1*m2, m1*m2) / g, m1*m2 / g);
9 }
10
11 // Chinese remainder theorem: find z such that
12 // z % m[i] = r[i] for all i. Note that the solution is
13 // unique modulo M = lcm_i (m[i]). Return (z, M). On
14 // failure, M = -1. Note that we do not require the a[i]'s
15 // to be relatively prime.
16 PII chinese_remainder_theorem(const VI &m, const VI &r) {
17     PII ret = make_pair(r[0], m[0]);
18     for (int i = 1; i < m.size(); i++) {
19         ret = chinese_remainder_theorem(ret.second, ret.first, m[i], r[i]);
20         if (ret.second == -1) break;
21     }

```

```

22     return ret;
23 }

```

## 6.6. Funciones de primos

Iterar mientras el  $p^2 \leq N$ . Revisar que  $N! = 1$ , en este caso  $N$  es primo. **NumDiv**: Producto (exponentes+1). **SumDiv**: Product suma geom. factores. **EulerPhi** (coprimos): Inicia  $\text{ans} = N$ . Para cada primo divisor:  $\text{ans} = \text{ans} / \text{primo}$  (una vez) y dividir luego  $N$  todo lo posible por  $p$ .

## 6.7. Phollard's Rho (rolando)

```

1 ll gcd(ll a, ll b){return a?gcd(b %a, a):b;}
2
3 ll mulmod (ll a, ll b, ll c) { //returns (a*b)%c, and minimize overflow
4     ll x = 0, y = a%c;
5     while (b > 0){
6         if (b % 2 == 1) x = (x+y) % c;
7         y = (y*2) % c;
8         b /= 2;
9     }
10    return x % c;
11 }
12
13 ll expmod (ll b, ll e, ll m){//O(log b)
14     if(!e) return 1;
15     ll q = expmod(b,e/2,m); q=mulmod(q,q,m);
16     return e%2? mulmod(b,q,m) : q;
17 }
18
19 bool es_primo_prob (ll n, int a)
20 {
21     if (n == a) return true;
22     ll s = 0, d = n-1;
23     while (d % 2 == 0) s++, d/=2;
24
25     ll x = expmod(a,d,n);
26     if ((x == 1) || (x+1 == n)) return true;
27
28     for (i, s-1){
29         x = mulmod(x, x, n);
30         if (x == 1) return false;
31         if (x+1 == n) return true;

```



```

32 }
33 return false;
34 }
35
36 bool rabin (ll n){ //devuelve true si n es primo
37     if (n == 1) return false;
38     const int ar[] = {2,3,5,7,11,13,17,19,23};
39     forn (j,9)
40         if (!es_primo_prob(n,ar[j]))
41             return false;
42     return true;
43 }
44
45 ll rho(ll n){
46     if( (n & 1) == 0 ) return 2;
47     ll x = 2 , y = 2 , d = 1;
48     ll c = rand() % n + 1;
49     while( d == 1 ){
50         x = (mulmod( x , x , n ) + c)%n;
51         y = (mulmod( y , y , n ) + c)%n;
52         y = (mulmod( y , y , n ) + c)%n;
53         if( x - y >= 0 ) d = gcd( x - y , n );
54         else d = gcd( y - x , n );
55     }
56     return d==n? rho(n):d;
57 }
58
59 map<ll,ll> prim;
60 void factRho (ll n){ //0 (lg n)^3. un solo numero
61     if (n == 1) return;
62     if (rabin(n)){
63         prim[n]++;
64         return;
65     }
66     ll factor = rho(n);
67     factRho(factor);
68     factRho(n/factor);
69 }

```

## 6.8. GCD

```

1 | tipo gcd(tipo a, tipo b){return a?gcd(b %a, a):b;}

```

## 6.9. Extended Euclid

```

1 void extendedEuclid (ll a, ll b){ //a * x + b * y = d
2     if (!b) { x = 1; y = 0; d = a; return;}
3     extendedEuclid (b, a%b);
4     ll x1 = y;
5     ll y1 = x - (a/b) * y;
6     x = x1; y = y1;
7 }

```

## 6.10. Polinomio

```

1     int m = sz(c), n = sz(o.c);
2     vector<tipo> res(max(m,n));
3     forn(i, m) res[i] += c[i];
4     forn(i, n) res[i] += o.c[i];
5     return poly(res);    }
6 poly operator*(const tipo cons) const {
7     vector<tipo> res(sz(c));
8     forn(i, sz(c)) res[i]=c[i]*cons;
9     return poly(res);    }
10 poly operator*(const poly &o) const {
11     int m = sz(c), n = sz(o.c);
12     vector<tipo> res(m+n-1);
13     forn(i, m) forn(j, n) res[i+j]+=c[i]*o.c[j];
14     return poly(res);    }
15 tipo eval(tipo v) {
16     tipo sum = 0;
17     dforn(i, sz(c)) sum=sum*v + c[i];
18     return sum; }
19 //poly contains only a vector<int> c (the coefficients)
20 //the following function generates the roots of the polynomial
21 //it can be easily modified to return float roots
22 set<tipo> roots(){
23     set<tipo> roots;
24     tipo a0 = abs(c[0]), an = abs(c[sz(c)-1]);
25     vector<tipo> ps,qs;
26     forr(p,1,sqrt(a0)+1) if (a0%p==0) ps.pb(p),ps.pb(a0/p);
27     forr(q,1,sqrt(an)+1) if (an%q==0) qs.pb(q),qs.pb(an/q);
28     forall(pt,ps)
29         forall(qt,qs) if ( (*pt) % (*qt)==0 ) {
30             tipo root = abs((*pt) / (*qt));
31             if (eval(root)==0) roots.insert(root);

```

```

32     }
33     return roots; }
34 };
35 pair<poly, tipo> ruffini(const poly p, tipo r) {
36     int n = sz(p.c) - 1;
37     vector<tipo> b(n);
38     b[n-1] = p.c[n];
39     dforn(k,n-1) b[k] = p.c[k+1] + r*b[k+1];
40     tipo resto = p.c[0] + r*b[0];
41     poly result(b);
42     return make_pair(result, resto);
43 }
44 poly interpolate(const vector<tipo>& x, const vector<tipo>& y) {
45     poly A; A.c.pb(1);
46     forn(i, sz(x)) { poly aux; aux.c.pb(-x[i]), aux.c.pb(1), A = A * aux;
47     }
48     poly S; S.c.pb(0);
49     forn(i, sz(x)) { poly Li;
50         Li = ruffini(A, x[i]).fst;
51         Li = Li * (1.0 / Li.eval(x[i])); // here put a multiple of the
52         // coefficients instead of 1.0 to avoid using double
53         S = S + Li * y[i]; }
54     return S;
55 }
56 int main(){
57     return 0;
58 }

```

## 6.11. FFT

```

1 //~ typedef complex<double> base; //menos codigo, pero mas lento
2 //elegir si usar complejos de c (lento) o estos
3 struct base{
4     double r,i;
5     base(double r=0, double i=0):r(r), i(i){}
6     double real()const{return r;}
7     void operator/=(const int c){r/=c, i/=c;}
8 };
9 base operator*(const base &a, const base &b){
10     return base(a.r*b.r-a.i*b.i, a.r*b.i+a.i*b.r);}
11 base operator+(const base &a, const base &b){
12     return base(a.r+b.r, a.i+b.i);}

```

```

13 base operator-(const base &a, const base &b){
14     return base(a.r-b.r, a.i-b.i);}
15 vector<int> rev; vector<base> wlen_pw;
16 inline static void fft(base a[], int n, bool invert) {
17     forn(i, n) if(i<rev[i]) swap(a[i], a[rev[i]]);
18     for (int len=2; len<=n; len<=1) {
19         double ang = 2*M_PI/len * (invert?-1:+1);
20         int len2 = len>>1;
21         base wlen (cos(ang), sin(ang));
22         wlen_pw[0] = base (1, 0);
23         forr(i, 1, len2) wlen_pw[i] = wlen_pw[i-1] * wlen;
24         for (int i=0; i<n; i+=len) {
25             base t, *pu = a+i, *pv = a+i+len2, *pu_end = a+i+len2, *pw = &
26                 wlen_pw[0];
27             for (; pu!=pu_end; ++pu, ++pv, ++pw)
28                 t = *pv * *pw, *pv = *pu - t, *pu = *pu + t;
29         }
30     }
31     if (invert) forn(i, n) a[i]/= n;}
32 inline static void calc_rev(int n){//precalculo: llamar antes de fft!!
33     wlen_pw.resize(n), rev.resize(n);
34     int lg=31-__builtin_clz(n);
35     forn(i, n){
36         rev[i] = 0;
37         forn(k, lg) if(i&(1<<k)) rev[i]|=1<<(lg-1-k);}
38 inline static void multiply(const vector<int> &a, const vector<int> &b,
39     vector<int> &res) {
40     vector<base> fa (a.begin(), a.end()), fb (b.begin(), b.end());
41     int n=1; while(n < max(sz(a), sz(b))) n <= 1; n <= 1;
42     calc_rev(n);
43     fa.resize (n), fb.resize (n);
44     fft (&fa[0], n, false), fft (&fb[0], n, false);
45     forn(i, n) fa[i] = fa[i] * fb[i];
46     fft (&fa[0], n, true);
47     res.resize(n);
48     forn(i, n) res[i] = int (fa[i].real() + 0.5); }
49 void toPoly(const string &s, vector<int> &P){//convierte un numero a
50     //polinomio
51     P.clear();
52     dforn(i, sz(s)) P.pb(s[i]-'0');}

```

## 7. Grafos

### 7.1. Bellman-Ford

```

1 vector<ii> G[MAX_N]; //ady. list with pairs (weight, dst)
2 int dist[MAX_N];
3 void bford(int src){ //O(VE)
4     dist[src]=0;
5     forn(i, N-1) forn(j, N) if(dist[j]!=INF) forall(it, G[j])
6         dist[it->snd]=min(dist[it->snd], dist[j]+it->fst);
7 }
8
9 bool hasNegCycle(){
10     forn(j, N) if(dist[j]!=INF) forall(it, G[j])
11         if(dist[it->snd]>dist[j]+it->fst) return true;
12     //inside if: all points reachable from it->snd will have -INF distance
13     //      (do bfs)
14     return false;
15 }

```

### 7.2. 2-SAT + Tarjan SCC

```

1 //We have a vertex representing a var and other for his negation.
2 //Every edge stored in G represents an implication. To add an equation
3 //of the form a||b, use addor(a, b)
4 //MAX=max cant var, n=cant var
5 #define addor(a, b) (G[neg(a)].pb(b), G[neg(b)].pb(a))
6 vector<int> G[MAX*2];
7 //idx[i]=index assigned in the dfs
8 //lw[i]=lowest index(closer from the root) reachable from i
9 int lw[MAX*2], idx[MAX*2], qidx;
10 stack<int> q;
11 int qcmp, cmp[MAX*2];
12 //verdad[cmp[i]]=valor de la variable i
13 bool verdad[MAX*2+1];
14
15 int neg(int x) { return x>=n? x-n : x+n;}
16 void tjn(int v){
17     lw[v]=idx[v]=++qidx;
18     q.push(v), cmp[v]=-2;
19     forall(it, G[v]){
20         if(!idx[*it] || cmp[*it]==-2){
21             if(!idx[*it]) tjn(*it);

```

```

21         lw[v]=min(lw[v], lw[*it]);
22     }
23 }
24 if(lw[v]==idx[v]){
25     int x;
26     do{x=q.top(); q.pop(); cmp[x]=qcmp;}while(x!=v);
27     verdad[qcmp]=(cmp[neg(v)]<0);
28     qcmp++;
29 }
30 }
31 //remember to CLEAR G!!!
32 bool satisf(){ //O(n)
33     memset(idx, 0, sizeof(idx)), qidx=0;
34     memset(cmp, -1, sizeof(cmp)), qcmp=0;
35     forn(i, n){
36         if(!idx[i]) tjn(i);
37         if(!idx[neg(i)]) tjn(neg(i));
38     }
39     forn(i, n) if(cmp[i]==cmp[neg(i)]) return false;
40     return true;
41 }

```

### 7.3. Puentes y Articulation Points

```

1 int dfsNumberCounter, dfsRoot, rootChildren;
2 vi dfs_num, dfs_low, dfs_parent, articulation_vertex;
3
4 void articulationPointAndBridge(int u) {
5     dfs_low[u] = dfs_num[u] = dfsNumberCounter++;
6     for (int j = 0; j < (int)AdjList[u].size(); j++) {
7         ii v = AdjList[u][j];
8         if (dfs_num[v.first] == -1) {
9             dfs_parent[v.first] = u;
10            if (u == dfsRoot) rootChildren++;
11            articulationPointAndBridge(v.first);
12            if (dfs_low[v.first] >= dfs_num[u])
13                articulation_vertex[u] = true;
14            if (dfs_low[v.first] > dfs_num[u])
15                printf("Edge (%d, %d) is a bridge\n", u, v.first);
16            dfs_low[u] = min(dfs_low[u], dfs_low[v.first]);
17        }
18        else if (v.first != dfs_parent[u])
19            dfs_low[u] = min(dfs_low[u], dfs_num[v.first]);

```

```

20 } }
21 // At main
22 dfsNumberCounter = 0; dfs_num.assign(V, -1); dfs_low.assign(V, 0);
23 dfs_parent.assign(V, -1); articulation_vertex.assign(V, 0);
24 printf("Bridges:\n");
25 for (int i = 0; i < V; i++)
26     if (dfs_num[i] == -1) {
27         dfsRoot = i; rootChildren = 0;
28         articulationPointAndBridge(i);
29         articulation_vertex[dfsRoot] = (rootChildren > 1); }
30 printf("Articulation Points:\n");
31 for (int i = 0; i < V; i++)
32     if (articulation_vertex[i])
33         printf("Vertex %d\n", i);

```

## 7.4. LCA + Climb

```

1 const int MAXN=100001;
2 const int LOGN=20;
3 //f[v][k] holds the 2^k father of v
4 //L[v] holds the level of v
5 int N, f[MAXN][LOGN], L[MAXN];
6 //call before build:
7 void dfs(int v, int fa=-1, int lvl=0){//generate required data
8     f[v][0]=fa, L[v]=lvl;
9     forall(it, G[v])if(*it!=fa) dfs(*it, v, lvl+1); }
10 void build(){//f[i][0] must be filled previously, 0(nlgn)
11     forn(k, LOGN-1) forn(i, N) f[i][k+1]=f[f[i][k]][k];}
12 #define lg(x) (31-__builtin_clz(x))//=floor(log2(x))
13 int climb(int a, int d){//0(lgn)
14     if(!d) return a;
15     dforn(i, lg(L[a])+1) if(1<=i<=d) a=f[a][i], d-=1<=i;
16     return a;}
17 int lca(int a, int b){//0(lgn)
18     if(L[a]<L[b]) swap(a, b);
19     a=climb(a, L[a]-L[b]);
20     if(a==b) return a;
21     dforn(i, lg(L[a])+1) if(f[a][i]!=f[b][i]) a=f[a][i], b=f[b][i];
22     return f[a][0]; }
23 int dist(int a, int b) {//returns distance between nodes
24     return L[a]+L[b]-2*L[lca(a, b)];}

```

## 7.5. Heavy Light Decomposition

```

1 int treesz[MAXN];//cantidad de nodos en el subarbol del nodo v
2 int dad[MAXN];//dad[v]=padre del nodo v
3 void dfs1(int v, int p=-1){//pre-dfs
4     dad[v]=p;
5     treesz[v]=1;
6     forall(it, G[v]) if(*it!=p){
7         dfs1(*it, v);
8         treesz[v]+=treesz[*it];
9     }
10 }
11 //PONER Q EN 0 !!!!
12 int pos[MAXN], q;//pos[v]=posicion del nodo v en el recorrido de la dfs
13 //Las cadenas aparecen continuas en el recorrido!
14 int canticad;
15 int homecad[MAXN];//dada una cadena devuelve su nodo inicial
16 int cad[MAXN];//cad[v]=cadena a la que pertenece el nodo
17 void heavylight(int v, int cur=-1){
18     if(cur==-1) homecad[cur=canticad++]=v;
19     pos[v]=q++;
20     cad[v]=cur;
21     int mx=-1;
22     forn(i, sz(G[v])) if(G[v][i]!=dad[v])
23         if(mx==-1 || treesz[G[v][mx]]<treesz[G[v][i]]) mx=i;
24     if(mx!=-1) heavylight(G[v][mx], cur);
25     forn(i, sz(G[v])) if(i!=mx && G[v][i]!=dad[v])
26         heavylight(G[v][i], -1);
27 }
28 //ejemplo de obtener el maximo numero en el camino entre dos nodos
29 //RTA: max(query(low, u), query(low, v)), con low=lca(u, v)
30 //esta funcion va trepando por las cadenas
31 int query(int an, int v){//0(logn)
32     //si estan en la misma cadena:
33     if(cad[an]==cad[v]) return rmq.get(pos[an], pos[v]+1);
34     return max(query(an, dad[homecad[cad[v]]]),
35               rmq.get(pos[homecad[cad[v]]], pos[v]+1));
36 }

```

## 7.6. Centroid Decomposition

```

1 int n;
2 vector<int> G[MAXN];
3 bool taken[MAXN];//poner todos en FALSE al principio!!
4 int padre[MAXN];//padre de cada nodo en el centroid tree

```

```

5
6 int szt[MAXN];
7 void calcsz(int v, int p) {
8     szt[v] = 1;
9     forall(it,G[v]) if (*it!=p && !taken[*it])
10         calcsz(*it,v), szt[v]+=szt[*it];
11 }
12 void centroid(int v=0, int f=-1, int lvl=0, int tam=-1) { //O(nlogn)
13     if(tam==-1) calcsz(v, -1), tam=szt[v];
14     forall(it, G[v]) if(!taken[*it] && szt[*it]>=tam/2)
15         {szt[v]=0; centroid(*it, f, lvl, tam); return;}
16     taken[v]=true;
17     padre[v]=f;
18     forall(it, G[v]) if(!taken[*it])
19         centroid(*it, v, lvl+1, -1);
20 }

```

## 7.7. Euler Cycle

```

1 int n,m,ars[MAXE], eq;
2 vector<int> G[MAXN]; //fill G,n,m,ars,eq
3 list<int> path;
4 int used[MAXN];
5 bool usede[MAXE];
6 queue<list<int>::iterator> q;
7 int get(int v){
8     while(used[v]<sz(G[v]) && usede[ G[v][used[v]] ]) used[v]++;
9     return used[v];
10 }
11 void explore(int v, int r, list<int>::iterator it){
12     int ar=G[v][get(v)]; int u=v^ars[ar];
13     usede[ar]=true;
14     list<int>::iterator it2=path.insert(it, u);
15     if(u!=r) explore(u, r, it2);
16     if(get(v)<sz(G[v])) q.push(it);
17 }
18 void euler(){
19     zero(used), zero(usede);
20     path.clear();
21     q=queue<list<int>::iterator>();
22     path.push_back(0); q.push(path.begin());
23     while(sz(q)){
24         list<int>::iterator it=q.front(); q.pop();

```

```

25         if(used[*it]<sz(G[*it])) explore(*it, *it, it);
26     }
27     reverse(path.begin(), path.end());
28 }
29 void addEdge(int u, int v){
30     G[u].pb(eq), G[v].pb(eq);
31     ars[eq++]=u^v;
32 }

```

## 7.8. Chu-liu

```

1 void visit(graph &h, int v, int s, int r,
2     vector<int> &no, vector< vector<int> > &comp,
3     vector<int> &prev, vector< vector<int> > &next, vector<weight> &mcost,
4     vector<int> &mark, weight &cost, bool &found) {
5     if (mark[v]) {
6         vector<int> temp = no;
7         found = true;
8         do {
9             cost += mcost[v];
10            v = prev[v];
11            if (v != s) {
12                while (comp[v].size() > 0) {
13                    no[comp[v].back()] = s;
14                    comp[s].push_back(comp[v].back());
15                    comp[v].pop_back();
16                }
17            }
18        } while (v != s);
19        forall(j,comp[s]) if (*j != r) forall(e,h[*j])
20            if (no[e->src] != s) e->w -= mcost[ temp[*j] ];
21    }
22    mark[v] = true;
23    forall(i,next[v]) if (no[*i] != no[v] && prev[no[*i]] == v)
24        if (!mark[no[*i]] || *i == s)
25            visit(h, *i, s, r, no, comp, prev, next, mcost, mark, cost, found)
26        ;
27 }
28 weight minimumSpanningArborescence(const graph &g, int r) {
29     const int n=sz(g);
30     graph h(n);
31     forn(u,n) forall(e,g[u]) h[e->dst].pb(*e);
32     vector<int> no(n);

```

```

32 vector<vector<int> > comp(n);
33 forn(u, n) comp[u].pb(no[u] = u);
34 for (weight cost = 0; ;) {
35     vector<int> prev(n, -1);
36     vector<weight> mcost(n, INF);
37     forn(j,n) if (j != r) forall(e,h[j])
38         if (no[e->src] != no[j])
39             if (e->w < mcost[ no[j] ])
40                 mcost[ no[j] ] = e->w, prev[ no[j] ] = no[e->src];
41     vector< vector<int> > next(n);
42     forn(u,n) if (prev[u] >= 0)
43         next[ prev[u] ].push_back(u);
44     bool stop = true;
45     vector<int> mark(n);
46     forn(u,n) if (u != r && !mark[u] && !comp[u].empty()) {
47         bool found = false;
48         visit(h, u, u, r, no, comp, prev, next, mcost, mark, cost, found);
49         if (found) stop = false;
50     }
51     if (stop) {
52         forn(u,n) if (prev[u] >= 0) cost += mcost[u];
53         return cost;
54     }
55 }
56 }

```

## 7.9. Hungarian

```

1 //Dado un grafo bipartito completo con costos no negativos, encuentra el
  matching perfecto de minimo costo.
2 tipo cost[N][N], lx[N], ly[N], slack[N]; //llenar: cost=matriz de
  adyacencia
3 int n, max_match, xy[N], yx[N], slackx[N],prev2[N]; //n=cantidad de nodos
4 bool S[N], T[N]; //sets S and T in algorithm
5 void add_to_tree(int x, int prevx) {
6     S[x] = true, prev2[x] = prevx;
7     forn(y, n) if (lx[x] + ly[y] - cost[x][y] < slack[y] - EPS)
8         slack[y] = lx[x] + ly[y] - cost[x][y], slackx[y] = x;
9 }
10 void update_labels(){
11     tipo delta = INF;
12     forn (y, n) if (!T[y]) delta = min(delta, slack[y]);
13     forn (x, n) if (S[x]) lx[x] -= delta;

```

```

14     forn (y, n) if (T[y]) ly[y] += delta; else slack[y] -= delta;
15 }
16 void init_labels(){
17     zero(lx), zero(ly);
18     forn (x,n) forn(y,n) lx[x] = max(lx[x], cost[x][y]);
19 }
20 void augment() {
21     if (max_match == n) return;
22     int x, y, root, q[N], wr = 0, rd = 0;
23     memset(S, false, sizeof(S)), memset(T, false, sizeof(T));
24     memset(prev2, -1, sizeof(prev2));
25     forn (x, n) if (xy[x] == -1){
26         q[wr++] = root = x, prev2[x] = -2;
27         S[x] = true; break; }
28     forn (y, n) slack[y] = lx[root] + ly[y] - cost[root][y], slackx[y] =
        root;
29     while (true){
30         while (rd < wr){
31             x = q[rd++];
32             for (y = 0; y < n; y++) if (cost[x][y] == lx[x] + ly[y] && !T[y]){
33                 if (yx[y] == -1) break; T[y] = true;
34                 q[wr++] = yx[y], add_to_tree(yx[y], x); }
35             if (y < n) break; }
36         if (y < n) break;
37         update_labels(), wr = rd = 0;
38         for (y = 0; y < n; y++) if (!T[y] && slack[y] == 0){
39             if (yx[y] == -1){x = slackx[y]; break;}
40             else{
41                 T[y] = true;
42                 if (!S[yx[y]]) q[wr++] = yx[y], add_to_tree(yx[y], slackx[y]);
43             }
44             if (y < n) break; }
45         if (y < n){
46             max_match++;
47             for (int cx = x, cy = y, ty; cx != -2; cx = prev2[cx], cy = ty)
48                 ty = xy[cx], yx[cy] = cx, xy[cx] = cy;
49             augment(); }
50     }
51     tipo hungarian(){
52         tipo ret = 0; max_match = 0, memset(xy, -1, sizeof(xy));
53         memset(yx, -1, sizeof(yx)), init_labels(), augment(); //steps 1-3
54         forn (x,n) ret += cost[x][xy[x]]; return ret;
55     }

```

## 7.10. Dynamic Connectivity

```

1 struct UnionFind {
2     int n, comp;
3     vector<int> pre, si, c;
4     UnionFind(int n=0):n(n), comp(n), pre(n), si(n, 1) {
5         forn(i,n) pre[i] = i; }
6     int find(int u){return u==pre[u]?u:find(pre[u]);}
7     bool merge(int u, int v) {
8         if((u=find(u))==v) return false;
9         if(si[u]<si[v]) swap(u, v);
10        si[u]+=si[v], pre[v]=u, comp--, c.pb(v);
11        return true;
12    }
13    int snap(){return sz(c);}
14    void rollback(int snap){
15        while(sz(c)>snap){
16            int v = c.back(); c.pop_back();
17            si[pre[v]] -= si[v], pre[v] = v, comp++;
18        }
19    }
20 };
21 enum {ADD,DEL,QUERY};
22 struct Query {int type,u,v};
23 struct DynCon {
24     vector<Query> q;
25     UnionFind dsu;
26     vector<int> match,res;
27     map<ii,int> last; //se puede no usar cuando hay identificador para
                        //cada arista (mejora poco)
28     DynCon(int n=0):dsu(n){}
29     void add(int u, int v) {
30         if(u>v) swap(u,v);
31         q.pb((Query){ADD, u, v}), match.pb(-1);
32         last[ii(u,v)] = sz(q)-1;
33     }
34     void remove(int u, int v) {
35         if(u>v) swap(u,v);
36         q.pb((Query){DEL, u, v});
37         int prev = last[ii(u,v)];
38         match[prev] = sz(q)-1;
39         match.pb(prev);
40     }

```

```

41     void query() { //podria pasarle un puntero donde guardar la respuesta
42         q.pb((Query){QUERY, -1, -1}), match.pb(-1);}
43     void process() {
44         forn(i,sz(q)) if (q[i].type == ADD && match[i] == -1) match[i] =
45             sz(q);
46         go(0,sz(q));
47     }
48     void go(int l, int r) {
49         if(l+1==r){
50             if (q[l].type == QUERY) //Aqui responder la query usando el
51                 dsu!
52                 res.pb(dsu.comp); //aqui query=cantidad de componentes
53                 conexas
54             return;
55         }
56         int s=dsu.snap(), m = (l+r) / 2;
57         forr(i,m,r) if(match[i]!=-1 && match[i]<l) dsu.merge(q[i].u, q[i]
58             ].v);
59         go(l,m);
60         dsu.rollback(s);
61         s = dsu.snap();
62         forr(i,l,m) if(match[i]!=-1 && match[i]>=r) dsu.merge(q[i].u, q[
63             i].v);
64         go(m,r);
65         dsu.rollback(s);
66     }
67 }dc;

```

## 8. Network Flow

## 8.1. Dinic

```

1
2 const int MAX = 300;
3 // Corte minimo: vertices con dist[v]>=0 (del lado de src) VS. dist[v]
4 // ==-1 (del lado del dst)
5 // Para el caso de la red de Bipartite Matching (Sean V1 y V2 los
6 // conjuntos mas proximos a src y dst respectivamente):
7 // Reconstruir matching: para todo v1 en V1 ver las aristas a vertices
8 // de V2 con it->f>0, es arista del Matching
9 // Min Vertex Cover: vertices de V1 con dist[v]==-1 + vertices de V2 con
10 // dist[v]>0
11 // Max Independent Set: tomar los vertices NO tomados por el Min Vertex

```



```

    Cover
8 // Max Clique: construir la red de G complemento (debe ser bipartito!) y
    encontrar un Max Independent Set
9 // Min Edge Cover: tomar las aristas del matching + para todo vertices
    no cubierto hasta el momento, tomar cualquier arista de el
10 int nodes, src, dst;
11 int dist[MAX], q[MAX], work[MAX];
12 struct Edge {
13     int to, rev;
14     ll f, cap;
15     Edge(int to, int rev, ll f, ll cap) : to(to), rev(rev), f(f), cap(
        cap) {}
16 };
17 vector<Edge> G[MAX];
18 void addEdge(int s, int t, ll cap){
19     G[s].pb(Edge(t, sz(G[t]), 0, cap)), G[t].pb(Edge(s, sz(G[s])-1, 0,
        0));}
20 bool dinic_bfs(){
21     fill(dist, dist+nodes, -1), dist[src]=0;
22     int qt=0; q[qt++]=src;
23     for(int qh=0; qh<qt; qh++){
24         int u = q[qh];
25         forall(e, G[u]){
26             int v=e->to;
27             if(dist[v]<0 && e->f < e->cap)
28                 dist[v]=dist[u]+1, q[qt++]=v;
29         }
30     }
31     return dist[dst]>=0;
32 }
33 ll dinic_dfs(int u, ll f){
34     if(u==dst) return f;
35     for(int &i=work[u]; i<sz(G[u]); i++){
36         Edge &e = G[u][i];
37         if(e.cap<=e.f) continue;
38         int v=e.to;
39         if(dist[v]==dist[u]+1){
40             ll df=dinic_dfs(v, min(f, e.cap-e.f));
41             if(df>0){
42                 e.f+=df, G[v][e.rev].f-= df;
43                 return df; }
44         }
45     }

```

```

46     return 0;
47 }
48 ll maxFlow(int _src, int _dst){
49     src=_src, dst=_dst;
50     ll result=0;
51     while(dinic_bfs()){
52         fill(work, work+nodes, 0);
53         while(ll delta=dinic_dfs(src, INF))
54             result+=delta;
55     }
56     // todos los nodos con dist[v]!=-1 vs los que tienen dist[v]==-1
    forman el min-cut
57     return result; }

```

## 8.2. Edmonds Karp's

```

1 #define MAX_V 1000
2 #define INF 1e9
3 //special nodes
4 #define SRC 0
5 #define SNK 1
6 map<int, int> G[MAX_V]; //limpiar esto
7 //To add an edge use
8 #define add(a, b, w) G[a][b]=w
9 int f, p[MAX_V];
10 void augment(int v, int minE){
11     if(v==SRC) f=minE;
12     else if(p[v]!=-1){
13         augment(p[v], min(minE, G[p[v]][v]));
14         G[p[v]][v]-=f, G[v][p[v]]+=f;
15     }
16 }
17 ll maxflow(){//O(VE^2)
18     ll Mf=0;
19     do{
20         f=0;
21         char used[MAX_V]; queue<int> q; q.push(SRC);
22         zero(used), memset(p, -1, sizeof(p));
23         while(sz(q)){
24             int u=q.front(); q.pop();
25             if(u==SNK) break;
26             forall(it, G[u])
27                 if(it->snd>0 && !used[it->fst])

```



```

28     used[it->fst]=true, q.push(it->fst), p[it->fst]=u;
29 }
30 augment(SNK, INF);
31 Mf+=f;
32 }while(f);
33 return Mf;
34 }

```

### 8.3. Max Matching

```

1  int LEFT, r[MAXV]; bool seen[MAXV]; VI AdjList[MAXV];
2  bool can_match(int u) {
3      for (auto & v : AdjList[u]) {
4          if (!seen[v]) {
5              seen[v] = true;
6              if (r[v] < 0 || can_match(r[v])) {
7                  r[v] = u; return true;
8              }
9          }
10     } return false;
11 }
12 int max_matching() {
13     memset(r, -1, sizeof r);
14     int ans = 0;
15     for (int u=0 ; u<LEFT ; u++) {
16         memset(seen, 0, sizeof seen);
17         if (can_match(u)) ans++;
18     } return ans;
19 }

```

### 8.4. Min-cost Max-flow

```

1  const int MAXN=10000;
2  typedef ll tf;
3  typedef ll tc;
4  const tf INFFLUJO = 1e14;
5  const tc INFCOSTO = 1e14;
6  struct edge {
7      int u, v;
8      tf cap, flow;
9      tc cost;
10     tf rem() { return cap - flow; }
11 };
12 int nodes; //numero de nodos

```

```

13 vector<int> G[MAXN]; // limpiar!
14 vector<edge> e; // limpiar!
15 void addEdge(int u, int v, tf cap, tc cost) {
16     G[u].pb(sz(e)); e.pb((edge){u,v,cap,0,cost});
17     G[v].pb(sz(e)); e.pb((edge){v,u,0,0,-cost});
18 }
19 tc dist[MAXN], mnCost;
20 int pre[MAXN];
21 tf cap[MAXN], mxFlow;
22 bool in_queue[MAXN];
23 void flow(int s, int t) {
24     zero(in_queue);
25     mxFlow=mnCost=0;
26     while(1){
27         fill(dist, dist+nodes, INFCOSTO); dist[s] = 0;
28         memset(pre, -1, sizeof(pre)); pre[s]=0;
29         zero(cap); cap[s] = INFFLUJO;
30         queue<int> q; q.push(s); in_queue[s]=1;
31         while(sz(q)){
32             int u=q.front(); q.pop(); in_queue[u]=0;
33             for(auto it:G[u]) {
34                 edge &E = e[it];
35                 if(E.rem() && dist[E.v] > dist[u] + E.cost + 1e-9){ // ojo EPS
36                     dist[E.v]=dist[u]+E.cost;
37                     pre[E.v] = it;
38                     cap[E.v] = min(cap[u], E.rem());
39                     if(!in_queue[E.v]) q.push(E.v), in_queue[E.v]=1;
40                 }
41             }
42         }
43         if (pre[t] == -1) break;
44         mxFlow +=cap[t];
45         mnCost +=cap[t]*dist[t];
46         for (int v = t; v != s; v = e[pre[v]].u) {
47             e[pre[v]].flow += cap[t];
48             e[pre[v]^1].flow -= cap[t];
49         }
50     }
51 }

```

## 9. Template y Otros

### Template

```

1 //touch {a..m}.in; tee {a..m}.cpp < template.cpp
2 #include <bits/stdc++.h>
3 using namespace std;
4 #define forr(i,a,b) for(int i=(a); i<(b); i++)
5 #define forn(i,n) forr(i,0,n)
6 #define sz(c) ((int)c.size())
7 #define zero(v) memset(v, 0, sizeof(v))
8 #define forall(it,v) for(auto it=v.begin();it!=v.end();++it)
9 #define pb push_back
10 #define fst first
11 #define snd second
12 typedef long long ll;
13 typedef pair<int,int> ii;
14 #define dforr(i,n) for(int i=n-1; i>=0; i--)
15 #define dprint(v) cout << #v<"< << v << endl //;)
16
17 const int MAXN=100100;
18 int n;
19
20 int main() {
21     freopen("input.in", "r", stdin);
22     ios::sync_with_stdio(0);
23     while(cin >> n){
24
25     }
26     return 0;
27 }
```

### Rellenar con espacios(para justificar)

```

1 #include <iomanip>
2 cout << setfill('␣') << setw(3) << 2 << endl;
```

### Aleatorios

```

1 #define RAND(a, b) (rand()%(b-a+1)+a)
2 srand(time(NULL));
```

### Doubles Comp.

```

1 const double EPS = 1e-9;
2 x == y  <=> fabs(x-y) < EPS, x > y  <=> x > y + EPS
3 x >= y  <=> x > y - EPS
```

### Expandir pila

```

1 #include <sys/resource.h>
2 rlimit rl;
3 getrlimit(RLIMIT_STACK, &rl);
4 rl.rlim_cur=1024L*1024L*256L;//256mb
5 setrlimit(RLIMIT_STACK, &rl);
```

### Iterar subconjunto

```

1 for(int sbm=bm; sbm; sbm=(sbm-1)&bm)
```