

# Índice

Ι.	1. Algorithm	2
2.	2. Estructuras	<b>2</b>
	2.1. RMQ (static)	 2
	2.2. RMQ (dynamic)	 2
	2.3. Fenwick Tree	3
	2.4. Union Find	 3
	2.5. Disjoint Intervals	 3
	2.6. RMQ (2D)	 3
	2.7. Big Int	 4
	2.8. Modnum	5
	2.9. Bittrie	 5
3.	3. Strings	6
	3.1. Trie	 6
	3.2. Suffix Array	 6
	3.3. String Matching With Suffix Array	 6
	3.4. LCP (Longest Common Prefix)	7
	3.5. Corasick	7

4.	Geometría	7
	4.1. Punto	7
	4.2. Line	8
	4.3. Segment	8
	4.4. Rectangle	8
	4.5. Polygon Area	8
	4.6. Circle	9
	4.7. Point in Poly	9
	4.8. Convex Check CHECK	9
	4.9. Convex Hull	9
	4.10. Cut Polygon	9
	4.11. Bresenham	10
	4.12. Rotate Matrix	10
<b>5.</b>	Math	<b>10</b>
	5.1. GCD	10
	5.2. LCM	10
	5.3. Simpson	10
	5.4. Fraction	10
	5.5. Polinomio	10
_		
6.	Grafos	11
	6.1. Dijkstra	11
	6.2. Bellman-Ford	12
	6.3. Floyd-Warshall	12
	6.4. 2-SAT + Tarjan SCC	12
	6.5. Articulation Points	13
	6.6. LCA + Climb	13
7	Network Flow	13
٠.	7.1. Edmonds Karp's	13
	7.2. Push-Relabel	13
	(.2. 1 usii-iveiauci	19
8.	Ayudamemoria	14

# 1. Algorithm

#include <algorithm> #include <numeric>

Algo	Params	Funcion
sort, stable_sort	f, 1	ordena el intervalo
$nth\_element$	f, nth, l	void ordena el n-esimo, y
		particiona el resto
fill, fill_n	f, l / n, elem	void llena [f, l) o [f,
		f+n) con elem
lower_bound, upper_bound	f, l, elem	it al primer / ultimo donde se
		puede insertar elem para que
		quede ordenada
binary_search	f, l, elem	bool esta elem en [f, l)
copy	f, l, resul	hace resul+ $i$ =f+ $i$ $\forall i$
find, find_if, find_first_of	f, l, elem	$it$ encuentra i $\in$ [f,l) tq. i=elem,
	/ pred / f2, l2	$\operatorname{pred}(i), i \in [f2,l2)$
count, count_if	f, l, elem/pred	cuenta elem, pred(i)
search	f, l, f2, l2	busca $[f2,l2) \in [f,l)$
replace, replace_if	f, l, old	cambia old / pred(i) por new
	/ pred, new	
reverse	f, 1	da vuelta
partition, stable_partition	f, l, pred	pred(i) ad, !pred(i) atras
min_element, max_element	f, l, [comp]	it min, max de [f,l]
lexicographical_compare	f1,l1,f2,l2	bool con [f1,l1];[f2,l2]
next/prev_permutation	f,l	deja en [f,l) la perm sig, ant
set_intersection,	f1, l1, f2, l2, res	[res,) la op. de conj
set_difference, set_union,		
set_symmetric_difference,		
push_heap, pop_heap,	f, l, e / e /	mete/saca e en heap [f,l),
make_heap		hace un heap de [f,l)
is_heap	f,l	bool es [f,l) un heap
accumulate	f,l,i,[op]	$T = \sum \text{oper de [f,l)}$
$inner\_product$	f1, l1, f2, i	$T = i + [f1, 11) \cdot [f2, \dots)$
partial_sum	f, l, r, [op]	$r+i = \sum /oper de [f,f+i] \forall i \in [f,l)$
_builtin_ffs	unsigned int	Pos. del primer 1 desde la derecha
_builtin_clz	unsigned int	Cant. de ceros desde la izquierda.
_builtin_ctz	unsigned int	Cant. de ceros desde la derecha.
_builtin_popcount	unsigned int	Cant. de 1's en x.
builtin nomiter	unsigned int	1 si x es par, 0 si es impar.
builtin_paritybuiltin_llXXXXXX	dibiglied int	I of it of part, o of of impart

#### 2. Estructuras

## 2.1. RMQ (static)

Dado un arreglo y una operación asociativa idempotente, get(i, j) opera sobre el rango [i, j). Restricción: LVL  $\geq 2$ \*ceil(logn); Usar [ ] para llenar arreglo y luego build().

```
1 struct RMQ{
     #define LVL 10
     tipo vec[LVL] [1<<(LVL+1)];
     tipo &operator[](int p){return vec[0][p];}
     tipo get(int i, int j) {//intervalo [i,j)
       int p = 31-__builtin_clz(j-i);
       return min(vec[p][i],vec[p][j-(1<<p)]);
7
8
     void build(int n) {//O(nlogn)
       int mp = 31-__builtin_clz(n);
       forn(p, mp) forn(x, n-(1 << p))
11
         vec[p+1][x] = min(vec[p][x], vec[p][x+(1<<p)]);
12
    }
13
14 };
```

## 2.2. RMQ (dynamic)

```
1 //Dado un arreglo y una operacion asociativa con neutro, get(i, j) opera
        sobre el rango [i, j).
2 #define MAXN 100000
   struct RMQ{
     int sz;
     tipo t[4*MAXN];
     tipo &operator[](int p){return t[sz+p];}
     void init(int n){//O(nlgn)
       sz = 1 \ll (32-\_builtin\_clz(n));
       fill(t, t+2*sz, 0); // O=elemento neutro
9
    }
10
     void updall(){//0(n)}
11
       dforsn(i, 0, sz) t[i]=max(t[2*i], t[2*i+1]);}
12
     tipo get(int i, int j){return get(i,j,1,0,sz);}
13
     tipo get(int i, int j, int n, int a, int b){//0(lgn)
14
       if(j<=a || i>=b) return 0;//neutro
15
       if(i<=a && b<=j) return t[n];
16
       int c=(a+b)/2;
17
       return max(get(i, j, 2*n, a, c), get(i, j, 2*n+1, c, b));
```

```
2
                                                                                        vector<int> f;//the array contains the parent of each node
19
     void set(int p, tipo val){//O(lgn)
                                                                                        void init(int n){f.clear(); f.insert(f.begin(), n, -1);}
                                                                                   3
20
       for(p+=sz; p>0 && t[p]!=val;){
                                                                                        int comp(int x){return (f[x]=-1?x:f[x]=comp(f[x]));}//0(1)
21
                                                                                   4
         t[p]=val;
                                                                                        bool join(int i, int j) {
^{22}
         p/=2;
                                                                                          bool con=comp(i)==comp(j);
                                                                                   6
23
         val=max(t[p*2], t[p*2+1]);
                                                                                          if(!con) f[comp(i)] = comp(j);
24
                                                                                          return con;
25
     }
                                                                                       }
                                                                                   9
26
                                                                                   10 };
   }rmq;
27
   //Usage:
                                                                                                            2.5. Disjoint Intervals
  cin >> n; rmq.init(n); forn(i, n) cin >> rmq[i]; rmq.updall();
                                                                                   1 | bool operator< (const ii &a, const ii &b) {return a.fst<b.fst;}
                           2.3. Fenwick Tree
                                                                                      //Stores intervals as [first, second]
1 //For 2D threat each column as a Fenwick tree, by adding a nested for in
                                                                                      //in case of a collision it joins them in a single interval
        each operation
                                                                                      struct disjoint_intervals {
  struct Fenwick{
                                                                                        set<ii>> segs;
                                                                                   5
     static const int sz=1000001;
                                                                                        void insert(ii v) {//O(lgn)
                                                                                   6
3
     tipo t[sz]:
                                                                                          if(v.snd-v.fst==0.) return;//0J0
4
     tipo sum(int a, int b){return sum(b)-sum(a-1);}
                                                                                   8
                                                                                          set<ii>>::iterator it,at;
5
                                                                                          at = it = segs.lower_bound(v);
     void adjust(int p, tipo v){//valid with p in [1, sz), O(lgn)
6
                                                                                   9
       for(; p<sz; p+=(p&-p)) t[p]+=v; }
                                                                                          if (at!=segs.begin() && (--at)->snd >= v.fst)
                                                                                   10
       tipo sum(int p){//cumulative sum in [1, p], O(lgn)
                                                                                            v.fst = at->fst, --it;
                                                                                   11
8
       tipo s=0:
                                                                                          for(; it!=segs.end() && it->fst <= v.snd; segs.erase(it++))</pre>
                                                                                   12
9
       for(; p; p-=(p&-p)) s+=t[p];
                                                                                            v.snd=max(v.snd, it->snd);
                                                                                   13
10
       return s;
                                                                                          segs.insert(v);
                                                                                   14
11
     }
                                                                                       }
                                                                                   15
^{12}
     //get largest value with cumulative sum less than or equal to x;
                                                                                   <sub>16</sub> | };
13
     //for smallest, pass x-1 and add 1 to result
14
                                                                                                                2.6. RMQ (2D)
     int getind(tipo x) {//O(lgn)
15
         int idx = 0, mask = N;
16
                                                                                   1 struct RMQ2D{
         while(mask && idx < N) {</pre>
17
                                                                                        static const int sz=1024;
                                                                                   2
           int t = idx + mask;
18
                                                                                        RMQ t[sz];
                                                                                   3
         if(x \ge tree[t])
19
                                                                                        RMQ &operator[](int p){return t[sz/2+p];}
                                                                                   4
             idx = t, x -= tree[t];
20
                                                                                        void build(int n, int m){\frac{1}{0}}(nm)
           mask >>= 1:
21
                                                                                          forr(y, sz/2, sz/2+m)
                                                                                   6
         }
22
                                                                                            t[v].build(m);
         return idx;
                                                                                          forr(y, sz/2+m, sz)
                                                                                   8
     }
24
                                                                                            forn(x, sz)
                                                                                   9
25 | };
                                                                                              t[y].t[x]=0;
                                                                                   10
                             2.4. Union Find
                                                                                          dforn(y, sz/2)
                                                                                  11
                                                                                            forn(x, sz)
                                                                                   12
struct UnionFind{
                                                                                              t[y].t[x]=max(t[y*2].t[x], t[y*2+1].t[x]);
                                                                                  13
```

```
tipo r=1;
14
                                                                                  19
     void set(int x, int y, tipo v){//O(lgm.lgn)
                                                                                              forn(i, sz(x)){
                                                                                  20
15
                                                                                                  n[i / BASEXP] += r * (x[x.size()-1-i]-'0');
       v += sz/2;
                                                                                  21
16
       t[y].set(x, v);
                                                                                                  r*=10; if(r==BASE)r=1;
                                                                                  22
17
       while(v/=2)
                                                                                              }
                                                                                  23
18
         t[y].set(x, max(t[y*2][x], t[y*2+1][x]));
                                                                                          }
                                                                                  24
19
     }
                                                                                          void out(){
                                                                                  25
20
                                                                                          cout << n[1-1];
     //0(lgm.lgn)
21
     int get(int x1, int y1, int x2, int y2, int n=1, int a=0, int b=sz/2){
                                                                                          dforn(i, 1-1) printf("%6.611u", n[i]);//6=BASEXP!
22
       if(y2<=a || y1>=b) return 0;
                                                                                  28
23
       if(y1<=a && b<=y2) return t[n].get(x1, x2);
                                                                                       void invar(){
                                                                                  29
24
       int c=(a+b)/2;
                                                                                         fill(n+1, n+LMAX, 0);
25
                                                                                  30
                                                                                         while(1>1 && !n[1-1]) 1--;
       return max(get(x1, y1, x2, y2, 2*n, a, c),
                                                                                  31
            get(x1, y1, x2, y2, 2*n+1, c, b));
                                                                                       }
                                                                                  32
27
     }
                                                                                     }:
                                                                                  33
28
                                                                                     bint operator+(const bint&a, const bint&b){
29
   //Example to initialize a grid of M rows and N columns:
                                                                                       bint c:
   RMQ2D rmq;
                                                                                         c.1 = max(a.1, b.1);
   forn(i, M)
                                                                                         tipo a = 0:
                                                                                         forn(i, c.l) q += a.n[i]+b.n[i], c.n[i]=q %BASE, q/=BASE;
    forn(j, N)
33
       cin >> rmq[i][j];
                                                                                         if(q) c.n[c.l++] = q;
34
                                                                                         c.invar();
35 rmq.build(N, M);
                                                                                         return c;
                                                                                  41
                               2.7. Big Int
                                                                                     pair<br/>bint, bool> lresta(const bint& a, const bint& b) // c = a - b
                                                                                     {
   #define BASEXP 6
                                                                                  44
                                                                                       bint c:
   #define BASE 1000000
                                                                                  45
                                                                                         c.1 = max(a.1, b.1);
   #define LMAX 1000
                                                                                         tipo q = 0;
   struct bint{
                                                                                  47
                                                                                         forn(i, c.1) q += a.n[i]-b.n[i], c.n[i]=(q+BASE) BASE, q=(q+BASE)/
       int 1;
5
                                                                                              BASE-1:
       tipo n[LMAX];
6
                                                                                          c.invar();
       bint(tipo x){
7
                                                                                          return mkp(c, !q);
           1=0:
                                                                                  50
8
           forn(i, LMAX){
                                                                                  51
9
                                                                                  52 bint& operator-= (bint& a, const bint& b){return a=lresta(a, b).fst;}
               n[i]=x \%BASE;
10
                                                                                     bint operator- (const bint&a, const bint&b){return lresta(a, b).fst;}
               x/=BASE;
11
                                                                                     bool operator (const bint&a, const bint&b) {return !lresta(a, b).snd;}
               1+=!!x||!i;
12
                                                                                     bool operator <= (const bint&a, const bint&b) {return lresta(b, a).snd;}
           }
13
                                                                                     bool operator == (const bint&a, const bint&b) {return a <= b && b <= a;}
14
                                                                                     bint operator*(const bint&a, tipo b){
       bint(){THIS = bint(0);}
15
                                                                                         bint c;
       bint(string x){
                                                                                  58
16
```

l=(x.size()-1)/BASEXP+1;

fill(n, n+LMAX, 0);

17

18

tipo q = 0;

forn(i, a.1) q += a.n[i]\*b, c.n[i] = q %BASE, q/=BASE;

59

```
tipo q = rm.n[b.1] * BASE + rm.n[b.1-1];
       c.1 = a.1:
61
                                                                                   103
       while(q) c.n[c.l++] = q %BASE, q/=BASE;
                                                                                               tipo u = q / (b.n[b.l-1] + 1);
                                                                                   104
62
                                                                                               tipo v = q / b.n[b.l-1] + 1;
       c.invar();
                                                                                   105
63
       return c;
                                                                                               while (u < v-1){
64
                                                                                   106
                                                                                                   tipo m = (u+v)/2;
                                                                                   107
65
   bint operator*(const bint&a, const bint&b){
                                                                                                   if (b*m \le rm) u = m;
                                                                                   108
       bint c;
                                                                                                   else v = m;
67
                                                                                   109
       c.l = a.l+b.l;
                                                                                  110
68
       fill(c.n, c.n+b.1, 0);
                                                                                               c.n[i]=u;
                                                                                  111
69
       forn(i, a.1){
                                                                                               rm-=b*u;
                                                                                  112
70
                                                                                          }
            tipo q = 0;
71
                                                                                  113
           forn(j, b.l) q += a.n[i]*b.n[j]+c.n[i+j], c.n[i+j] = q \text{BASE}, q
                                                                                        c.l=a.l;
72
                                                                                  114
                /=BASE:
                                                                                  115
                                                                                          c.invar():
           c.n[i+b.1] = a:
                                                                                          return mkp(c, rm);
                                                                                   116
73
       }
                                                                                   117 }
74
       c.invar();
                                                                                      bint operator/(const bint&a, const bint&b){return ldiv(a, b).fst;}
75
                                                                                   bint operator %(const bint&a, const bint&b) {return ldiv(a, b).snd;}
       return c;
76
77
                                                                                                                  2.8. Modnum
    pair<birt, tipo > ldiv(const bint& a, tipo b){// c = a / b : rm = a %b
78
   bint c;
79
                                                                                    1 struct mnum{
      tipo rm = 0;
80
                                                                                         static const tipo mod=12582917;
    dforn(i, a.1){
81
                                                                                         tipo v:
                rm = rm * BASE + a.n[i];
82
                                                                                         mnum(tipo v=0): v(v mod) {}
                c.n[i] = rm / b;
83
                                                                                         mnum operator+(mnum b){return v+b.v;}
                rm % b;
84
                                                                                         mnum operator-(mnum b){return v>=b.v? v-b.v : mod-b.v+v;}
       }
85
                                                                                         mnum operator*(mnum b){return v*b.v;}
       c.1 = a.1;
86
                                                                                         mnum operator^(int n){
       c.invar();
87
                                                                                          if(!n) return 1;
                                                                                    9
       return mkp(c, rm);
88
                                                                                          return n\%2? (*this)^(n/2)*(this) : (*this)^(n/2);}
                                                                                   10
89
                                                                                   11 };
   bint operator/(const bint&a, tipo b){return ldiv(a, b).fst;}
    tipo operator %(const bint&a, tipo b) {return ldiv(a, b).snd;}
                                                                                                                   2.9. Bittrie
    pair<bint, bint> ldiv(const bint& a, const bint& b){
92
     bint c:
                                                                                    struct bitrie{
93
       bint rm = 0:
                                                                                         static const int sz=1<<5;//5=ceil(log(n))
94
       dforn(i, a.1){
                                                                                        int V://valor del nodo
95
           if (rm.l==1 && !rm.n[0])
96
                                                                                        vector<bitrie> ch;//childs
                rm.n[0] = a.n[i]:
                                                                                        bitrie():V(0){}//NEUTRO
97
           else{
                                                                                        void set(int p, int v, int bit=sz>>1){//0(log sz)
98
                dforn(j, rm.l) rm.n[j+1] = rm.n[j];
                                                                                          if(bit){
99
                                                                                    7
                rm.n[0] = a.n[i];
                                                                                             ch.resize(2):
100
                                                                                    8
                rm.l++;
                                                                                             ch[(p&bit)>0].set(p, v, bit>>1);
101
                                                                                    9
           }
102
                                                                                             V=max(ch[0].V, ch[1].V);
                                                                                   10
```

18

```
}
11
        else V=v;
12
13
      int get(int i, int j, int a=0, int b=sz){\frac{1}{0}} \log sz
14
        if(j<=a || i>=b) return 0;//NEUTRO
15
       if(i<=a && b<=j) return V;</pre>
16
       if(!sz(ch)) return V;
17
       int c=(a+b)/2;
18
       return max(ch[0].get(i, j, a, c), ch[1].get(i, j, c, b));
19
20
21 | };
```

# Strings

#### 3.1. Trie

```
struct trie{
     map<char, trie> m;
2
     void add(const string &s, int p=0){
       if(s[p]) m[s[p]].add(s, p+1);
4
     }
5
     void dfs(){
6
       //Do stuff
       forall(it, m)
         it->second.dfs();
9
    }
10
11 | };
```

## 3.2. Suffix Array

```
#define MAX_N 1000
   #define RABOUND(x) (x<n? RA[x] : 0)</pre>
   //SA will hold the suffixes in order.
   int SA[MAX_N], RA[MAX_N], n;
   string s; //input string, n=sz(s)
6
   void countingSort(int k){
     int f[MAX_N], tmpSA[MAX_N];
     zero(f);
9
    forn(i, n) f[RABOUND(i+k)]++;
     int sum=0:
11
    forn(i, max(255, n)){
12
       int t=f[i]; f[i]=sum; sum+=t;}
13
```

```
forn(i, n)
14
       tmpSA[f[RABOUND(SA[i]+k)]++]=SA[i];
15
     memcpy(SA, tmpSA, sizeof(SA));
16
17
   void constructSA(){\frac{1}{0} \text{ (n log n)}}
18
     n=sz(s);
     forn(i, n) SA[i]=i, RA[i]=s[i];
     for(int k=1; k<n; k<<=1){</pre>
21
       countingSort(k), countingSort(0);
22
        int r, tmpRA[MAX_N];
23
        tmpRA[SA[O]]=r=0;
24
       forr(i, 1, n)
25
          tmpRA[SA[i]] = (RA[SA[i]] = RA[SA[i-1]] & RA[SA[i]+k] = RA[SA[i-1]+k]
26
               )? r : ++r:
        memcpy(RA, tmpRA, sizeof(RA));
27
        if(RA[SA[n-1]]==n-1) break:
28
     }
29
30
   void print(){//for debug
31
     forn(i, n)
        cout << i << ''' <<
33
        s.substr(SA[i], s.find( '$', SA[i])-SA[i]) << endl;}</pre>
```

### 3.3. String Matching With Suffix Array

```
1 //returns (lowerbound, upperbound) of the search
   ii stringMatching(string P){ //O(sz(P)lgn)
     int lo=0, hi=n-1, mid=lo;
     while(lo<hi){</pre>
       mid=(lo+hi)/2;
5
       int res=s.compare(SA[mid], sz(P), P);
6
       if(res>=0) hi=mid;
       else lo=mid+1;
8
9
     if(s.compare(SA[lo], sz(P), P)!=0) return ii(-1, -1);
10
     ii ans; ans.fst=lo;
11
     lo=0, hi=n-1, mid;
12
     while(lo<hi){
13
       mid=(lo+hi)/2:
14
       int res=s.compare(SA[mid], sz(P), P);
15
       if(res>0) hi=mid;
16
       else lo=mid+1;
17
    }
```

```
if(s.compare(SA[hi], sz(P), P)!=0) hi--;
     ans.snd=hi;
20
     return ans;
^{21}
22 }
                3.4. LCP (Longest Common Prefix)
    /Calculates the LCP between consecutives suffixes in the Suffix Array.
    //LCP[i] is the length of the LCP between SA[i] and SA[i-1]
   int LCP[MAX_N];
   void computeLCP(){//0(n)}
     int phi[MAX_N], PLCP[MAX_N];
     phi[SA[0]]=-1;
6
     forr(i, 1, n) phi[SA[i]]=SA[i-1];
     int L=0;
8
     forn(i, n){
9
       if(phi[i]==-1) {PLCP[i]=0; continue;}
10
       while(s[i+L] == s[phi[i]+L]) L++;
11
       PLCP[i]=L:
12
       L=max(L-1, 0);
13
14
     forn(i, n) LCP[i]=PLCP[SA[i]];
15
16 }
                              3.5. Corasick
1
   struct trie{
     map<char, trie> next;
3
     trie* tran[256];//transiciones del automata
     int idhoja, szhoja;//id de la hoja o 0 si no lo es
     //link lleva al sufijo mas largo, nxthoja lleva al mas largo pero que
6
         es hoja
     trie *padre, *link, *nxthoja;
7
     char pch;//caracter que conecta con padre
8
     trie(): tran(), idhoja(), padre(), link() {}
9
     void insert(const string &s, int id=1, int p=0){//id>0!!!
10
       if(p \le z(s)){
11
         trie &ch=next[s[p]];
12
         tran[(int)s[p]]=&ch;
13
         ch.padre=this, ch.pch=s[p];
14
         ch.insert(s, id, p+1);
15
       }
16
       else idhoja=id, szhoja=sz(s);
```

17

```
18
     trie* get_link() {
19
       if(!link){
20
         if(!padre) link=this;//es la raiz
21
         else if(!padre->padre) link=padre;//hijo de la raiz
22
         else link=padre->get_link()->get_tran(pch);
23
       }
24
       return link;
25
     }
26
     trie* get_tran(int c) {
27
       if(!tran[c])
28
         tran[c] = !padre? this : this->get_link()->get_tran(c);
29
       return tran[c]:
30
     }
31
     trie *get_nxthoja(){
32
       if(!nxthoja) nxthoja = get_link()->idhoja? link : link->nxthoja;
33
       return nxthoja;
34
     }
35
     void print(int p){
36
       if(idhoja)
         cout << "found" << idhoja << "LUatuposition" << p-szhoja << endl
38
       if(get_nxthoja()) get_nxthoja()->print(p);
39
40
     void matching(const string &s, int p=0){
41
       print(p);
42
       if(p<sz(s)) get_tran(s[p])->matching(s, p+1);
43
```

### Geometría

#define EPS 1e-9

#### 4.1. Punto

```
1 struct pto{
    tipo x, v;
2
    pto(tipo x=0, tipo y=0):x(x),y(y){}
    pto operator+(pto a){return pto(x+a.x, y+a.y);}
    pto operator-(pto a){return pto(x-a.x, y-a.y);}
    pto operator+(tipo a){return pto(x+a, y+a);}
    pto operator*(tipo a){return pto(x*a, y*a);}
    pto operator/(tipo a){return pto(x/a, y/a);}
    //dot product, producto interno:
```

```
tipo operator*(pto a){return x*a.x+y*a.y;}
     //module of the cross product or vectorial product:
11
     //if a is less than 180 clockwise from b, a^b>0
12
     tipo operator^(pto a){return x*a.y-y*a.x;}
13
     //returns true if this is at the left side of line gr
14
     bool left(pto q, pto r){return ((q-*this)^(r-*this))>0;}
15
     bool operator<(const pto &a) const{return x<a.x || (abs(x-a.x)<EPS &&
         v<a.v);}
   bool operator==(pto a){return abs(x-a.x)<EPS && abs(y-a.y)<EPS;}
17
     double norm(){return sqrt(x*x+v*v);}
18
     tipo norm_sq(){return x*x+y*y;}
20
   double dist(pto a, pto b){return (b-a).norm();}
   typedef pto vec;
   double angle(pto a, pto o, pto b){
     vec oa=a-o, ob=b-o;
25
     return acos((oa*ob) / sqrt(oa.norm_sq()*ob.norm_sq()));}
26
27
   //rotate p by theta rads CCW w.r.t. origin (0,0)
   pto rotate(pto p, double theta){
29
     return pto(p.x*cos(theta)-p.y*sin(theta),
30
        p.x*sin(theta)+p.y*cos(theta));
31
32 }
                                4.2. Line
  struct line{
     line() {}
     double a,b,c;//Ax+By=C
   //pto MUST store float coordinates!
    line(double a, double b, double c):a(a),b(b),c(c){}
    line(pto p, pto q): a(q.y-p.y), b(p.x-q.x), c(a*p.x+b*p.y) {}
6
7
   bool parallels(line 11, line 12){return abs(11.a*12.b-12.a*11.b)<EPS;}
   pto inter(line 11, line 12){//intersection
     double det=11.a*12.b-12.a*11.b;
```

### 4.3. Segment

if(abs(det) < EPS) return pto(INF, INF); //parallels</pre>

return pto(12.b\*11.c-11.b\*12.c, 11.a\*12.c-12.a\*11.c)/det;

```
struct segm{
```

13 }

```
pto s.f:
2
     segm(pto s, pto f):s(s), f(f) {}
3
     pto closest(pto p) {//use for dist to point
        double 12 = dist_sq(s, f);
        if(12==0.) return s;
6
        double t = ((p-s)*(f-s))/12;
        if (t<0.) return s;//not write if is a line
        else if(t>1.)return f;//not write if is a line
        return s+((f-s)*t);
10
    }
11
     bool inside(pto p){
12
   return ((s-p)^(f-p))==0 \&\& min(s, f)<*this&&*this<max(s, f);}
   };
14
15
   bool insidebox(pto a, pto b, pto p) {
     return (a.x-p.x)*(p.x-b.x)>-EPS && (a.y-p.y)*(p.y-b.y)>-EPS;
   }
18
   pto inter(segm s1, segm s2){
     pto r=inter(line(s1.s, s1.f), line(s2.s, s2.f));
20
     if(insidebox(s1.s,s1.f,p) && insidebox(s2.s,s2.f,p))
         return r;
22
     return pto(INF, INF);
24 }
                             4.4. Rectangle
1 | struct rect{
     //lower-left and upper-right corners
     pto lw, up;
   };
4
   //returns if there's an intersection and stores it in r
   bool inter(rect a, rect b, rect &r){
     r.lw=pto(max(a.lw.x, b.lw.x), max(a.lw.y, b.lw.y));
     r.up=pto(min(a.up.x, b.up.x), min(a.up.y, b.up.y));
   //check case when only a edge is common
     return r.lw.x<r.up.x && r.lw.y<r.up.y;</pre>
11 }
                           4.5. Polygon Area
| double area(vector<tipo> &p){//0(sz(p))
     double area=0:
    forn(i, sz(p)) area+=p[i]^p[(i+1) %z(p)];
     //if points are in clockwise order then area is negative
```

```
return abs(area)/2;
  }
6
  //Area ellipse = M_PI*a*b where a and b are the semi axis lengths
8 //Area triangle = sqrt(s*(s-a)(s-b)(s-c)) where s=(a+b+c)/2
                               4.6. Circle
   vec perp(vec v){return vec(-v.y, v.x);}
   line bisector(pto x, pto y){
     line l=line(x, y); pto m=(x+y)/2;
     return line(-1.b, 1.a, -1.b*m.x+1.a*m.y);
4
5
   struct Circle{
     pto o;
7
     double r;
8
    //circle determined by three points, uses line
     Circle(pto x, pto y, pto z){
10
       o=inter(bisector(x, y), bisector(y, z));
11
       r=dist(o, x);
12
     }
13
     pair<pto, pto> ptosTang(pto p){
14
       pto m=(p+o)/2;
15
       tipo d=dist(o, m);
16
       tipo a=r*r/(2*d):
17
       tipo h=sqrt(r*r-a*a);
18
       pto m2=o+(m-o)*a/d;
19
       vec per=perp(m-o)/d;
20
       return mkp(m2-per*h, m2+per*h);
21
22
23
    //finds the center of the circle containing p1 and p2 with radius r
   //as there may be two solutions swap p1, p2 to get the other
   bool circle2PtsRad(pto p1, pto p2, double r, pto &c){
26
           double d2=(p1-p2).norm_sq(), det=r*r/d2-0.25;
27
           if(det<0) return false;</pre>
28
           c=(p1+p2)/2+perp(p2-p1)*sqrt(det);
29
           return true;
31 }
                           4.7. Point in Poly
1 //checks if v is inside of P, using ray casting
  //works with convex and concave.
3 //excludes boundaries, handle it separately using segment.inside()
```

```
bool inPolygon(pto v, vector<pto>& P) {
     bool c = false;
5
     forn(i, sz(P)){
6
       int j=(i+1) %z(P);
       if((P[j].y>v.y) != (P[i].y>v.y) &&
     (v.x < (P[i].x - P[j].x) * (v.y-P[j].y) / (P[i].y - P[j].y) + P[j].x))
         c = !c;
10
    }
11
     return c;
13 }
                     4.8. Convex Check CHECK
  | bool isConvex(vector<int> &p){//O(N)
     int N=sz(p);
     if(N<3) return false;
     bool isLeft=p[0].left(p[1], p[2]);
     forr(i, 1, N)
5
       if(p[i].left(p[(i+1) \( \mathbb{N} \)], p[(i+2) \( \mathbb{N} \)])!=isLeft)
         return false;
7
     return true; }
                            4.9. Convex Hull
   //stores convex hull of P in S, CCW order
  void CH(vector<pto>& P, vector<pto> &S){
     S.clear():
     sort(P.begin(), P.end());
     forn(i, sz(P)){
       while(sz(S) \ge 2 \&\& S[sz(S)-1].left(S[sz(S)-2], P[i])) S.pop_back();
       S.pb(P[i]);
7
     }
8
     S.pop_back();
9
     int k=sz(S);
10
     dforn(i, sz(P)){
11
       while(sz(S) \ge k+2 \&\& S[sz(S)-1].left(S[sz(S)-2], P[i])) S.pop_back
12
           ();
       S.pb(P[i]);
13
14
     S.pop_back();
15
16 }
                           4.10. Cut Polygon
```

```
//cuts polygon Q along the line ab
//stores the left side (swap a, b for the right one) in P
void cutPolygon(pto a, pto b, vector<pto> Q, vector<pto> &P){
P.clear();
forn(i, sz(Q)){
    double left1=(b-a)^(Q[i]-a), left2=(b-a)^(Q[(i+1)%z(Q)]-a);
    if(left1>=0) P.pb(Q[i]);
    if(left1*left2<0)
        P.pb(inter(line(Q[i], Q[(i+1)%z(Q)]), line(a, b)));
}
</pre>
```

#### 4.11. Bresenham

```
//plot a line approximation in a 2d map
   void bresenham(pto a, pto b){
     pto d=b-a; d.x=abs(d.x), d.y=abs(d.y);
     pto s(a.x<b.x? 1: -1, a.y<b.y? 1: -1);
     int err=d.x-d.y;
     while(1){
       m[a.x][a.y]=1;//plot
       if(a==b) break;
       int e2=2*err;
9
       if(e2 > -d.y){
         err-=d.y, a.x+=s.x;
       if(e2 < d.x)
12
         err+= d.x, a.y+= s.y;
13
     }
14
15 | }
```

#### 4.12. Rotate Matrix

```
//rotates matrix t 90 degrees clockwise
//using auxiliary matrix t2(faster)
void rotate(){
forn(x, n) forn(y, n)
t2[n-y-1][x]=t[x][y];
memcpy(t, t2, sizeof(t));
}
```

## 5. Math

### 5.1. GCD

```
1 | tipo gcd(tipo a, tipo b){return a?gcd(b %a, a):b;}
                                5.2. LCM
 tipo lcm(tipo a, tipo b) {return a / gcd(a,b) * b;}
                              5.3. Simpson
double integral (double a, double b, int n=10000) {//O(n), n=cantdiv
     double area=0, h=(b-a)/n, fa=f(a), fb;
     forn(i, n){
       fb=f(a+h*(i+1));
4
       area+=fa+ 4*f(a+h*(i+0.5)) +fb, fa=fb;
5
    }
6
     return area*h/6.;}
7
                              5.4. Fraction
struct frac{
     tipo p,q;
     frac(tipo p=0, tipo q=1):p(p),q(q) {norm();}
     tipo mcd(tipo a, tipo b){return a?mcd(b \( \frac{1}{2} \), a):b;}
     void norm(){
       tipo a = mcd(p,q);
       if(a) p/=a, q/=a;
7
       else q=1;
8
       if (q<0) q=-q, p=-p;}
     frac operator+(const frac& o){
10
       tipo a = mcd(q, o.q);
       return frac(p*(o.q/a)+o.p*(q/a), q*(o.q/a));}
12
     frac operator-(const frac& o){
13
       tipo a = mcd(q, o.q);
14
       return frac(p*(o.q/a)-o.p*(q/a), q*(o.q/a));}
15
     frac operator*(frac o){
16
       tipo a = mcd(q,o.q), b = mcd(o.q,p);
17
       return frac((p/b)*(o.p/a), (q/a)*(o.q/b));}
18
     frac operator/(frac o){
19
       tipo a = mcd(q,o.q), b = mcd(o.p,p);
20
       return frac((p/b)*(o.q/a),(q/a)*(o.p/b));}
21
     bool operator<(frac o){return (*this-o).p<0;}</pre>
     bool operator==(frac o){return p==o.p&kq==o.q;}
24 };
```

#### 5.5. Polinomio

}

}

20

21

```
#define MAX GR 20
   struct poly {
     tipo p[MAX_GR];//guarda los coeficientes del polinomio
     poly(){zero(p);}
4
     int gr(){//calculates grade of the polynomial
       dforn(i,MAX_GR) if(p[i]) return i;
       return 0; }
7
     bool isnull() {return gr()==0 && !p[0];}
8
     poly operator+(poly b) {// - is analogous
       poly c=THIS;
10
       forn(i,MAX_GR) c.p[i]+=b.p[i];
11
       return c;
12
     }
13
     poly operator*(poly b) {
14
       poly c;
15
       forn(i,MAX_GR) forn(k,i+1) c.p[i]+=p[k]*b.p[i-k];
16
       return c;
17
     }
18
     tipo eval(tipo v) {
19
       tipo sum = 0;
20
       dforsn(i, 0, MAX_GR) sum=sum*v + p[i];
21
       return sum;
22
23
     //the following function generates the roots of the polynomial
24
    //it can be easily modified to return float roots
25
     set<tipo> roots(){
26
       set<tipo> roots;
27
       tipo a0 = abs(p[0]), an = abs(p[gr()]);
28
       vector<tipo> ps,qs;
29
       forr(p,1,sqrt(a0)+1) if (a0\%p==0) ps.pb(p),ps.pb(a0/p);
30
       forr(q,1,sqrt(an)+1) if (an)(q=0) qs.pb(q),qs.pb(an/q);
31
       forall(pt,ps)
32
         forall(qt,qs) if ( (*pt) % (*qt)==0 ) {
33
           tipo root = abs((*pt) / (*qt));
34
           if (eval(root)==0) roots.insert(root);
35
         }
36
       return roots;
37
38
39
   //the following functions allows parsing an expression like
   //34+150+4*45
   //into a polynomial(el numero en funcion de la base)
  #define LAST(s) (sz(s)? s[sz(s)-1]:0)
```

```
#define POP(s) s.erase(--s.end()):
   poly D(string &s) {
     poly d;
46
     for(int i=0; isdigit(LAST(s)); i++) d.p[i]=LAST(s)-'0', POP(s);
47
     return d;}
48
49
   poly T(string &s) {
     poly t=D(s);
     if (LAST(s)=='*'){POP(s); return T(s)*t;}
     return t;
54
   //main function, call this to parse
   poly E(string &s) {
     poly e=T(s);
     if (LAST(s)=='+')\{POP(s); return E(s)+e;\}
     return e;
60 }
                                    Grafos
                              6.1. Dijkstra
1 #define INF 1e9
   int N:
   #define MAX V 250001
   vector<ii> G[MAX V]:
   //To add an edge use
   #define add(a, b, w) G[a].pb(mkp(w, b))
   ll dijkstra(int s, int t){\frac{1}{0}(|E| \log |V|)}
     priority_queue<ii, vector<ii>, greater<ii> > Q;
9
     vector<ll> dist(N, INF); vector<int> dad(N, -1);
10
     Q.push(mkp(0, s)); dist[s] = 0;
11
     while(sz(Q)){
12
       ii p = Q.top(); Q.pop();
13
       if(p.snd == t) break;
14
       forall(it, G[p.snd])
15
         if(dist[p.snd]+it->first < dist[it->snd]){
16
           dist[it->snd] = dist[p.snd] + it->fst;
17
           dad[it->snd] = p.snd;
18
           Q.push(mkp(dist[it->snd], it->snd));
19
```

```
return dist[t];
22
     if(dist[t]<INF)//path generator</pre>
23
       for(int i=t; i!=-1; i=dad[i])
^{24}
         printf("%%", i, (i==s?'\n':'\_'));
25
26 }
                          6.2. Bellman-Ford
   vector<ii> G[MAX_N];//ady. list with pairs (weight, dst)
   int dist[MAX_N];
   void bford(int src){//O(VE)
     dist[src]=0;
     forn(i, N-1) forn(j, N) if(dist[j]!=INF) forall(it, G[j])
5
       dist[it->snd]=min(dist[it->snd], dist[j]+it->fst);
6
7
8
   bool hasNegCycle(){
9
    forn(j, N) if(dist[j]!=INF) forall(it, G[j])
10
       if(dist[it->snd]>dist[j]+it->fst) return true;
11
     //inside if: all points reachable from it->snd will have -INF distance
         (do bfs)
     return false:
13
14 }
                         6.3. Floyd-Warshall
1 //G[i][j] contains weight of edge (i, j) or INF
  //G[i][i]=0
  int G[MAX_N][MAX_N];
  void floyd(){//0(N^3)}
  forn(k, N) forn(i, N) if(G[i][k]!=INF) forn(j, N) if(G[k][j]!=INF)
    G[i][j]=min(G[i][j], G[i][k]+G[k][j]);
   }
7
  bool inNegCycle(int v){
    return G[v][v]<0;}
   //checks if there's a neg. cycle in path from a to b
   bool hasNegCycle(int a, int b){
    forn(i, N) if(G[a][i]!=INF && G[i][i]<0 && G[i][b]!=INF)
12
       return true;
13
     return false;
14
15 }
                     6.4. 2-SAT + Tarjan SCC
```

```
1 //We have a vertex representing a var and other for his negation.
2 //Every edge stored in G represents an implication. To add an equation
       of the form a | |b, use addor(a, b)
3 //MAX=max cant var, n=cant var
   #define addor(a, b) (G[neg(a)].pb(b), G[neg(b)].pb(a))
  vector<int> G[MAX*2];
  //idx[i]=index assigned in the dfs
  //lw[i]=lowest index(closer from the root) reachable from i
   int lw[MAX*2], idx[MAX*2], qidx;
   stack<int> q;
   int qcmp, cmp[MAX*2];
   //verdad[cmp[i]]=valor de la variable i
   bool verdad[MAX*2+1]:
13
   int neg(int x) { return x>=n? x-n : x+n;}
   void tjn(int v){
     lw[v]=idx[v]=++qidx;
     q.push(v), cmp[v]=-2;
     forall(it, G[v]){
18
       if(!idx[*it] || cmp[*it]==-2){
19
         if(!idx[*it]) tjn(*it);
20
         lw[v]=min(lw[v], lw[*it]);
21
       }
22
     }
23
     if(lw[v]==idx[v]){
24
       qcmp++;
25
       int x;
26
       do{x=q.top(); q.pop(); cmp[x]=qcmp;}while(x!=v);
27
       verdad[qcmp] = (cmp[neg(v)] < 0);</pre>
28
     }
29
30
   //remember to CLEAR G!!!
   bool satisf(){//O(n)
     memset(idx, 0, sizeof(idx)), qidx=0;
33
     memset(cmp, -1, sizeof(cmp)), qcmp=0;
34
     forn(i, n){
35
       if(!idx[i]) tjn(i);
36
       if(!idx[neg(i)]) tjn(neg(i));
37
38
     forn(i, n) if(cmp[i]==cmp[neg(i)]) return false;
39
     return true:
40
41 }
```

#### 6.5. Articulation Points

```
1 int N;
  vector<int> G[1000000];
   //V[i]=node number(if visited), L[i]= lowest V[i] reachable from i
   int qV, V[1000000], L[1000000], P[1000000];
   void dfs(int v, int f){
     L[v]=V[v]=++qV;
     forall(it, G[v])
       if(!V[*it]){
         dfs(*it, v);
9
         L[v] = min(L[v], L[*it]);
         P[v] += L[*it] >= V[v];
11
12
       else if(*it!=f)
13
         L[v]=min(L[v], V[*it]);
14
15
   int cantart() { //0(n)
16
     qV=0;
17
     zero(V), zero(P);
     dfs(1, 0); P[1]--;
     int q=0;
     forn(i, N) if(P[i]) q++;
   return q;
23 }
```

- 6.6. LCA + Climb
- 7. Network Flow
- 7.1. Edmonds Karp's

```
#define MAX_V 1000
#define INF 1e9

//special nodes
#define SRC 0

#define SNK 1
map<int, int> G[MAX_V];//limpiar esto
//To add an edge use
#define add(a, b, w) G[a][b]=w
int f, p[MAX_V];
void augment(int v, int minE){
if(v==SRC) f=minE;
```

```
else if(p[v]!=-1){
12
       augment(p[v], min(minE, G[p[v]][v]));
13
       G[p[v]][v]-=f, G[v][p[v]]+=f;
14
     }
15
16
   ll maxflow(){//O(VE^2)
     11 Mf=0;
18
     do{
19
       f=0;
20
       char used[MAX_V]; queue<int> q; q.push(SRC);
       zero(used), memset(p, -1, sizeof(p));
22
       while(sz(q)){
23
         int u=q.front(); q.pop();
24
         if(u==SNK) break;
         forall(it, G[u])
           if(it->snd>0 && !used[it->fst])
             used[it->fst]=true, q.push(it->fst), p[it->fst]=u;
28
       }
       augment(SNK, INF);
30
       Mf+=f;
     }while(f);
     return Mf;
34 }
```

#### 7.2. Push-Relabel

```
1 #define MAX_V 1000
   int N;//valid nodes are [0...N-1]
   #define INF 1e9
   //special nodes
   #define SRC 0
   #define SNK 1
   map<int, int> G[MAX_V];
   //To add an edge use
   #define add(a, b, w) G[a][b]=w
   11 excess[MAX_V];
   int height[MAX_V], active[MAX_V], count[2*MAX_V+1];
   queue<int> Q;
   void enqueue(int v) {
    if (!active[v] && excess[v] > 0) active[v]=true, Q.push(v); }
   void push(int a, int b) {
    int amt = min(excess[a], ll(G[a][b]));
16
     if(height[a] <= height[b] || amt == 0) return;</pre>
```

```
G[a][b]-=amt, G[b][a]+=amt;
     excess[b] += amt, excess[a] -= amt;
19
     enqueue(b);
20
^{21}
   void gap(int k) {
22
     forn(v, N){
23
       if (height[v] < k) continue;</pre>
24
       count[height[v]]--;
25
       height[v] = max(height[v], N+1);
26
       count[height[v]]++;
27
       enqueue(v);
28
29
30
    void relabel(int v) {
     count[height[v]]--;
32
     height[v] = 2*N;
33
     forall(it, G[v])
34
       if(it->snd)
35
         height[v] = min(height[v], height[it->fst] + 1);
36
     count[height[v]]++;
37
     enqueue(v);
38
39
    ll maxflow() \{//0(V^3)
40
     zero(height), zero(active), zero(count), zero(excess);
41
     count[0] = N-1;
42
     count[N] = 1;
43
     height[SRC] = N;
44
     active[SRC] = active[SNK] = true;
45
     forall(it, G[SRC]){
46
       excess[SRC] += it->snd;
47
       push(SRC, it->fst);
48
49
     while(sz(Q)) {
50
       int v = Q.front(); Q.pop();
51
       active[v]=false;
52
     forall(it, G[v]) push(v, it->fst);
53
     if(excess[v] > 0)
54
       count[height[v]] == 1? gap(height[v]):relabel(v);
55
     }
56
     ll mf=0;
57
     forall(it, G[SRC]) mf+=G[it->fst][SRC];
58
     return mf;
59
60 }
```

# 8. Ayudamemoria

```
Límites
1 | #include <climits> //INT_MIN, LONG_MAX, ULLONG_MAX, etc.
                            Cant. decimales
#include <iomanip>
cout << setprecision(2) << fixed;</pre>
               Rellenar con espacios(para justificar)
1 #include <iomanip>
cout << setfill(',') << setw(3) << 2 << endl;</pre>
                        Leer hasta fin de línea
1 #include <sstream>
  //hacer cin.ignore() antes de getline()
  while(getline(cin, line)){
       istringstream is(line);
4
       while(is >> X)
5
         cout << X << "";
6
       cout << endl;</pre>
7
8 }
                                Aleatorios
1 | #define RAND(a, b) (rand() %(b-a+1)+a)
srand(time(NULL));
                            Doubles Comp.
const double EPS = 1e-9:
_2 | x == y <=> fabs(x-y) < EPS
_3 | x > y <=> x > y + EPS
_{4} | x >= y <=> x > y - EPS
                                 Límites
1 #include <limits>
2 | numeric_limits<T>
    ::max()
    ::min()
4
```

::epsilon()

## Muahaha

```
#include <signal.h>
void divzero(int p){
while(true);}

void segm(int p){
exit(0);}

//in main
signal(SIGFPE, divzero);
signal(SIGSEGV, segm);

Mejorar velocidad

| ios::sync_with_stdio(false);

Leer del teclado
| freopen("/dev/tty", "a", stdin);

File setup

//tambien se pueden usar comas: {a, x, m, l}
for i in {a..k}; do cp template.cpp $i.cpp; touch $i.in; done
```