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# 1. algorithm

#include <algorithm> #include <numeric>

Algo	Params	Funcion
sort, stable_sort	f, 1	ordena el intervalo
nth_element	f, nth, l	void ordena el n-esimo, y
		particiona el resto
fill, fill_n	f, l / n, elem	void llena [f, l) o [f,
		f+n) con elem
lower_bound, upper_bound	f, l, elem	it al primer / ultimo donde se
		puede insertar elem para que
		quede ordenada
binary_search	f, l, elem	bool esta elem en [f, l)
copy	f, l, resul	hace resul+ $i$ =f+ $i$ $\forall i$
find, find_if, find_first_of	f, l, elem	$it$ encuentra i $\in$ [f,l) tq. i=elem,
	/ pred / f2, l2	$\operatorname{pred}(i), i \in [f2, l2)$
count, count_if	f, l, elem/pred	cuenta elem, pred(i)
search	f, l, f2, l2	busca $[f2,l2) \in [f,l)$
replace, replace_if	f, l, old	cambia old / pred(i) por new
	/ pred, new	
reverse	f, l	da vuelta
partition, stable_partition	f, l, pred	pred(i) ad, !pred(i) atras
min_element, max_element	f, l, [comp]	it min, max de [f,l]
lexicographical_compare	f1,l1,f2,l2	bool con [f1,l1];[f2,l2]
next/prev_permutation	f,l	deja en [f,l) la perm sig, ant
$set\_intersection,$	f1, l1, f2, l2, res	[res,) la op. de conj
set_difference, set_union,		
set_symmetric_difference,		
push_heap, pop_heap,	f, l, e / e /	mete/saca e en heap [f,l),
make_heap		hace un heap de [f,l)
is_heap	f,l	bool es [f,l) un heap
accumulate	f,l,i,[op]	$T = \sum /\text{oper de [f,l)}$
inner_product	f1, l1, f2, i	$T = i + [f1, 11) \cdot [f2, \dots)$
partial_sum	f, l, r, [op]	$r+i = \sum /oper de [f,f+i] \forall i \in [f,l)$
builtin_ffs	unsigned int	Pos. del primer 1 desde la derecha
_builtin_clz	unsigned int	Cant. de ceros desde la izquierda.
_builtin_ctz	unsigned int	Cant. de ceros desde la derecha.
_builtin_popcount	unsigned int	Cant. de 1's en x.
_builtin_parity	unsigned int	1 si x es par, 0 si es impar.
builtin_XXXXXXIl	unsigned ll	= pero para long long's.

## 2. Estructuras

# 2.1. RMQ (static)

Dado un arreglo y una operacion asociativa *idempotente*, get(i, j) opera sobre el rango [i, j). Restriccion: LVL ≥ ceil(logn); Usar [] para llenar arreglo y luego build().

```
1 | struct RMQ{
     #define LVL 10
2
     tipo vec[LVL] [1<<(LVL+1)];
     tipo &operator[](int p){return vec[0][p];}
     tipo get(int i, int j) {//intervalo [i,j)
5
       int p = 31-_builtin_clz(j-i);
6
       return min(vec[p][i],vec[p][j-(1<<p)]);</pre>
7
     }
8
     void build(int n) {//O(nlogn)
9
       int mp = 31-__builtin_clz(n);
10
       forn(p, mp) forn(x, n-(1<<p))
11
         vec[p+1][x] = min(vec[p][x], vec[p][x+(1<<p)]);
12
     }};
13
```

# 2.2. RMQ (dynamic)

```
1 //Dado un arreglo y una operacion asociativa con neutro, get(i, j) opera
        sobre el rango [i, j).
   #define MAXN 100000
   #define operacion(x, y) max(x, y)
   const int neutro=0;
   struct RMQ{
     int sz;
6
     tipo t[4*MAXN];
7
     tipo &operator[](int p){return t[sz+p];}
8
     void init(int n){//O(nlgn)
9
       sz = 1 \ll (32-\_builtin\_clz(n));
10
       forn(i, 2*sz) t[i]=neutro;
11
12
     void updall(){\frac{}{0}}
13
       dforn(i, sz) t[i]=operacion(t[2*i], t[2*i+1]);}
14
     tipo get(int i, int j){return get(i,j,1,0,sz);}
15
     tipo get(int i, int j, int n, int a, int b){\frac{1}{0}}
16
       if(j<=a || i>=b) return neutro;
17
       if(i<=a && b<=j) return t[n];
18
       int c=(a+b)/2;
19
```

```
return operacion(get(i, j, 2*n, a, c), get(i, j, 2*n+1, c, b));
20
21
     void set(int p, tipo val){//O(lgn)
22
       for(p+=sz; p>0 && t[p]!=val;){
23
         t[p]=val;
24
         p/=2;
25
         val=operacion(t[p*2], t[p*2+1]);
26
27
     }
28
   }rma;
   //Usage:
31 | cin >> n; rmg.init(n); forn(i, n) cin >> rmg[i]; rmg.updall();
                            2.3. RMQ (lazy)
1 //Dado un arreglo y una operacion asociativa con neutro, get(i, j) opera
        sobre el rango [i, j).
   typedef int Elem; //Elem de los elementos del arreglo
   typedef int Alt;//Elem de la alteracion
   #define operacion(x,y) x+y
   const Elem neutro=0; const Alt neutro2=0;
   #define MAXN 100000
   struct RMQ{
     int sz:
     Elem t[4*MAXN]:
     Alt dirty[4*MAXN];//las alteraciones pueden ser de distinto Elem
10
     Elem &operator[](int p){return t[sz+p];}
11
     void init(int n){//O(nlgn)
12
       sz = 1 \ll (32-\_builtin\_clz(n));
13
       forn(i, 2*sz) t[i]=neutro;
14
       forn(i, 2*sz) dirty[i]=neutro2;
15
16
     void push(int n, int a, int b){//propaga el dirty a sus hijos
17
       if(dirty[n]!=0){
18
         t[n]+=dirty[n]*(b-a);//altera el nodo
19
         if(n<sz){
20
           dirty[2*n]+=dirty[n];
21
           dirty[2*n+1]+=dirty[n];
22
23
         dirty[n]=0;
24
25
26
     }
     Elem get(int i, int j, int n, int a, int b){\frac{1}{0}}
```

```
if(j<=a || i>=b) return neutro;
28
       push(n, a, b);//corrige el valor antes de usarlo
29
       if(i<=a && b<=j) return t[n];</pre>
30
       int c=(a+b)/2;
31
       return operacion(get(i, j, 2*n, a, c), get(i, j, 2*n+1, c, b));
32
33
     Elem get(int i, int j){return get(i,j,1,0,sz);}
34
     //altera los valores en [i, j) con una alteración de val
35
     void alterar(Alt val, int i, int j, int n, int a, int b)\frac{1}{0}
36
       push(n, a, b);
37
       if(j<=a || i>=b) return;
38
       if(i<=a && b<=j){
39
         dirty[n]+=val;
40
         push(n, a, b);
41
         return;
42
       }
43
       int c=(a+b)/2:
44
       alterar(val, i, j, 2*n, a, c), alterar(val, i, j, 2*n+1, c, b);
45
       t[n]=operacion(t[2*n], t[2*n+1]);//por esto es el push de arriba
46
47
     void alterar(Alt val, int i, int j){alterar(val,i,j,1,0,sz);}
48
49 |}rmq;
```

#### 2.4. RMQ (persistente)

```
typedef int tipo;
   tipo oper(const tipo &a, const tipo &b){
       return a+b;
3
4
  struct node{
5
     tipo v; node *1,*r;
6
     node(tipo v):v(v), 1(NULL), r(NULL) {}
7
       node(node *1, node *r) : 1(1), r(r){
8
           if(!1) v=r->v;
9
           else if(!r) v=l->v;
10
           else v=oper(1->v, r->v);
11
       }
12
   };
13
   node *build (tipo *a, int tl, int tr) {//modificar para que tome tipo a
     if (tl+1==tr) return new node(a[tl]);
15
     int tm=(tl + tr)>>1:
16
     return new node(build(a, tl, tm), build(a, tm, tr));
17
18 }
```

```
node *update(int pos, int new_val, node *t, int tl, int tr){
     if (tl+1==tr) return new node(new_val);
20
     int tm=(tl+tr)>>1;
21
     if(pos < tm) return new node(update(pos, new_val, t->1, tl, tm), t->r)
22
     else return new node(t->1, update(pos, new_val, t->r, tm, tr));
23
24
   tipo get(int 1, int r, node *t, int tl, int tr){
25
       if(l==tl && tr==r) return t->v;
26
     int tm=(tl + tr)>>1;
27
       if(r<=tm) return get(1, r, t->1, t1, tm);
28
       else if(1>=tm) return get(1, r, t->r, tm, tr);
29
    return oper(get(1, tm, t->1, t1, tm), get(tm, r, t->r, tm, tr));
31 }
```

#### 2.5. Fenwick Tree

```
1 //For 2D threat each column as a Fenwick tree, by adding a nested for in
        each operation
2 struct Fenwick{
     static const int sz=1000001;
     tipo t[sz];
     void adjust(int p, tipo v){//valid with p in [1, sz), O(lgn)
       for(int i=p; i<sz; i+=(i&-i)) t[i]+=v; }</pre>
6
     tipo sum(int p){//cumulative sum in [1, p], O(lgn)
7
       tipo s=0:
8
       for(int i=p; i; i-=(i&-i)) s+=t[i];
       return s;
10
11
     tipo sum(int a, int b){return sum(b)-sum(a-1);}
12
     //get largest value with cumulative sum less than or equal to x;
13
     //for smallest, pass x-1 and add 1 to result
14
     int getind(tipo x) {//O(lgn)
15
         int idx = 0, mask = N;
16
         while(mask && idx < N) {</pre>
17
           int t = idx + mask;
18
         if(x >= tree[t])
19
             idx = t, x -= tree[t];
20
           mask >>= 1:
21
22
         return idx:
23
    }};
24
```

}

23

#### 2.6. Union Find

```
struct UnionFind{
    vector<int> f;//the array contains the parent of each node
2
    void init(int n){f.clear(); f.insert(f.begin(), n, -1);}
3
    int comp(int x){return (f[x]=-1?x:f[x]=comp(f[x]));}//0(1)
4
    bool join(int i, int j) {
5
      bool con=comp(i)==comp(j);
6
      if(!con) f[comp(i)] = comp(j);
      return con;
8
    }};
9
```

## 2.7. Disjoint Intervals

```
| bool operator< (const ii &a, const ii &b) {return a.fst<b.fst;}
   //Stores intervals as [first, second]
   //in case of a collision it joins them in a single interval
   struct disjoint_intervals {
     set<ii>> segs;
5
     void insert(ii v) {//O(lgn)
6
       if(v.snd-v.fst==0.) return;//0J0
       set<ii>>::iterator it,at;
8
       at = it = segs.lower_bound(v);
9
       if (at!=segs.begin() && (--at)->snd >= v.fst)
10
         v.fst = at->fst, --it;
11
       for(; it!=segs.end() && it->fst <= v.snd; segs.erase(it++))</pre>
12
         v.snd=max(v.snd, it->snd);
13
       segs.insert(v);
14
15
<sub>16</sub> | };
```

## 2.8. RMQ (2D)

```
struct RMQ2D{//n filas x m columnas
     int sz;
2
     RMQ t[4*MAXN];
3
     RMQ &operator[](int p){return t[sz/2+p];}//t[i][j]=i fila, j col
4
     void init(int n, int m){\frac{}{\frac{1}{0}(n*m)}}
5
       sz = 1 \ll (32-\_builtin\_clz(n));
6
       forn(i, 2*sz) t[i].init(m); }
7
     void set(int i, int j, tipo val){//O(lgm.lgn)
       for(i+=sz; i>0;){
9
         t[i].set(j, val);
10
         i/=2;
11
```

```
val=operacion(t[i*2][j], t[i*2+1][j]);
12
       } }
13
     tipo get(int i1, int j1, int i2, int j2){return get(i1, j1, i2, j2, 1, 0,
14
     //O(lgm.lgn), rangos cerrado abierto
15
     int get(int i1, int j1, int i2, int j2, int n, int a, int b){
16
       if(i2<=a || i1>=b) return 0;
17
       if(i1<=a && b<=i2) return t[n].get(j1, j2);</pre>
18
       int c=(a+b)/2;
19
       return operacion(get(i1, j1, i2, j2, 2*n, a, c),
            get(i1, j1, i2, j2, 2*n+1, c, b));
21
     }
22
   } rma:
23
   //Example to initialize a grid of M rows and N columns:
   RMQ2D rmq; rmq.init(n,m);
   forn(i, n) forn(j, m){
    int v; cin >> v; rmq.set(i, j, v);}
                               2.9. Big Int
 1 #define BASEXP 6
   #define BASE 1000000
   #define LMAX 1000
   struct bint{
       int 1;
5
       11 n[LMAX];
6
       bint(11 x=0){
7
           1=1;
8
           forn(i, LMAX){
9
                if (x) l=i+1;
10
                n[i]=x %BASE;
11
                x/=BASE;
12
13
           }
14
15
       bint(string x){
16
       l=(x.size()-1)/BASEXP+1;
17
           fill(n, n+LMAX, 0);
18
           ll r=1;
19
           forn(i, sz(x)){
20
               n[i / BASEXP] += r * (x[x.size()-1-i]-'0');
21
                r*=10; if(r==BASE)r=1;
22
```

```
}
24
       void out(){
25
       cout << n[1-1];
26
       dforn(i, l-1) printf("%6.6llu", n[i]);//6=BASEXP!
27
28
     void invar(){
29
       fill(n+l, n+LMAX, 0);
30
       while(1>1 && !n[1-1]) 1--;
31
    }
32
33
   bint operator+(const bint&a, const bint&b){
34
     bint c;
35
       c.1 = max(a.1, b.1):
36
       11 q = 0;
37
       forn(i, c.1) q += a.n[i]+b.n[i], c.n[i]=q %BASE, q/=BASE;
38
       if(q) c.n[c.l++] = q;
39
       c.invar();
40
       return c;
41
42
   pair<bint, bool> lresta(const bint& a, const bint& b) // c = a - b
44
     bint c;
45
       c.1 = max(a.1, b.1);
46
       11 q = 0;
47
       forn(i, c.l) q += a.n[i]-b.n[i], c.n[i]=(q+BASE) %BASE, q=(q+BASE)/
48
           BASE-1;
       c.invar();
49
       return make_pair(c, !q);
50
51
   bint& operator-= (bint& a, const bint& b){return a=lresta(a, b).first;}
   bint operator- (const bint&a, const bint&b) {return lresta(a, b).first;}
   bool operator< (const bint&a, const bint&b){return !lresta(a, b).second
       :}
   bool operator<= (const bint&a, const bint&b){return lresta(b, a).second
       :}
   bool operator==(const bint&a, const bint&b){return a <= b && b <= a;}
   bint operator*(const bint&a, 11 b){
57
       bint c:
58
59
       forn(i, a.1) q += a.n[i]*b, c.n[i] = q \text{BASE}, q/=BASE;
60
61
       while(q) c.n[c.l++] = q %BASE, q/=BASE;
62
       c.invar();
63
```

```
return c;
64
   }
65
   bint operator*(const bint&a, const bint&b){
66
        bint c;
        c.1 = a.1+b.1;
       fill(c.n, c.n+b.1, 0);
       forn(i, a.1){
           11 q = 0;
            forn(j, b.1) q += a.n[i]*b.n[j]+c.n[i+j], c.n[i+j] = q BASE, q
                /=BASE;
            c.n[i+b.1] = q;
73
74
       c.invar();
75
       return c;
77
    pair<br/>
\frac{1}{c} = a / b; rm = a % b
     bint c:
     11 \text{ rm} = 0;
     dforn(i, a.1){
                rm = rm * BASE + a.n[i];
                c.n[i] = rm / b;
83
                rm %= b;
84
       }
       c.1 = a.1;
        c.invar();
       return make_pair(c, rm);
89
    bint operator/(const bint&a, ll b){return ldiv(a, b).first;}
    11 operator%(const bint&a, 11 b){return ldiv(a, b).second;}
    pair<bint, bint> ldiv(const bint& a, const bint& b){
     bint c;
93
       bint rm = 0;
94
        dforn(i, a.l){
95
            if (rm.l==1 && !rm.n[0])
96
                rm.n[0] = a.n[i];
97
            elsef
98
                dforn(j, rm.l) rm.n[j+1] = rm.n[j];
99
                rm.n[0] = a.n[i]:
100
                rm.l++;
101
102
            ll q = rm.n[b.1] * BASE + rm.n[b.1-1];
103
            ll u = q / (b.n[b.l-1] + 1);
104
           ll v = q / b.n[b.l-1] + 1;
105
```

```
while (u < v-1)
106
                11 m = (u+v)/2:
107
                if (b*m \le rm) u = m;
108
                else v = m;
109
           }
110
           c.n[i]=u;
111
           rm-=b*u;
112
       }
113
     c.l=a.l;
114
       c.invar();
115
       return make_pair(c, rm);
116
117
   bint operator/(const bint&a, const bint&b) {return ldiv(a, b).first;}
   bint operator %(const bint&a, const bint&b) {return ldiv(a, b).second;}
                            2.10. HashTables
   //Compilar: g++ --std=c++11
   struct Hash{
 2
     size_t operator()(const ii &a)const{
       size_t s=hash<int>()(a.fst);
 4
       return hash<int>()(a.snd)+0x9e3779b9+(s<<6)+(s>>2);
 5
 6
     size_t operator()(const vector<int> &v)const{
       size_t s=0;
 8
       for(auto &e : v)
 9
         s = hash<int>()(e)+0x9e3779b9+(s<<6)+(s>>2);
10
       return s;
11
     }
12
13
    unordered_set<ii, Hash> s;
   unordered_map<ii, int, Hash> m;//map<key, value, hasher>
                              2.11. Modnum
   struct mnum{
     static const tipo mod=12582917;
 2
 3
     mnum(tipo v=0): v(v mod) {}
     mnum operator+(mnum b){return v+b.v;}
 5
     mnum operator-(mnum b){return v>=b.v? v-b.v : mod-b.v+v;}
     mnum operator*(mnum b){return v*b.v;}
     mnum operator^(int n){
```

if(!n) return 1;

9

```
return n \% ? (*this)^(n/2) * (this) : (*this)^(n/2);}
11 };
                           2.12. Treap para set
typedef int Key;
   typedef struct node *pnode;
   struct node{
       Key key;
       int prior, size;
       pnode l,r;
       node(Key key=0): key(key), prior(rand()), size(1), 1(0), r(0) {}
7
   };
8
   static int size(pnode p) { return p ? p->size : 0; }
   void push(pnode p) {
     // modificar v propagar el dirty a los hijos aca(para lazy)
11
12
   // Update function and size from children's Value
   void pull(pnode p) {//recalcular valor del nodo aca (para rmg)
     p->size = 1 + size(p->1) + size(p->r);
16
   //junta dos arreglos
   pnode merge(pnode 1, pnode r) {
     if (!1 || !r) return 1 ? 1 : r;
     push(1), push(r);
     pnode t;
21
     if (1->prior < r->prior) 1->r=merge(1->r, r), t = 1;
     else r\rightarrow l=merge(1, r\rightarrow 1), t = r;
     pull(t);
24
     return t;
25
26
   //parte el arreglo en dos, l<key<=r
   void split(pnode t, Key key, pnode &1, pnode &r) {
       if (!t) return void(1 = r = 0);
29
       push(t);
30
       if (\text{key} \leftarrow \text{t->key}) split(\text{t->l}, \text{key}, l, \text{t->l}), r = t;
31
       else split(t->r, key, t->r, r), l = t;
32
       pull(t):
33
   }
34
35
   void erase(pnode &t, Key key) {
       if (!t) return;
37
       push(t);
38
```

```
if (key == t->key) t=merge(t->1, t->r);
                                                                                         int prior, size;
39
                                                                                   5
       else if (key < t->key) erase(t->1, key);
                                                                                         pnode l,r,parent;
                                                                                   6
40
                                                                                         node(Value val): val(val), mini(val), dirty(0), prior(rand()), size
       else erase(t->r, key);
41
       if(t) pull(t);
                                                                                              (1), 1(0), r(0), parent(0) {}
42
                                                                                     };
                                                                                   8
43
                                                                                     static int size(pnode p) { return p ? p->size : 0; }
44
                                                                                     void push(pnode p) {//propagar dirty a los hijos(aca para lazy)
   ostream& operator<<(ostream &out, const pnode &t) {
45
     if(!t) return out;
                                                                                       p->val.fst+=p->dirty;
46
       return out << t->l << t->key << ''_' << t->r;
                                                                                       p->mini.fst+=p->dirty;
47
                                                                                  12
                                                                                       if(p->1) p->1->dirty+=p->dirty;
48
   pnode find(pnode t, Key key) {
                                                                                       if(p->r) p->r->dirty+=p->dirty;
                                                                                  14
       if (!t) return 0;
                                                                                       p->dirty=0;
                                                                                  15
50
       if (key == t->key) return t;
                                                                                  16
                                                                                     }
51
       if (key < t->key) return find(t->1, key);
                                                                                     static Value mini(pnode p) { return p ? push(p), p->mini : ii(1e9, -1);
52
       return find(t->r, key);
53
                                                                                     // Update function and size from children's Value
54
                                                                                     void pull(pnode p) {//recalcular valor del nodo aca (para rmq)
   struct treap {
55
                                                                                       p->size = 1 + size(p->1) + size(p->r);
       pnode root;
56
       treap(pnode root=0): root(root) {}
                                                                                       p->mini = min(min(p->val, mini(p->l)), mini(p->r));//operacion del rmq
57
       int size() { return ::size(root); }
58
       void insert(Key key) {
                                                                                       p->parent=0;
                                                                                  22
59
                                                                                       if(p->1) p->1->parent=p;
           pnode t1, t2; split(root, key, t1, t2);
60
           t1=::merge(t1,new node(key));
                                                                                       if(p->r) p->r->parent=p;
                                                                                  24
61
           root=::merge(t1,t2);
                                                                                  25
62
       }
                                                                                     //junta dos arreglos
                                                                                  26
63
                                                                                     pnode merge(pnode 1, pnode r) {
       void erase(Key key1, Key key2) {
64
           pnode t1,t2,t3;
                                                                                       if (!1 || !r) return 1 ? 1 : r;
65
                                                                                       push(1), push(r);
           split(root,key1,t1,t2);
                                                                                  29
66
           split(t2,key2, t2, t3);
                                                                                       pnode t;
                                                                                  30
67
           root=merge(t1,t3);
                                                                                       if (1->prior < r->prior) 1->r=merge(1->r, r), t = 1;
68
                                                                                  31
                                                                                       else r\rightarrow l=merge(1, r\rightarrow 1), t = r;
                                                                                  32
69
       void erase(Key key) {::erase(root, key);}
                                                                                       pull(t);
70
       pnode find(Key key) { return ::find(root, key); }
                                                                                       return t:
                                                                                  34
71
       Key &operator[](int pos){return find(pos)->key;}//ojito
                                                                                  35
72
  };
                                                                                     //parte el arreglo en dos, sz(l)==tam
73
                                                                                  36
treap merge(treap a, treap b) {return treap(merge(a.root, b.root));}
                                                                                     void split(pnode t, int tam, pnode &1, pnode &r) {
                                                                                       if (!t) return void(1 = r = 0);
                                                                                  38
                       2.13. Treap para arreglo
                                                                                       push(t):
                                                                                  39
                                                                                       if (tam \le size(t->1)) split(t->1, tam, 1, t->1), r = t;
                                                                                       else split(t->r, tam - 1 - size(t->l), t->r, r), l = t;
  typedef struct node *pnode;
                                                                                       pull(t);
  struct node{
                                                                                  ^{42}
       Value val, mini;
                                                                                  43
3
                                                                                  pnode at(pnode t, int pos) {
       int dirty;
4
```

```
if(!t) exit(1);
45
     push(t);
46
     if(pos == size(t->1)) return t;
47
     if(pos < size(t->1)) return at(t->1, pos);
48
     return at(t->r, pos - 1 - size(t->1));
49
50
   int getpos(pnode t){//inversa de at
     if(!t->parent) return size(t->1);
52
     if(t==t->parent->l) return getpos(t->parent)-size(t->r)-1;
53
     return getpos(t->parent)+size(t->1)+1;
54
55
   void split(pnode t, int i, int j, pnode &l, pnode &m, pnode &r) {
     split(t, i, l, t), split(t, j-i, m, r);}
   Value get(pnode &p, int i, int j){//like rmq
     pnode 1,m,r;
59
       split(p, i, j, l, m, r);
60
       Value ret=mini(m);
61
       p=merge(1, merge(m, r));
62
       return ret:
63
64
   void print(const pnode &t) {//for debugging
65
     if(!t) return;
66
       push(t);
67
       print(t->1);
68
       cout << t->val.fst << '';
69
       print(t->r);
70
71 }
```

#### 2.14. Convex Hull Trick

```
struct Line{tipo m,h;};
  tipo inter(Line a, Line b){
       tipo x=b.h-a.h, y=a.m-b.m;
       return x/y+(x\%?!((x>0)^(y>0)):0);//==ceil(x/y)
4
5
   struct CHT {
6
     vector<Line> c;
     bool mx:
8
     int pos;
9
     CHT(bool mx=0):mx(mx),pos(0){}//mx=1 si las query devuelven el max
     inline Line acc(int i){return c[c[0].m>c.back().m? i : sz(c)-1-i];}
11
     inline bool irre(Line x, Line y, Line z){
12
       return c[0].m>z.m? inter(y, z) <= inter(x, y)
13
```

```
: inter(y, z) >= inter(x, y);
14
15
     void add(tipo m, tipo h) {//O(1), los m tienen que entrar ordenados
16
           if (mx) m*=-1, h*=-1;
17
       Line l=(Line){m, h};
18
           if(sz(c) && m==c.back().m) { 1.h=min(h, c.back().h), c.pop_back
19
                (); if(pos) pos--; }
           while(sz(c) \ge 2 \&\& irre(c[sz(c)-2], c[sz(c)-1], 1)) { c.pop_back
20
                (); if(pos) pos--; }
           c.pb(1);
21
22
     inline bool fbin(tipo x, int m) {return inter(acc(m), acc(m+1))>x:}
23
     tipo eval(tipo x){
24
       int n = sz(c);
25
       //query con x no ordenados O(lgn)
26
       int a=-1, b=n-1;
27
       while(b-a>1) { int m = (a+b)/2;
         if(fbin(x, m)) b=m;
         else a=m:
30
       return (acc(b).m*x+acc(b).h)*(mx?-1:1);
32
           //query 0(1)
       while(pos>0 && fbin(x, pos-1)) pos--;
34
       while(pos<n-1 && !fbin(x, pos)) pos++;
       return (acc(pos).m*x+acc(pos).h)*(mx?-1:1);
36
37
38 } ch;
```

### 2.15. Convex Hull Trick (Dynamic)

```
const ll is_query = -(1LL<<62);</pre>
  struct Line {
       ll m, b;
       mutable multiset<Line>::iterator it;
4
       const Line *succ(multiset<Line>::iterator it) const;
5
       bool operator<(const Line& rhs) const {</pre>
6
            if (rhs.b != is_query) return m < rhs.m;</pre>
7
            const Line *s=succ(it):
8
           if(!s) return 0;
           11 x = rhs.m:
10
            return b - s->b < (s->m - m) * x:
11
       }
12
13 };
```

```
14 struct HullDynamic : public multiset<Line>{ // will maintain upper hull
       for maximum
       bool bad(iterator y) {
15
           iterator z = next(y);
16
           if (v == begin()) {
17
               if (z == end()) return 0;
18
               return y->m == z->m && y->b <= z->b;
19
20
           iterator x = prev(y);
21
           if (z == end()) return y->m == x->m && y->b <= x->b;
22
           return (x-b - y-b)*(z-m - y-m) >= (y-b - z-b)*(y-m - x-m)
23
               );
       }
24
       iterator next(iterator y){return ++y;}
25
       iterator prev(iterator y){return --y;}
26
       void insert_line(ll m, ll b) {
27
           iterator y = insert((Line) { m, b });
28
           y->it=y;
29
           if (bad(y)) { erase(y); return; }
30
           while (next(y) != end() && bad(next(y))) erase(next(y));
31
           while (y != begin() && bad(prev(y))) erase(prev(y));
32
       }
33
       ll eval(ll x) {
34
           Line l = *lower_bound((Line) { x, is_query });
35
           return 1.m * x + 1.b;
36
37
   }h;
38
   const Line *Line::succ(multiset<Line>::iterator it) const{
       return (++it==h.end()? NULL : &*it);}
```

#### 2.16. Gain-Cost Set

```
//esta estructura mantiene pairs(beneficio, costo)
//de tal manera que en el set quedan ordenados
//por beneficio Y COSTO creciente. (va borrando los que no son optimos)
struct V{
   int gain, cost;
   bool operator<(const V &b)const{return gain<b.gain;}
};
set<V> s;
void add(V x){
   set<V>::iterator p=s.lower_bound(x);//primer elemento mayor o igual
   if(p!=s.end() && p->cost <= x.cost) return;//ya hay uno mejor</pre>
```

```
p=s.upper_bound(x);//primer elemento mayor
     if(p!=s.begin()){//borro todos los peores (<=beneficio y >=costo)
13
       --p;//ahora es ultimo elemento menor o igual
14
       while(p->cost >= x.cost){
15
         if(p==s.begin()){s.erase(p); break;}
16
         s.erase(p--);
18
    }
19
     s.insert(x);
20
21
   int get(int gain){//minimo costo de obtener tal ganancia
22
     set<V>::iterator p=s.lower_bound((V){gain, 0});
     return p==s.end()? INF : p->cost;}
24
```

#### 2.17. Set con busq binaria

```
#include <ext/pb_ds/assoc_container.hpp>
#include <ext/pb_ds/tree_policy.hpp>
using namespace __gnu_pbds;
typedef tree<int,null_type,less<int>,//key,mapped type, comparator

rb_tree_tag,tree_order_statistics_node_update> set_t;
//find_by_order(i) devuelve iterador al i-esimo elemento
//order_of_key(k): devuelve la pos del lower bound de k
//Ej: 12, 100, 505, 1000, 10000.
//order_of_key(10) == 0, order_of_key(100) == 1,
//order_of_key(707) == 3, order_of_key(9999999) == 5
```

# 3. Algos

## 3.1. Longest Increasing Subsecuence

```
//Para non-increasing, cambiar comparaciones y revisar busq binaria
//Given an array, paint it in the least number of colors so that each
color turns to a non-increasing subsequence.
//Solution:Min number of colors=Length of the longest increasing
subsequence
int N, a[MAXN];//secuencia y su longitud
ii d[MAXN+1];//d[i]=ultimo valor de la subsecuencia de tamanio i
int p[MAXN];//padres
vector<int> R;//respuesta
void rec(int i){
if(i==-1) return;
R.push_back(a[i]);
```

```
rec(p[i]);
11
   }
^{12}
   int lis(){//O(nlogn)
13
     d[0] = ii(-INF, -1); forn(i, N) d[i+1]=ii(INF, -1);
     forn(i, N){
15
       int j = upper_bound(d, d+N+1, ii(a[i], INF))-d;
16
       if (d[j-1].first < a[i]&&a[i] < d[j].first){</pre>
17
         p[i]=d[j-1].second;
18
         d[j] = ii(a[i], i);
19
20
     }
21
     R.clear();
22
     dforn(i, N+1) if(d[i].first!=INF){
23
       rec(d[i].second)://reconstruir
24
       reverse(R.begin(), R.end());
25
       return i;//longitud
26
     }
27
     return 0;
28
29 }
                      3.2. Alpha-Beta prunning
       INF, 11 beta = INF) { //player = true -> Maximiza
```

```
1 | 11 alphabeta(State &s, bool player = true, int depth = 1e9, 11 alpha = -
       if(s.isFinal()) return s.score;
2
     //~ if (!depth) return s.heuristic();
3
       vector<State> children;
4
       s.expand(player, children);
5
       int n = children.size();
6
       forn(i, n) {
7
           ll v = alphabeta(children[i], !player, depth-1, alpha, beta);
8
           if(!player) alpha = max(alpha, v);
9
           else beta = min(beta, v);
10
           if(beta <= alpha) break;</pre>
11
12
       return !player ? alpha : beta;}
13
```

## 3.3. Mo's algorithm

```
6 int ans[MAXN], curans; //ans[i] = ans to ith query
   bool bymos(const Qu &a, const Qu &b){
       if(a.l/sq!=b.l/sq) return a.l<b.l;</pre>
       return (a.1/sq)&1? a.r<b.r : a.r>b.r;
9
   }
10
   void mos(){
       forn(i, t) qs[i].id=i;
12
       sort(qs, qs+t, bymos);
       int cl=0, cr=0;
       sq=sqrt(n);
       curans=0;
16
       forn(i, t){ //intervalos cerrado abiertos !!! importante!!
            Qu &q=qs[i];
18
            while(cl>q.1) add(--cl);
19
            while(cr<q.r) add(cr++);</pre>
20
            while(cl<q.1) remove(cl++);</pre>
21
            while(cr>q.r) remove(--cr);
22
            ans[q.id]=curans;
23
       }
24
25 }
```

#### 3.4. Binary search

```
1 | #include <functional>
   //C++11
   //Busca el minimo x (minX \leq x \leq maxX) tal que f(x) >= value
   int lowerBound(int minX, int maxX, function<int(int)> f, int value){
     int low = minX, hi = maxX+1, mid;
     while(low < hi){</pre>
6
       mid = (low+hi)/2;
       int v = f(mid);
8
       if (v >= value) hi = mid;
9
       else low = mid+1;
10
     }
11
     return low;
12
13
14
    //Busca el minimo x (minX \leq x \leq maxX) tal que f(x) > value
   int upperBound(int minX, int maxX, function<int(int)> f, int value){
     int low = minX, hi = maxX+1, mid;
17
     while(low < hi){</pre>
18
       mid = (low+hi)/2;
19
       int v = f(mid);
20
```

```
if (v <= value) low = mid+1;</pre>
21
       else hi = mid:
^{22}
     }
23
     return low;
24
25
   #define SEARCH_ERROR 1e-9
   //Busca el minimo x (minX <= x <= maxX) tal que x cumpla el predicado p(
       x) con error menor a SEARCH_ERROR
   double realLowerBound(double minX, double maxX, function<bool(double)> p
       ) {
     double lo = minX, hi = maxX, mid = 0.0, ans = 0.0;
29
     int iterations = (int)floor(log2((maxX-minX)/SEARCH_ERROR)+7); //
30
         Calcula iteraciones para cumplir con SEARCH_ERROR
     forn (i. iterations) {
31
       mid = (lo + hi) / 2.0;
32
       if (p(mid)) { ans = mid; hi = mid; }
33
       else lo = mid;
34
    }
35
     return ans;
36
37 }
```

# 4. Strings

#### 4.1. Manacher

```
int d1[MAXN];//d1[i]=long del maximo palindromo impar con centro en i
  int d2[MAXN];//d2[i]=analogo pero para longitud par
  //0 1 2 3 4
   //a a b c c <--d1[2]=3
   //a a b b <--d2[2]=2 (estan uno antes)
   void manacher(){
     int l=0, r=-1, n=sz(s);
7
     forn(i, n){
8
       int k=(i>r? 1 : min(d1[l+r-i], r-i));
9
       while(i+k<n && i-k>=0 && s[i+k]==s[i-k]) ++k;
10
       d1[i] = k--;
11
       if(i+k > r) l=i-k, r=i+k;
12
     }
13
     1=0, r=-1;
14
     forn(i, n){
15
       int k=(i>r? 0 : min(d2[1+r-i+1], r-i+1))+1;
16
       while(i+k-1<n && i-k>=0 && s[i+k-1]==s[i-k]) k++;
17
       d2[i] = --k;
18
```

```
if(i+k-1 > r) l=i-k, r=i+k-1;
20
                                 4.2. KMP
string T;//cadena donde buscar(where)
  string P;//cadena a buscar(what)
   int b[MAXLEN];//back table b[i] maximo borde de [0..i)
   void kmppre(){//by gabina with love
       int i =0, j=-1; b[0]=-1;
5
       while(i<sz(P)){</pre>
6
           while(j>=0 && P[i] != P[j]) j=b[j];
7
           i++, j++, b[i] = j;
8
       }
9
   }
10
   void kmp(){
12
       int i=0, j=0;
       while(i<sz(T)){</pre>
           while(j>=0 && T[i]!=P[j]) j=b[j];
14
           i++, j++;
15
           if(j==sz(P)) printf("P<sub>i</sub> is found at index Main T\n", i-j), j=b[j
16
                ];
       }
17
   }
18
19
   int main(){
       cout << "T=";
21
       cin >> T;
22
       cout << "P=";
23
                                  4.3. Trie
1 | struct trie{
     map<char, trie> m;
     void add(const string &s, int p=0){
       if(s[p]) m[s[p]].add(s, p+1);
4
5
     void dfs(){
6
       //Do stuff
7
       forall(it, m)
8
         it->second.dfs();
9
    }
10
11 | };
```

## 4.4. Suffix Array (largo, nlogn)

```
#define MAX_N 1000
   #define rBOUND(x) (x<n? r[x] : 0)
   //sa will hold the suffixes in order.
   int sa[MAX_N], r[MAX_N], n;
   string s; //input string, n=sz(s)
   int f[MAX_N], tmpsa[MAX_N];
   void countingSort(int k){
     zero(f):
     forn(i, n) f[rBOUND(i+k)]++;
     int sum=0:
11
     forn(i, max(255, n)){
12
       int t=f[i]; f[i]=sum; sum+=t;}
13
     forn(i, n)
14
       tmpsa[f[rBOUND(sa[i]+k)]++]=sa[i];
15
     memcpy(sa, tmpsa, sizeof(sa));
16
17
   void constructsa(){\frac{1}{0}} n log n
18
     n=sz(s);
19
     forn(i, n) sa[i]=i, r[i]=s[i];
20
     for(int k=1; k<n; k<<=1){
21
       countingSort(k), countingSort(0);
22
       int rank, tmpr[MAX_N];
23
       tmpr[sa[0]]=rank=0;
24
       forr(i, 1, n)
25
         tmpr[sa[i]]=(r[sa[i]]==r[sa[i-1]] && r[sa[i]+k]==r[sa[i-1]+k] )?
26
             rank : ++rank;
       memcpy(r, tmpr, sizeof(r));
27
       if(r[sa[n-1]]==n-1) break;
28
     }
29
30
   void print(){//for debug
31
     forn(i, n)
32
       cout << i << ''' <<
33
       s.substr(sa[i], s.find( '$', sa[i])-sa[i]) << endl;}
34
```

## 4.5. String Matching With Suffix Array

```
//returns (lowerbound, upperbound) of the search
ii stringMatching(string P){ //O(sz(P)lgn)
int lo=0, hi=n-1, mid=lo;
```

```
while(lo<hi){
4
       mid=(lo+hi)/2;
5
       int res=s.compare(sa[mid], sz(P), P);
6
       if(res>=0) hi=mid;
       else lo=mid+1;
    }
9
     if(s.compare(sa[lo], sz(P), P)!=0) return ii(-1, -1);
10
     ii ans; ans.fst=lo;
11
     lo=0, hi=n-1, mid;
12
     while(lo<hi){</pre>
       mid=(lo+hi)/2;
14
       int res=s.compare(sa[mid], sz(P), P);
15
       if(res>0) hi=mid:
16
       else lo=mid+1;
17
18
     if(s.compare(sa[hi], sz(P), P)!=0) hi--;
19
     ans.snd=hi:
21
     return ans;
22 }
                4.6. LCP (Longest Common Prefix)
 1 //Calculates the LCP between consecutives suffixes in the Suffix Array.
  //LCP[i] is the length of the LCP between sa[i] and sa[i-1]
   int LCP[MAX_N], phi[MAX_N], PLCP[MAX_N];
   void computeLCP(){//0(n)}
     phi[sa[0]]=-1;
    forr(i, 1, n) phi[sa[i]]=sa[i-1];
     int L=0;
     forn(i, n){
       if(phi[i]==-1) {PLCP[i]=0; continue;}
       while(s[i+L]==s[phi[i]+L]) L++;
       PLCP[i]=L;
       L=max(L-1, 0);
12
13
     forn(i, n) LCP[i]=PLCP[sa[i]];
14
15 }
                              4.7. Corasick
1
2
  struct trie{
     map<char, trie> next;
```

trie\* tran[256];//transiciones del automata

```
int idhoja, szhoja;//id de la hoja o 0 si no lo es
5
     //link lleva al sufijo mas largo, nxthoja lleva al mas largo pero que
6
         es hoja
     trie *padre, *link, *nxthoja;
                                                                                    4
     char pch;//caracter que conecta con padre
                                                                                       };
                                                                                    5
     trie(): tran(), idhoja(), padre(), link() {}
9
     void insert(const string &s, int id=1, int p=0){//id>0!!!
10
       if(p \le z(s)){
11
         trie &ch=next[s[p]];
12
         tran[(int)s[p]]=&ch;
13
         ch.padre=this, ch.pch=s[p];
14
         ch.insert(s, id, p+1);
15
       }
16
       else idhoja=id, szhoja=sz(s);
                                                                                         ++sz;
                                                                                   14
17
                                                                                      1
                                                                                    15
18
     trie* get_link() {
19
       if(!link){
20
         if(!padre) link=this;//es la raiz
21
         else if(!padre->padre) link=padre;//hijo de la raiz
22
         else link=padre->get_link()->get_tran(pch);
23
       }
24
       return link; }
25
     trie* get_tran(int c) {
26
       if(!tran[c]) tran[c] = !padre? this : this->get_link()->get_tran(c);
27
       return tran[c]; }
28
     trie *get_nxthoja(){
                                                                                   23
29
       if(!nxthoja) nxthoja = get_link()->idhoja? link : link->nxthoja;
                                                                                   24
30
       return nxthoja; }
31
     void print(int p){
                                                                                   25
32
       if(idhoja) cout << "found," << idhoja << ",, at, position," << p-
33
           szhoja << endl;</pre>
       if(get_nxthoja()) get_nxthoja()->print(p); }
34
     void matching(const string &s, int p=0){
                                                                                   27
35
       print(p); if(p<sz(s)) get_tran(s[p])->matching(s, p+1); }
                                                                                   28
36
   }tri:
                                                                                   29
37
                                                                                   30
38
39
                                                                                   31
   int main(){
     tri=trie();//clear
                                                                                   33
41
     tri.insert("ho", 1);
                                                                                   34
42
     tri.insert("hoho", 2);
                                                                                   35
```

#### 4.8. Suffix Automaton

```
1 struct state {
     int len, link;
     map<char,int> next;
     state() { }
   const int MAXLEN = 10010;
   state st[MAXLEN*2];
   int sz, last;
   void sa_init() {
    forn(i,sz) st[i].next.clear();
   sz = last = 0:
   st[0].len = 0;
    st[0].link = -1;
   // Es un DAG de una sola fuente y una sola hoja
17 // cantidad de endpos = cantidad de apariciones = cantidad de caminos de
        la clase al nodo terminal
18 // cantidad de miembros de la clase = st[v].len-st[st[v].link].len (v>0)
        = caminos del inicio a la clase
19 // El arbol de los suffix links es el suffix tree de la cadena invertida
       . La string de la arista link(v)->v son los caracteres que difieren
   void sa_extend (char c) {
    int cur = sz++;
     st[cur].len = st[last].len + 1;
    // en cur agregamos la posicion que estamos extendiendo
    //podria agregar tambien un identificador de las cadenas a las cuales
         pertenece (si hay varias)
     int p;
     for (p=last; p!=-1 && !st[p].next.count(c); p=st[p].link) // modificar
          esta linea para hacer separadores unicos entre varias cadenas (c
         =='$')
       st[p].next[c] = cur;
     if (p == -1)
       st[cur].link = 0;
     else {
       int q = st[p].next[c];
       if (st[p].len + 1 == st[q].len)
         st[cur].link = q;
       else {
         int clone = sz++;
         // no le ponemos la posicion actual a clone sino indirectamente
36
             por el link de cur
```

int p\_length;

```
st[clone].len = st[p].len + 1;
                                                                                       int t_length;
37
                                                                                  14
         st[clone].next = st[q].next;
                                                                                       f_getter t_getter;
                                                                                  15
38
         st[clone].link = st[q].link;
                                                                                       f_matcher matcher;
39
                                                                                  16
         for (; p!=-1 && st[p].next.count(c) && st[p].next[c]==q; p=st[p].
                                                                                     };
                                                                                  17
40
             link)
                                                                                  18
           st[p].next[c] = clone;
                                                                                     11 rk_pot[MAX_LENGTH];
41
         st[q].link = st[cur].link = clone;
                                                                                     ll rk_p = 257, rk_M = 1000000007, rk_p_inv = 70038911; //pow
42
                                                                                          (257,10**9+7-2,10**9+7)
43
     }
                                                                                     void initRK(){
44
                                                                                       11 p = 1;
     last = cur;
                                                                                       for (int i = 0; i < MAX_LENGTH; i++, p=(p*rk_p) \frac{1}{r}k_M){
  |}
46
                                                                                         rk_pot[i]=p:
                             4.9. Z Function
                                                                                       }
                                                                                  25
                                                                                     }
                                                                                  26
  char s[MAXN];
                                                                                  27
  [int z[MAXN]; // z[i] = i==0 ? 0 : max k tq s[0,k) match with s[i,i+k)
                                                                                     11 calcHashRK(int start, int offset, f_getter getter){
   void z_function(char s[],int z[]) {
                                                                                       11 r = 0:
                                                                                  29
       int n = strlen(s):
4
                                                                                       for (int i = start; i < start+offset; i++) r=(r+rk_pot[i]*getter(i))%</pre>
       forn(i, n) z[i]=0;
5
                                                                                            rk M:
       for (int i = 1, l = 0, r = 0; i < n; ++i) {
6
                                                                                       return r;
                                                                                  31
           if (i \le r) z[i] = min (r - i + 1, z[i - 1]);
7
                                                                                  32
           while (i + z[i] < n \&\& s[z[i]] == s[i + z[i]]) ++z[i]:
                                                                                  33
           if (i + z[i] - 1 > r) l = i, r = i + z[i] - 1;
9
                                                                                      void RKSearch(RobinKarpMatchSetting &ms){
                                                                                  34
       }
10
                                                                                       11 h = calcHashRK(0,ms.p_length,ms.t_getter);
11
                                                                                       ms.matcher(h);
                                                                                  36
12
                                                                                       for (int i = ms.p_length; i < ms.t_length; i++){</pre>
   int main() {
13
                                                                                         h = ((h-ms.t_getter(i-ms.p_length)) %rk_M+rk_M) %rk_M;
       ios::sync_with_stdio(0);
14
                                                                                         h = (h * rk_p_inv) % rk_M;
                                                                                         h = (h + ms.t_getter(i)*rk_pot[ms.p_length-1]) % rk_M;
                  4.10. Rabin Karp Fixed Length
                                                                                         ms.matcher(h);
                                                                                  41
                                                                                       }
   #include <bits/stdc++.h>
                                                                                  42
                                                                                     }
   #include <functional>
                                                                                  43
   using namespace std;
                                                                                  44
                                                                                      //Search lowest x such that f(x) >= value
                                                                                     int lowerBound(int minX, int maxX, function<int(int)> f, int value){
   typedef long long 11;
                                                                                       int low = minX, hi = maxX+1, mid;
   typedef function<char(int)> f_getter;
                                                                                       while(low < hi){</pre>
   typedef function<void(11)> f_matcher;
                                                                                         mid = (low+hi)/2:
                                                                                        int v = f(mid);
                                                                                         if (v >= value) hi = mid;
   #define MAX_LENGTH 100005
                                                                                         else low = mid+1;
                                                                                  52
11
  struct RobinKarpMatchSetting {
                                                                                  53
12
```

return low;

```
55 |}
56
    //Search lowest x such that f(x) > value
57
    int upperBound(int minX, int maxX, function<int(int)> f, int value){
     int low = minX, hi = maxX+1, mid;
59
     while(low < hi){</pre>
60
       mid = (low+hi)/2;
61
       int v = f(mid);
62
       if (v <= value) low = mid+1;
63
       else hi = mid;
64
65
     return low;
66
67
68
   string text[35];
   int N;
70
71
    //Return 2 if not shared, 1 if shared
   int evalLength(int length){
73
     set<ll> shared;
74
     RobinKarpMatchSetting ms;
75
     ms.t_length = text[0].size();
76
     ms.t_getter = [](int j)->char{return text[0][j];};
77
     ms.p_length = length;
78
     ms.matcher = [&shared](ll h){shared.insert(h);};
79
     RKSearch(ms);
80
     for (int i = 1; i < N; i++){
81
       set<11> newShared;
82
       ms.matcher = [&shared,&newShared](11 h){if (shared.count(h))
83
           newShared.insert(h);};
       ms.t_getter = [i](int j)->char{return text[i][j];};
84
       ms.t_length = text[i].size();
85
       RKSearch(ms);
86
       if (newShared.size() == 0) return 2:
87
       shared = newShared:
88
     }
89
     return 1;
90
91
92
   int main() {
93
     ios_base::sync_with_stdio(false);
94
     cin.tie(0);
95
     initRK();
96
```

```
while (cin >> N){
    int minLength = 100005;
    for (int i = 0; i < N; i++) {
        cin >> text[i];
        minLength=min(minLength,(int)text[i].size());
    }
    cout << (lowerBound(1,minLength,evalLength,2) - 1) << "\n";
}
</pre>
```

#### 5. Geometria

#### 5.1. Punto

```
1 struct pto{
     double x, v;
     pto(double x=0, double y=0):x(x),y(y){}
     pto operator+(pto a){return pto(x+a.x, y+a.y);}
     pto operator-(pto a){return pto(x-a.x, y-a.y);}
     pto operator+(double a){return pto(x+a, y+a);}
     pto operator*(double a){return pto(x*a, y*a);}
     pto operator/(double a){return pto(x/a, y/a);}
     //dot product, producto interno:
9
     double operator*(pto a){return x*a.x+y*a.y;}
10
     //module of the cross product or vectorial product:
11
     //if a is less than 180 clockwise from b, a^b>0
12
     double operator^(pto a){return x*a.y-y*a.x;}
13
     //returns true if this is at the left side of line qr
14
     bool left(pto q, pto r){return ((q-*this)^(r-*this))>0;}
15
     bool operator<(const pto &a) const{return x<a.x-EPS || (abs(x-a.x)<EPS
16
          && v<a.v-EPS);}
   bool operator==(pto a){return abs(x-a.x)<EPS && abs(y-a.y)<EPS;}
17
     double norm(){return sqrt(x*x+y*y);}
18
     double norm_sq(){return x*x+y*y;}
19
   };
20
   double dist(pto a, pto b){return (b-a).norm();}
   typedef pto vec;
23
   double angle(pto a, pto o, pto b){
     pto oa=a-o, ob=b-o;
25
     return atan2(oa^ob, oa*ob);}
26
27
   //rotate p by theta rads CCW w.r.t. origin (0,0)
```

15 }

#### 5.2. Orden radial de puntos

```
struct Cmp{//orden total de puntos alrededor de un punto r
2
     pto r;
     Cmp(pto r):r(r) {}
3
     int cuad(const pto &a) const{
4
       if(a.x > 0 && a.y >= 0)return 0;
       if(a.x <= 0 && a.y > 0)return 1;
6
       if(a.x < 0 \&\& a.y <= 0)return 2;
       if(a.x >= 0 \&\& a.y < 0)return 3;
8
       assert(a.x ==0 && a.y==0);
9
       return -1:
10
11
     bool cmp(const pto&p1, const pto&p2)const{
12
       int c1 = cuad(p1), c2 = cuad(p2);
13
       if(c1==c2) return p1.y*p2.x<p1.x*p2.y;
14
           else return c1 < c2:
15
16
       bool operator()(const pto&p1, const pto&p2) const{
17
       return cmp(pto(p1.x-r.x,p1.y-r.y),pto(p2.x-r.x,p2.y-r.y));
18
19
20 | };
```

#### 5.3. Line

```
int sgn(ll x){return x<0? -1 : !!x;}
  struct line{
     line() {}
     double a,b,c;//Ax+By=C
   //pto MUST store float coordinates!
     line(double a, double b, double c):a(a),b(b),c(c){}
6
    line(pto p, pto q): a(q.y-p.y), b(p.x-q.x), c(a*p.x+b*p.y) {}
     int side(pto p){return sgn(ll(a) * p.x + ll(b) * p.y - c);}
8
9
   bool parallels(line 11, line 12){return abs(11.a*12.b-12.a*11.b)<EPS;}
   pto inter(line 11, line 12){//intersection
     double det=11.a*12.b-12.a*11.b;
12
     if(abs(det) < EPS) return pto(INF, INF); //parallels</pre>
13
     return pto(12.b*11.c-11.b*12.c, 11.a*12.c-12.a*11.c)/det;
```

# 5.4. Segment

```
1 struct segm{
     pto s,f;
     segm(pto s, pto f):s(s), f(f) {}
     pto closest(pto p) {//use for dist to point
        double 12 = dist_sq(s, f);
5
        if(12==0.) return s:
6
        double t = ((p-s)*(f-s))/12;
        if (t<0.) return s;//not write if is a line
        else if(t>1.)return f;//not write if is a line
        return s+((f-s)*t);
10
    }
11
       bool inside(pto p){return abs(dist(s, p)+dist(p, f)-dist(s, f))<EPS
12
           : }
  };
13
   pto inter(segm s1, segm s2){
15
     pto r=inter(line(s1.s, s1.f), line(s2.s, s2.f));
       if(s1.inside(r) && s2.inside(r)) return r;
17
     return pto(INF, INF);
18
19 }
```

## 5.5. Rectangle

```
struct rect{
//lower-left and upper-right corners
pto lw, up;
};

//returns if there's an intersection and stores it in r
bool inter(rect a, rect b, rect &r){
r.lw=pto(max(a.lw.x, b.lw.x), max(a.lw.y, b.lw.y));
r.up=pto(min(a.up.x, b.up.x), min(a.up.y, b.up.y));
//check case when only a edge is common
return r.lw.x<r.up.x && r.lw.y<r.up.y;
}</pre>
```

## 5.6. Polygon Area

```
double area(vector<pto> &p){//O(sz(p))
double area=0;
forn(i, sz(p)) area+=p[i]^p[(i+1)%sz(p)];
```

```
//if points are in clockwise order then area is negative
    return abs(area)/2;
5
  }
6
  //Area ellipse = M_PI*a*b where a and b are the semi axis lengths
 //Area triangle = sqrt(s*(s-a)(s-b)(s-c)) where s=(a+b+c)/2
```

#### 5.7. Circle

```
vec perp(vec v){return vec(-v.y, v.x);}
  line bisector(pto x, pto y){
     line l=line(x, y); pto m=(x+y)/2;
     return line(-1.b, 1.a, -1.b*m.x+1.a*m.y);
5
   struct Circle{
     pto o;
7
     double r;
8
     Circle(pto x, pto y, pto z){
9
       o=inter(bisector(x, y), bisector(y, z));
10
       r=dist(o, x);
11
12
     pair<pto, pto> ptosTang(pto p){
13
       pto m=(p+o)/2;
14
       tipo d=dist(o, m);
15
       tipo a=r*r/(2*d):
16
       tipo h=sqrt(r*r-a*a);
17
       pto m2=o+(m-o)*a/d;
18
       vec per=perp(m-o)/d;
19
       return make_pair(m2-per*h, m2+per*h);
20
21
22
   //finds the center of the circle containing p1 and p2 with radius r
   //as there may be two solutions swap p1, p2 to get the other
   bool circle2PtsRad(pto p1, pto p2, double r, pto &c){
25
           double d2=(p1-p2).norm_sq(), det=r*r/d2-0.25;
26
           if(det<0) return false;
27
           c=(p1+p2)/2+perp(p2-p1)*sqrt(det);
28
           return true;
29
30
   #define sqr(a) ((a)*(a))
   #define feq(a,b) (fabs((a)-(b))<EPS)</pre>
   pair<tipo, tipo > ecCuad(tipo a, tipo b, tipo c){//a*x*x+b*x+c=0
     tipo dx = sqrt(b*b-4.0*a*c);
34
     return make_pair((-b + dx)/(2.0*a), (-b - dx)/(2.0*a));
35
```

```
36 }
   pair<pto, pto> interCL(Circle c, line 1){
     bool sw=false;
38
     if((sw=feq(0,1.b))){
39
     swap(l.a, l.b);
40
     swap(c.o.x, c.o.y);
41
42
     pair<tipo, tipo> rc = ecCuad(
43
     sqr(1.a)+sqr(1.b),
44
     2.0*1.a*1.b*c.o.y-2.0*(sqr(1.b)*c.o.x+1.c*1.a),
     sqr(1.b)*(sqr(c.o.x)+sqr(c.o.y)-sqr(c.r))+sqr(1.c)-2.0*1.c*1.b*c.o.y
46
47
     pair<pto, pto> p( pto(rc.first, (1.c - 1.a * rc.first) / 1.b),
48
               pto(rc.second, (l.c - l.a * rc.second) / l.b) );
49
     if(sw){
50
     swap(p.first.x, p.first.y);
     swap(p.second.x, p.second.y);
53
54
     return p;
55
   pair<pto, pto> interCC(Circle c1, Circle c2){
    line 1;
    1.a = c1.o.x-c2.o.x;
    1.b = c1.o.v-c2.o.v;
    1.c = (sqr(c2.r)-sqr(c1.r)+sqr(c1.o.x)-sqr(c2.o.x)+sqr(c1.o.y)
60
     -sgr(c2.o.v))/2.0;
61
     return interCL(c1, 1);
62
63 }
                           5.8. Point in Poly
1 //checks if v is inside of P, using ray casting
   //works with convex and concave.
   //excludes boundaries, handle it separately using segment.inside()
   bool inPolygon(pto v, vector<pto>& P) {
     bool c = false;
    forn(i, sz(P)){
6
```

```
int j=(i+1) \%z(P);
7
       if((P[j].y>v.y) != (P[i].y > v.y) &&
8
     (v.x < (P[i].x - P[j].x) * (v.y-P[j].y) / (P[i].y - P[j].y) + P[j].x))
         c = !c;
10
     }
11
     return c;
12
```

```
13 }
                  5.9. Point in Convex Poly log(n)
   void normalize(vector<pto> &pt){//delete collinear points first!
     //this makes it clockwise:
2
       if(pt[2].left(pt[0], pt[1])) reverse(pt.begin(), pt.end());
3
     int n=sz(pt), pi=0;
     forn(i, n)
5
       if(pt[i].x<pt[pi].x || (pt[i].x==pt[pi].x && pt[i].y<pt[pi].y))</pre>
6
7
     vector<pto> shift(n);//puts pi as first point
8
       forn(i, n) shift[i]=pt[(pi+i) %n];
9
       pt.swap(shift);
10
11
   bool inPolygon(pto p, const vector<pto> &pt){
12
     //call normalize first!
13
     if(p.left(pt[0], pt[1]) || p.left(pt[sz(pt)-1], pt[0])) return false;
14
     int a=1, b=sz(pt)-1;
15
     while(b-a>1){
16
       int c=(a+b)/2:
17
       if(!p.left(pt[0], pt[c])) a=c;
18
       else b=c:
19
     }
20
     return !p.left(pt[a], pt[a+1]);
21
22 }
                    5.10. Convex Check CHECK
  | bool isConvex(vector<int> &p){//O(N), delete collinear points!
     int N=sz(p):
     if(N<3) return false;
     bool isLeft=p[0].left(p[1], p[2]);
     forr(i, 1, N)
       if(p[i].left(p[(i+1) \( \mathbb{N} \)], p[(i+2) \( \mathbb{N} \)])!=isLeft)
6
         return false:
7
     return true: }
                           5.11. Convex Hull
1 //stores convex hull of P in S, CCW order
   //left must return >=0 to delete collinear points!
  void CH(vector<pto>& P, vector<pto> &S){
     S.clear();
```

```
sort(P.begin(), P.end());//first x, then y
5
     forn(i, sz(P)){//lower hull
6
       while(sz(S) \ge 2 \&\& S[sz(S)-1].left(S[sz(S)-2], P[i])) S.pop_back();
       S.pb(P[i]);
     }
9
     S.pop_back();
     int k=sz(S);
11
     dforn(i, sz(P)){//upper hull
12
       while(sz(S) \ge k+2 \&\& S[sz(S)-1].left(S[sz(S)-2], P[i])) S.pop_back
13
           ();
       S.pb(P[i]);
14
    }
15
16
     S.pop_back();
17 }
                          5.12. Cut Polygon
1 //cuts polygon Q along the line ab
  //stores the left side (swap a, b for the right one) in P
   void cutPolygon(pto a, pto b, vector<pto> Q, vector<pto> &P){
    P.clear();
    forn(i, sz(Q)){
       double left1=(b-a)^(Q[i]-a), left2=(b-a)^(Q[(i+1) \%z(Q)]-a);
       if(left1>=0) P.pb(Q[i]);
       if(left1*left2<0)</pre>
8
         P.pb(inter(line(Q[i], Q[(i+1) %z(Q)]), line(a, b)));
9
    }
10
11 }
                            5.13. Bresenham
1 //plot a line approximation in a 2d map
  void bresenham(pto a, pto b){
    pto d=b-a; d.x=abs(d.x), d.y=abs(d.y);
     pto s(a.x<b.x? 1: -1, a.y<b.y? 1: -1);
    int err=d.x-d.v;
     while(1){
       m[a.x][a.y]=1;//plot
       if(a==b) break:
8
       int e2=err:
       if(e2 >= 0) err-=2*d.y, a.x+=s.x;
       if(e2 <= 0) err+= 2*d.x, a.y+= s.y;
11
12
13 }
```

#### 5.14. Rotate Matrix

```
1 //rotates matrix t 90 degrees clockwise
   //using auxiliary matrix t2(faster)
  void rotate(){
    forn(x, n) forn(y, n)
       t2[n-y-1][x]=t[x][y];
    memcpy(t, t2, sizeof(t));
7 | }
            5.15. Interseccion de Circulos en n3log(n)
  struct event {
       double x; int t;
       event(double xx, int tt) : x(xx), t(tt) {}
       bool operator <(const event &o) const { return x < o.x; }</pre>
5
   typedef vector<Circle> VC;
   typedef vector<event> VE;
   int n:
   double cuenta(VE &v, double A, double B) {
       sort(v.begin(), v.end());
10
       double res = 0.0, lx = ((v.empty())?0.0:v[0].x);
11
       int contador = 0:
12
       forn(i.sz(v)) {
13
           //interseccion de todos (contador == n), union de todos (
14
               contador > 0)
           //conjunto de puntos cubierto por exacta k Circulos (contador ==
15
           if (contador == n) res += v[i].x - lx;
16
           contador += v[i].t, lx = v[i].x;
17
       }
18
       return res;
19
20
   // Primitiva de sgrt(r*r - x*x) como funcion double de una variable x.
21
   inline double primitiva(double x,double r) {
22
       if (x \ge r) return r*r*M_PI/4.0;
23
       if (x \le -r) return -r*r*M PI/4.0:
24
       double raiz = sqrt(r*r-x*x);
25
       return 0.5 * (x * raiz + r*r*atan(x/raiz));
26
27
   double interCircle(VC &v) {
28
```

vector<double> p; p.reserve(v.size() \* (v.size() + 2));

29

```
forn(i,sz(v)) p.push_back(v[i].c.x + v[i].r), p.push_back(v[i].c.x
           - v[i].r);
       forn(i,sz(v)) forn(j,i) {
31
           Circle &a = v[i], b = v[j];
32
           double d = (a.c - b.c).norm();
33
           if (fabs(a.r - b.r) < d \&\& d < a.r + b.r) {
               double alfa = acos((sqr(a.r) + sqr(d) - sqr(b.r)) / (2.0 * d)
35
                     * a.r)):
               pto vec = (b.c - a.c) * (a.r / d);
               p.pb((a.c + rotate(vec, alfa)).x), p.pb((a.c + rotate(vec, -
37
                    alfa)).x);
           }
38
       }
39
       sort(p.begin(), p.end());
       double res = 0.0:
       forn(i,sz(p)-1) {
42
           const double A = p[i], B = p[i+1];
43
           VE ve; ve.reserve(2 * v.size());
           forn(j,sz(v)) {
               const Circle &c = v[j];
               double arco = primitiva(B-c.c.x,c.r) - primitiva(A-c.c.x,c.r
47
                    );
               double base = c.c.y * (B-A);
48
               ve.push_back(event(base + arco,-1));
                ve.push_back(event(base - arco, 1));
50
51
           res += cuenta(ve,A,B);
52
       }
       return res;
54
55 }
```

#### 6. Math

#### 6.1. Identidades

```
\sum_{i=0}^{n} {n \choose i} = 2^n
\sum_{i=0}^{n} i {n \choose i} = n * 2^{n-1}
\sum_{i=m}^{n} i = \frac{n(n+1)}{2} - \frac{m(m-1)}{2} = \frac{(n+1-m)(n+m)}{2}
\sum_{i=0}^{n} i = \sum_{i=1}^{n} i = \frac{n(n+1)}{2}
\sum_{i=0}^{n} i^2 = \frac{n(n+1)(2n+1)}{6} = \frac{n^3}{3} + \frac{n^2}{2} + \frac{n}{6}
\sum_{i=0}^{n} i(i-1) = \frac{8}{6} (\frac{n}{2})(\frac{n}{2} + 1)(n+1) \text{ (doubles)} \rightarrow \text{Sino ver caso impar y par}
\sum_{i=0}^{n} i^3 = \left(\frac{n(n+1)}{2}\right)^2 = \frac{n^4}{4} + \frac{n^3}{2} + \frac{n^2}{4} = \left[\sum_{i=1}^{n} i\right]^2
```

32

```
\sum_{i=0}^{n} i^4 = \frac{n(n+1)(2n+1)(3n^2+3n-1)}{30} = \frac{n^5}{5} + \frac{n^4}{2} + \frac{n^3}{3} - \frac{n}{30}\sum_{i=0}^{n} i^p = \frac{(n+1)^{p+1}}{p+1} + \sum_{k=1}^{p} \frac{B_k}{p-k+1} \binom{p}{k} (n+1)^{p-k+1}r = e - v + k + 1
```

Teorema de Pick: (Area, puntos interiores y puntos en el borde)  $A = I + \frac{B}{2} - 1$ 

#### 6.2. Ec. Caracteristica

$$\begin{aligned} a_0T(n) + a_1T(n-1) + \ldots + a_kT(n-k) &= 0 \\ p(x) = a_0x^k + a_1x^{k-1} + \ldots + a_k \end{aligned}$$
 Sean  $r_1, r_2, \ldots, r_q$  las raíces distintas, de mult.  $m_1, m_2, \ldots, m_q$  
$$T(n) = \sum_{i=1}^q \sum_{j=0}^{m_i-1} c_{ij} n^j r_i^n$$
 Las constantes  $c_{ij}$  se determinan por los casos base.

#### 6.3. Combinatorio

```
forn(i, MAXN+1){//comb[i][k]=i tomados de a k
    comb[i][0]=comb[i][i]=1;
    forr(k, 1, i) comb[i][k]=(comb[i-1][k]+comb[i-1][k-1]) MOD;
}
ll lucas (ll n, ll k, int p){ //Calcula (n,k) %p teniendo comb[p][p]
    precalculado.
ll aux = 1;
while (n + k) aux = (aux * comb[n%p][k%p]) %p, n/=p, k/=p;
return aux;
}
```

## 6.4. Exp. de Numeros Mod.

# 6.5. Exp. de Matrices y Fibonacci en log(n)

```
#define SIZE 350
int NN;
double tmp[SIZE] [SIZE];
void mul(double a[SIZE] [SIZE], double b[SIZE] [SIZE]){ zero(tmp);
forn(i, NN) forn(j, NN) forn(k, NN) res[i][j]+=a[i][k]*b[k][j];
forn(i, NN) forn(j, NN) a[i][j]=res[i][j];
```

```
7 }
   void powmat(double a[SIZE][SIZE], int n, double res[SIZE][SIZE]){
       forn(i, NN) forn(j, NN) res[i][j]=(i==j);
       while(n){
10
           if(n&1) mul(res, a), n--;
11
           else mul(a, a), n/=2;
12
       } }
13
                6.6. Matrices y determinante O(n^3)
 1 | struct Mat {
       vector<vector<double> > vec;
       Mat(int n): vec(n, vector<double>(n) ) {}
       Mat(int n, int m): vec(n, vector<double>(m) ) {}
       vector<double> &operator[](int f){return vec[f];}
       const vector<double> &operator[](int f) const {return vec[f];}
6
       int size() const {return sz(vec);}
7
       Mat operator+(Mat &b) { //this de n x m entonces b de n x m
8
           Mat m(sz(b), sz(b[0]));
           forn(i,sz(vec)) forn(j,sz(vec[0])) m[i][j] = vec[i][j] + b[i][j
10
           return m;
11
       Mat operator*(const Mat &b) { ///this de n x m entonces b de m x t
12
           int n = sz(vec), m = sz(vec[0]), t = sz(b[0]);
13
           Mat mat(n.t):
14
           forn(i,n) forn(j,t) forn(k,m) mat[i][j] += vec[i][k] * b[k][j];
15
           return mat:
16
       double determinant(){//sacado de e maxx ru
17
           double det = 1;
18
           int n = sz(vec);
19
           Mat m(*this);
20
           forn(i, n){//para cada columna
21
               int k = i;
22
               forr(j, i+1, n)//busco la fila con mayor val abs
23
                   if(abs(m[i][i])>abs(m[k][i])) k = j;
24
               if(abs(m[k][i])<1e-9) return 0;
25
               m[i].swap(m[k]);//la swapeo
26
               if(i!=k) det = -det:
27
               det *= m[i][i];
28
               forr(j, i+1, n) m[i][j] /= m[i][i];
29
               //hago 0 todas las otras filas
30
               forn(j, n) if (j!= i && abs(m[j][i])>1e-9)
31
```

forr(k, i+1, n) m[j][k]-=m[i][k]\*m[j][i];

28

```
}
33
           return det;
34
       }
35
36
37
   int n;
   int main() {
    //DETERMINANTE:
   //https://uva.onlinejudge.org/index.php?option=com_onlinejudge&Itemid=8&
       page=show_problem&problem=625
     freopen("input.in", "r", stdin);
42
       ios::sync_with_stdio(0);
43
       while(cin >> n && n){
44
           Mat m(n):
45
           forn(i, n) forn(j, n) cin >> m[i][j];
46
           cout << (11)round(m.determinant()) << endl;</pre>
47
       }
48
       cout << "*" << endl;
     return 0:
50
51 }
```

#### 6.7. Teorema Chino del Resto

$$y = \sum_{j=1}^{n} (x_j * (\prod_{i=1, i \neq j}^{n} m_i)_{m_j}^{-1} * \prod_{i=1, i \neq j}^{n} m_i)$$

## 6.8. Criba

```
#define MAXP 100000 //no necesariamente primo
   int criba[MAXP+1];
   void crearcriba(){
     int w[] = \{4,2,4,2,4,6,2,6\};
4
     for(int p=25;p<=MAXP;p+=10) criba[p]=5;</pre>
5
     for(int p=9;p<=MAXP;p+=6) criba[p]=3;</pre>
6
     for(int p=4;p<=MAXP;p+=2) criba[p]=2;</pre>
     for(int p=7,cur=0;p*p<=MAXP;p+=w[cur+&7]) if (!criba[p])
       for(int j=p*p;j<=MAXP;j+=(p<<1)) if(!criba[j]) criba[j]=p;</pre>
9
10
   vector<int> primos;
   void buscarprimos(){
12
     crearcriba();
13
```

```
forr (i,2,MAXP+1) if (!criba[i]) primos.push_back(i);
15
   //~ Useful for bit trick: #define SET(i) ( criba[(i)>>5] |=1<<((i)&31) ),
        #define INDEX(i) ( (criba[i>>5]>>((i)&31))&1 ), unsigned int criba[
       MAXP/32+1;
17
   int main() {
     freopen("primos", "w", stdout);
     buscarprimos();
                       6.9. Funciones de primos
       Sea n = \prod p_i^{k_i}, fact(n) genera un map donde a cada p_i le asocia su k_i
1 //factoriza bien numeros hasta MAXP^2
   map<11,11> fact(11 n){ //0 (cant primos)
     map<ll,ll> ret;
     forall(p, primos){
       while(!(n \(\frac{1}{2}\)\)){
         ret[*p]++;//divisor found
         n/=*p;
7
8
     if(n>1) ret[n]++:
     return ret:
11
12
    //factoriza bien numeros hasta MAXP
   map<11,11> fact2(11 n){ //0 (lg n)
     map<ll,ll> ret;
     while (criba[n]){
16
       ret[criba[n]]++;
       n/=criba[n];
18
19
     if(n>1) ret[n]++;
20
     return ret;
21
22
   //Usar asi: divisores(fac, divs, fac.begin()); NO ESTA ORDENADO
   void divisores(const map<11,11> &f, vector<11> &divs, map<11,11>::
       iterator it, ll n=1){
       if(it==f.begin()) divs.clear();
25
       if(it==f.end()) { divs.pb(n); return; }
26
       ll p=it->fst, k=it->snd; ++it;
27
```

forn(\_, k+1) divisores(f, divs, it, n), n\*=p;

```
29 |}
   ll sumDiv (ll n){
30
     ll rta = 1;
31
     map<11,11> f=fact(n);
32
     forall(it, f) {
     11 \text{ pot} = 1, \text{ aux} = 0;
     forn(i, it->snd+1) aux += pot, pot *= it->fst;
     rta*=aux;
36
     }
37
     return rta;
38
39
   ll eulerPhi (ll n){ // con criba: O(lg n)
     11 \text{ rta} = n:
     map<ll,ll> f=fact(n);
     forall(it, f) rta -= rta / it->first;
     return rta;
45
   11 eulerPhi2 (11 n){ // 0 (sqrt n)
     11 r = n:
47
     forr (i,2,n+1){
48
       if ((11)i*i > n) break;
49
       if (n \% i == 0){
50
         while (n\% == 0) n/=i;
51
         r = r/i; }
52
     }
53
     if (n != 1) r= r/n;
     return r;
55
56
57
    int main() {
58
     buscarprimos();
59
     forr (x,1, 500000){
60
        cout << ||x|| = || || << x << endl;
61
        cout << "Numero | de | factores | primos: | | " << numPrimeFactors(x) << endl;</pre>
62
        cout << "Numero de distintos factores primos: " <<
63
            numDiffPrimeFactors(x) << endl;</pre>
        cout << "Suma, de, factores, primos:, " << sumPrimeFactors(x) << endl;
64
        cout << "Numero de divisores:" << numDiv(x) << endl;</pre>
65
        cout << "Suma_de_divisores:__" << sumDiv(x) << endl;</pre>
66
        cout << "Phi_de_Euler:_" << eulerPhi(x) << endl;</pre>
67
     }
68
     return 0;
69
70 |}
```

#### 6.10. Phollard's Rho (rolando)

```
1 | 11 gcd(11 a, 11 b){return a?gcd(b %a, a):b;}
3 | 11 mulmod (11 a, 11 b, 11 c) { //returns (a*b) %, and minimize overfloor
    11 x = 0, y = a\%;
     while (b > 0){
       if (b \% 2 == 1) x = (x+y) \% c;
       y = (y*2) \% c;
       b /= 2;
     return x %c;
10
11
   ll expmod (ll b, ll e, ll m){\frac{1}{0}} \log b
     if(!e) return 1;
    11 q= expmod(b,e/2,m); q=mulmod(q,q,m);
     return e %2? mulmod(b,q,m) : q;
17
   bool es_primo_prob (ll n, int a)
20
     if (n == a) return true;
     11 s = 0.d = n-1:
     while (d \% 2 == 0) s++, d/=2;
24
     ll x = expmod(a,d,n);
     if ((x == 1) \mid | (x+1 == n)) return true;
    forn (i, s-1){
28
    x = mulmod(x, x, n);
      if (x == 1) return false;
       if (x+1 == n) return true;
31
32
     return false;
33
34
35
   bool rabin (ll n){ //devuelve true si n es primo
     if (n == 1) return false:
37
    const int ar[] = \{2,3,5,7,11,13,17,19,23\};
38
     forn (j,9)
39
       if (!es_primo_prob(n,ar[j]))
40
         return false;
41
```

```
tipo lcm(tipo a, tipo b){return a / gcd(a,b) * b;}
     return true;
42
  }
43
                                                                                                             6.14. Inversos
44
   ll rho(ll n){
45
                                                                                 1 #define MAXMOD 15485867
       if( (n & 1) == 0 ) return 2;
46
                                                                                   ll inv[MAXMOD];//inv[i]*i=1 mod MOD
       11 x = 2 , y = 2 , d = 1;
47
                                                                                   void calc(int p){\frac{1}{0}}
       ll c = rand() % n + 1;
48
                                                                                     inv[1]=1:
       while(d == 1){
49
                                                                                     forr(i, 2, p) inv[i] = p-((p/i)*inv[p%i])%p;
          x = (mulmod(x, x, n) + c) n;
50
                                                                                6
           y = (mulmod( y , y , n ) + c) n;
51
                                                                                   int inverso(int x){\frac{1}{0}(\log x)}
           y = (mulmod(y, y, n) + c) n;
52
                                                                                     return expmod(x, eulerphi(MOD)-2);//si mod no es primo(sacar a mano)
          if(x - y \ge 0) d = gcd(x - y, n);
53
                                                                                     return expmod(x, MOD-2);//si mod es primo
           else d = gcd(y - x, n);
54
                                                                                10 }
       }
55
       return d==n? rho(n):d;
56
                                                                                                             6.15. Simpson
57
58
                                                                                 double integral (double a, double b, int n=10000) {//O(n), n=cantdiv
   map<ll,ll> prim;
                                                                                     double area=0, h=(b-a)/n, fa=f(a), fb;
   void factRho (ll n){ //O (lg n)^3. un solo numero
                                                                                     forn(i, n){
     if (n == 1) return;
61
                                                                                       fb=f(a+h*(i+1));
     if (rabin(n)){
62
                                                                                       area+=fa+ 4*f(a+h*(i+0.5)) +fb, fa=fb;
                                                                                5
       prim[n]++;
63
                                                                                 6
       return;
64
                                                                                     return area*h/6.;}
65
     11 factor = rho(n);
                                                                                                             6.16. Fraction
66
     factRho(factor);
67
     factRho(n/factor);
68
                                                                                 tipo mcd(tipo a, tipo b){return a?mcd(b%a, a):b;}
69 }
                                                                                   struct frac{
                                                                                     tipo p,q;
                               6.11. GCD
                                                                                     frac(tipo p=0, tipo q=1):p(p),q(q) {norm();}
                                                                                     void norm(){
                                                                                5
tipo gcd(tipo a, tipo b){return a?gcd(b %a, a):b;}
                                                                                       tipo a = mcd(p,q);
                                                                                6
                        6.12. Extended Euclid
                                                                                       if(a) p/=a, q/=a;
                                                                                7
                                                                                       else q=1;
                                                                                8
  void extendedEuclid (ll a, ll b) \{ //a * x + b * y = d \}
                                                                                       if (q<0) q=-q, p=-p;}
     if (!b) { x = 1; y = 0; d = a; return;}
                                                                                     frac operator+(const frac& o){
2
                                                                                10
     extendedEuclid (b, a%);
                                                                                       tipo a = mcd(q, o.q);
                                                                                11
    11 x1 = y;
                                                                                       return frac(p*(o.q/a)+o.p*(q/a), q*(o.q/a));}
                                                                                12
    11 y1 = x - (a/b) * y;
                                                                                     frac operator-(const frac& o){
                                                                                13
    x = x1; y = y1;
                                                                                       tipo a = mcd(q, o.q);
                                                                                14
                                                                                       return frac(p*(o.q/a)-o.p*(q/a), q*(o.q/a));}
                                                                                15
                                                                                     frac operator*(frac o){
                                                                                16
                               6.13. LCM
                                                                                       tipo a = mcd(q,o.p), b = mcd(o.q,p);
                                                                                17
```

return roots; }

33

13

```
return frac((p/b)*(o.p/a), (q/a)*(o.q/b));}
                                                                                 34 };
18
     frac operator/(frac o){
                                                                                   pair<poly,tipo> ruffini(const poly p, tipo r) {
19
       tipo a = mcd(q,o.q), b = mcd(o.p,p);
                                                                                      int n = sz(p.c) - 1;
20
       return frac((p/b)*(o.q/a),(q/a)*(o.p/b));}
                                                                                      vector<tipo> b(n);
                                                                                37
21
     bool operator<(const frac &o) const{return p*o.g < o.p*g;}</pre>
                                                                                      b[n-1] = p.c[n];
22
                                                                                38
     bool operator==(frac o){return p==o.p&kq==o.q;}
                                                                                      dforn(k,n-1) b[k] = p.c[k+1] + r*b[k+1];
23
                                                                                39
                                                                                      tipo resto = p.c[0] + r*b[0];
24 };
                                                                                 40
                                                                                      poly result(b);
                                                                                 41
                            6.17. Polinomio
                                                                                      return make_pair(result,resto);
                                                                                 42
                                                                                 43
                                                                                    poly interpolate(const vector<tipo>& x,const vector<tipo>& y) {
           int m = sz(c), n = sz(o.c);
                                                                                 44
                                                                                        poly A; A.c.pb(1);
           vector<tipo> res(max(m,n));
                                                                                 45
2
                                                                                        forn(i,sz(x)) { poly aux; aux.c.pb(-x[i]), aux.c.pb(1), A = A * aux;
           forn(i, m) res[i] += c[i];
                                                                                 46
           forn(i, n) res[i] += o.c[i];
                                                                                      poly S; S.c.pb(0);
           return poly(res); }
5
                                                                                      forn(i,sz(x)) { poly Li;
      polv operator*(const tipo cons) const {
6
                                                                                       Li = ruffini(A,x[i]).fst;
       vector<tipo> res(sz(c));
7
                                                                                        Li = Li * (1.0 / Li.eval(x[i])); // here put a multiple of the
           forn(i, sz(c)) res[i]=c[i]*cons;
8
                                                                                            coefficients instead of 1.0 to avoid using double
           return poly(res); }
9
                                                                                        S = S + Li * v[i];
       poly operator*(const poly &o) const {
10
                                                                                      return S;
           int m = sz(c), n = sz(o.c);
                                                                                 52
11
                                                                                    }
           vector<tipo> res(m+n-1);
                                                                                 53
12
           forn(i, m) forn(j, n) res[i+j]+=c[i]*o.c[j];
                                                                                 54
13
                                                                                    int main(){
           return poly(res); }
14
                                                                                      return 0;
     tipo eval(tipo v) {
                                                                                 56
15
                                                                                 57 }
       tipo sum = 0;
16
       dforn(i, sz(c)) sum=sum*v + c[i];
17
                                                                                                            6.18. Ec. Lineales
       return sum; }
18
       //poly contains only a vector<int> c (the coeficients)
19
     //the following function generates the roots of the polynomial
                                                                                 bool resolver_ev(Mat a, Vec v, Vec &x, Mat &ev){
20
                                                                                      int n = a.size(), m = n?a[0].size():0, rw = min(n, m);
    //it can be easily modified to return float roots
21
                                                                                      vector<int> p; forn(i,m) p.push_back(i);
     set<tipo> roots(){
                                                                                 3
22
       set<tipo> roots;
                                                                                      forn(i, rw) {
                                                                                 4
23
       tipo a0 = abs(c[0]), an = abs(c[sz(c)-1]);
                                                                                        int uc=i, uf=i;
                                                                                 5
24
                                                                                        forr(f, i, n) forr(c, i, m) if(fabs(a[f][c])>fabs(a[uf][uc])) {uf=f;
       vector<tipo> ps.qs;
                                                                                 6
25
      forr(p,1,sqrt(a0)+1) if (a0\%p==0) ps.pb(p),ps.pb(a0/p);
                                                                                            uc=c;}
26
      forr(q,1,sqrt(an)+1) if (an \%q==0) qs.pb(q),qs.pb(an/q);
                                                                                        if (feq(a[uf][uc], 0)) { rw = i; break; }
                                                                                 7
27
                                                                                        forn(j, n) swap(a[j][i], a[j][uc]);
       forall(pt.ps)
                                                                                 8
28
        forall(qt,qs) if ( (*pt) % (*qt)==0 ) {
                                                                                        swap(a[i], a[uf]); swap(y[i], y[uf]); swap(p[i], p[uc]);
                                                                                 9
29
                                                                                        tipo inv = 1 / a[i][i]; //aca divide
           tipo root = abs((*pt) / (*qt));
                                                                                 10
30
           if (eval(root)==0) roots.insert(root);
                                                                                        forr(j, i+1, n) {
                                                                                11
31
                                                                                          tipo v = a[j][i] * inv;
         }
                                                                                 12
32
                                                                                          forr(k, i, m) a[j][k]-=v * a[i][k];
```

```
y[j] -= v*y[i];
14
15
     } // rw = rango(a), aca la matriz esta triangulada
16
     forr(i, rw, n) if (!feq(y[i],0)) return false; // checkeo de
17
         compatibilidad
     x = vector < tipo > (m, 0);
18
     dforn(i, rw){
19
       tipo s = v[i];
20
       forr(j, i+1, rw) s -= a[i][j]*x[p[j]];
21
       x[p[i]] = s / a[i][i]; //aca divide
22
23
     ev = Mat(m-rw, Vec(m, 0)); // Esta parte va SOLO si se necesita el ev
24
     forn(k, m-rw) {
25
       ev[k][p[k+rw]] = 1;
26
       dforn(i, rw){
27
         tipo s = -a[i][k+rw];
28
         forr(j, i+1, rw) s -= a[i][j]*ev[k][p[j]];
29
         ev[k][p[i]] = s / a[i][i]; //aca divide
30
       }
31
    }
32
     return true;
33
34 }
```

#### 6.19. FFT

```
//~ typedef complex<double> base; //menos codigo, pero mas lento
   //elegir si usar complejos de c (lento) o estos
  struct base{
3
       double r,i;
4
       base(double r=0, double i=0):r(r), i(i){}
5
       double real()const{return r;}
6
       void operator/=(const int c){r/=c, i/=c;}
7
8
   base operator*(const base &a, const base &b){
9
       return base(a.r*b.r-a.i*b.i, a.r*b.i+a.i*b.r);}
10
   base operator+(const base &a, const base &b){
11
       return base(a.r+b.r, a.i+b.i);}
12
   base operator-(const base &a, const base &b){
13
       return base(a.r-b.r, a.i-b.i);}
14
   vector<int> rev; vector<base> wlen_pw;
   inline static void fft(base a[], int n, bool invert) {
       forn(i, n) if(i<rev[i]) swap(a[i], a[rev[i]]);</pre>
17
    for (int len=2; len<=n; len<<=1) {
```

```
double ang = 2*M_PI/len * (invert?-1:+1);
19
       int len2 = len >> 1;
20
       base wlen (cos(ang), sin(ang));
21
       wlen_pw[0] = base(1, 0);
22
           forr(i, 1, len2) wlen_pw[i] = wlen_pw[i-1] * wlen;
23
       for (int i=0; i<n; i+=len) {
24
         base t, *pu = a+i, *pv = a+i+len2, *pu_end = a+i+len2, *pw = &
25
             wlen_pw[0];
         for (; pu!=pu_end; ++pu, ++pv, ++pw)
           t = *pv * *pw, *pv = *pu - t,*pu = *pu + t;
       }
28
     }
29
     if (invert) forn(i, n) a[i]/= n:}
   inline static void calc_rev(int n){//precalculo: llamar antes de fft!!
       wlen_pw.resize(n), rev.resize(n);
32
       int lg=31-__builtin_clz(n);
33
       forn(i, n){
       rev[i] = 0;
           forn(k, lg) if(i\&(1<< k)) rev[i]|=1<<(lg-1-k);
   inline static void multiply(const vector<int> &a, const vector<int> &b,
       vector<int> &res) {
     vector<base> fa (a.begin(), a.end()), fb (b.begin(), b.end());
       int n=1; while(n < max(sz(a), sz(b))) n <<= 1; n <<= 1;
       calc_rev(n);
41
     fa.resize (n), fb.resize (n);
42
     fft (&fa[0], n, false), fft (&fb[0], n, false);
43
     forn(i, n) fa[i] = fa[i] * fb[i];
44
     fft (&fa[0], n, true);
45
     res.resize(n);
       forn(i, n) res[i] = int (fa[i].real() + 0.5); }
47
   void toPoly(const string &s, vector<int> &P){//convierte un numero a
       polinomio
       P.clear():
49
       dforn(i, sz(s)) P.pb(s[i]-'0');}
50
```

6.20. Tablas y cotas (Primos, Divisores, Factoriales, etc)

**Factoriales** 

```
0! = 1
                  11! = 39.916.800
1! = 1
                  12! = 479.001.600 \ (\in int)
2! = 2
                  13! = 6.227.020.800
3! = 6
                  14! = 87.178.291.200
4! = 24
                  15! = 1.307.674.368.000
5! = 120
                  16! = 20.922.789.888.000
6! = 720
                  17! = 355.687.428.096.000
7! = 5.040
                  18! = 6.402.373.705.728.000
8! = 40.320
                  19! = 121.645.100.408.832.000
9! = 362.880
                  20! = 2.432.902.008.176.640.000 (\in tint)
10! = 3.628.800 \mid 21! = 51.090.942.171.709.400.000
       max signed tint = 9.223.372.036.854.775.807
     max unsigned tint = 18.446.744.073.709.551.615
```

#### Primos

2 3 5 7 11 13 17 19 23 29 31 37 41 43 47 53 59 61 67 71 73 79 83 89 97 101 103 107 109 113 127 131 137 139 149 151 157 163 167 173 179 181 191 193 197 199 211 223 227 229 233 239 241 251 257 263 269 271 277 281 283 293 307 311 313 317 331 337 347  $349\ 353\ 359\ 367\ 373\ 379\ 383\ 389\ 397\ 401\ 409\ 419\ 421\ 431\ 433\ 439\ 443\ 449\ 457\ 461$  $463\ 467\ 479\ 487\ 491\ 499\ 503\ 509\ 521\ 523\ 541\ 547\ 557\ 563\ 569\ 571\ 577\ 587\ 593\ 599$  $601\ 607\ 613\ 617\ 619\ 631\ 641\ 643\ 647\ 653\ 659\ 661\ 673\ 677\ 683\ 691\ 701\ 709\ 719\ 727$  $733\ 739\ 743\ 751\ 757\ 761\ 769\ 773\ 787\ 797\ 809\ 811\ 821\ 823\ 827\ 829\ 839\ 853\ 857\ 859$ 863 877 881 883 887 907 911 919 929 937 941 947 953 967 971 977 983 991 997 1009 1013 1019 1021 1031 1033 1039 1049 1051 1061 1063 1069 1087 1091 1093 1097 1103  $1109\ 1117\ 1123\ 1129\ 1151\ 1153\ 1163\ 1171\ 1181\ 1187\ 1193\ 1201\ 1213\ 1217\ 1223\ 1229$ 1231 1237 1249 1259 1277 1279 1283 1289 1291 1297 1301 1303 1307 1319 1321 1327  $1361\ 1367\ 1373\ 1381\ 1399\ 1409\ 1423\ 1427\ 1429\ 1433\ 1439\ 1447\ 1451\ 1453\ 1459\ 1471$ 1481 1483 1487 1489 1493 1499 1511 1523 1531 1543 1549 1553 1559 1567 1571 1579  $1583\ 1597\ 1601\ 1607\ 1609\ 1613\ 1619\ 1621\ 1627\ 1637\ 1657\ 1663\ 1667\ 1669\ 1693\ 1697$ 1699 1709 1721 1723 1733 1741 1747 1753 1759 1777 1783 1787 1789 1801 1811 1823 1831 1847 1861 1867 1871 1873 1877 1879 1889 1901 1907 1913 1931 1933 1949 1951 1973 1979 1987 1993 1997 1999 2003 2011 2017 2027 2029 2039 2053 2063 2069 2081

#### Primos cercanos a $10^n$

9941 9949 9967 9973 10007 10009 10037 10039 10061 10067 10069 10079 99961 99971 99989 99991 100003 100019 100043 100049 100057 100069 999959 999961 999979 999983 1000003 1000033 1000037 1000039 9999943 9999971 9999991 10000019 10000079 10000103 10000121 99999941 99999959 9999971 99999989 100000007 100000037 100000039 100000049 99999893 99999929 99999937 1000000007 100000009 1000000021 1000000033

## Cantidad de primos menores que $10^n$

```
\pi(10^{1}) = 4 \; ; \; \pi(10^{2}) = 25 \; ; \; \pi(10^{3}) = 168 \; ; \; \pi(10^{4}) = 1229 \; ; \; \pi(10^{5}) = 9592
\pi(10^{6}) = 78.498 \; ; \; \pi(10^{7}) = 664.579 \; ; \; \pi(10^{8}) = 5.761.455 \; ; \; \pi(10^{9}) = 50.847.534
\pi(10^{10}) = 455.052,511 \; ; \; \pi(10^{11}) = 4.118.054.813 \; ; \; \pi(10^{12}) = 37.607.912.018
\textbf{Divisores}
Cantidad de divisores (\sigma_{0}) para algunos n/\neg \exists n' < n, \sigma_{0}(n') \geqslant \sigma_{0}(n)
```

```
\sigma_0(60) = 12; \sigma_0(120) = 16; \sigma_0(180) = 18; \sigma_0(240) = 20; \sigma_0(360) = 24
   \sigma_0(720) = 30; \sigma_0(840) = 32; \sigma_0(1260) = 36; \sigma_0(1680) = 40; \sigma_0(10080) = 72
       \sigma_0(15120) = 80; \sigma_0(50400) = 108; \sigma_0(83160) = 128; \sigma_0(110880) = 144
   \sigma_0(498960) = 200; \sigma_0(554400) = 216; \sigma_0(1081080) = 256; \sigma_0(1441440) = 288
                           \sigma_0(4324320) = 384; \sigma_0(8648640) = 448
            Suma de divisores (\sigma_1) para algunos n/\neg \exists n' < n, \sigma_1(n') \ge \sigma_1(n)
   \sigma_1(96) = 252; \sigma_1(108) = 280; \sigma_1(120) = 360; \sigma_1(144) = 403; \sigma_1(168) = 480
        \sigma_1(960) = 3048; \sigma_1(1008) = 3224; \sigma_1(1080) = 3600; \sigma_1(1200) = 3844
     \sigma_1(4620) = 16128; \sigma_1(4680) = 16380; \sigma_1(5040) = 19344; \sigma_1(5760) = 19890
   \sigma_1(8820) = 31122; \sigma_1(9240) = 34560; \sigma_1(10080) = 39312; \sigma_1(10920) = 40320
\sigma_1(32760) = 131040; \sigma_1(35280) = 137826; \sigma_1(36960) = 145152; \sigma_1(37800) = 148800
\sigma_1(60480) = 243840; \sigma_1(64680) = 246240; \sigma_1(65520) = 270816; \sigma_1(70560) = 280098
            \sigma_1(95760) = 386880; \sigma_1(98280) = 403200; \sigma_1(100800) = 409448
        \sigma_1(491400) = 2083200 : \sigma_1(498960) = 2160576 : \sigma_1(514080) = 2177280
        \sigma_1(982800) = 4305280; \sigma_1(997920) = 4390848; \sigma_1(1048320) = 4464096
    \sigma_1(4979520) = 22189440; \sigma_1(4989600) = 22686048; \sigma_1(5045040) = 23154768
    \sigma_1(9896040) = 44323200; \sigma_1(9959040) = 44553600; \sigma_1(9979200) = 45732192
```

## 7. Grafos

# 7.1. Dijkstra

```
#define INF 1e9
  int N;
   #define MAX_V 250001
   vector<ii> G[MAX_V];
   //To add an edge use
   #define add(a, b, w) G[a].pb(make_pair(w, b))
   ll dijkstra(int s, int t){\frac{}{|0|}} \log |V|
     priority_queue<ii, vector<ii>, greater<ii> > Q;
     vector<ll> dist(N, INF): vector<int> dad(N, -1);
9
     Q.push(make_pair(0, s)); dist[s] = 0;
10
     while(sz(Q)){
11
       ii p = Q.top(); Q.pop();
12
       if(p.snd == t) break;
13
       forall(it, G[p.snd])
14
```

return false;

14

1 //We have a vertex representing a var and other for his negation.

```
15 }
         if(dist[p.snd]+it->first < dist[it->snd]){
15
           dist[it->snd] = dist[p.snd] + it->fst;
16
                                                                                                               7.4. Kruskal
           dad[it->snd] = p.snd;
17
           Q.push(make_pair(dist[it->snd], it->snd)); }
18
                                                                                 struct Ar{int a,b,w;};
     }
19
                                                                                   | bool operator<(const Ar& a, const Ar &b){return a.w<b.w;}
     return dist[t];
20
                                                                                    vector<Ar> E;
     if(dist[t]<INF)//path generator</pre>
21
                                                                                   ll kruskal(){
       for(int i=t; i!=-1; i=dad[i])
22
                                                                                        11 cost=0:
         printf("%d%", i, (i==s?'\n':'\_'));}
23
                                                                                        sort(E.begin(), E.end());//ordenar aristas de menor a mayor
                                                                                 6
                          7.2. Bellman-Ford
                                                                                        uf.init(n);
                                                                                 7
                                                                                        forall(it, E){
                                                                                 8
   vector<ii> G[MAX_N];//ady. list with pairs (weight, dst)
                                                                                            if(uf.comp(it->a)!=uf.comp(it->b)){//si no estan conectados
                                                                                 9
   int dist[MAX_N];
                                                                                                uf.unir(it->a, it->b);//conectar
                                                                                 10
   void bford(int src){//O(VE)
                                                                                                cost+=it->w;
                                                                                11
     dist[src]=0:
                                                                                            }
                                                                                12
    forn(i, N-1) forn(j, N) if(dist[j]!=INF) forall(it, G[j])
                                                                                        }
5
                                                                                 13
       dist[it->snd]=min(dist[it->snd], dist[j]+it->fst);
6
                                                                                        return cost;
                                                                                 14
7
                                                                                 15 }
8
                                                                                                                 7.5. Prim
   bool hasNegCycle(){
9
    forn(j, N) if(dist[j]!=INF) forall(it, G[j])
10
                                                                                 bool taken[MAXN];
       if(dist[it->snd]>dist[j]+it->fst) return true;
11
                                                                                 priority_queue<ii, vector<ii>, greater<ii>> pq;//min heap
     //inside if: all points reachable from it->snd will have -INF distance
12
                                                                                    void process(int v){
         (do bfs)
                                                                                        taken[v]=true;
     return false;
13
                                                                                        forall(e, G[v])
14 }
                                                                                 5
                                                                                            if(!taken[e->second]) pg.push(*e);
                                                                                 6
                          7.3. Floyd-Warshall
                                                                                 7
   //G[i][j] contains weight of edge (i, j) or INF
                                                                                    ll prim(){
   //G[i][i]=0
2
                                                                                        zero(taken);
   int G[MAX_N] [MAX_N];
                                                                                        process(0);
                                                                                11
   void floyd(){//0(N^3)}
                                                                                        11 cost=0;
                                                                                 12
  forn(k, N) forn(i, N) if(G[i][k]!=INF) forn(j, N) if(G[k][j]!=INF)
                                                                                        while(sz(pq)){
                                                                                13
     G[i][j]=min(G[i][j], G[i][k]+G[k][j]);
6
                                                                                            ii e=pq.top(); pq.pop();
                                                                                14
7
                                                                                            if(!taken[e.second]) cost+=e.first, process(e.second);
                                                                                15
   bool inNegCycle(int v){
                                                                                        }
                                                                                 16
    return G[v][v]<0:}
                                                                                        return cost;
                                                                                17
   //checks if there's a neg. cycle in path from a to b
                                                                                 18 }
   bool hasNegCycle(int a, int b){
                                                                                                       7.6. 2-SAT + Tarjan SCC
    forn(i, N) if(G[a][i]!=INF && G[i][i]<0 && G[i][b]!=INF)
12
       return true:
13
```

```
2 //Every edge stored in G represents an implication. To add an equation
       of the form allb, use addor(a, b)
   //MAX=max cant var, n=cant var
   #define addor(a, b) (G[neg(a)].pb(b), G[neg(b)].pb(a))
   vector<int> G[MAX*2];
   //idx[i]=index assigned in the dfs
   //lw[i]=lowest index(closer from the root) reachable from i
   int lw[MAX*2], idx[MAX*2], qidx;
   stack<int> q;
   int gcmp, cmp[MAX*2];
   //verdad[cmp[i]]=valor de la variable i
   bool verdad[MAX*2+1];
   int neg(int x) { return x>=n? x-n : x+n;}
   void tjn(int v){
15
     lw[v]=idx[v]=++qidx;
16
     q.push(v), cmp[v]=-2;
17
     forall(it, G[v]){
18
       if(!idx[*it] || cmp[*it]==-2){
19
         if(!idx[*it]) tjn(*it);
20
         lw[v]=min(lw[v], lw[*it]);
21
       }
22
     }
23
     if(lw[v]==idx[v]){
24
       int x;
25
       do{x=q.top(); q.pop(); cmp[x]=qcmp;}while(x!=v);
26
       verdad[qcmp] = (cmp[neg(v)] < 0);</pre>
27
       qcmp++;
28
     }
29
30
    //remember to CLEAR G!!!
31
   bool satisf(){\frac{}{0(n)}}
     memset(idx, 0, sizeof(idx)), qidx=0;
33
     memset(cmp, -1, sizeof(cmp)), qcmp=0;
34
     forn(i, n){
35
       if(!idx[i]) tjn(i);
36
       if(!idx[neg(i)]) tjn(neg(i));
37
38
     forn(i, n) if(cmp[i]==cmp[neg(i)]) return false;
39
     return true;
40
41 }
```

#### 7.7. Articulation Points

```
1 | int N;
   vector<int> G[1000000];
   //V[i]=node number(if visited), L[i]= lowest V[i] reachable from i
   int qV, V[1000000], L[1000000], P[1000000];
   void dfs(int v, int f){
     L[v]=V[v]=++qV;
     forall(it, G[v])
       if(!V[*it]){
         dfs(*it, v);
         L[v] = min(L[v], L[*it]);
         P[v] += L[*it] >= V[v];
11
12
       else if(*it!=f)
13
         L[v]=min(L[v], V[*it]);
14
15
   int cantart() { //0(n)
     aV=0:
     zero(V), zero(P);
     dfs(1, 0); P[1]--;
19
     int q=0;
     forn(i, N) if(P[i]) q++;
21
   return q;
23 }
```

#### 7.8. Comp. Biconexas y Puentes

```
struct edge {
     int u, v, comp;
     bool bridge;
   };
4
   vector<edge> e;
   void addEdge(int u, int v) {
    G[u].pb(sz(e)), G[v].pb(sz(e));
     e.pb((edge){u,v,-1,false});
   }
9
   //d[i]=id de la dfs
   //b[i]=lowest id reachable from i
  int d[MAXN], b[MAXN], t;
   int nbc;//cant componentes
  int comp[MAXN];//comp[i]=cant comp biconexas a la cual pertenece i
   void initDfs(int n) {
     zero(G), zero(comp);
16
     e.clear();
```

```
forn(i,n) d[i]=-1;
     nbc = t = 0;
19
   }
20
   stack<int> st;
   void dfs(int u, int pe) \{//0(n + m)\}
     b[u] = d[u] = t++;
23
     comp[u] = (pe != -1);
     forall(ne, G[u]) if (*ne != pe){
25
       int v = e[*ne].u ^e[*ne].v ^u;
26
       if (d[v] == -1) {
27
         st.push(*ne);
28
         dfs(v,*ne);
29
         if (b[v] > d[u])
30
            e[*ne].bridge = true; // bridge
31
         }
32
         if (b[v] >= d[u]) \{ // art \}
33
           int last:
34
           do {
35
             last = st.top(); st.pop();
36
              e[last].comp = nbc;
37
           } while (last != *ne);
38
           nbc++;
39
            comp[u]++;
40
41
         b[u] = min(b[u], b[v]);
42
43
       else if (d[v] < d[u]) \{ // back edge
44
         st.push(*ne);
45
         b[u] = min(b[u], d[v]);
46
       }
47
     }
48
49
```

#### 7.9. LCA + Climb

```
const int MAXN=100001;
const int LOGN=20;

//f[v][k] holds the 2^k father of v

//L[v] holds the level of v

int N, f[MAXN][LOGN], L[MAXN];

//call before build:
void dfs(int v, int fa=-1, int lvl=0){//generate required data}

f[v][0]=fa, L[v]=lvl;
```

```
forall(it, G[v])if(*it!=fa) dfs(*it, v, lvl+1); }
   void build(){//f[i][0] must be filled previously, O(nlgn)
    forn(k, LOGN-1) forn(i, N) f[i][k+1]=f[f[i][k]][k];}
   #define lg(x) (31-__builtin_clz(x))//=floor(log2(x))
   int climb(int a, int d){\frac{1}{0}}
    if(!d) return a;
     dforn(i, lg(L[a])+1) if(1<<i<=d) a=f[a][i], d-=1<<i;
      return a;}
   int lca(int a, int b){\frac{1}{0}}
    if(L[a]<L[b]) swap(a, b);
    a=climb(a, L[a]-L[b]);
19
    if(a==b) return a;
     dforn(i, lg(L[a])+1) if(f[a][i]!=f[b][i]) a=f[a][i], b=f[b][i];
    return f[a][0]; }
   int dist(int a, int b) {//returns distance between nodes
    return L[a]+L[b]-2*L[lca(a, b)];}
                 7.10. Heavy Light Decomposition
int treesz[MAXN];//cantidad de nodos en el subarbol del nodo v
  int dad[MAXN];//dad[v]=padre del nodo v
   void dfs1(int v, int p=-1){//pre-dfs
     dad[v]=p;
     treesz[v]=1:
     forall(it, G[v]) if(*it!=p){
       dfs1(*it, v);
7
       treesz[v]+=treesz[*it];
8
9
10
   //PONER Q EN O !!!!!
   int pos[MAXN], q;//pos[v]=posicion del nodo v en el recorrido de la dfs
   //Las cadenas aparecen continuas en el recorrido!
   int cantcad;
   int homecad[MAXN];//dada una cadena devuelve su nodo inicial
   int cad[MAXN];//cad[v]=cadena a la que pertenece el nodo
   void heavylight(int v, int cur=-1){
    if(cur==-1) homecad[cur=cantcad++]=v;
     pos[v]=a++:
19
     cad[v]=cur;
20
     int mx=-1;
21
     forn(i, sz(G[v])) if(G[v][i]!=dad[v])
22
       if(mx==-1 || treesz[G[v][mx]]<treesz[G[v][i]]) mx=i;</pre>
23
```

if(mx!=-1) heavylight(G[v][mx], cur);

24

```
forn(i, sz(G[v])) if(i!=mx && G[v][i]!=dad[v])
25
       heavylight(G[v][i], -1);
^{26}
   }
27
    //ejemplo de obtener el maximo numero en el camino entre dos nodos
   //RTA: max(query(low, u), query(low, v)), con low=lca(u, v)
   //esta funcion va trepando por las cadenas
   int query(int an, int v){//0(logn)
     //si estan en la misma cadena:
     if(cad[an] == cad[v]) return rmq.get(pos[an], pos[v]+1);
33
     return max(query(an, dad[homecad[cad[v]]]),
34
            rmq.get(pos[homecad[cad[v]]], pos[v]+1));
35
36
                    7.11. Centroid Decomposition
  int n;
1
   vector<int> G[MAXN];
   bool taken[MAXN];//poner todos en FALSE al principio!!
   int padre [MAXN]; //padre de cada nodo en el centroid tree
5
   int szt[MAXN];
   void calcsz(int v, int p) {
     szt[v] = 1:
     forall(it,G[v]) if (*it!=p && !taken[*it])
       calcsz(*it,v), szt[v]+=szt[*it];
10
11
   void centroid(int v=0, int f=-1, int lvl=0, int tam=-1) {//0(nlogn)
     if(tam==-1) calcsz(v, -1), tam=szt[v];
13
     forall(it, G[v]) if(!taken[*it] && szt[*it]>=tam/2)
14
       {szt[v]=0; centroid(*it, f, lvl, tam); return;}
15
     taken[v]=true;
16
     padre[v]=f;
17
     forall(it, G[v]) if(!taken[*it])
18
       centroid(*it, v, lvl+1, -1);
19
20 | }
                           7.12. Euler Cycle
int n,m,ars[MAXE], eq;
  vector<int> G[MAXN];//fill G,n,m,ars,eq
  list<int> path;
   int used[MAXN];
  bool usede[MAXE];
```

queue<list<int>::iterator> q;

```
7 int get(int v){
     while(used[v]<sz(G[v]) && usede[ G[v][used[v]] ]) used[v]++;</pre>
     return used[v];
10
   void explore(int v, int r, list<int>::iterator it){
11
     int ar=G[v][get(v)]; int u=v^ars[ar];
12
     usede[ar]=true;
13
     list<int>::iterator it2=path.insert(it, u);
14
     if(u!=r) explore(u, r, it2);
15
     if(get(v)<sz(G[v])) q.push(it);</pre>
16
17
   void euler(){
     zero(used), zero(usede):
     path.clear();
20
     q=queue<list<int>::iterator>();
21
     path.push_back(0); q.push(path.begin());
     while(sz(q)){
23
       list<int>::iterator it=q.front(); q.pop();
24
       if(used[*it] < sz(G[*it])) explore(*it, *it, it);</pre>
25
26
     reverse(path.begin(), path.end());
27
28
   void addEdge(int u, int v){
29
     G[u].pb(eq), G[v].pb(eq);
     ars[eq++]=u^v;
31
32 }
                         7.13. Diametro árbol
   vector<int> G[MAXN]; int n,m,p[MAXN],d[MAXN],d2[MAXN];
   int bfs(int r, int *d) {
     queue<int> q;
     d[r]=0; q.push(r);
     int v;
5
     while(sz(q)) { v=q.front(); q.pop();
6
       forall(it,G[v]) if (d[*it]==-1)
         d[*it]=d[v]+1, p[*it]=v, q.push(*it);
8
     }
9
     return v;//ultimo nodo visitado
10
11
   vector<int> diams; vector<ii> centros;
   void diametros(){
     memset(d,-1,sizeof(d));
```

mark[v] = true;

```
memset(d2,-1,sizeof(d2));
15
     diams.clear(), centros.clear();
16
     forn(i, n) if(d[i]==-1){
17
       int v,c;
18
       c=v=bfs(bfs(i, d2), d);
19
       forn(_,d[v]/2) c=p[c];
20
       diams.pb(d[v]);
21
       if(d[v]&1) centros.pb(ii(c, p[c]));
22
       else centros.pb(ii(c, c));
23
24
25
26
   int main() {
27
     freopen("in", "r", stdin);
28
     while(cin >> n >> m)
29
       forn(i,m) { int a,b; cin >> a >> b; a--, b--;
30
         G[a].pb(b);
31
         G[b].pb(a);
32
                              7.14. Chu-liu
   void visit(graph &h, int v, int s, int r,
     vector<int> &no, vector< vector<int> > &comp,
     vector<int> &prev, vector< vector<int> > &next, vector<weight> &mcost,
3
     vector<int> &mark, weight &cost, bool &found) {
4
     if (mark[v]) {
5
       vector<int> temp = no;
6
       found = true;
7
       do {
8
         cost += mcost[v];
9
         v = prev[v];
10
         if (v != s) {
11
           while (comp[v].size() > 0) {
12
             no[comp[v].back()] = s;
13
             comp[s].push_back(comp[v].back());
14
             comp[v].pop_back();
15
16
         }
17
       } while (v != s);
18
       forall(j,comp[s]) if (*j != r) forall(e,h[*j])
19
         if (no[e->src] != s) e->w -= mcost[ temp[*j] ];
20
     }
21
```

```
forall(i,next[v]) if (no[*i] != no[v] && prev[no[*i]] == v)
       if (!mark[no[*i]] || *i == s)
24
         visit(h, *i, s, r, no, comp, prev, next, mcost, mark, cost, found)
25
26
   weight minimumSpanningArborescence(const graph &g, int r) {
       const int n=sz(g);
28
     graph h(n);
29
     forn(u,n) forall(e,g[u]) h[e->dst].pb(*e);
     vector<int> no(n);
     vector<vector<int> > comp(n);
32
     forn(u, n) comp[u].pb(no[u] = u);
33
     for (weight cost = 0; ;) {
34
       vector<int> prev(n, -1);
35
       vector<weight> mcost(n, INF);
36
       forn(j,n) if (j != r) forall(e,h[j])
         if (no[e->src] != no[j])
           if (e->w < mcost[ no[j] ])</pre>
             mcost[no[j]] = e->w, prev[no[j]] = no[e->src];
40
       vector< vector<int> > next(n);
       forn(u,n) if (prev[u] >= 0)
42
         next[ prev[u] ].push_back(u);
       bool stop = true;
44
       vector<int> mark(n);
45
       forn(u,n) if (u != r && !mark[u] && !comp[u].empty()) {
46
         bool found = false;
47
         visit(h, u, u, r, no, comp, prev, next, mcost, mark, cost, found);
         if (found) stop = false;
49
       }
50
       if (stop) {
         forn(u,n) if (prev[u] >= 0) cost += mcost[u];
         return cost:
       }
54
    }
55
56 }
```

## 7.15. Hungarian

```
//Dado un grafo bipartito completo con costos no negativos, encuentra el
matching perfecto de minimo costo.

tipo cost[N][N], lx[N], ly[N], slack[N]; //llenar: cost=matriz de
adyacencia
int n, max_match, xy[N], yx[N], slackx[N], prev2[N]; //n=cantidad de nodos
```

```
4 bool S[N], T[N]; //sets S and T in algorithm
                                                                                         max match++:
                                                                                  46
  void add_to_tree(int x, int prevx) {
                                                                                         for (int cx = x, cy = y, ty; cx != -2; cx = prev2[cx], cy = ty)
                                                                                  47
     S[x] = true, prev2[x] = prevx;
                                                                                           ty = xy[cx], yx[cy] = cx, xy[cx] = cy;
                                                                                  48
    forn(y, n) if (lx[x] + ly[y] - cost[x][y] < slack[y] - EPS)
                                                                                         augment(); }
                                                                                  49
       slack[y] = lx[x] + ly[y] - cost[x][y], slackx[y] = x;
                                                                                     }
                                                                                  50
                                                                                     tipo hungarian(){
9
   void update_labels(){
                                                                                       tipo ret = 0; max_match = 0, memset(xy, -1, sizeof(xy));
                                                                                  52
                                                                                       memset(yx, -1, sizeof(yx)), init_labels(), augment(); //steps 1-3
     tipo delta = INF;
                                                                                       forn (x,n) ret += cost[x][xy[x]]; return ret;
     forn (y, n) if (!T[y]) delta = min(delta, slack[y]);
     form (x, n) if (S[x]) lx[x] -= delta;
                                                                                  55 }
13
     forn (y, n) if (T[y]) ly[y] += delta; else slack[y] -= delta;
14
                                                                                                        7.16. Dynamic Conectivity
15
   void init_labels(){
     zero(lx), zero(ly);
                                                                                   1 | struct UnionFind {
     form (x,n) form(y,n) lx[x] = max(lx[x], cost[x][y]);
                                                                                         int n, comp;
18
19
                                                                                         vector<int> pre,si,c;
   void augment() {
20
                                                                                         UnionFind(int n=0):n(n), comp(n), pre(n), si(n, 1) {
     if (max_match == n) return;
                                                                                             forn(i,n) pre[i] = i; }
21
                                                                                  5
     int x, y, root, q[N], wr = 0, rd = 0;
                                                                                         int find(int u){return u==pre[u]?u:find(pre[u]);}
22
     memset(S, false, sizeof(S)), memset(T, false, sizeof(T));
                                                                                         bool merge(int u, int v) {
23
     memset(prev2, -1, sizeof(prev2));
                                                                                             if((u=find(u))==(v=find(v))) return false;
24
     forn (x, n) if (xy[x] == -1){
25
                                                                                             if(si[u]<si[v]) swap(u, v);</pre>
       q[wr++] = root = x, prev2[x] = -2;
                                                                                             si[u]+=si[v], pre[v]=u, comp--, c.pb(v);
26
       S[x] = true; break; }
27
                                                                                             return true:
                                                                                  11
     form (y, n) slack[y] = lx[root] + ly[y] - cost[root][y], slackx[y] =
28
                                                                                  12
         root;
                                                                                         int snap(){return sz(c);}
                                                                                  13
     while (true){
                                                                                         void rollback(int snap){
29
                                                                                  14
       while (rd < wr){
                                                                                             while(sz(c)>snap){
30
                                                                                  15
         x = q[rd++];
                                                                                                 int v = c.back(); c.pop_back();
31
                                                                                  16
        for (y = 0; y < n; y++) if (cost[x][y] == lx[x] + ly[y] && !T[y]){
                                                                                                 si[pre[v]] -= si[v], pre[v] = v, comp++;
32
                                                                                  17
           if (yx[y] == -1) break; T[y] = true;
33
                                                                                  18
           q[wr++] = yx[y], add_to_tree(yx[y], x); }
34
                                                                                  19
         if (y < n) break; }
                                                                                     };
35
                                                                                  20
       if (v < n) break:
                                                                                     enum {ADD,DEL,QUERY};
36
       update_labels(), wr = rd = 0;
                                                                                     struct Query {int type,u,v;};
37
       for (y = 0; y < n; y++) if (!T[y] \&\& slack[y] == 0){
                                                                                     struct DynCon {
38
         if (yx[y] == -1)\{x = slackx[y]; break;\}
39
                                                                                         vector<Query> q;
                                                                                  24
         else{
                                                                                         UnionFind dsu:
40
                                                                                  25
           T[v] = true;
                                                                                         vector<int> match,res;
41
                                                                                  26
           if (!S[yx[y]]) q[wr++] = yx[y], add_to_tree(yx[y], slackx[y]);
                                                                                         map<ii,int> last;//se puede no usar cuando hay identificador para
42
43
                                                                                             cada arista (mejora poco)
       if (y < n) break; }</pre>
                                                                                         DynCon(int n=0):dsu(n){}
                                                                                  28
     if (y < n){
                                                                                         void add(int u, int v) {
                                                                                  29
```

```
if(u>v) swap(u,v);
30
           q.pb((Query){ADD, u, v}), match.pb(-1);
31
           last[ii(u,v)] = sz(q)-1;
32
       }
33
       void remove(int u, int v) {
34
           if(u>v) swap(u,v);
35
           q.pb((Query){DEL, u, v});
36
           int prev = last[ii(u,v)];
37
           match[prev] = sz(q)-1;
38
           match.pb(prev);
39
       }
40
       void query() {//podria pasarle un puntero donde guardar la respuesta
41
           q.pb((Query){QUERY, -1, -1}), match.pb(-1);}
42
       void process() {
43
           forn(i,sz(q)) if (q[i].type == ADD && match[i] == -1) match[i] =
44
                 sz(q);
           go(0,sz(q));
45
       }
46
       void go(int 1, int r) {
47
           if(l+1==r){
48
               if (q[1].type == QUERY)//Aqui responder la query usando el
49
                    dsu!
                    res.pb(dsu.comp);//aqui query=cantidad de componentes
50
                        conexas
               return;
51
52
           int s=dsu.snap(), m = (l+r) / 2;
53
           forr(i,m,r) if(match[i]!=-1 && match[i]<1) dsu.merge(q[i].u, q[i</pre>
54
               ].v);
           go(1,m);
55
           dsu.rollback(s);
56
           s = dsu.snap();
57
           forr(i,1,m) if(match[i]!=-1 && match[i]>=r) dsu.merge(q[i].u, q[
58
               il.v):
           go(m,r);
59
           dsu.rollback(s):
60
       }
61
62 | }dc;
```

#### 8. Network Flow

## 8.1. Dinic

```
1
2 const int MAX = 300;
3 // Corte minimo: vertices con dist[v]>=0 (del lado de src) VS. dist[v
       ]==-1 (del lado del dst)
4 // Para el caso de la red de Bipartite Matching (Sean V1 y V2 los
       conjuntos mas proximos a src y dst respectivamente):
5 // Reconstruir matching: para todo v1 en V1 ver las aristas a vertices
       de V2 con it->f>0, es arista del Matching
6 // Min Vertex Cover: vertices de V1 con dist[v]==-1 + vertices de V2 con
        dist[v]>0
7 // Max Independent Set: tomar los vertices NO tomados por el Min Vertex
s // Max Clique: construir la red de G complemento (debe ser bipartito!) y
        encontrar un Max Independet Set
9 // Min Edge Cover: tomar las aristas del matching + para todo vertices
       no cubierto hasta el momento, tomar cualquier arista de el
  int nodes, src, dst;
int dist[MAX], q[MAX], work[MAX];
struct Edge {
       int to, rev;
       ll f, cap;
14
       Edge(int to, int rev, ll f, ll cap) : to(to), rev(rev), f(f), cap(
15
           cap) {}
   };
16
   vector<Edge> G[MAX];
   void addEdge(int s, int t, ll cap){
       G[s].pb(Edge(t, sz(G[t]), 0, cap)), G[t].pb(Edge(s, sz(G[s])-1, 0,
19
           0)):}
   bool dinic_bfs(){
       fill(dist, dist+nodes, -1), dist[src]=0;
       int qt=0; q[qt++]=src;
22
       for(int qh=0; qh<qt; qh++){</pre>
23
           int u =q[qh];
24
           forall(e, G[u]){
25
               int v=e->to;
26
               if(dist[v]<0 && e->f < e->cap)
27
                   dist[v]=dist[u]+1, q[qt++]=v;
28
           }
29
30
       return dist[dst]>=0;
31
32
   11 dinic_dfs(int u, ll f){
33
       if(u==dst) return f;
34
```

```
for(int &i=work[u]; i<sz(G[u]); i++){</pre>
35
           Edge &e = G[u][i];
36
           if(e.cap<=e.f) continue;</pre>
37
           int v=e.to;
38
           if(dist[v]==dist[u]+1){
39
                    11 df=dinic_dfs(v, min(f, e.cap-e.f));
40
                    if(df>0){
41
                             e.f+=df, G[v][e.rev].f-= df;
42
                             return df; }
43
44
       }
45
       return 0;
46
47
   ll maxFlow(int _src, int _dst){
       src=_src, dst=_dst;
49
       11 result=0;
50
       while(dinic_bfs()){
51
           fill(work, work+nodes, 0);
52
           while(ll delta=dinic_dfs(src,INF))
53
                result+=delta;
54
       }
55
       // todos los nodos con dist[v]!=-1 vs los que tienen dist[v]==-1
56
           forman el min-cut
       return result; }
57
```

# **8.2.** Konig

```
// asume que el dinic YA ESTA tirado
   // asume que nodes-1 y nodes-2 son la fuente y destino
  int match[maxnodes]; // match[v] = u si u-v esta en el matching, -1 si v
       no esta matcheado
  int s[maxnodes]; // numero de la bfs del koning
   queue<int> kq;
   // s[e] %2==1 o si e esta en V1 y s[e]==-1-> lo agarras
   void koning() \{//0(n)\}
     forn(v,nodes-2) s[v] = match[v] = -1;
8
     forn(v,nodes-2) forall(it,g[v]) if (it->to < nodes-2 && it->f>0)
9
       { match[v]=it->to; match[it->to]=v;}
10
     form(v,nodes-2) if (match[v]==-1) {s[v]=0;kq.push(v);}
11
     while(!kq.empty()) {
12
       int e = kq.front(); kq.pop();
13
       if (s[e] %2==1) {
14
         s[match[e]] = s[e]+1;
15
```

```
kq.push(match[e]);
16
       } else {
17
18
         forall(it,g[e]) if (it->to < nodes-2 && s[it->to]==-1) {
19
            s[it->to] = s[e]+1;
20
            kq.push(it->to);
21
         }
22
23
     }
24
25 | }
```

#### 8.3. Edmonds Karp's

```
#define MAX_V 1000
   #define INF 1e9
   //special nodes
   #define SRC 0
   #define SNK 1
   map<int, int> G[MAX_V];//limpiar esto
   //To add an edge use
   #define add(a, b, w) G[a][b]=w
   int f, p[MAX_V];
   void augment(int v, int minE){
    if(v==SRC) f=minE:
     else if(p[v]!=-1){
       augment(p[v], min(minE, G[p[v]][v]));
13
       G[p[v]][v]-=f, G[v][p[v]]+=f;
14
     }
15
   }
16
   ll maxflow(){//O(VE^2)
     ll Mf=0;
18
     do{
19
       f=0;
20
       char used[MAX_V]; queue<int> q; q.push(SRC);
21
       zero(used), memset(p, -1, sizeof(p));
22
       while(sz(q)){
23
         int u=q.front(); q.pop();
24
         if(u==SNK) break;
25
         forall(it, G[u])
26
           if(it->snd>0 && !used[it->fst])
27
             used[it->fst]=true, q.push(it->fst), p[it->fst]=u;
28
       }
29
       augment(SNK, INF);
30
```

```
Mf+=f:
31
     }while(f);
                                                                                    38
32
     return Mf;
                                                                                    39
33
34 }
                                                                                    40
                                                                                    41
                       8.4. Push-Relabel O(N3)
                                                                                    42
                                                                                    43
   #define MAX V 1000
                                                                                    44
  int N;//valid nodes are [0...N-1]
   #define INF 1e9
   //special nodes
                                                                                    47
   #define SRC 0
                                                                                    48
                                                                                         }
   #define SNK 1
                                                                                    49
   map<int, int> G[MAX_V];
   //To add an edge use
   #define add(a, b, w) G[a][b]=w
                                                                                    52
   11 excess[MAX_V];
   int height[MAX_V], active[MAX_V], count[2*MAX_V+1];
   queue<int> Q;
                                                                                    55
                                                                                         }
   void enqueue(int v) {
                                                                                    56
                                                                                         ll mf=0;
     if (!active[v] && excess[v] > 0) active[v]=true, Q.push(v); }
                                                                                    57
   void push(int a, int b) {
     int amt = min(excess[a], ll(G[a][b]));
                                                                                    59
16
                                                                                    60 }
     if(height[a] <= height[b] || amt == 0) return;</pre>
17
     G[a][b]-=amt, G[b][a]+=amt;
18
     excess[b] += amt, excess[a] -= amt;
19
     enqueue(b);
20
21
    void gap(int k) {
22
     forn(v, N){
23
       if (height[v] < k) continue;
24
       count[height[v]]--;
25
       height[v] = max(height[v], N+1);
26
       count[height[v]]++;
27
       enqueue(v);
28
                                                                                         tc cost;
29
                                                                                    10
30
   void relabel(int v) {
                                                                                    11
31
     count[height[v]]--;
32
     height[v] = 2*N;
33
     forall(it, G[v])
34
       if(it->snd)
35
         height[v] = min(height[v], height[it->fst] + 1);
36
```

```
count[height[v]]++;
37
     enqueue(v);
   ll maxflow() {\frac{}{0}(V^3)}
     zero(height), zero(active), zero(count), zero(excess);
     count[0] = N-1;
     count[N] = 1;
     height[SRC] = N;
     active[SRC] = active[SNK] = true;
     forall(it, G[SRC]){
       excess[SRC] += it->snd;
       push(SRC, it->fst);
     while(sz(Q)) {
       int v = Q.front(); Q.pop();
       active[v]=false;
     forall(it, G[v]) push(v, it->fst);
     if(excess[v] > 0)
       count[height[v]] == 1? gap(height[v]):relabel(v);
     forall(it, G[SRC]) mf+=G[it->fst][SRC];
     return mf;
```

#### 8.5. Min-cost Max-flow

```
const int MAXN=10000;
typedef ll tf;
typedef ll tc;
const tf INFFLUJO = 1e14;
const tc INFCOSTO = 1e14;
struct edge {
  int u, v;
  tf cap, flow;
  tc cost;
  tf rem() { return cap - flow; }
};
int nodes; //numero de nodos
vector<int> G[MAXN]; // limpiar!
vector<edge> e; // limpiar!
void addEdge(int u, int v, tf cap, tc cost) {
  G[u].pb(sz(e)); e.pb((edge){u,v,cap,0,cost});
```

```
G[v].pb(sz(e)); e.pb((edge){v,u,0,0,-cost});
18
   tc dist[MAXN], mnCost;
   int pre[MAXN];
   tf cap[MAXN], mxFlow;
   bool in_queue[MAXN];
   void flow(int s, int t) {
     zero(in_queue);
     mxFlow=mnCost=0;
25
     while(1){
26
       fill(dist, dist+nodes, INFCOSTO); dist[s] = 0;
27
       memset(pre, -1, sizeof(pre)); pre[s]=0;
28
       zero(cap); cap[s] = INFFLUJO;
29
       queue<int> q; q.push(s); in_queue[s]=1;
30
       while(sz(q)){
31
         int u=q.front(); q.pop(); in_queue[u]=0;
32
         for(auto it:G[u]) {
33
           edge &E = e[it];
34
           if(E.rem() \&\& dist[E.v] > dist[u] + E.cost + 1e-9){ // ojo EPS}
35
             dist[E.v] = dist[u] + E.cost;
36
             pre[E.v] = it;
37
             cap[E.v] = min(cap[u], E.rem());
38
             if(!in_queue[E.v]) q.push(E.v), in_queue[E.v]=1;
39
40
         }
41
42
       if (pre[t] == -1) break;
43
       mxFlow +=cap[t];
44
       mnCost +=cap[t]*dist[t];
45
       for (int v = t; v != s; v = e[pre[v]].u) {
46
         e[pre[v]].flow += cap[t];
47
         e[pre[v]^1].flow -= cap[t];
48
49
     }
50
  |}
51
```

# 9. Template

```
//touch {a..m}.in; tee {a..m}.cpp < template.cpp
#include <bits/stdc++.h>
using namespace std;
#define forr(i,a,b) for(int i=(a); i<(b); i++)
#define forn(i,n) forr(i,0,n)</pre>
```

```
6 #define sz(c) ((int)c.size())
   #define zero(v) memset(v, 0, sizeof(v))
   #define forall(it,v) for(auto it=v.begin();it!=v.end();++it)
   #define pb push_back
   #define fst first
   #define snd second
   typedef long long 11;
   typedef pair<int,int> ii;
   #define dforn(i,n) for(int i=n-1; i>=0; i--)
   #define dprint(v) cout << #v"=" << v << endl //;)</pre>
16
   const int MAXN=100100;
   int n;
18
19
   int main() {
       freopen("input.in", "r", stdin);
21
       ios::sync_with_stdio(0);
22
       while(cin >> n){
23
24
       }
25
       return 0;
26
27 }
```

# 10. Ayudamemoria

Cant. decimales

```
#include <iomanip>
cout << setprecision(2) << fixed;

Rellenar con espacios(para justificar)

#include <iomanip>
cout << setfill('u') << setw(3) << 2 << endl;

Leer hasta fin de linea

#include <sstream>
```

//hacer cin.ignore() antes de getline()

istringstream is(line);

cout << X << "";

while(getline(cin, line)){

while(is >> X)

cout << endl;</pre>

4

5

6

```
8 | }
                               Aleatorios
#define RAND(a, b) (rand()%(b-a+1)+a)
srand(time(NULL));
                           Doubles Comp.
 const double EPS = 1e-9;
  x == y \iff fabs(x-y) \iff EPS
 |x > y <=> x > y + EPS
_4 | x >= y <=> x > y - EPS
                                Limites
  #include <limits>
  numeric limits<T>
    ::max()
    ::min()
4
    ::epsilon()
                               Muahaha
1 #include <signal.h>
 void divzero(int p){
    while(true);}
  void segm(int p){
    exit(0);}
   //in main
  signal(SIGFPE, divzero);
8 | signal(SIGSEGV, segm);
                          Mejorar velocidad
i lios::sync_with_stdio(false);
                         Mejorar velocidad 2
 //Solo para enteros positivos
  inline void Scanf(int& a){
    char c = 0;
    while(c<33) c = getc(stdin);</pre>
    while(c>33) a = a*10 + c - '0', c = getc(stdin);
7 | }
```

## Expandir pila

```
#include <sys/resource.h>
rlimit rl;
getrlimit(RLIMIT_STACK, &rl);
rl.rlim_cur=1024L*1024L*256L;//256mb
setrlimit(RLIMIT_STACK, &rl);

C++11

Leer del teclado

lefreopen("/dev/tty", "a", stdin);

Iterar subconjunto

for(int sbm=bm; sbm; sbm=(sbm-1)&bm)

File setup

//tambien se pueden usar comas: {a, x, m, 1}
touch {a..l}.in; tee {a..l}.cpp < template.cpp</pre>
```