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1. algorithm

#include <algorithm> #include <numeric>

Algo	Params	Funcion
sort, stable_sort	f, l	ordena el intervalo
nth_element	f, nth, l	<i>void</i> ordena el n-esimo, y particiona el resto
fill, fill_n	f, l / n, elem	<i>void</i> llena [f, l) o [f, f+n) con elem
lower_bound, upper_bound	f, l, elem	<i>it</i> al primer / ultimo donde se puede insertar elem para que quede ordenada
binary_search	f, l, elem	<i>bool</i> esta elem en [f, l)
copy	f, l, resul	hace resul+i=f+i $\forall i$
find, find_if, find_first_of	f, l, elem / pred / f2, l2	<i>it</i> encuentra i $\in$ [f,l) tq. i=elem, pred(i), i $\in$ [f2,l2)
count, count_if	f, l, elem/pred	cuenta elem, pred(i)
search	f, l, f2, l2	busca [f2,l2) $\in$ [f,l)
replace, replace_if	f, l, old / pred, new	cambia old / pred(i) por new
reverse	f, l	da vuelta
partition, stable_partition	f, l, pred	pred(i) ad, !pred(i) atras
min_element, max_element	f, l, [comp]	<i>it</i> min, max de [f,l]
lexicographical_compare	f1,l1,f2,l2	<i>bool</i> con [f1,l1] <sub>j</sub> [f2,l2]
next/prev_permutation	f,l	deja en [f,l) la perm sig, ant
set_intersection, set_difference, set_union, set_symmetric_difference,	f1, l1, f2, l2, res	[res, ...) la op. de conj
push_heap, pop_heap, make_heap	f, l, e / e /	mete/saca e en heap [f,l), hace un heap de [f,l)
is_heap	f,l	<i>bool</i> es [f,l) un heap
accumulate	f,l,i,[op]	$T = \sum$ /oper de [f,l)
inner_product	f1, l1, f2, i	$T = i + [f1, l1) \cdot [f2, \dots)$
partial_sum	f, l, r, [op]	r+i = $\sum$ /oper de [f,f+i] $\forall i \in$ [f,l)
__builtin_ffs	unsigned int	Pos. del primer 1 desde la derecha
__builtin_clz	unsigned int	Cant. de ceros desde la izquierda.
__builtin_ctz	unsigned int	Cant. de ceros desde la derecha.
__builtin_popcount	unsigned int	Cant. de 1's en x.
__builtin_parity	unsigned int	1 si x es par, 0 si es impar.
__builtin_XXXXXXll	unsigned ll	= pero para long long's.

## 2. Estructuras

### 2.1. Easy segment

```

1 const int N = 1e5; // limit for array size
2 int n; // array size
3 int t[2 * N];
4
5 void build() { // build the tree
6     for (int i = n - 1; i > 0; --i) t[i] = t[i<<1] + t[i<<1|1];
7 }
8
9 void modify(int p, int value) { // set value at position p
10    for (t[p += n] = value; p > 1; p >>= 1) t[p>>1] = t[p] + t[p^1];
11 }
12
13 int query(int l, int r) { // sum on interval [l, r)
14    int res = 0;
15    for (l += n, r += n; l < r; l >>= 1, r >>= 1) {
16        if (l&1) res += t[l++];
17        if (r&1) res += t[--r];
18    }
19    return res;
20 }
21
22 int main() {
23    scanf("%d", &n);
24    for (int i = 0; i < n; ++i) scanf("%d", t + n + i);
25    build();
26    modify(0, 1);
27    printf("%d\n", query(3, 11));
28    return 0;
29 }

```

### 2.2. RMQ (static)

Dado un arreglo y una operacion asociativa *idempotente*,  $\text{get}(i, j)$  opera sobre el rango  $[i, j)$ . Restriccion:  $\text{LVL} \geq \text{ceil}(\log n)$ ; Usar `[]` para llenar arreglo y luego `build()`.

```

1 struct RMQ{
2     #define LVL 10
3     tipo vec[LVL][1<<(LVL+1)];
4     tipo &operator[] (int p){return vec[0][p];}
5     tipo get(int i, int j) { //intervalo [i,j)

```

```

6         int p = 31-__builtin_clz(j-i);
7         return min(vec[p][i], vec[p][j-(1<<p)]);
8     }
9     void build(int n) { //O(nlogn)
10        int mp = 31-__builtin_clz(n);
11        forn(p, mp) forn(x, n-(1<<p))
12            vec[p+1][x] = min(vec[p][x], vec[p][x+(1<<p)]);
13    };

```

### 2.3. RMQ (dynamic)

```

1 //Dado un arreglo y una operacion asociativa con neutro, get(i, j) opera
2 //sobre el rango [i, j).
3 #define MAXN 100000
4 #define operacion(x, y) max(x, y)
5 const int neutro=0;
6 struct RMQ{
7     int sz;
8     tipo t[4*MAXN];
9     tipo &operator[] (int p){return t[sz+p];}
10    void init(int n){ //O(nlgn)
11        sz = 1 << (32-__builtin_clz(n));
12        forn(i, 2*sz) t[i]=neutro;
13    }
14    void updall(){ //O(n)
15        dfor(n, sz) t[i]=operacion(t[2*i], t[2*i+1]);
16    }
17    tipo get(int i, int j){return get(i,j,1,0,sz);}
18    tipo get(int i, int j, int n, int a, int b){ //O(lgn)
19        if(j<=a || i>=b) return neutro;
20        if(i<=a && b<=j) return t[n];
21        int c=(a+b)/2;
22        return operacion(get(i, j, 2*n, a, c), get(i, j, 2*n+1, c, b));
23    }
24    void set(int p, tipo val){ //O(lgn)
25        for(p+=sz; p>0 && t[p]!=val;){
26            t[p]=val;
27            p/=2;
28            val=operacion(t[p*2], t[p*2+1]);
29        }
30    }
31 }rmq;
32 //Usage:

```

```
31 | cin >> n; rmq.init(n); for(n,i, n) cin >> rmq[i]; rmq.updall();
```

## 2.4. RMQ (lazy)

```
1 //Dado un arreglo y una operacion asociativa con neutro, get(i, j) opera
  sobre el rango [i, j].
2 typedef int Elem; //Elem de los elementos del arreglo
3 typedef int Alt; //Elem de la alteracion
4 #define operacion(x,y) x+y
5 const Elem neutro=0; const Alt neutro2=0;
6 #define MAXN 100000
7 struct RMQ{
8     int sz;
9     Elem t[4*MAXN];
10    Alt dirty[4*MAXN]; //las alteraciones pueden ser de distinto Elem
11    Elem &operator[](int p){return t[sz+p];}
12    void init(int n){//O(nlgn)
13        sz = 1 << (32-__builtin_clz(n));
14        for(n,i, 2*sz) t[i]=neutro;
15        for(n,i, 2*sz) dirty[i]=neutro2;
16    }
17    void push(int n, int a, int b){//propaga el dirty a sus hijos
18        if(dirty[n]!=0){
19            t[n]+=dirty[n]*(b-a); //altera el nodo
20            if(n<sz){
21                dirty[2*n]+=dirty[n];
22                dirty[2*n+1]+=dirty[n];
23            }
24            dirty[n]=0;
25        }
26    }
27    Elem get(int i, int j, int n, int a, int b){//O(lgn)
28        if(j<=a || i>=b) return neutro;
29        push(n, a, b); //corrige el valor antes de usarlo
30        if(i<=a && b<=j) return t[n];
31        int c=(a+b)/2;
32        return operacion(get(i, j, 2*n, a, c), get(i, j, 2*n+1, c, b));
33    }
34    Elem get(int i, int j){return get(i,j,1,0,sz);}
35    //altera los valores en [i, j] con una alteracion de val
36    void alterar(Alt val, int i, int j, int n, int a, int b){//O(lgn)
37        push(n, a, b);
38        if(j<=a || i>=b) return;
```

```
39        if(i<=a && b<=j){
40            dirty[n]+=val;
41            push(n, a, b);
42            return;
43        }
44        int c=(a+b)/2;
45        alterar(val, i, j, 2*n, a, c), alterar(val, i, j, 2*n+1, c, b);
46        t[n]=operacion(t[2*n], t[2*n+1]); //por esto es el push de arriba
47    }
48    void alterar(Alt val, int i, int j){alterar(val,i,j,1,0,sz);}
49 }rmq;
```

## 2.5. RMQ (persistente)

```
1 typedef int tipo;
2 tipo oper(const tipo &a, const tipo &b){
3     return a+b;
4 }
5 struct node{
6     tipo v; node *l,*r;
7     node(tipo v):v(v), l(NULL), r(NULL) {}
8     node(node *l, node *r) : l(l), r(r){
9         if(!l) v=r->v;
10        else if(!r) v=l->v;
11        else v=oper(l->v, r->v);
12    }
13 };
14 node *build (tipo *a, int tl, int tr) { //modificar para que tome tipo a
15     if (tl+1==tr) return new node(a[tl]);
16     int tm=(tl + tr)>>1;
17     return new node(build(a, tl, tm), build(a, tm, tr));
18 }
19 node *update(int pos, int new_val, node *t, int tl, int tr){
20     if (tl+1==tr) return new node(new_val);
21     int tm=(tl+tr)>>1;
22     if(pos < tm) return new node(update(pos, new_val, t->l, tl, tm), t->r);
23     ;
24     else return new node(t->l, update(pos, new_val, t->r, tm, tr));
25 }
26 tipo get(int l, int r, node *t, int tl, int tr){
27     if(l==tl && tr==r) return t->v;
28     int tm=(tl + tr)>>1;
29     if(r<=tm) return get(l, r, t->l, tl, tm);
```

```

29     else if(l>=tm) return get(l, r, t->r, tm, tr);
30     return oper(get(l, tm, t->l, tl, tm), get(tm, r, t->r, tm, tr));
31 }

```

## 2.6. Union Find

```

1 struct UnionFind{
2     vector<int> f; //the array contains the parent of each node
3     void init(int n){f.clear(); f.insert(f.begin(), n, -1);}
4     int comp(int x){return (f[x]==-1?x:f[x]=comp(f[x]));} //O(1)
5     bool join(int i, int j) {
6         bool con=comp(i)==comp(j);
7         if(!con) f[comp(i)] = comp(j);
8         return con;
9     };

```

## 2.7. Disjoint Intervals

```

1 bool operator< (const ii &a, const ii &b) {return a.fst<b.fst;}
2 //Stores intervals as [first, second]
3 //in case of a collision it joins them in a single interval
4 struct disjoint_intervals {
5     set<ii> segs;
6     void insert(ii v) { //O(lgn)
7         if(v.snd-v.fst==0.) return; //OJO
8         set<ii>::iterator it, at;
9         at = it = segs.lower_bound(v);
10        if (at!=segs.begin() && (--at)->snd >= v.fst)
11            v.fst = at->fst, --it;
12        for(; it!=segs.end() && it->fst <= v.snd; segs.erase(it++))
13            v.snd=max(v.snd, it->snd);
14        segs.insert(v);
15    }
16 };

```

## 2.8. RMQ (2D)

```

1 struct RMQ2D{//n filas x m columnas
2     int sz;
3     RMQ t[4*MAXN];
4     RMQ &operator[] (int p){return t[sz/2+p];} //t[i][j]=i fila, j col
5     void init(int n, int m){ //O(n*m)
6         sz = 1 << (32-__builtin_clz(n));
7         forn(i, 2*sz) t[i].init(m); }

```

```

8     void set(int i, int j, tipo val){ //O(lgm.lgn)
9         for(i+=sz; i>0;){
10             t[i].set(j, val);
11             i/=2;
12             val=operacion(t[i*2][j], t[i*2+1][j]);
13         } }
14     tipo get(int i1, int j1, int i2, int j2){return get(i1,j1,i2,j2,1,0,
15         sz);}
16     //O(lgm.lgn), rangos cerrado abierto
17     int get(int i1, int j1, int i2, int j2, int n, int a, int b){
18         if(i2<=a || i1>=b) return 0;
19         if(i1<=a && b<=i2) return t[n].get(j1, j2);
20         int c=(a+b)/2;
21         return operacion(get(i1, j1, i2, j2, 2*n, a, c),
22             get(i1, j1, i2, j2, 2*n+1, c, b));
23     }
24 } rmq;
25 //Example to initialize a grid of M rows and N columns:
26 RMQ2D rmq; rmq.init(n,m);
27 forn(i, n) forn(j, m){
28     int v; cin >> v; rmq.set(i, j, v);}

```

## 2.9. Treap para set

```

1 typedef int Key;
2 typedef struct node *pnode;
3 struct node{
4     Key key;
5     int prior, size;
6     pnode l,r;
7     node(Key key=0): key(key), prior(rand()), size(1), l(0), r(0) {}
8 };
9 static int size(pnode p) { return p ? p->size : 0; }
10 void push(pnode p) {
11     // modificar y propagar el dirty a los hijos aca(para lazy)
12 }
13 // Update function and size from children's Value
14 void pull(pnode p) { //recalcular valor del nodo aca (para rmq)
15     p->size = 1 + size(p->l) + size(p->r);
16 }
17 //junta dos arreglos
18 pnode merge(pnode l, pnode r) {
19     if (!l || !r) return l ? l : r;

```

```

20 push(l), push(r);
21 pnode t;
22 if (l->prior < r->prior) l->r=merge(l->r, r), t = l;
23 else r->l=merge(l, r->l), t = r;
24 pull(t);
25 return t;
26 }
27 //parte el arreglo en dos, l<key<=r
28 void split(pnode t, Key key, pnode &l, pnode &r) {
29     if (!t) return void(l = r = 0);
30     push(t);
31     if (key <= t->key) split(t->l, key, l, t->l), r = t;
32     else split(t->r, key, t->r, r), l = t;
33     pull(t);
34 }
35
36 void erase(pnode &t, Key key) {
37     if (!t) return;
38     push(t);
39     if (key == t->key) t=merge(t->l, t->r);
40     else if (key < t->key) erase(t->l, key);
41     else erase(t->r, key);
42     if(t) pull(t);
43 }
44
45 pnode find(pnode t, Key key) {
46     if (!t) return 0;
47     if (key == t->key) return t;
48     if (key < t->key) return find(t->l, key);
49     return find(t->r, key);
50 }
51 struct treap {
52     pnode root;
53     treap(pnode root=0): root(root) {}
54     int size() { return ::size(root); }
55     void insert(Key key) {
56         pnode t1, t2; split(root, key, t1, t2);
57         t1=::merge(t1,new node(key));
58         root=::merge(t1,t2);
59     }
60     void erase(Key key1, Key key2) {
61         pnode t1,t2,t3;
62         split(root,key1,t1,t2);

```

```

63         split(t2,key2, t2, t3);
64         root=merge(t1,t3);
65     }

```

## 2.10. Treap para arreglo

```

1  typedef struct node *pnode;
2  struct node{
3      Value val, mini;
4      int dirty;
5      int prior, size;
6      pnode l,r,parent;
7      node(Value val): val(val), mini(val), dirty(0), prior(rand()), size
          (1), l(0), r(0), parent(0) {}
8  };
9  static int size(pnode p) { return p ? p->size : 0; }
10 void push(pnode p) { //propagar dirty a los hijos(aca para lazy)
11     p->val.fst+=p->dirty;
12     p->mini.fst+=p->dirty;
13     if(p->l) p->l->dirty+=p->dirty;
14     if(p->r) p->r->dirty+=p->dirty;
15     p->dirty=0;
16 }
17 static Value mini(pnode p) { return p ? push(p), p->mini : ii(1e9, -1);
    }
18 // Update function and size from children's Value
19 void pull(pnode p) { //recalcular valor del nodo aca (para rmq)
20     p->size = 1 + size(p->l) + size(p->r);
21     p->mini = min(min(p->val, mini(p->l)), mini(p->r)); //operacion del rmq
22     !
23     p->parent=0;
24     if(p->l) p->l->parent=p;
25     if(p->r) p->r->parent=p;
26 }
27 //junta dos arreglos
28 pnode merge(pnode l, pnode r) {
29     if (!l || !r) return l ? l : r;
30     push(l), push(r);
31     pnode t;
32     if (l->prior < r->prior) l->r=merge(l->r, r), t = l;
33     else r->l=merge(l, r->l), t = r;
34     pull(t);
35     return t;

```

```

35 }
36 //parte el arreglo en dos, sz(l)==tam
37 void split(pnode t, int tam, pnode &l, pnode &r) {
38     if (!t) return void(l = r = 0);
39     push(t);
40     if (tam <= size(t->l)) split(t->l, tam, l, t->l), r = t;
41     else split(t->r, tam - 1 - size(t->l), t->r, r), l = t;
42     pull(t);
43 }
44 pnode at(pnode t, int pos) {
45     if(!t) exit(1);
46     push(t);
47     if(pos == size(t->l)) return t;
48     if(pos < size(t->l)) return at(t->l, pos);
49     return at(t->r, pos - 1 - size(t->l));
50 }
51 int getpos(pnode t){//inversa de at
52     if(!t->parent) return size(t->l);
53     if(t==t->parent->l) return getpos(t->parent)-size(t->r)-1;
54     return getpos(t->parent)+size(t->l)+1;
55 }
56 void split(pnode t, int i, int j, pnode &l, pnode &m, pnode &r) {
57     split(t, i, l, t), split(t, j-i, m, r);}
58 Value get(pnode &p, int i, int j){//like rmq
59     pnode l,m,r;
60     split(p, i, j, l, m, r);
61     Value ret=mini(m);
62     p=merge(l, merge(m, r));
63     return ret;
64 }
65
66 //Sample program: C. LCA Online from Petrozavodsk Summer-2012.
67 //Petrozavodsk SU Contest
68 //Available at http://opentrains.snarknews.info/~ejudge
69 const int MAXN=300100;
70 int n;
71 pnode beg[MAXN], fin[MAXN];
72 pnode lista;

```

## 2.11. Gain-Cost Set

```

1 //esta estructura mantiene pairs(beneficio, costo)
2 //de tal manera que en el set quedan ordenados

```

```

3 //por beneficio Y COSTO creciente. (va borrando los que no son optimos)
4 struct V{
5     int gain, cost;
6     bool operator<(const V &b)const{return gain<b.gain;}
7 };
8 set<V> s;
9 void add(V x){
10     set<V>::iterator p=s.lower_bound(x);//primer elemento mayor o igual
11     if(p!=s.end() && p->cost <= x.cost) return;//ya hay uno mejor
12     p=s.upper_bound(x);//primer elemento mayor
13     if(p!=s.begin()){//borro todos los peores (<=beneficio y >=costo)
14         --p;//ahora es ultimo elemento menor o igual
15         while(p->cost >= x.cost){
16             if(p==s.begin()){s.erase(p); break;}
17             s.erase(p--);
18         }
19     }
20     s.insert(x);
21 }
22 int get(int gain){//minimo costo de obtener tal ganancia
23     set<V>::iterator p=s.lower_bound((V){gain, 0});
24     return p==s.end()? INF : p->cost;}

```

## 2.12. Set con busq binaria

```

1 #include <ext/pb_ds/assoc_container.hpp>
2 #include <ext/pb_ds/tree_policy.hpp>
3 using namespace __gnu_pbds;
4 typedef tree<int,null_type,less<int>,//key,mapped type, comparator
5     rb_tree_tag,tree_order_statistics_node_update> set_t;
6 //find_by_order(i) devuelve iterador al i-esimo elemento
7 //order_of_key(k): devuelve la pos del lower bound de k
8 //Ej: 12, 100, 505, 1000, 10000.
9 //order_of_key(10) == 0, order_of_key(100) == 1,
10 //order_of_key(707) == 3, order_of_key(9999999) == 5

```

## 2.13. Wavelet tree/matrix

```

1 ==> bitmap.hpp <==
2 #ifndef BITMAP_HPP
3 #define BITMAP_HPP
4 #include <vector>
5 #include "utils.hpp"
6 using namespace std;

```

```

7
8 // Indices start from 0
9 struct BitmapRank {
10     const int bits = sizeof(int)*8;
11     vector<int> vec;
12     vector<int> count;
13
14     BitmapRank() {}
15
16     void resize(int n) {
17         vec.resize((n+bits-1)/bits);
18         count.resize(vec.size());
19     }
20
21     void set(int i, bool b) {
22         set_bit(vec[i/bits], i % bits, b);
23     }
24
25     void build_rank() {
26         for (int i = 1; i < (int)vec.size(); ++i)
27             count[i] = count[i-1] + popcnt(vec[i-1]);
28     }
29
30     int rank1(int i) const {
31         return i < 0 ? 0 : count[i/bits] + popcnt(vec[i/bits] << (bits - i %
32             bits - 1));
33     }
34
35     int rank1(int i, int j) const {
36         return rank1(j) - rank1(i-1);
37     }
38
39     int rank0(int i) const {
40         return i < 0 ? 0 : i - rank1(i) + 1;
41     }
42
43     int rank0(int i, int j) const {
44         return rank0(j) - rank0(i-1);
45     }
46 };
47 #endif
48

```

```

49 ==> utils.hpp <==
50 #ifndef UTILS_HPP
51 #define UTILS_HPP
52
53 #define log2(x) (sizeof(uint)*8 - __builtin_clz(x))
54
55 #define popcnt(x) __builtin_popcount(x)
56
57 #define set_bit(v, i, b) v |= ((b) << (i))
58 #define get_bit(v, i) ((v) & (1 << (i)))
59
60 #endif
61
62 ==> wavelet-matrix.cpp <==
63 /*
64  *
65  * -----
66  * "THE BEER-WARE LICENSE" (Revision 42):
67  * <nlehmann@dcc.uchile.cl> wrote this file. As long as you retain this
68  * notice
69  * you can do whatever you want with this stuff. If we meet some day,
70  * and you
71  * think this stuff is worth it, you can buy me a beer in return Nicol'
72  * as Lehmann
73  *
74  * -----
75  */
76 #include <vector>
77 #include <cstdio>
78 #include <algorithm>
79 #include "utils.hpp"
80 #include "bitmap.hpp"
81 using namespace std;
82
83 typedef unsigned int uint;
84
85 // Wavelet Matrix with succinct representation of bitmaps
86 struct WaveMatrixSucc {
87     uint height;
88     vector<BitmapRank> B;
89     vector<int> z;
90

```



```

85
86 WaveMatrixSucc(vector<int> &A) :
87     WaveMatrixSucc(A, *max_element(A.begin(), A.end()) + 1) {}
88
89 // sigma = size of the alphabet, ie., one more than the maximum
90 // element
91 // in A.
92 WaveMatrixSucc(vector<int> &A, int sigma)
93 : height(log2(sigma - 1)),
94   B(height), z(height) {
95     for (uint l = 0; l < height; ++l) {
96       B[l].resize(A.size());
97       for (uint i = 0; i < A.size(); ++i)
98         B[l].set(i, get_bit(A[i], height - l - 1));
99       B[l].build_rank();
100
101       auto it = stable_partition(A.begin(), A.end(), [=] (int c) {
102         return not get_bit(c, height - l - 1);
103       });
104       z[l] = distance(A.begin(), it);
105     }
106
107 // Count occurrences of number c until position i.
108 // ie, occurrences of c in positions [i,j]
109 int rank(int c, int i) const {
110     int p = -1;
111     for (uint l = 0; l < height; ++l) {
112       if (get_bit(c, height - l - 1)) {
113         p = z[l] + B[l].rank1(p) - 1;
114         i = z[l] + B[l].rank1(i) - 1;
115       } else {
116         p = B[l].rank0(p) - 1;
117         i = B[l].rank0(i) - 1;
118       }
119     }
120     return i - p;
121 }
122
123 // Find the k-th smallest element in positions [i,j].
124 // The smallest element is k=1
125 int quantile(int k, int i, int j) const {
126     int element = 0;

```

```

127     for (uint l = 0; l < height; ++l) {
128       int r = B[l].rank0(i, j);
129       if (r >= k) {
130         i = B[l].rank0(i-1);
131         j = B[l].rank0(j) - 1;
132       } else {
133         i = z[l] + B[l].rank1(i-1);
134         j = z[l] + B[l].rank1(j) - 1;
135         k -= r;
136         set_bit(element, height - l - 1, 1);
137       }
138     }
139     return element;
140 }
141
142 // Count number of occurrences of numbers in the range [a, b]
143 // present in the sequence in positions [i, j], ie, if representing a
144 // grid it
145 // counts number of points in the specified rectangle.
146 int range(int i, int j, int a, int b) const {
147     return range(i, j, a, b, 0, (1 << height)-1, 0);
148 }
149
150 int range(int i, int j, int a, int b, int L, int U, int l) const {
151     if (b < L || U < a)
152         return 0;
153
154     int M = L + (U-L)/2;
155     if (a <= L && U <= b)
156         return j - i + 1;
157     else {
158         int left = range(B[l].rank0(i-1), B[l].rank0(j) - 1,
159                         a, b, L, M, l + 1);
160         int right = range(z[l] + B[l].rank1(i-1), z[l] + B[l].rank1(j) -
161                          1,
162                          a, b, M+1, U, l+1);
163         return left + right;
164     }
165 }
166 };
167
168 ==> wavelet-tree.cpp <==
169 #include<vector>

```

```

168 #include<algorithm>
169 #include "bitmap.hpp"
170 using namespace std;
171 typedef vector<int>::iterator iter;
172
173 //Wavelet tree with succinct representation of bitmaps
174 struct WaveTreeSucc {
175     vector<vector<int> > C; int s;
176
177     // sigma = size of the alphabet, ie., one more than the maximum
178     // element
179     // in S.
180     WaveTreeSucc(vector<int> &A, int sigma) : C(sigma*2), s(sigma) {
181         build(A.begin(), A.end(), 0, s-1, 1);
182     }
183
184     void build(iter b, iter e, int L, int U, int u) {
185         if (L == U)
186             return;
187         int M = (L+U)/2;
188
189         // C[u][i] contains number of zeros until position i-1: [0,i)
190         C[u].reserve(e-b+1); C[u].push_back(0);
191         for (iter it = b; it != e; ++it)
192             C[u].push_back(C[u].back() + (*it<=M));
193
194         iter p = stable_partition(b, e, [=](int i){return i<=M;});
195
196         build(b, p, L, M, u*2);
197         build(p, e, M+1, U, u*2+1);
198     }
199
200     // Count occurrences of number c until position i.
201     // ie, occurrences of c in positions [i,j]
202     int rank(int c, int i) const {
203         // Internally we consider an interval open on the left: [0, i)
204         i++;
205         int L = 0, U = s-1, u = 1, M, r;
206         while (L != U) {
207             M = (L+U)/2;
208             r = C[u][i]; u*=2;
209             if (c <= M)

```

```

210             else
211                 i -= r, L = M+1, ++u;
212         }
213         return i;
214     }
215
216     // Find the k-th smallest element in positions [i,j].
217     // The smallest element is k=1
218     int quantile(int k, int i, int j) const {
219         // internally we consider an interval open on the left: [i, j)
220         j++;
221         int L = 0, U = s-1, u = 1, M, ri, rj;
222         while (L != U) {
223             M = (L+U)/2;
224             ri = C[u][i]; rj = C[u][j]; u*=2;
225             if (k <= rj-ri)
226                 i = ri, j = rj, U = M;
227             else
228                 k -= rj-ri, i -= ri, j -= rj,
229                 L = M+1, ++u;
230         }
231         return U;
232     }
233
234     // Count number of occurrences of numbers in the range [a, b]
235     // present in the sequence in positions [i, j], ie, if representing a
236     // grid it
237     // counts number of points in the specified rectangle.
238     mutable int L, U;
239     int range(int i, int j, int a, int b) const {
240         if (b < a or j < i)
241             return 0;
242         L = a; U = b;
243         return range(i, j+1, 0, s-1, 1);
244     }
245
246     int range(int i, int j, int a, int b, int u) const {
247         if (b < L or U < a)
248             return 0;
249         if (L <= a and b <= U)
250             return j-i;
251         int M = (a+b)/2, ri = C[u][i], rj = C[u][j];
252         return range(ri, rj, a, M, u*2) +

```

```

252     range(i-ri, j-rj, M+1, b, u*2+1);
253 }
254 };

```

### 3. Algos

#### 3.1. Longest Increasing Subsequence

```

1 //Para non-increasing, cambiar comparaciones y revisar busq binaria
2 //Given an array, paint it in the least number of colors so that each
  color turns to a non-increasing subsequence.
3 //Solution:Min number of colors=Length of the longest increasing
  subsequence
4 int N, a[MAXN]; //secuencia y su longitud
5 ii d[MAXN+1]; //d[i]=ultimo valor de la subsecuencia de tamaño i
6 int p[MAXN]; //padres
7 vector<int> R; //respuesta
8 void rec(int i){
9     if(i== -1) return;
10    R.push_back(a[i]);
11    rec(p[i]);
12 }
13 int lis(){ //O(nlogn)
14     d[0] = ii(-INF, -1); for(i, N) d[i+1]=ii(INF, -1);
15     for(i, N){
16         int j = upper_bound(d, d+N+1, ii(a[i], INF))-d;
17         if (d[j-1].first < a[i] && a[i] < d[j].first){
18             p[i]=d[j-1].second;
19             d[j] = ii(a[i], i);
20         }
21     }
22     R.clear();
23     dforn(i, N+1) if(d[i].first!=INF){
24         rec(d[i].second); //reconstruir
25         reverse(R.begin(), R.end());
26         return i; //longitud
27     }
28     return 0;
29 }

```

#### 3.2. Alpha-Beta pruning

```

1 ll alphabeta(State &s, bool player = true, int depth = 1e9, ll alpha = -
  INF, ll beta = INF) { //player = true -> Maximiza

```

```

2     if(s.isFinal()) return s.score;
3     //~ if (!depth) return s.heuristic();
4     vector<State> children;
5     s.expand(player, children);
6     int n = children.size();
7     forn(i, n) {
8         ll v = alphabeta(children[i], !player, depth-1, alpha, beta);
9         if(!player) alpha = max(alpha, v);
10        else beta = min(beta, v);
11        if(beta <= alpha) break;
12    }
13    return !player ? alpha : beta;

```

#### 3.3. Mo's algorithm

```

1 int n, sq;
2 struct Qu{ //queries [l, r]
3     //intervalos cerrado abiertos !!! importante!!
4     int l, r, id;
5 }qs[MAXN];
6 int ans[MAXN], curans; //ans[i]=ans to ith query
7 bool bymos(const Qu &a, const Qu &b){
8     if(a.l/sq!=b.l/sq) return a.l<b.l;
9     return (a.l/sq)&1? a.r<b.r : a.r>b.r;
10 }
11 void mos(){
12     forn(i, t) qs[i].id=i;
13     sort(qs, qs+t, bymos);
14     int cl=0, cr=0;
15     sq=sqrt(n);
16     curans=0;
17     forn(i, t){ //intervalos cerrado abiertos !!! importante!!
18         Qu &q=qs[i];
19         while(cl>q.l) add(--cl);
20         while(cr<q.r) add(cr++);
21         while(cl<q.l) remove(cl++);
22         while(cr>q.r) remove(--cr);
23         ans[q.id]=curans;
24     }
25 }

```

#### 3.4. Ternary search

```

1 #include <functional>

```

```

2 //Retorna argmax de una funcion unimodal 'f' en el rango [left,right]
3 double ternarySearch(double l, double r, function<double(double)> f){
4     for(int i = 0; i < 300; i++){
5         double m1 = l+(r-l)/3, m2 = r-(r-l)/3;
6         if (f(m1) < f(m2)) l = m1; else r = m2;
7     }
8     return (left + right)/2;
9 }

```

## 4. Strings

### 4.1. Manacher

```

1 int d1[MAXN]; //d1[i]=long del maximo palindromo impar con centro en i
2 int d2[MAXN]; //d2[i]=analogo pero para longitud par
3 //0 1 2 3 4
4 //a a b c c <--d1[2]=3
5 //a a b b <--d2[2]=2 (estan uno antes)
6 void manacher(){
7     int l=0, r=-1, n=sz(s);
8     forn(i, n){
9         int k=(i>r? 1 : min(d1[l+r-i], r-i));
10        while(i+k<n && i-k>=0 && s[i+k]==s[i-k]) ++k;
11        d1[i] = k--;
12        if(i+k > r) l=i-k, r=i+k;
13    }
14    l=0, r=-1;
15    forn(i, n){
16        int k=(i>r? 0 : min(d2[l+r-i+1], r-i+1))+1;
17        while(i+k-1<n && i-k>=0 && s[i+k-1]==s[i-k]) k++;
18        d2[i] = --k;
19        if(i+k-1 > r) l=i-k, r=i+k-1;
20    }

```

### 4.2. KMP

```

1 string T; //cadena donde buscar(what)
2 string P; //cadena a buscar(what)
3 int b[MAXLEN]; //back table b[i] maximo borde de [0..i]
4 void kmppre(){ //by gabina with love
5     int i = 0, j = -1; b[0] = -1;
6     while(i < sz(P)){
7         while(j >= 0 && P[i] != P[j]) j = b[j];

```

```

8         i++, j++, b[i] = j;
9     }
10 }
11 void kmp(){
12     int i=0, j=0;
13     while(i < sz(T)){
14         while(j >= 0 && T[i] != P[j]) j = b[j];
15         i++, j++;
16         if(j == sz(P)) printf("P is found at index %d in T\n", i-j), j = b[j];
17     }
18 }
19
20 int main(){
21     cout << "T=";
22     cin >> T;
23     cout << "P=";

```

### 4.3. Trie

```

1 struct trie{
2     map<char, trie> m;
3     void add(const string &s, int p=0){
4         if(s[p]) m[s[p]].add(s, p+1);
5     }
6     void dfs(){
7         //Do stuff
8         forall(it, m)
9             it->second.dfs();
10    }
11 };

```

### 4.4. Suffix Array (largo, nlogn)

```

1 #define MAX_N 1000
2 #define rBOUND(x) (x<n? r[x] : 0)
3 //sa will hold the suffixes in order.
4 int sa[MAX_N], r[MAX_N], n;
5 string s; //input string, n=sz(s)
6
7 int f[MAX_N], tmpsa[MAX_N];
8 void countingSort(int k){
9     zero(f);
10    forn(i, n) f[rBOUND(i+k)]++;

```

```

11  int sum=0;
12  forn(i, max(255, n)){
13      int t=f[i]; f[i]=sum; sum+=t;}
14  forn(i, n)
15      tmpsa[f[rBOUND(sa[i]+k)]++] = sa[i];
16  memcpy(sa, tmpsa, sizeof(sa));
17  }
18  void constructsa(){//O(n log n)
19      n=sz(s);
20      forn(i, n) sa[i]=i, r[i]=s[i];
21      for(int k=1; k<n; k<=1){
22          countingSort(k), countingSort(0);
23          int rank, tmpr[MAX_N];
24          tmpr[sa[0]]=rank=0;
25          forr(i, 1, n)
26              tmpr[sa[i]]=(r[sa[i]]==r[sa[i-1]] && r[sa[i]+k]==r[sa[i-1]+k])?
                  rank : ++rank;
27          memcpy(r, tmpr, sizeof(r));
28          if(r[sa[n-1]]==n-1) break;
29      }
30  }
31  void print(){//for debug
32      forn(i, n)
33          cout << i << ' ' <<
34          s.substr(sa[i], s.find( '$', sa[i])-sa[i]) << endl;}

```

#### 4.5. String Matching With Suffix Array

```

1  //returns (lowerbound, upperbound) of the search
2  ii stringMatching(string P){ //O(sz(P)lgn)
3      int lo=0, hi=n-1, mid=lo;
4      while(lo<hi){
5          mid=(lo+hi)/2;
6          int res=s.compare(sa[mid], sz(P), P);
7          if(res>=0) hi=mid;
8          else lo=mid+1;
9      }
10     if(s.compare(sa[lo], sz(P), P)!=0) return ii(-1, -1);
11     ii ans; ans.fst=lo;
12     lo=0, hi=n-1, mid;
13     while(lo<hi){
14         mid=(lo+hi)/2;
15         int res=s.compare(sa[mid], sz(P), P);

```

```

16         if(res>0) hi=mid;
17         else lo=mid+1;
18     }
19     if(s.compare(sa[hi], sz(P), P)!=0) hi--;
20     ans.snd=hi;
21     return ans;
22 }

```

#### 4.6. LCP (Longest Common Prefix)

```

1  //Calculates the LCP between consecutives suffixes in the Suffix Array.
2  //LCP[i] is the length of the LCP between sa[i] and sa[i-1]
3  int LCP[MAX_N], phi[MAX_N], PLCP[MAX_N];
4  void computeLCP(){//O(n)
5      phi[sa[0]]=-1;
6      forr(i, 1, n) phi[sa[i]]=sa[i-1];
7      int L=0;
8      forn(i, n){
9          if(phi[i]==-1) {PLCP[i]=0; continue;}
10         while(s[i+L]==s[phi[i]+L]) L++;
11         PLCP[i]=L;
12         L=max(L-1, 0);
13     }
14     forn(i, n) LCP[i]=PLCP[sa[i]];
15 }

```

#### 4.7. Corasick

```

1
2  struct trie{
3      map<char, trie> next;
4      trie* tran[256]; //transiciones del automata
5      int idhoja, szhoja; //id de la hoja o 0 si no lo es
6      //link lleva al sufijo mas largo, nxthoja lleva al mas largo pero que
        es hoja
7      trie *padre, *link, *nxthoja;
8      char pch; //caracter que conecta con padre
9      trie(): tran(), idhoja(), padre(), link() {}
10     void insert(const string &s, int id=1, int p=0){ //id>0!!!
11         if(p<sz(s)){
12             trie &ch=next[s[p]];
13             tran[(int)s[p]]=&ch;
14             ch.padre=this, ch.pch=s[p];
15             ch.insert(s, id, p+1);

```

```

16     }
17     else idhoja=id, szhoja=sz(s);
18 }
19 trie* get_link() {
20     if(!link){
21         if(!padre) link=this; //es la raiz
22         else if(!padre->padre) link=padre; //hijo de la raiz
23         else link=padre->get_link()->get_tran(pch);
24     }
25     return link; }
26 trie* get_tran(int c) {
27     if(!tran[c]) tran[c] = !padre? this : this->get_link()->get_tran(c);
28     return tran[c]; }
29 trie *get_nxthoja(){
30     if(!nxthoja) nxthoja = get_link()->idhoja? link : link->nxthoja;
31     return nxthoja; }
32 void print(int p){
33     if(idhoja) cout << "found_" << idhoja << "_at_position_" << p-
34         szhoja << endl;
35     if(get_nxthoja()) get_nxthoja()->print(p); }
36 void matching(const string &s, int p=0){
37     print(p); if(p<sz(s)) get_tran(s[p])->matching(s, p+1); }
38 }tri;
39
40 int main(){
41     tri=trie(); //clear
42     tri.insert("ho", 1);
43     tri.insert("hoho", 2);

```

#### 4.8. Suffix Automaton

```

1 struct state {
2     int len, link;
3     map<char,int> next;
4     state() { }
5 };
6 const int MAXLEN = 10010;
7 state st[MAXLEN*2];
8 int sz, last;
9 void sa_init() {
10     forn(i,sz) st[i].next.clear();
11     sz = last = 0;

```

```

12     st[0].len = 0;
13     st[0].link = -1;
14     ++sz;
15 }
16 // Es un DAG de una sola fuente y una sola hoja
17 // cantidad de endpos = cantidad de apariciones = cantidad de caminos de
18 // la clase al nodo terminal
19 // cantidad de miembros de la clase = st[v].len-st[st[v].link].len (v>0)
20 // = caminos del inicio a la clase
21 // El arbol de los suffix links es el suffix tree de la cadena invertida
22 // La string de la arista link(v)->v son los caracteres que difieren
23 void sa_extend (char c) {
24     int cur = sz++;
25     st[cur].len = st[last].len + 1;
26     // en cur agregamos la posicion que estamos extendiendo
27     //podria agregar tambien un identificador de las cadenas a las cuales
28     //pertenece (si hay varias)
29     int p;
30     for (p=last; p!=-1 && !st[p].next.count(c); p=st[p].link) // modificar
31         esta linea para hacer separadores unicos entre varias cadenas (c
32         =='$')
33     st[p].next[c] = cur;
34     if (p == -1)
35         st[cur].link = 0;
36     else {
37         int q = st[p].next[c];
38         if (st[p].len + 1 == st[q].len)
39             st[cur].link = q;
40         else {
41             int clone = sz++;
42             // no le ponemos la posicion actual a clone sino indirectamente
43             // por el link de cur
44             st[clone].len = st[p].len + 1;
45             st[clone].next = st[q].next;
46             st[clone].link = st[q].link;
47             for (; p!=-1 && st[p].next.count(c) && st[p].next[c]==q; p=st[p].
48                 link)
49                 st[p].next[c] = clone;
50             st[q].link = st[cur].link = clone;
51         }
52     }
53     last = cur;
54 }

```

## 4.9. Z Function

```

1 char s[MAXN];
2 int z[MAXN]; // z[i] = i==0 ? 0 : max k tq s[0,k) match with s[i,i+k)
3 void z_function(char s[],int z[]) {
4     int n = strlen(s);
5     forn(i, n) z[i]=0;
6     for (int i = 1, l = 0, r = 0; i < n; ++i) {
7         if (i <= r) z[i] = min (r - i + 1, z[i - l]);
8         while (i + z[i] < n && s[z[i]] == s[i + z[i]]) ++z[i];
9         if (i + z[i] - 1 > r) l = i, r = i + z[i] - 1;
10    }
11 }
12
13 int main() {
14     ios::sync_with_stdio(0);

```

## 4.10. Palindromic tree

```

1 const int maxn = 10100100;
2
3 int len[maxn];
4 int suffLink[maxn];
5 int to[maxn][2];
6 int cnt[maxn];
7 int numV;
8 char str[maxn];
9
10 int v;
11
12 void addLetter(int n) {
13     while (str[n - len[v] - 1] != str[n] )
14         v = suffLink[v];
15     int u = suffLink[v];
16     while (str[n - len[u] - 1] != str[n] )
17         u = suffLink[u];
18     int u_ = to[u][str[n] - 'a'];
19     int v_ = to[v][str[n] - 'a'];
20     if (v_ == -1)
21     {
22         v_ = to[v][str[n] - 'a'] = numV;
23         len[numV++] = len[v] + 2;
24         suffLink[v_] = u_;

```

```

25     }
26     v = v_;
27     cnt[v]++;
28 }
29
30 void init() {
31     memset(to, -1, sizeof to);
32     str[0] = '#';
33     len[0] = -1;
34     len[1] = 0;
35     len[2] = len[3] = 1;
36     suffLink[1] = 0;
37     suffLink[0] = 0;
38     suffLink[2] = 1;
39     suffLink[3] = 1;
40     to[0][0] = 2;
41     to[0][1] = 3;
42     numV = 4;
43 }
44
45 int main() {
46     init();
47     scanf("%s", str + 1);
48     int n = strlen(str);
49     for (int i = 1; i < n; i++)
50         addLetter(i);
51     long long ans = 0;
52     for (int i = numV - 1; i > 0; i--) {
53         cnt[suffLink[i]] += cnt[i];
54         ans = max(ans, cnt[i] * 1LL * len[i] );
55         // fprintf(stderr, "i = %d, cnt = %d, len = %d\n", i, cnt[i]
56         ], len[i] );
57     }
58     printf("%lld\n", ans);
59     return 0;

```

## 4.11. Rabin Karp - Distinct Substrings

```

1 int count_unique_substrings(string const& s) {
2     int n = s.size();
3
4     const int p = 31;

```

```

5   const int m = 1e9 + 9;
6   vector<long long> p_pow(n);
7   p_pow[0] = 1;
8   for (int i = 1; i < n; i++)
9       p_pow[i] = (p_pow[i-1] * p) % m;

10
11   vector<long long> h(n + 1, 0);
12   for (int i = 0; i < n; i++)
13       h[i+1] = (h[i] + (s[i] - 'a' + 1) * p_pow[i]) % m;

14
15   int cnt = 0;
16   for (int l = 1; l <= n; l++) {
17       set<long long> hs;
18       for (int i = 0; i <= n - l; i++) {
19           long long cur_h = (h[i + l] + m - h[i]) % m;
20           cur_h = (cur_h * p_pow[n-i-1]) % m;
21           hs.insert(cur_h);
22       }
23       cnt += hs.size();
24   }
25   return cnt;
26 }

```

## 5. Geometria

### 5.1. Punto

```

1 struct pto{
2     double x, y;
3     pto(double x=0, double y=0):x(x),y(y){}
4     pto operator+(pto a){return pto(x+a.x, y+a.y);}
5     pto operator-(pto a){return pto(x-a.x, y-a.y);}
6     pto operator+(double a){return pto(x+a, y+a);}
7     pto operator*(double a){return pto(x*a, y*a);}
8     pto operator/(double a){return pto(x/a, y/a);}
9     //dot product, producto interno:
10    double operator*(pto a){return x*a.x+y*a.y;}
11    //module of the cross product or vectorial product:
12    //if a is less than 180 clockwise from b, a^b>0
13    double operator^(pto a){return x*a.y-y*a.x;}
14    //returns true if this is at the left side of line qr
15    bool left(pto q, pto r){return ((q-*this)^(r-*this))>0;}
16    bool operator<(const pto &a) const{return x<a.x-EPS || (abs(x-a.x)<EPS

```

```

    && y<a.y-EPS);}
17 bool operator==(pto a){return abs(x-a.x)<EPS && abs(y-a.y)<EPS;}
18 double norm(){return sqrt(x*x+y*y);}
19 double norm_sq(){return x*x+y*y;}
20 };
21 double dist(pto a, pto b){return (b-a).norm();}
22 typedef pto vec;
23
24 double angle(pto a, pto o, pto b){
25     pto oa=a-o, ob=b-o;
26     return atan2(oa^ob, oa*ob);}
27
28 //rotate p by theta rads CCW w.r.t. origin (0,0)
29 pto rotate(pto p, double theta){
30     return pto(p.x*cos(theta)-p.y*sin(theta),
31               p.x*sin(theta)+p.y*cos(theta));
32 }

```

### 5.2. Orden radial de puntos

```

1 struct Cmp{//orden total de puntos alrededor de un punto r
2     pto r;
3     Cmp(pto r):r(r) {}
4     int cuad(const pto &a) const{
5         if(a.x > 0 && a.y >= 0)return 0;
6         if(a.x <= 0 && a.y > 0)return 1;
7         if(a.x < 0 && a.y <= 0)return 2;
8         if(a.x >= 0 && a.y < 0)return 3;
9         assert(a.x ==0 && a.y==0);
10        return -1;
11    }
12    bool cmp(const pto&p1, const pto&p2)const{
13        int c1 = cuad(p1), c2 = cuad(p2);
14        if(c1==c2) return p1.y*p2.x<p1.x*p2.y;
15        else return c1 < c2;
16    }
17    bool operator()(const pto&p1, const pto&p2) const{
18        return cmp(pto(p1.x-r.x,p1.y-r.y),pto(p2.x-r.x,p2.y-r.y));
19    }
20 };

```

### 5.3. Line

```

1 int sgn(ll x){return x<0? -1 : !!x;}

```



```

2 struct line{
3     line() {}
4     double a,b,c;//Ax+By=C
5     //pto MUST store float coordinates!
6     line(double a, double b, double c):a(a),b(b),c(c){}
7     line(pto p, pto q): a(q.y-p.y), b(p.x-q.x), c(a*p.x+b*p.y) {}
8     int side(pto p){return sgn(l1(a) * p.x + l1(b) * p.y - c);}
9 };
10 bool parallels(line l1, line l2){return abs(l1.a*l2.b-l2.a*l1.b)<EPS;}
11 pto inter(line l1, line l2){//intersection
12     double det=l1.a*l2.b-l2.a*l1.b;
13     if(abs(det)<EPS) return pto(INF, INF);//parallels
14     return pto(l2.b*l1.c-l1.b*l2.c, l1.a*l2.c-l2.a*l1.c)/det;
15 }

```

## 5.4. Segment

```

1 struct segm{
2     pto s,f;
3     segm(pto s, pto f):s(s), f(f) {}
4     pto closest(pto p) {//use for dist to point
5         double l2 = dist_sq(s, f);
6         if(l2==0.) return s;
7         double t=((p-s)*(f-s))/l2;
8         if (t<0.) return s;//not write if is a line
9         else if(t>1.)return f;//not write if is a line
10        return s+((f-s)*t);
11    }
12    bool inside(pto p){return abs(dist(s, p)+dist(p, f)-dist(s, f))<EPS;
13        ;}
14 };
15 pto inter(segm s1, segm s2){
16     pto r=inter(line(s1.s, s1.f), line(s2.s, s2.f));
17     if(s1.inside(r) && s2.inside(r)) return r;
18     return pto(INF, INF);
19 }

```

## 5.5. Rectangle

```

1 struct rect{
2     //lower-left and upper-right corners
3     pto lw, up;
4 };

```

```

5 //returns if there's an intersection and stores it in r
6 bool inter(rect a, rect b, rect &r){
7     r.lw=pto(max(a.lw.x, b.lw.x), max(a.lw.y, b.lw.y));
8     r.up=pto(min(a.up.x, b.up.x), min(a.up.y, b.up.y));
9     //check case when only a edge is common
10    return r.lw.x<r.up.x && r.lw.y<r.up.y;
11 }

```

## 5.6. Polygon Area

```

1 double area(vector<pto> &p){//0(sz(p))
2     double area=0;
3     forn(i, sz(p)) area+=p[i]^p[(i+1)%sz(p)];
4     //if points are in clockwise order then area is negative
5     return abs(area)/2;
6 }
7 //Area ellipse = M_PI*a*b where a and b are the semi axis lengths
8 //Area triangle = sqrt(s*(s-a)(s-b)(s-c)) where s=(a+b+c)/2

```

## 5.7. Circle

```

1 vec perp(vec v){return vec(-v.y, v.x);}
2 line bisector(pto x, pto y){
3     line l=line(x, y); pto m=(x+y)/2;
4     return line(-l.b, l.a, -l.b*m.x+l.a*m.y);
5 }
6 struct Circle{
7     pto o;
8     double r;
9     Circle(pto x, pto y, pto z){
10        o=inter(bisector(x, y), bisector(y, z));
11        r=dist(o, x);
12    }
13    pair<pto, pto> ptosTang(pto p){
14        pto m=(p+o)/2;
15        tipo d=dist(o, m);
16        tipo a=r*r/(2*d);
17        tipo h=sqrt(r*r-a*a);
18        pto m2=o+(m-o)*a/d;
19        vec per=perp(m-o)/d;
20        return make_pair(m2-per*h, m2+per*h);
21    }
22 };
23 //finds the center of the circle containing p1 and p2 with radius r

```

```

24 //as there may be two solutions swap p1, p2 to get the other
25 bool circle2PtsRad(pto p1, pto p2, double r, pto &c){
26     double d2=(p1-p2).norm_sq(), det=r*r/d2-0.25;
27     if(det<0) return false;
28     c=(p1+p2)/2+perp(p2-p1)*sqrt(det);
29     return true;
30 }
31 #define sqr(a) ((a)*(a))
32 #define feq(a,b) (fabs((a)-(b))<EPS)
33 pair<tipo, tipo> ecCuad(tipo a, tipo b, tipo c){//a*x*x+b*x+c=0
34     tipo dx = sqrt(b*b-4.0*a*c);
35     return make_pair((-b + dx)/(2.0*a), (-b - dx)/(2.0*a));
36 }
37 pair<pto, pto> interCL(Circle c, line l){
38     bool sw=false;
39     if((sw=feq(0,l.b))){
40         swap(l.a, l.b);
41         swap(c.o.x, c.o.y);
42     }
43     pair<tipo, tipo> rc = ecCuad(
44         sqr(l.a)+sqr(l.b),
45         2.0*l.a*l.b*c.o.y-2.0*(sqr(l.b)*c.o.x+l.c*l.a),
46         sqr(l.b)*(sqr(c.o.x)+sqr(c.o.y)-sqr(c.r))+sqr(l.c)-2.0*l.c*l.b*c.o.y
47     );
48     pair<pto, pto> p( pto(rc.first, (l.c - l.a * rc.first) / l.b),
49         pto(rc.second, (l.c - l.a * rc.second) / l.b) );
50     if(sw){
51         swap(p.first.x, p.first.y);
52         swap(p.second.x, p.second.y);
53     }
54     return p;
55 }
56 pair<pto, pto> interCC(Circle c1, Circle c2){
57     line l;
58     l.a = c1.o.x-c2.o.x;
59     l.b = c1.o.y-c2.o.y;
60     l.c = (sqr(c2.r)-sqr(c1.r)+sqr(c1.o.x)-sqr(c2.o.x)+sqr(c1.o.y)
61         -sqr(c2.o.y))/2.0;
62     return interCL(c1, l);
63 }

```

## 5.8. Point in Poly

```

1 //checks if v is inside of P, using ray casting
2 //works with convex and concave.
3 //excludes boundaries, handle it separately using segment.inside()
4 bool inPolygon(pto v, vector<pto>& P) {
5     bool c = false;
6     forn(i, sz(P)){
7         int j=(i+1)%sz(P);
8         if((P[j].y>v.y) != (P[i].y > v.y) &&
9             (v.x < (P[i].x - P[j].x) * (v.y-P[j].y) / (P[i].y - P[j].y) + P[j].x))
10            c = !c;
11     }
12     return c;
13 }

```

## 5.9. Point in Convex Poly log(n)

```

1 void normalize(vector<pto> &pt){//delete collinear points first!
2     //this makes it clockwise:
3     if(pt[2].left(pt[0], pt[1])) reverse(pt.begin(), pt.end());
4     int n=sz(pt), pi=0;
5     forn(i, n)
6         if(pt[i].x<pt[pi].x || (pt[i].x==pt[pi].x && pt[i].y<pt[pi].y))
7             pi=i;
8     vector<pto> shift(n);//puts pi as first point
9     forn(i, n) shift[i]=pt[(pi+i)%n];
10    pt.swap(shift);
11 }
12 bool inPolygon(pto p, const vector<pto> &pt){
13     //call normalize first!
14     if(p.left(pt[0], pt[1]) || p.left(pt[sz(pt)-1], pt[0])) return false;
15     int a=1, b=sz(pt)-1;
16     while(b-a>1){
17         int c=(a+b)/2;
18         if(!p.left(pt[0], pt[c])) a=c;
19         else b=c;
20     }
21     return !p.left(pt[a], pt[a+1]);
22 }

```

## 5.10. Convex Check CHECK

```

1 bool isConvex(vector<int> &p){//O(N), delete collinear points!
2     int N=sz(p);
3     if(N<3) return false;

```

```

4 bool isLeft=p[0].left(p[1], p[2]);
5 forr(i, 1, N)
6     if(p[i].left(p[(i+1)%N], p[(i+2)%N])!=isLeft)
7         return false;
8 return true; }

```

### 5.11. Convex Hull

```

1 //stores convex hull of P in S, CCW order
2 //left must return >=0 to delete collinear points!
3 void CH(vector<pto>& P, vector<pto> &S){
4     S.clear();
5     sort(P.begin(), P.end()); //first x, then y
6     forn(i, sz(P)){ //lower hull
7         while(sz(S)>= 2 && S[sz(S)-1].left(S[sz(S)-2], P[i])) S.pop_back();
8         S.pb(P[i]);
9     }
10    S.pop_back();
11    int k=sz(S);
12    dforn(i, sz(P)){ //upper hull
13        while(sz(S) >= k+2 && S[sz(S)-1].left(S[sz(S)-2], P[i])) S.pop_back();
14        S.pb(P[i]);
15    }
16    S.pop_back();
17 }

```

### 5.12. Cut Polygon

```

1 //cuts polygon Q along the line ab
2 //stores the left side (swap a, b for the right one) in P
3 void cutPolygon(pto a, pto b, vector<pto> Q, vector<pto> &P){
4     P.clear();
5     forn(i, sz(Q)){
6         double left1=(b-a)^(Q[i]-a), left2=(b-a)^(Q[(i+1)%sz(Q)]-a);
7         if(left1>=0) P.pb(Q[i]);
8         if(left1*left2<0)
9             P.pb(inter(line(Q[i], Q[(i+1)%sz(Q)]), line(a, b)));
10    }
11 }

```

### 5.13. Bresenham

```

1 //plot a line approximation in a 2d map

```

```

2 void bresenham(pto a, pto b){
3     pto d=b-a; d.x=abs(d.x), d.y=abs(d.y);
4     pto s(a.x<b.x? 1: -1, a.y<b.y? 1: -1);
5     int err=d.x-d.y;
6     while(1){
7         m[a.x][a.y]=1; //plot
8         if(a==b) break;
9         int e2=err;
10        if(e2 >= 0) err-=2*d.y, a.x+=s.x;
11        if(e2 <= 0) err+= 2*d.x, a.y+= s.y;
12    }
13 }

```

### 5.14. Rotate Matrix

```

1 //rotates matrix t 90 degrees clockwise
2 //using auxiliary matrix t2(faster)
3 void rotate(){
4     forn(x, n) forn(y, n)
5         t2[n-y-1][x]=t[x][y];
6     memcpy(t, t2, sizeof(t));
7 }

```

### 5.15. Interseccion de Circulos en $n^3 \log(n)$

```

1 struct event {
2     double x; int t;
3     event(double xx, int tt) : x(xx), t(tt) {}
4     bool operator <(const event &o) const { return x < o.x; }
5 };
6 typedef vector<Circle> VC;
7 typedef vector<event> VE;
8 int n;
9 double cuenta(VE &v, double A, double B) {
10    sort(v.begin(), v.end());
11    double res = 0.0, lx = ((v.empty())?0.0:v[0].x);
12    int contador = 0;
13    forn(i, sz(v)) {
14        //interseccion de todos (contador == n), union de todos (
15            contador > 0)
16        //conjunto de puntos cubierto por exacta k Circulos (contador ==
17            k)
18        if (contador == n) res += v[i].x - lx;
19        contador += v[i].t, lx = v[i].x;

```

```

18     }
19     return res;
20 }
21 // Primitiva de sqrt(r*r - x*x) como funcion double de una variable x.
22 inline double primitiva(double x,double r) {
23     if (x >= r) return r*r*M_PI/4.0;
24     if (x <= -r) return -r*r*M_PI/4.0;
25     double raiz = sqrt(r*r-x*x);
26     return 0.5 * (x * raiz + r*r*atan(x/raiz));
27 }
28 double interCircle(VC &v) {
29     vector<double> p; p.reserve(v.size() * (v.size() + 2));
30     forn(i,sz(v)) p.push_back(v[i].c.x + v[i].r), p.push_back(v[i].c.x
31         - v[i].r);
32     forn(i,sz(v)) forn(j,i) {
33         Circle &a = v[i], b = v[j];
34         double d = (a.c - b.c).norm();
35         if (fabs(a.r - b.r) < d && d < a.r + b.r) {
36             double alfa = acos((sqr(a.r) + sqr(d) - sqr(b.r)) / (2.0 * d
37                 * a.r));
38             pto vec = (b.c - a.c) * (a.r / d);
39             p.pb((a.c + rotate(vec, alfa)).x), p.pb((a.c + rotate(vec, -
40                 alfa)).x);
41         }
42     }
43     sort(p.begin(), p.end());
44     double res = 0.0;
45     forn(i,sz(p)-1) {
46         const double A = p[i], B = p[i+1];
47         VE ve; ve.reserve(2 * v.size());
48         forn(j,sz(v)) {
49             const Circle &c = v[j];
50             double arco = primitiva(B-c.c.x,c.r) - primitiva(A-c.c.x,c.r
51                 );
52             double base = c.c.y * (B-A);
53             ve.push_back(event(base + arco,-1));
54             ve.push_back(event(base - arco, 1));
55         }
56     }
57     res += cuenta(ve,A,B);
58 }
59 return res;
60 }

```

## 6. Math

### 6.1. Identidades

$\sum_{i=0}^n i \binom{n}{i} = n * 2^{n-1}$   
 $\sum_{i=m}^n i = \frac{n(n+1)}{2} - \frac{m(m-1)}{2} = \frac{(n+1-m)(n+m)}{2}$   
 $\sum_{i=0}^n i^2 = \frac{n(n+1)(2n+1)}{6} = \frac{n^3}{3} + \frac{n^2}{2} + \frac{n}{6}$   
 $\sum_{i=0}^n i(i-1) = \frac{8}{6} \left(\frac{n}{2}\right) \left(\frac{n}{2} + 1\right) (n+1)$  (doubles)  $\rightarrow$  Sino ver caso impar y par  
 $\sum_{i=0}^n i^3 = \left(\frac{n(n+1)}{2}\right)^2 = \frac{n^4}{4} + \frac{n^3}{2} + \frac{n^2}{4} = \left[\sum_{i=1}^n i\right]^2$   
 $\sum_{i=0}^n i^4 = \frac{n(n+1)(2n+1)(3n^2+3n-1)}{30} = \frac{n^5}{5} + \frac{n^4}{2} + \frac{n^3}{3} - \frac{n}{30}$   
 $\sum_{i=0}^n i^p = \frac{(n+1)^{p+1}}{p+1} + \sum_{k=1}^p \frac{B_k}{p-k+1} \binom{p}{k} (n+1)^{p-k+1}$   
 $r = e - v + k + 1$   
 Teorema de Pick: (Area, puntos interiores y puntos en el borde)  
 $A = I + \frac{B}{2} - 1$   
 $\left\{ \begin{smallmatrix} n+1 \\ k \end{smallmatrix} \right\} = k \left\{ \begin{smallmatrix} n \\ k \end{smallmatrix} \right\} + \left\{ \begin{smallmatrix} n \\ k-1 \end{smallmatrix} \right\}$  for  $k > 0$  with initial conditions  $\left\{ \begin{smallmatrix} 0 \\ 0 \end{smallmatrix} \right\} = 1$  and  $\left\{ \begin{smallmatrix} n \\ 0 \end{smallmatrix} \right\} = \left\{ \begin{smallmatrix} 0 \\ n \end{smallmatrix} \right\} = 0$  for  $n > 0$ . Same as  $\frac{1}{k!} \sum_{j=0}^k (-1)^{k-j} \binom{k}{j} j^n$   
 $\left[ \begin{smallmatrix} n+1 \\ k \end{smallmatrix} \right] = n \left[ \begin{smallmatrix} n \\ k \end{smallmatrix} \right] + \left[ \begin{smallmatrix} n \\ k-1 \end{smallmatrix} \right]$  for  $k > 0$ , with the initial conditions  $\left[ \begin{smallmatrix} 0 \\ 0 \end{smallmatrix} \right] = 1$  and  $\left[ \begin{smallmatrix} n \\ 0 \end{smallmatrix} \right] = \left[ \begin{smallmatrix} 0 \\ n \end{smallmatrix} \right] = 0$  for  $n > 0$ .

### 6.2. Ec. Caracteristica

$a_0 T(n) + a_1 T(n-1) + \dots + a_k T(n-k) = 0$   
 $p(x) = a_0 x^k + a_1 x^{k-1} + \dots + a_k$   
 Sean  $r_1, r_2, \dots, r_q$  las raíces distintas, de mult.  $m_1, m_2, \dots, m_q$   
 $T(n) = \sum_{i=1}^q \sum_{j=0}^{m_i-1} c_{ij} n^j r_i^n$   
 Las constantes  $c_{ij}$  se determinan por los casos base.

### 6.3. Combinatorio

```

1  forn(i, MAXN+1){ //comb[i][k]=i tomados de a k
2      comb[i][0]=comb[i][i]=1;
3      forr(k, 1, i) comb[i][k]=(comb[i-1][k]+comb[i-1][k-1])%MOD;
4  }
5  ll lucas (ll n, ll k, int p){ //Calcula (n,k)%p teniendo comb[p][p]
6      //precalculado.
7      ll aux = 1;
8      while (n + k) aux = (aux * comb[n%p][k%p]) %p, n/=p, k/=p;
9      return aux;
10 }

```

6.4. Gauss Jordan, Determinante  $O(n^3)$ 

```

1 // Gauss-Jordan elimination with full pivoting.
2 //
3 // Uses:
4 //   (1) solving systems of linear equations (AX=B)
5 //   (2) inverting matrices (AX=I)
6 //   (3) computing determinants of square matrices
7 //
8 // Running time:  $O(n^3)$ 
9 //
10 // INPUT:   a[] [] = an nxn matrix
11 //          b[] [] = an nxm matrix
12 //
13 // OUTPUT:  X      = an nxm matrix (stored in b[] [])
14 //          A^{-1} = an nxn matrix (stored in a[] [])
15 //          returns determinant of a[] []
16
17 #include <iostream>
18 #include <vector>
19 #include <cmath>
20
21 using namespace std;
22
23 const double EPS = 1e-10;
24
25 typedef vector<int> VI;
26 typedef double T;
27 typedef vector<T> VT;
28 typedef vector<VT> VVT;
29
30 T GaussJordan(VVT &a, VVT &b) {
31     const int n = a.size();
32     const int m = b[0].size();
33     VI irow(n), icol(n), ipiv(n);
34     T det = 1;
35
36     for (int i = 0; i < n; i++) {
37         int pj = -1, pk = -1;
38         for (int j = 0; j < n; j++) if (!ipiv[j])
39             for (int k = 0; k < n; k++) if (!ipiv[k])
40                 if (pj == -1 || fabs(a[j][k]) > fabs(a[pj][pk])) { pj = j; pk = k; }
41         if (fabs(a[pj][pk]) < EPS) { cerr << "Matrix is singular." << endl;

```

```

        exit(0); }
42     ipiv[pk]++;
43     swap(a[pj], a[pk]);
44     swap(b[pj], b[pk]);
45     if (pj != pk) det *= -1;
46     irow[i] = pj;
47     icol[i] = pk;
48
49     T c = 1.0 / a[pk][pk];
50     det *= a[pk][pk];
51     a[pk][pk] = 1.0;
52     for (int p = 0; p < n; p++) a[pk][p] *= c;
53     for (int p = 0; p < m; p++) b[pk][p] *= c;
54     for (int p = 0; p < n; p++) if (p != pk) {
55         c = a[p][pk];
56         a[p][pk] = 0;
57         for (int q = 0; q < n; q++) a[p][q] -= a[pk][q] * c;
58         for (int q = 0; q < m; q++) b[p][q] -= b[pk][q] * c;
59     }
60 }
61
62 for (int p = n-1; p >= 0; p--) if (irow[p] != icol[p]) {
63     for (int k = 0; k < n; k++) swap(a[k][irow[p]], a[k][icol[p]]);
64 }
65
66 return det;
67 }
68
69 int main() {
70     const int n = 4;
71     const int m = 2;
72     double A[n][n] = { {1,2,3,4},{1,0,1,0},{5,3,2,4},{6,1,4,6} };
73     double B[n][m] = { {1,2},{4,3},{5,6},{8,7} };
74     VVT a(n), b(n);
75     for (int i = 0; i < n; i++) {
76         a[i] = VT(A[i], A[i] + n);
77         b[i] = VT(B[i], B[i] + m);
78     }
79
80     double det = GaussJordan(a, b);
81
82     // expected: 60
83     cout << "Determinant: " << det << endl;

```

```

84
85 // expected: -0.233333 0.166667 0.133333 0.0666667
86 //           0.166667 0.166667 0.333333 -0.333333
87 //           0.233333 0.833333 -0.133333 -0.0666667
88 //           0.05 -0.75 -0.1 0.2
89 cout << "Inverse:␣" << endl;
90 for (int i = 0; i < n; i++) {
91     for (int j = 0; j < n; j++)
92         cout << a[i][j] << '␣';
93     cout << endl;
94 }
95
96 // expected: 1.63333 1.3
97 //           -0.166667 0.5
98 //           2.36667 1.7
99 //           -1.85 -1.35
100 cout << "Solution:␣" << endl;
101 for (int i = 0; i < n; i++) {
102     for (int j = 0; j < m; j++)
103         cout << b[i][j] << '␣';
104     cout << endl;
105 }
106 }

```

## 6.5. Teorema Chino del Resto

$$y = \sum_{j=1}^n (x_j * (\prod_{i=1, i \neq j}^n m_i)^{-1} * \prod_{i=1, i \neq j}^n m_i)$$

```

1 // Chinese remainder theorem (special case): find z such that
2 // z % m1 = r1, z % m2 = r2. Here, z is unique modulo M = lcm(m1, m2).
3 // Return (z, M). On failure, M = -1.
4 PII chinese_remainder_theorem(int m1, int r1, int m2, int r2) {
5     int s, t;
6     int g = extended_euclid(m1, m2, s, t);
7     if (r1 % g != r2 % g) return make_pair(0, -1);
8     return make_pair(mod(s*r2*m1 + t*r1*m2, m1*m2) / g, m1*m2 / g);
9 }
10
11 // Chinese remainder theorem: find z such that
12 // z % m[i] = r[i] for all i. Note that the solution is
13 // unique modulo M = lcm_i (m[i]). Return (z, M). On

```

```

14 // failure, M = -1. Note that we do not require the a[i]'s
15 // to be relatively prime.
16 PII chinese_remainder_theorem(const VI &m, const VI &r) {
17     PII ret = make_pair(r[0], m[0]);
18     for (int i = 1; i < m.size(); i++) {
19         ret = chinese_remainder_theorem(ret.second, ret.first, m[i], r[i]);
20         if (ret.second == -1) break;
21     }
22     return ret;
23 }

```

## 6.6. Funciones de primos

Iterar mientras el  $p^2 \leq N$ . Revisar que  $N! = 1$ , en este caso  $N$  es primo. **NumDiv**: Producto (exponentes+1). **SumDiv**: Product suma geom. factores. **EulerPhi** (coprimos): Inicia ans=N. Para cada primo divisor: ans=ans/primo (una vez) y dividir luego  $N$  todo lo posible por  $p$ .

## 6.7. Phollard's Rho (rolando)

```

1 ll gcd(ll a, ll b){return a?gcd(b %a, a):b;}
2
3 ll mulmod (ll a, ll b, ll c) { //returns (a*b)%c, and minimize overflow
4     ll x = 0, y = a%c;
5     while (b > 0){
6         if (b % 2 == 1) x = (x+y) % c;
7         y = (y*2) % c;
8         b /= 2;
9     }
10    return x % c;
11 }
12
13 ll expmod (ll b, ll e, ll m){//O(log b)
14     if(!e) return 1;
15     ll q= expmod(b,e/2,m); q=mulmod(q,q,m);
16     return e%2? mulmod(b,q,m) : q;
17 }
18
19 bool es_primo_prob (ll n, int a)
20 {
21     if (n == a) return true;
22     ll s = 0, d = n-1;

```

```

23 while (d % 2 == 0) s++,d/=2;
24
25 ll x = expmod(a,d,n);
26 if ((x == 1) || (x+1 == n)) return true;
27
28 forn (i, s-1){
29     x = mulmod(x, x, n);
30     if (x == 1) return false;
31     if (x+1 == n) return true;
32 }
33 return false;
34 }
35
36 bool rabin (ll n){ //devuelve true si n es primo
37     if (n == 1) return false;
38     const int ar[] = {2,3,5,7,11,13,17,19,23};
39     forn (j,9)
40         if (!es_primo_prob(n,ar[j]))
41             return false;
42     return true;
43 }
44
45 ll rho(ll n){
46     if( (n & 1) == 0 ) return 2;
47     ll x = 2 , y = 2 , d = 1;
48     ll c = rand() % n + 1;
49     while( d == 1 ){
50         x = (mulmod( x , x , n ) + c)%n;
51         y = (mulmod( y , y , n ) + c)%n;
52         y = (mulmod( y , y , n ) + c)%n;
53         if( x - y >= 0 ) d = gcd( x - y , n );
54         else d = gcd( y - x , n );
55     }
56     return d==n? rho(n):d;
57 }
58
59 map<ll,ll> prim;
60 void factRho (ll n){ //0 (lg n)^3. un solo numero
61     if (n == 1) return;
62     if (rabin(n)){
63         prim[n]++;
64         return;
65     }

```

```

66 ll factor = rho(n);
67 factRho(factor);
68 factRho(n/factor);
69 }

```

## 6.8. GCD

```

1 | tipo gcd(tipo a, tipo b){return a?gcd(b %a, a):b;}

```

## 6.9. Extended Euclid

```

1 void extendedEuclid (ll a, ll b){ //a * x + b * y = d
2     if (!b) { x = 1; y = 0; d = a; return;}
3     extendedEuclid (b, a%b);
4     ll x1 = y;
5     ll y1 = x - (a/b) * y;
6     x = x1; y = y1;
7 }

```

## 6.10. Polinomio

```

1     int m = sz(c), n = sz(o.c);
2     vector<tipo> res(max(m,n));
3     forn(i, m) res[i] += c[i];
4     forn(i, n) res[i] += o.c[i];
5     return poly(res);    }
6 poly operator*(const tipo cons) const {
7     vector<tipo> res(sz(c));
8     forn(i, sz(c)) res[i]=c[i]*cons;
9     return poly(res);    }
10 poly operator*(const poly &o) const {
11     int m = sz(c), n = sz(o.c);
12     vector<tipo> res(m+n-1);
13     forn(i, m) forn(j, n) res[i+j]+=c[i]*o.c[j];
14     return poly(res);    }
15 tipo eval(tipo v) {
16     tipo sum = 0;
17     dforn(i, sz(c)) sum=sum*v + c[i];
18     return sum; }
19 //poly contains only a vector<int> c (the coeficients)
20 //the following function generates the roots of the polynomial
21 //it can be easily modified to return float roots
22 set<tipo> roots(){
23     set<tipo> roots;

```



```

24     tipo a0 = abs(c[0]), an = abs(c[sz(c)-1]);
25     vector<tipo> ps,qs;
26     forr(p,1,sqrt(a0)+1) if (a0%p==0) ps.pb(p),ps.pb(a0/p);
27     forr(q,1,sqrt(an)+1) if (an%q==0) qs.pb(q),qs.pb(an/q);
28     forall(pt,ps)
29         forall(qt,qs) if ( (*pt) % (*qt)==0 ) {
30             tipo root = abs((*pt) / (*qt));
31             if (eval(root)==0) roots.insert(root);
32         }
33     return roots; }
34 };
35 pair<poly,tipo> ruffini(const poly p, tipo r) {
36     int n = sz(p.c) - 1 ;
37     vector<tipo> b(n);
38     b[n-1] = p.c[n];
39     dform(k,n-1) b[k] = p.c[k+1] + r*b[k+1];
40     tipo resto = p.c[0] + r*b[0];
41     poly result(b);
42     return make_pair(result,resto);
43 }
44 poly interpolate(const vector<tipo>& x,const vector<tipo>& y) {
45     poly A; A.c.pb(1);
46     for(i,sz(x)) { poly aux; aux.c.pb(-x[i]), aux.c.pb(1), A = A * aux;
47     }
48     poly S; S.c.pb(0);
49     for(i,sz(x)) { poly Li;
50         Li = ruffini(A,x[i]).fst;
51         Li = Li * (1.0 / Li.eval(x[i])); // here put a multiple of the
52         // coefficients instead of 1.0 to avoid using double
53         S = S + Li * y[i]; }
54     return S;
55 }
56 int main(){
57     return 0;
58 }

```

## 6.11. FFT

```

1 //~ typedef complex<double> base; //menos codigo, pero mas lento
2 //elegir si usar complejos de c (lento) o estos
3 struct base{
4     double r,i;

```

```

5     base(double r=0, double i=0):r(r), i(i){}
6     double real()const{return r;}
7     void operator/=(const int c){r/=c, i/=c;}
8 };
9 base operator*(const base &a, const base &b){
10     return base(a.r*b.r-a.i*b.i, a.r*b.i+a.i*b.r);}
11 base operator+(const base &a, const base &b){
12     return base(a.r+b.r, a.i+b.i);}
13 base operator-(const base &a, const base &b){
14     return base(a.r-b.r, a.i-b.i);}
15 vector<int> rev; vector<base> wlen_pw;
16 inline static void fft(base a[], int n, bool invert) {
17     for(i, n) if(i<rev[i]) swap(a[i], a[rev[i]]);
18     for (int len=2; len<=n; len<=1) {
19         double ang = 2*M_PI/len * (invert?-1:+1);
20         int len2 = len>>1;
21         base wlen (cos(ang), sin(ang));
22         wlen_pw[0] = base (1, 0);
23         for(i, 1, len2) wlen_pw[i] = wlen_pw[i-1] * wlen;
24         for (int i=0; i<n; i+=len) {
25             base t, *pu = a+i, *pv = a+i+len2, *pu_end = a+i+len2, *pw = &
26             wlen_pw[0];
27             for (; pu!=pu_end; ++pu, ++pv, ++pw)
28                 t = *pv * *pw, *pv = *pu - t,*pu = *pu + t;
29         }
30         if (invert) for(i, n) a[i]/= n;}
31 inline static void calc_rev(int n){//precalculo: llamar antes de fft!!
32     wlen_pw.resize(n), rev.resize(n);
33     int lg=31-__builtin_clz(n);
34     for(i, n){
35         rev[i] = 0;
36         for(k, lg) if(i&(1<<k)) rev[i] |= 1<<(lg-1-k);
37     }}
38 inline static void multiply(const vector<int> &a, const vector<int> &b,
39     vector<int> &res) {
40     vector<base> fa (a.begin(), a.end()), fb (b.begin(), b.end());
41     int n=1; while(n < max(sz(a), sz(b))) n <= 1; n <= 1;
42     calc_rev(n);
43     fa.resize (n), fb.resize (n);
44     fft (&fa[0], n, false), fft (&fb[0], n, false);
45     for(i, n) fa[i] = fa[i] * fb[i];
46     fft (&fa[0], n, true);

```



```

46 res.resize(n);
47     forn(i, n) res[i] = int (fa[i].real() + 0.5); }
48 void toPoly(const string &s, vector<int> &P){//convierte un numero a
    polinomio
49     P.clear();
50     dforn(i, sz(s)) P.pb(s[i]-'0');}

```

## 7. Grafos

### 7.1. Dijkstra

```

1 #define INF 1e9
2 int N;
3 #define MAX_V 250001
4 vector<ii> G[MAX_V];
5 //To add an edge use
6 #define add(a, b, w) G[a].pb(make_pair(w, b))
7 ll dijkstra(int s, int t){//O(|E| log |V|)
8     priority_queue<ii, vector<ii>, greater<ii> > Q;
9     vector<ll> dist(N, INF); vector<int> dad(N, -1);
10    Q.push(make_pair(0, s)); dist[s] = 0;
11    while(sz(Q)){
12        ii p = Q.top(); Q.pop();
13        if(p.snd == t) break;
14        forall(it, G[p.snd])
15            if(dist[p.snd]+it->first < dist[it->snd]){
16                dist[it->snd] = dist[p.snd] + it->fst;
17                dad[it->snd] = p.snd;
18                Q.push(make_pair(dist[it->snd], it->snd)); }
19    }
20    return dist[t];
21    if(dist[t]<INF)//path generator
22        for(int i=t; i!=-1; i=dad[i])
23            printf("%d%c", i, (i==s?'\\n':' '));}

```

### 7.2. Bellman-Ford

```

1 vector<ii> G[MAX_N]; //ady. list with pairs (weight, dst)
2 int dist[MAX_N];
3 void bford(int src){//O(VE)
4     dist[src]=0;
5     forn(i, N-1) forn(j, N) if(dist[j]!=INF) forall(it, G[j])
6         dist[it->snd]=min(dist[it->snd], dist[j]+it->fst);

```

```

7 }
8
9 bool hasNegCycle(){
10     forn(j, N) if(dist[j]!=INF) forall(it, G[j])
11         if(dist[it->snd]>dist[j]+it->fst) return true;
12     //inside if: all points reachable from it->snd will have -INF distance
13     (do bfs)
14     return false;
15 }

```

### 7.3. Floyd-Warshall

```

1 //G[i][j] contains weight of edge (i, j) or INF
2 //G[i][i]=0
3 int G[MAX_N][MAX_N];
4 void floyd(){//O(N^3)
5     forn(k, N) forn(i, N) if(G[i][k]!=INF) forn(j, N) if(G[k][j]!=INF)
6         G[i][j]=min(G[i][j], G[i][k]+G[k][j]);
7 }
8 bool inNegCycle(int v){
9     return G[v][v]<0;}
10 //checks if there's a neg. cycle in path from a to b
11 bool hasNegCycle(int a, int b){
12     forn(i, N) if(G[a][i]!=INF && G[i][i]<0 && G[i][b]!=INF)
13         return true;
14     return false;
15 }

```

### 7.4. 2-SAT + Tarjan SCC

```

1 //We have a vertex representing a var and other for his negation.
2 //Every edge stored in G represents an implication. To add an equation
3 //of the form a|b, use addor(a, b)
4 //MAX=max cant var, n=cant var
5 #define addor(a, b) (G[neg(a)].pb(b), G[neg(b)].pb(a))
6 vector<int> G[MAX*2];
7 //idx[i]=index assigned in the dfs
8 //lw[i]=lowest index(closer to the root) reachable from i
9 int lw[MAX*2], idx[MAX*2], qidx;
10 stack<int> q;
11 int qcmp, cmp[MAX*2];
12 //verdad[cmp[i]]=valor de la variable i
13 bool verdad[MAX*2+1];

```

```

14 int neg(int x) { return x>=n? x-n : x+n;}
15 void tjn(int v){
16     lw[v]=idx[v]++qidx;
17     q.push(v), cmp[v]=-2;
18     forall(it, G[v]){
19         if(!idx[*it] || cmp[*it]==-2){
20             if(!idx[*it]) tjn(*it);
21             lw[v]=min(lw[v], lw[*it]);
22         }
23     }
24     if(lw[v]==idx[v]){
25         int x;
26         do{x=q.top(); q.pop(); cmp[x]=qcmp;}while(x!=v);
27         verdad[qcmp]=(cmp[neg(v)]<0);
28         qcmp++;
29     }
30 }
31 //remember to CLEAR G!!!
32 bool satisf(){//O(n)
33     memset(idx, 0, sizeof(idx)), qidx=0;
34     memset(cmp, -1, sizeof(cmp)), qcmp=0;
35     forn(i, n){
36         if(!idx[i]) tjn(i);
37         if(!idx[neg(i)]) tjn(neg(i));
38     }
39     forn(i, n) if(cmp[i]==cmp[neg(i)]) return false;
40     return true;
41 }

```

## 7.5. Articulation Points

```

1 int N;
2 vector<int> G[1000000];
3 //V[i]=node number(if visited), L[i]= lowest V[i] reachable from i
4 int qV, V[1000000], L[1000000], P[1000000];
5 void dfs(int v, int f){
6     L[v]=V[v]++qV;
7     forall(it, G[v])
8         if(!V[*it]){
9             dfs(*it, v);
10            L[v] = min(L[v], L[*it]);
11            P[v]+= L[*it]>=V[v];
12        }

```

```

13     else if(*it!=f)
14         L[v]=min(L[v], V[*it]);
15 }
16 int cantart(){ //O(n)
17     qV=0;
18     zero(V), zero(P);
19     dfs(1, 0); P[1]--;
20     int q=0;
21     forn(i, N) if(P[i]) q++;
22     return q;
23 }

```

## 7.6. Comp. Biconexas y Puentes

```

1 struct edge {
2     int u,v, comp;
3     bool bridge;
4 };
5 vector<edge> e;
6 void addEdge(int u, int v) {
7     G[u].pb(sz(e)), G[v].pb(sz(e));
8     e.pb((edge){u,v,-1,false});
9 }
10 //d[i]=id de la dfs
11 //b[i]=lowest id reachable from i
12 int d[MAXN], b[MAXN], t;
13 int nbc;//cant componentes
14 int comp[MAXN]; //comp[i]=cant comp biconexas a la cual pertenece i
15 void initDfs(int n) {
16     zero(G), zero(comp);
17     e.clear();
18     forn(i,n) d[i]=-1;
19     nbc = t = 0;
20 }
21 stack<int> st;
22 void dfs(int u, int pe) { //O(n + m)
23     b[u] = d[u] = t++;
24     comp[u] = (pe != -1);
25     forall(ne, G[u]) if (*ne != pe){
26         int v = e[*ne].u ^ e[*ne].v ^ u;
27         if (d[v] == -1) {
28             st.push(*ne);
29             dfs(v,*ne);

```

```

30     if (b[v] > d[u]){
31         e[*ne].bridge = true; // bridge
32     }
33     if (b[v] >= d[u]){ // art
34         int last;
35         do {
36             last = st.top(); st.pop();
37             e[last].comp = nbc;
38         } while (last != *ne);
39         nbc++;
40         comp[u]++;
41     }
42     b[u] = min(b[u], b[v]);
43 }
44 else if (d[v] < d[u]) { // back edge
45     st.push(*ne);
46     b[u] = min(b[u], d[v]);
47 }
48 }
49 }

```

## 7.7. LCA + Climb

```

1  const int MAXN=100001;
2  const int LOGN=20;
3  //f[v][k] holds the 2^k father of v
4  //L[v] holds the level of v
5  int N, f[MAXN][LOGN], L[MAXN];
6  //call before build:
7  void dfs(int v, int fa=-1, int lvl=0){//generate required data
8      f[v][0]=fa, L[v]=lvl;
9      forall(it, G[v])if(*it!=fa) dfs(*it, v, lvl+1); }
10 void build(){//f[i][0] must be filled previously, 0(nlgn)
11     forn(k, LOGN-1) forn(i, N) f[i][k+1]=f[f[i][k]][k];}
12 #define lg(x) (31-__builtin_clz(x))//=floor(log2(x))
13 int climb(int a, int d){//O(lgn)
14     if(!d) return a;
15     dforn(i, lg(L[a])+1) if(1<<i<=d) a=f[a][i], d-=1<<i;
16     return a;}
17 int lca(int a, int b){//O(lgn)
18     if(L[a]<L[b]) swap(a, b);
19     a=climb(a, L[a]-L[b]);
20     if(a==b) return a;

```

```

21     dforn(i, lg(L[a])+1) if(f[a][i]!=f[b][i]) a=f[a][i], b=f[b][i];
22     return f[a][0]; }
23 int dist(int a, int b) {//returns distance between nodes
24     return L[a]+L[b]-2*L[lca(a, b)];}

```

## 7.8. Heavy Light Decomposition

```

1  int treesz[MAXN];//cantidad de nodos en el subarbol del nodo v
2  int dad[MAXN];//dad[v]=padre del nodo v
3  void dfs1(int v, int p=-1){//pre-dfs
4      dad[v]=p;
5      treesz[v]=1;
6      forall(it, G[v]) if(*it!=p){
7          dfs1(*it, v);
8          treesz[v]+=treesz[*it];
9      }
10 }
11 //PONER Q EN 0 !!!!!
12 int pos[MAXN], q;//pos[v]=posicion del nodo v en el recorrido de la dfs
13 //Las cadenas aparecen continuas en el recorrido!
14 int cantcad;
15 int homecad[MAXN];//dada una cadena devuelve su nodo inicial
16 int cad[MAXN];//cad[v]=cadena a la que pertenece el nodo
17 void heavylight(int v, int cur=-1){
18     if(cur==-1) homecad[cur=cantcad++]=v;
19     pos[v]=q++;
20     cad[v]=cur;
21     int mx=-1;
22     forn(i, sz(G[v])) if(G[v][i]!=dad[v])
23         if(mx==-1 || treesz[G[v][mx]]<treesz[G[v][i]]) mx=i;
24     if(mx!=-1) heavylight(G[v][mx], cur);
25     forn(i, sz(G[v])) if(i!=mx && G[v][i]!=dad[v])
26         heavylight(G[v][i], -1);
27 }
28 //ejemplo de obtener el maximo numero en el camino entre dos nodos
29 //RTA: max(query(low, u), query(low, v)), con low=lca(u, v)
30 //esta funcion va trepando por las cadenas
31 int query(int an, int v){//O(logn)
32     //si estan en la misma cadena:
33     if(cad[an]==cad[v]) return rmq.get(pos[an], pos[v]+1);
34     return max(query(an, dad[homecad[cad[v]]]),
35               rmq.get(pos[homecad[cad[v]]], pos[v]+1));
36 }

```

## 7.9. Centroid Decomposition

```

1 int n;
2 vector<int> G[MAXN];
3 bool taken[MAXN]; //poner todos en FALSE al principio!!
4 int padre[MAXN]; //padre de cada nodo en el centroid tree
5
6 int szt[MAXN];
7 void calcsz(int v, int p) {
8     szt[v] = 1;
9     forall(it, G[v]) if (*it!=p && !taken[*it])
10         calcsz(*it, v), szt[v] += szt[*it];
11 }
12 void centroid(int v=0, int f=-1, int lvl=0, int tam=-1) { //O(nlogn)
13     if(tam== -1) calcsz(v, -1), tam=szt[v];
14     forall(it, G[v]) if(!taken[*it] && szt[*it]>=tam/2)
15         {szt[v]=0; centroid(*it, f, lvl, tam); return;}
16     taken[v]=true;
17     padre[v]=f;
18     forall(it, G[v]) if(!taken[*it])
19         centroid(*it, v, lvl+1, -1);
20 }

```

## 7.10. Euler Cycle

```

1 int n,m,ars[MAXE], eq;
2 vector<int> G[MAXN]; //fill G,n,m,ars,eq
3 list<int> path;
4 int used[MAXN];
5 bool usede[MAXE];
6 queue<list<int>::iterator> q;
7 int get(int v){
8     while(used[v]<sz(G[v]) && usede[ G[v][used[v]] ]) used[v]++;
9     return used[v];
10 }
11 void explore(int v, int r, list<int>::iterator it){
12     int ar=G[v][get(v)]; int u=v^ars[ar];
13     usede[ar]=true;
14     list<int>::iterator it2=path.insert(it, u);
15     if(u!=r) explore(u, r, it2);
16     if(get(v)<sz(G[v])) q.push(it);
17 }
18 void euler(){

```

```

19     zero(used), zero(usede);
20     path.clear();
21     q=queue<list<int>::iterator>();
22     path.push_back(0); q.push(path.begin());
23     while(sz(q)){
24         list<int>::iterator it=q.front(); q.pop();
25         if(used[*it]<sz(G[*it])) explore(*it, *it, it);
26     }
27     reverse(path.begin(), path.end());
28 }
29 void addEdge(int u, int v){
30     G[u].pb(eq), G[v].pb(eq);
31     ars[eq++]=u^v;
32 }

```

## 7.11. Diametro árbol

```

1 vector<int> G[MAXN]; int n,m,p[MAXN],d[MAXN],d2[MAXN];
2 int bfs(int r, int *d) {
3     queue<int> q;
4     d[r]=0; q.push(r);
5     int v;
6     while(sz(q)) { v=q.front(); q.pop();
7         forall(it, G[v]) if (d[*it]==-1)
8             d[*it]=d[v]+1, p[*it]=v, q.push(*it);
9     }
10    return v; //ultimo nodo visitado
11 }
12 vector<int> diams; vector<ii> centros;
13 void diametros(){
14     memset(d,-1,sizeof(d));
15     memset(d2,-1,sizeof(d2));
16     diams.clear(), centros.clear();
17     forn(i, n) if(d[i]==-1){
18         int v,c;
19         c=bfs(bfs(i, d2), d);
20         forn(_, d[v]/2) c=p[c];
21         diams.pb(d[v]);
22         if(d[v]&1) centros.pb(ii(c, p[c]));
23         else centros.pb(ii(c, c));
24     }
25 }
26

```

```

27 int main() {
28     freopen("in", "r", stdin);
29     while(cin >> n >> m){
30         forn(i,m) { int a,b; cin >> a >> b; a--, b--;
31             G[a].pb(b);
32             G[b].pb(a);

```

## 7.12. Chu-liu

```

1 void visit(graph &h, int v, int s, int r,
2     vector<int> &no, vector< vector<int> > &comp,
3     vector<int> &prev, vector< vector<int> > &next, vector<weight> &mcost,
4     vector<int> &mark, weight &cost, bool &found) {
5     if (mark[v]) {
6         vector<int> temp = no;
7         found = true;
8         do {
9             cost += mcost[v];
10            v = prev[v];
11            if (v != s) {
12                while (comp[v].size() > 0) {
13                    no[comp[v].back()] = s;
14                    comp[s].push_back(comp[v].back());
15                    comp[v].pop_back();
16                }
17            }
18        } while (v != s);
19        forall(j,comp[s]) if (*j != r) forall(e,h[*j])
20            if (no[e->src] != s) e->w -= mcost[ temp[*j] ];
21    }
22    mark[v] = true;
23    forall(i,next[v]) if (no[*i] != no[v] && prev[no[*i]] == v)
24        if (!mark[no[*i]] || *i == s)
25            visit(h, *i, s, r, no, comp, prev, next, mcost, mark, cost, found)
26    ;
27 }
28 weight minimumSpanningArborescence(const graph &g, int r) {
29     const int n=sz(g);
30     graph h(n);
31     forn(u,n) forall(e,g[u]) h[e->dst].pb(*e);
32     vector<int> no(n);
33     vector<vector<int> > comp(n);
34     forn(u, n) comp[u].pb(no[u] = u);

```

```

34     for (weight cost = 0; ;) {
35         vector<int> prev(n, -1);
36         vector<weight> mcost(n, INF);
37         forn(j,n) if (j != r) forall(e,h[j])
38             if (no[e->src] != no[j])
39                 if (e->w < mcost[ no[j] ])
40                     mcost[ no[j] ] = e->w, prev[ no[j] ] = no[e->src];
41         vector< vector<int> > next(n);
42         forn(u,n) if (prev[u] >= 0)
43             next[ prev[u] ].push_back(u);
44         bool stop = true;
45         vector<int> mark(n);
46         forn(u,n) if (u != r && !mark[u] && !comp[u].empty()) {
47             bool found = false;
48             visit(h, u, u, r, no, comp, prev, next, mcost, mark, cost, found);
49             if (found) stop = false;
50         }
51         if (stop) {
52             forn(u,n) if (prev[u] >= 0) cost += mcost[u];
53             return cost;
54         }
55     }
56 }

```

## 7.13. Hungarian

```

1 //Dado un grafo bipartito completo con costos no negativos, encuentra el
2 //matching perfecto de minimo costo.
3 tipo cost[N][N], lx[N], ly[N], slack[N]; //llenar: cost=matriz de
4 //adyacencia
5 int n, max_match, xy[N], yx[N], slackx[N],prev2[N]; //n=cantidad de nodos
6 bool S[N], T[N]; //sets S and T in algorithm
7 void add_to_tree(int x, int prevx) {
8     S[x] = true, prev2[x] = prevx;
9     forn(y, n) if (lx[x] + ly[y] - cost[x][y] < slack[y] - EPS)
10         slack[y] = lx[x] + ly[y] - cost[x][y], slackx[y] = x;
11 }
12 void update_labels(){
13     tipo delta = INF;
14     forn (y, n) if (!T[y]) delta = min(delta, slack[y]);
15     forn (x, n) if (S[x]) lx[x] -= delta;
16     forn (y, n) if (T[y]) ly[y] += delta; else slack[y] -= delta;
17 }

```

```

16 void init_labels(){
17     zero(lx), zero(ly);
18     forn (x,n) forn(y,n) lx[x] = max(lx[x], cost[x][y]);
19 }
20 void augment() {
21     if (max_match == n) return;
22     int x, y, root, q[N], wr = 0, rd = 0;
23     memset(S, false, sizeof(S)), memset(T, false, sizeof(T));
24     memset(prev2, -1, sizeof(prev2));
25     forn (x, n) if (xy[x] == -1){
26         q[wr++] = root = x, prev2[x] = -2;
27         S[x] = true; break; }
28     forn (y, n) slack[y] = lx[root] + ly[y] - cost[root][y], slackx[y] =
        root;
29     while (true){
30         while (rd < wr){
31             x = q[rd++];
32             for (y = 0; y < n; y++) if (cost[x][y] == lx[x] + ly[y] && !T[y]){
33                 if (yx[y] == -1) break; T[y] = true;
34                 q[wr++] = yx[y], add_to_tree(yx[y], x); }
35             if (y < n) break; }
36         if (y < n) break;
37         update_labels(), wr = rd = 0;
38         for (y = 0; y < n; y++) if (!T[y] && slack[y] == 0){
39             if (yx[y] == -1){x = slackx[y]; break;}
40             else{
41                 T[y] = true;
42                 if (!S[yx[y]]) q[wr++] = yx[y], add_to_tree(yx[y], slackx[y]);
43             }
44             if (y < n) break; }
45         if (y < n){
46             max_match++;
47             for (int cx = x, cy = y, ty; cx != -2; cx = prev2[cx], cy = ty)
48                 ty = xy[cx], yx[cy] = cx, xy[cx] = cy;
49             augment(); }
50 }
51 tipo hungarian(){
52     tipo ret = 0; max_match = 0, memset(xy, -1, sizeof(xy));
53     memset(yx, -1, sizeof(yx)), init_labels(), augment(); //steps 1-3
54     forn (x,n) ret += cost[x][xy[x]]; return ret;
55 }

```

## 7.14. Dynamic Connectivity

```

1 struct UnionFind {
2     int n, comp;
3     vector<int> pre, si, c;
4     UnionFind(int n=0):n(n), comp(n), pre(n), si(n, 1) {
5         forn(i,n) pre[i] = i; }
6     int find(int u){return u==pre[u]?u:find(pre[u]);}
7     bool merge(int u, int v) {
8         if((u=find(u))==v) return false;
9         if(si[u]<si[v]) swap(u, v);
10        si[u]+=si[v], pre[v]=u, comp--, c.pb(v);
11        return true;
12    }
13    int snap(){return sz(c);}
14    void rollback(int snap){
15        while(sz(c)>snap){
16            int v = c.back(); c.pop_back();
17            si[pre[v]] -= si[v], pre[v] = v, comp++;
18        }
19    }
20 };
21 enum {ADD,DEL,QUERY};
22 struct Query {int type,u,v};
23 struct DynCon {
24     vector<Query> q;
25     UnionFind dsu;
26     vector<int> match,res;
27     map<ii,int> last; //se puede no usar cuando hay identificador para
                        cada arista (mejora poco)
28     DynCon(int n=0):dsu(n){}
29     void add(int u, int v) {
30         if(u>v) swap(u,v);
31         q.pb((Query){ADD, u, v}), match.pb(-1);
32         last[ii(u,v)] = sz(q)-1;
33     }
34     void remove(int u, int v) {
35         if(u>v) swap(u,v);
36         q.pb((Query){DEL, u, v});
37         int prev = last[ii(u,v)];
38         match[prev] = sz(q)-1;
39         match.pb(prev);
40     }

```

```

41 void query() {//podria pasarle un puntero donde guardar la respuesta
42   q.pb((Query){QUERY, -1, -1}), match.pb(-1);}
43 void process() {
44   forn(i,sz(q)) if (q[i].type == ADD && match[i] == -1) match[i] =
45     sz(q);
46   go(0,sz(q));
47 }
48 void go(int l, int r) {
49   if(l+1==r){
50     if (q[l].type == QUERY)//Aqui responder la query usando el dsu!
51       res.pb(dsu.comp);//aqui query=cantidad de componentes conexas
52     return;
53   }
54   int s=dsu.snap(), m = (l+r) / 2;
55   forr(i,m,r) if(match[i]!=-1 && match[i]<l) dsu.merge(q[i].u, q[i]
56     ].v);
57   go(l,m);
58   dsu.rollback(s);
59   s = dsu.snap();
60   forr(i,l,m) if(match[i]!=-1 && match[i]>=r) dsu.merge(q[i].u, q[
61     i].v);
62   go(m,r);
63   dsu.rollback(s);
64 }
65 }dc;

```

## 8. Network Flow

### 8.1. Dinic

```

1 const int MAX = 300;
2 // Corte minimo: vertices con dist[v]>=0 (del lado de src) VS. dist[v]
3 ]==-1 (del lado del dst)
4 // Para el caso de la red de Bipartite Matching (Sean V1 y V2 los
5 conjuntos mas proximos a src y dst respectivamente):
6 // Reconstruir matching: para todo v1 en V1 ver las aristas a vertices
7 de V2 con it->f>0, es arista del Matching
8 // Min Vertex Cover: vertices de V1 con dist[v]==-1 + vertices de V2 con
9 dist[v]>0
10 // Max Independent Set: tomar los vertices NO tomados por el Min Vertex

```

```

Cover
8 // Max Clique: construir la red de G complemento (debe ser bipartito!) y
9 encontrar un Max Independent Set
10 // Min Edge Cover: tomar las aristas del matching + para todo vertices
11 no cubierto hasta el momento, tomar cualquier arista de el
12 int nodes, src, dst;
13 int dist[MAX], q[MAX], work[MAX];
14 struct Edge {
15   int to, rev;
16   ll f, cap;
17   Edge(int to, int rev, ll f, ll cap) : to(to), rev(rev), f(f), cap(
18     cap) {}
19 };
20 vector<Edge> G[MAX];
21 void addEdge(int s, int t, ll cap){
22   G[s].pb(Edge(t, sz(G[t]), 0, cap)), G[t].pb(Edge(s, sz(G[s])-1, 0,
23     0));}
24 bool dinic_bfs(){
25   fill(dist, dist+nodes, -1), dist[src]=0;
26   int qt=0; q[qt++]=src;
27   for(int qh=0; qh<qt; qh++){
28     int u =q[qh];
29     forall(e, G[u]){
30       int v=e->to;
31       if(dist[v]<0 && e->f < e->cap)
32         dist[v]=dist[u]+1, q[qt++]=v;
33     }
34   }
35   return dist[dst]>=0;
36 }
37 ll dinic_dfs(int u, ll f){
38   if(u==dst) return f;
39   for(int &i=work[u]; i<sz(G[u]); i++){
40     Edge &e = G[u][i];
41     if(e.cap<=e.f) continue;
42     int v=e.to;
43     if(dist[v]==dist[u]+1){
44       ll df=dinic_dfs(v, min(f, e.cap-e.f));
45       if(df>0){
46         e.f+=df, G[v][e.rev].f-= df;
47         return df; }
48     }
49   }
50   return 0;
51 }

```



```

46     return 0;
47 }
48 ll maxFlow(int _src, int _dst){
49     src=_src, dst=_dst;
50     ll result=0;
51     while(dinic_bfs()){
52         fill(work, work+nodes, 0);
53         while(ll delta=dinic_dfs(src,INF))
54             result+=delta;
55     }
56     // todos los nodos con dist[v]!=-1 vs los que tienen dist[v]==-1
57     // forman el min-cut
58     return result; }

```

## 8.2. Konig

```

1 // asume que el dinic YA ESTA tirado
2 // asume que nodes-1 y nodes-2 son la fuente y destino
3 int match[maxnodes]; // match[v]=u si u-v esta en el matching, -1 si v
4 // no esta matcheado
5 int s[maxnodes]; // numero de la bfs del koning
6 queue<int> kq;
7 // s[e]%2==1 o si e esta en V1 y s[e]==-1-> lo agarras
8 void koning() { // O(n)
9     forn(v,nodes-2) s[v] = match[v] = -1;
10    forn(v,nodes-2) forall(it,g[v]) if (it->to < nodes-2 && it->f>0)
11        { match[v]=it->to; match[it->to]=v;}
12    forn(v,nodes-2) if (match[v]==-1) {s[v]=0;kq.push(v);}
13    while(!kq.empty()) {
14        int e = kq.front(); kq.pop();
15        if (s[e]%2==1) {
16            s[match[e]] = s[e]+1;
17            kq.push(match[e]);
18        } else {
19            forall(it,g[e]) if (it->to < nodes-2 && s[it->to]==-1) {
20                s[it->to] = s[e]+1;
21                kq.push(it->to);
22            }
23        }
24    }
25 }

```

## 8.3. Edmonds Karp's

```

1 #define MAX_V 1000
2 #define INF 1e9
3 //special nodes
4 #define SRC 0
5 #define SNK 1
6 map<int, int> G[MAX_V]; //limpiar esto
7 //To add an edge use
8 #define add(a, b, w) G[a][b]=w
9 int f, p[MAX_V];
10 void augment(int v, int minE){
11     if(v==SRC) f=minE;
12     else if(p[v]!=-1){
13         augment(p[v], min(minE, G[p[v]][v]));
14         G[p[v]][v]-=f, G[v][p[v]]+=f;
15     }
16 }
17 ll maxflow(){ //O(VE^2)
18     ll Mf=0;
19     do{
20         f=0;
21         char used[MAX_V]; queue<int> q; q.push(SRC);
22         zero(used), memset(p, -1, sizeof(p));
23         while(sz(q)){
24             int u=q.front(); q.pop();
25             if(u==SNK) break;
26             forall(it, G[u])
27                 if(it->snd>0 && !used[it->fst])
28                     used[it->fst]=true, q.push(it->fst), p[it->fst]=u;
29         }
30         augment(SNK, INF);
31         Mf+=f;
32     }while(f);
33     return Mf;
34 }

```

## 8.4. Push-Relabel O(N<sup>3</sup>)

```

1 #define MAX_V 1000
2 int N; //valid nodes are [0...N-1]
3 #define INF 1e9
4 //special nodes

```



```

5  #define SRC 0
6  #define SNK 1
7  map<int, int> G[MAX_V];
8  //To add an edge use
9  #define add(a, b, w) G[a][b]=w
10 ll excess[MAX_V];
11 int height[MAX_V], active[MAX_V], count[2*MAX_V+1];
12 queue<int> Q;
13 void enqueue(int v) {
14     if (!active[v] && excess[v] > 0) active[v]=true, Q.push(v); }
15 void push(int a, int b) {
16     int amt = min(excess[a], ll(G[a][b]));
17     if(height[a] <= height[b] || amt == 0) return;
18     G[a][b]-=amt, G[b][a]+=amt;
19     excess[b] += amt, excess[a] -= amt;
20     enqueue(b);
21 }
22 void gap(int k) {
23     forn(v, N){
24         if (height[v] < k) continue;
25         count[height[v]]--;
26         height[v] = max(height[v], N+1);
27         count[height[v]]++;
28         enqueue(v);
29     }
30 }
31 void relabel(int v) {
32     count[height[v]]--;
33     height[v] = 2*N;
34     forall(it, G[v])
35         if(it->snd)
36             height[v] = min(height[v], height[it->fst] + 1);
37     count[height[v]]++;
38     enqueue(v);
39 }
40 ll maxflow() { //O(V^3)
41     zero(height), zero(active), zero(count), zero(excess);
42     count[0] = N-1;
43     count[N] = 1;
44     height[SRC] = N;
45     active[SRC] = active[SNK] = true;
46     forall(it, G[SRC]){
47         excess[SRC] += it->snd;

```

```

48     push(SRC, it->fst);
49 }
50 while(sz(Q)) {
51     int v = Q.front(); Q.pop();
52     active[v]=false;
53     forall(it, G[v]) push(v, it->fst);
54     if(excess[v] > 0)
55         count[height[v]] == 1? gap(height[v]):relabel(v);
56 }
57 ll mf=0;
58 forall(it, G[SRC]) mf+=G[it->fst][SRC];
59 return mf;
60 }

```

## 8.5. Min-cost Max-flow

```

1  const int MAXN=10000;
2  typedef ll tf;
3  typedef ll tc;
4  const tf INFFLUJO = 1e14;
5  const tc INFCOSTO = 1e14;
6  struct edge {
7      int u, v;
8      tf cap, flow;
9      tc cost;
10     tf rem() { return cap - flow; }
11 };
12 int nodes; //numero de nodos
13 vector<int> G[MAXN]; // limpiar!
14 vector<edge> e; // limpiar!
15 void addEdge(int u, int v, tf cap, tc cost) {
16     G[u].pb(sz(e)); e.pb((edge){u,v,cap,0,cost});
17     G[v].pb(sz(e)); e.pb((edge){v,u,0,0,-cost});
18 }
19 tc dist[MAXN], mnCost;
20 int pre[MAXN];
21 tf cap[MAXN], mxFlow;
22 bool in_queue[MAXN];
23 void flow(int s, int t) {
24     zero(in_queue);
25     mxFlow=mnCost=0;
26     while(1){
27         fill(dist, dist+nodes, INFCOSTO); dist[s] = 0;

```

```

28  memset(pre, -1, sizeof(pre)); pre[s]=0;
29  zero(cap); cap[s] = INFFLUJO;
30  queue<int> q; q.push(s); in_queue[s]=1;
31  while(sz(q)){
32      int u=q.front(); q.pop(); in_queue[u]=0;
33      for(auto it:G[u]) {
34          edge &E = e[it];
35          if(E.rem() && dist[E.v] > dist[u] + E.cost + 1e-9){ // ojo EPS
36              dist[E.v]=dist[u]+E.cost;
37              pre[E.v] = it;
38              cap[E.v] = min(cap[u], E.rem());
39              if(!in_queue[E.v]) q.push(E.v), in_queue[E.v]=1;
40          }
41      }
42  }
43  if (pre[t] == -1) break;
44  mxFlow +=cap[t];
45  mnCost +=cap[t]*dist[t];
46  for (int v = t; v != s; v = e[pre[v]].u) {
47      e[pre[v]].flow += cap[t];
48      e[pre[v]^1].flow -= cap[t];
49  }
50  }
51  }

```

## 9. Template

```

1  //touch {a..m}.in; tee {a..m}.cpp < template.cpp
2  #include <bits/stdc++.h>
3  using namespace std;
4  #define forr(i,a,b) for(int i=(a); i<(b); i++)
5  #define forn(i,n) forr(i,0,n)
6  #define sz(c) ((int)c.size())
7  #define zero(v) memset(v, 0, sizeof(v))
8  #define forall(it,v) for(auto it=v.begin();it!=v.end();++it)
9  #define pb push_back
10 #define fst first
11 #define snd second
12 typedef long long ll;
13 typedef pair<int,int> ii;
14 #define dforn(i,n) for(int i=n-1; i>=0; i--)
15 #define dprint(v) cout << #v<<" << v << endl //;)
16

```

```

17 const int MAXN=100100;
18 int n;
19
20 int main() {
21     freopen("input.in", "r", stdin);
22     ios::sync_with_stdio(0);
23     while(cin >> n){
24
25     }
26     return 0;
27 }

```

## 10. Ayudamemoria

### Rellenar con espacios(para justificar)

```

1  #include <iomanip>
2  cout << setfill('␣') << setw(3) << 2 << endl;

```

### Aleatorios

```

1  #define RAND(a, b) (rand()%(b-a+1)+a)
2  srand(time(NULL));

```

### Doubles Comp.

```

1  const double EPS = 1e-9;
2  x == y  <=> fabs(x-y) < EPS, x > y  <=> x > y + EPS
3  x >= y  <=> x > y - EPS

```

### Expandir pila

```

1  #include <sys/resource.h>
2  rlimit rl;
3  getrlimit(RLIMIT_STACK, &rl);
4  rl.rlim_cur=1024L*1024L*256L;//256mb
5  setrlimit(RLIMIT_STACK, &rl);

```

### Iterar subconjunto

```

1  for(int sbm=bm; sbm; sbm=(sbm-1)&bm)

```