Modeling and Analysis of McPherson quarter-car suspension system

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Abstract: This paper aims to analyze McPherson suspension system of a quarter car model and draw a simple animation simulation of the system. McPherson strut is a type of suspension systems used in cars. Kinematics of the system is studied and derived. For the dynamic equations, Hamilton's Principal and Lagrangian dynamics are studied to be used for the derivation of the motion equations. The animation is conducted on MATLAB software, where the motion equations were numerically solved using Runge-Kutta 4th order method to obtain the generalized variables values. The system is tested on several input signals, and the animation included a half-car view for shape reasons only. Input signal, generalized variables plots are shown from the simulation results.

I. Introduction

Suspension systems play a vital role in vehicle dynamics, influencing ride comfort, handling stability, and overall safety. Among various suspension designs, the McPherson strut is one of the most widely used in modern automobiles due to its simplicity, compactness, and cost-effectiveness. This system provides an efficient balance between performance and affordability, making it a preferred choice many cars.

Accurate simulation and modeling of suspension systems are essential for optimizing vehicle performance, reducing development costs, and improving safety by testing various designs and parameters.

This paper focuses on the simulation and modeling of the McPherson strut, aiming to provide insights into its dynamic behavior and performance characteristics. Through numerical analysis and virtual prototyping, the study enhances the understanding of the system's response under different operating conditions.

II. Keywords of suspension system

Strut: a system of a spring and a damper inside it.

Camber Angle: the angle of the wheel from the vertical line.

Sprung mass: the mass lifted over the suspension system, i.e., chassis (frame).

Unsprung mass: the mass that is not lifted on the system, or the mass that lifts the system itself, i.e., wheels.

III. Suspension Systems

The following diagram represents a simplified cross-section view of McPherson suspension system.

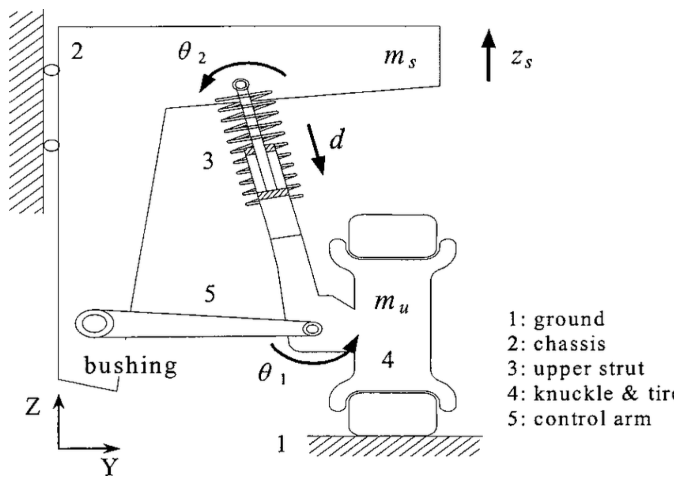

Figure 1: 2D view of McPherson suspension system of a quarter car [2]

As it can be noticed, the wheel is not directly connected to the chassis. This disconnection achieves the desired smooth ride and handles movements caused by the vehicle acceleration, braking and cornering through absorbing road bumps and tire vibration. The absorption of unwanted movements is due to three major reasons: the flexibility of the tire material, the damped oscillation of the strut, and the dispersion of the vibrations in the overall mechanical parts. Tire flexibility is only capable of handling small bumps, so when more serious bumps and vibrations are encountered, they will cause the system to be unstable and change its configuration.

Before starting building up the model of the system, further explanations and clarifications for the last figure must be made. To simplify the system view, MATLAB is used to draw an equivalent illustration for the initial configure of the system, in figure (2).

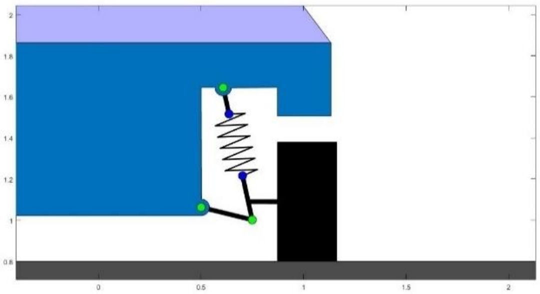

Figure 2 Simplified view of McPherson System

Blue block represents the sprung mass. Later on, it will be denoted as m_s. Because the model is a quarter car model, the sprung mass is only capable of moving vertically, on the z-axis, due to symmetry in the car. The three small green circles represent three revolute joints, while blue circles are point-fixed joints. The strut is clearly represented by the spring shaped part. The last block, the black one, is the tire, the unsprung mass m_u. It is connected to the sprung mass through revolute joints and a strut so that vertical movements, e.g., bumps and vibrations, are transformed to small angles revolutions and most of their energy is dissipated in the strut instead of the sprung mass.

IV. Motion Study and Dynamics:

This section will be mainly presenting 3 ideas: assumptions of the study, motion study and kinematics of the system, and dynamics of the system and its equations of motion.

A. Assumptions

various assumptions were adopted to relatively maintain the model simple and linearizable. It is assumed that:

- Horizontal velocity is constant however the road profile is changing. This will make the model time invariant and avoid extra calculations for the velocity.
- The model is a quarter car model. When moving to the full car animation, it will be assumed that each axle is independent. In other words, it can be considered as two half cars model. This assumption will avoid extra calculations for mass distribution and it considers the car as two independent masses.
- Camber angle and control arm angle are small angles, this will set a path for the linearization of sinusoids.
- Links are solid links, i.e., no strain or deflection in any link.
- All joints are ideal.
- The values of spring, damper, and wheel stiffness behave linearly.
- Masses of control arm, knuckle, and strut are negligible.

B. Kinematics

The first step of modelling any mechanical system is to study its Kinematics. The main purpose is to extract the equations that describe the motion of the links, i.e., each link has equations for its angle, velocity, and acceleration in the plane of study. In the studied system, the main concern is to describe the following *goal variables*:

- Control arm angle θ
- Camber angle φ
- Wheel vertical displacement z_u
- Chassis vertical displacement z_s
- Strut extension Δs

Determining values of each variable at an instant will draw a new configuration for the system, and putting configurations of consecutive instants together gives the animation of the model over a specific period of time.

Some of these variables are functions of the others, so we need to determine the degrees of freedom for the system to know how many generalized coordinated do we have.

Grubler's Formula states that Degrees of Freedom, DOF, of a system of links and joints are the number of total DOF (3 DOF in the plan, 6 in the 3D space) of all links minus the number of independent constraints provided by joints. [4]

$$\#DOF = m * (N - 1) - \Sigma c_j \qquad (1)$$

Where m is $\#DOF$ of a rigid body, N is the number of links including ground, and c_j is the independent constraints provided by joint j.

Furthermore, number of independent constraints provided by a joint j can be represented as the number of DOF of a rigid body minus the number of DOF provided by joint j. Developing on Eq. (1),

$$\#DOF = m * (N - 1 - J) + \Sigma f_j \qquad (2)$$

Where J is the number of joints in the system and f_j is the number of DOF provided by joint j.

Projecting Eq. (2) on McPherson system, it consists of 5 links (without ground), 3 revolute joints, and 2 point-fixed joints. The vertical movement of the sprung mass can be considered as a prismatic joint between the sprung mass and the ground, so this will be taken into consideration in Grubler's Formula. $f_j = 1$ for each revolute or prismatic joint, while $f_j = 0$ for each point-fixed joint. $m = 3$ for planar motion. Substituting in Eq. (2) $\#DOF = 1$. So, the system has 1 DOF without considering the strut. In total, McPherson system has 2 DOF.

The concept of DOF refers to the minimum number of quantities, e.g., coordinates, angles, variables…, that must be determined for a system to describe a configuration. Moreover, specific coordinates must be chosen to be the *Generalized Coordinates*. *Generalized Coordinates* are any set of quantities that completely specifies the configuration of the system. They are not unique, so the choice of the set is dependent on the simplicity of interpretation of equations of motion using this set.[6] In McPherson system, $\{z_s, z_u\}$ will be chosen as the generalized coordinates as they have obvious visual meaning and will result in straightforward equations and analysis. The rest of the subsection tries to write the *goal variables* as functions of the generalized coordinates. Kinematics equations are derived using one of the most useful and powerful representation of rotational movements and displacement of a rigid body called *Transformation Matrices*. *Transformation Matrix* is a combination of *Rotation Matrix R* and *Displacement Vector p* as the following,

$$T = \begin{bmatrix} R & p \\ 0 & 1 \end{bmatrix} \qquad (3)$$

where Rotational Matrix R in 2D is 2x2 matrix describing the angle of rotation of the body, p is 2x1 vector describing the displacement on both z and y axes, 0 is 1x2 zero vector, and 1 is 1x1.[4] Transformation Matrix is used for representing a rigid body configuration. It will be used to write the equations of motion of point B. But first, point C will be used for finding T.

$$\begin{bmatrix} C \\ 1 \end{bmatrix} = T \begin{bmatrix} C_0 \\ 1 \end{bmatrix} \qquad (4)$$

$$\begin{bmatrix} y_C \\ z_C \\ 1 \end{bmatrix} = \begin{bmatrix} \cos(\varphi) & \sin(\varphi) & p_1 \\ -\sin(\varphi) & \cos(\varphi) & p_2 \\ 0 & 0 & 1 \end{bmatrix} . \begin{bmatrix} y_{C_0} \\ z_{C_0} \\ 1 \end{bmatrix} (5)$$

Where φ is camber angle. From Eq. (5),

$$p_1 = y_C - (\cos(\varphi)\, y_{C_0} + \sin(\varphi) z_{C_0}) \qquad (6)$$

$$p_2 = z_C - (-\sin(\varphi)\, y_{C_0} + \cos(\varphi) z_{C_0}) \qquad (7)$$

Using Transformation Matrix for point B,

$$\begin{bmatrix} y_B \\ z_B \\ 1 \end{bmatrix} = \begin{bmatrix} \cos(\varphi) & \sin(\varphi) & p_1 \\ -\sin(\varphi) & \cos(\varphi) & p_2 \\ 0 & 0 & 1 \end{bmatrix} . \begin{bmatrix} y_{B_0} \\ z_{B_0} \\ 1 \end{bmatrix} (8)$$

$$y_B = y_C + \left(y_{B_0} - y_{C_0}\right)\cos(\varphi) + \left(z_{B_0} - z_{C_0}\right)\sin(\varphi) \quad (9)$$

$$z_B = z_C + \left(z_{B_0} - z_{C_0}\right)\cos(\varphi) - \left(y_{B_0} - y_{C_0}\right)\sin(\varphi) (10)$$

For small angles φ and θ,

$$y_B = y_C + \left(y_{B_0} - y_{C_0}\right) + \left(z_{B_0} - z_{C_0}\right)\varphi (11)$$

$$z_B = z_C + \left(z_{B_0} - z_{C_0}\right) - \left(y_{B_0} - y_{C_0}\right)\varphi (12)$$

From geometry, and considering initial θ_0 and small angles,

$$y_B = L_1 \cos(\theta_0 + \theta)$$

$$y_B = L_1 \cos(\theta_0)\cos(\theta) - L_1 \sin(\theta_0)\sin(\theta)$$

$$y_B = L_1 \cos(\theta_0) - L_1 \sin(\theta_0)\, \theta$$

$$y_B = y_{B_0} - z_{B_0}\theta \qquad (13)$$

Where L_1 is the length of control arm.

The same process for z_B,

$$z_B = z_{B_0} + y_{B_0}\theta + z_s \qquad (14)$$

Equations (11) through (14) are 4 equations that have 7 variables including only z_s. 2 more equations of some of the previous variables adding to them z_u are needed to be capable of representing the variables as functions of the generalized coordinates.

First equation comes from the geometry of the system considering that φ is small,

$$\varphi = \frac{y_{B_0} - y_B}{L_{02}} \quad (15)$$

Where L_{02} is the initial length of link BD, i.e., rest case length.

Second equation is a trivial one,

$$z_C = z_{C_0} + z_u \quad (16)$$

Having 6 equations of 8 variables, θ and φ can be written as functions of generalized coordinates. Starting from Eq. (14), substituting form Eq. (12), then substituting from Eqs. (15) and (16), after that, substituting from Eq. (13), and with some mathematical manipulation

$$\theta = \frac{z_B - z_{B_0} - z_s}{y_{B_0}}$$

$$\theta = \frac{z_C - z_{C_0} - \left(y_{B_0} - y_{C_0}\right) \varphi - z_s}{y_{B_0}}$$

$$\theta = \frac{z_u - z_s - \left(y_{B_0} - y_{C_0}\right)\left(\frac{y_{B_0} - y_B}{L_{02}}\right)}{y_{B_0}}$$

$$\theta = \frac{z_u - z_s - \left(y_{B_0} - y_{C_0}\right)\left(\frac{z_{B_0}\theta}{L_{02}}\right)}{y_{B_0}}$$

$$\theta = \frac{L_{02}}{y_{B_0}L_{02} + y_{B_0}z_{B_0} - y_{C_0}z_{B_0}}(z_u - z_s) \quad (17)$$

Assuming a constant $k_1 = \frac{L_{02}}{y_{B_0}L_{02} + y_{B_0}z_{B_0} - y_{C_0}z_{B_0}}$

$$\theta = k_1(z_u - z_s) \quad (18)$$

For φ, starting from Eq. (15) and substituting from (13) then (18),

$$\varphi = \frac{z_{B_0}\theta}{L_{02}}$$

$$\varphi = \frac{z_{B_0}}{y_{B_0}L_{02} + y_{B_0}z_{B_0} - y_{C_0}z_{B_0}}(z_u - z_s)$$

$$\varphi = k_2(z_u - z_s) \quad (19)$$

For y_C, starting from Eq. (11), substituting from Eqs. (13), (18), and (19),

$$y_C = y_{C_0} + \left(y_B - y_{B_0}\right) + (y_{C_0} - y_{B_0})\varphi$$

$$y_C = y_{C_0} - z_{B_0}\theta + (y_{C_0} - y_{B_0})\frac{z_{B_0}}{L_{02}}\theta$$

$$y_C = y_{C_0} + \frac{z_{B_0}\left(z_{C_0} - z_{B_0} - L_{02}\right)}{y_{B_0}L_{02} + y_{B_0}z_{B_0} - y_{C_0}z_{B_0}}(z_u - z_s)$$

$$y_C = y_{C_0} + k_3(z_u - z_s) \quad (20)$$

For the points fixed on the sprung mass,

$$y_A = y_{A_0}$$

$$z_A = z_{A_0} + z_s$$

$$y_D = y_{D_0}$$

$$z_D = z_{D_0} + z_s$$

For the extension of the strut Δs,

$$\Delta s = L_2 - L_{02}$$

$$\Delta s = \sqrt{(y_B - y_D)^2 + (z_B - z_D)^2} - L_{02}$$

$$\Delta s = \sqrt{(y_{B_0} - z_{B_0}\theta - y_{D_0})^2 + (z_{B_0} + y_{B_0}\theta + z_s - z_{D_0} - z_s)^2} - L_{02}$$

$$\Delta s = \sqrt{({z_{B_0}}^2 + {y_{B_0}}^2)\theta^2 + 2\left(y_{D_0}z_{B_0} - z_{D_0}y_{B_0}\right)\theta + {L_{02}}^2} - L_{02}$$

θ is a small angle, so, $\theta^2 \approx 0$,

$$\Delta s = \sqrt{2\left(y_{D_0}z_{B_0} - z_{D_0}y_{B_0}\right)\theta + {L_{02}}^2} - L_{02} \quad (21)$$

At last, there are 2 variables that are not mentioned yet, which are the deflections of the tire and its responses to compression and extension. Tire has two deflection responses, a vertical deflection, and a lateral one. They are represented as a mass-spring-damper systems [5] The equations of tire deflections for the current study will only assume that they are responses to mass-spring-damper systems.

$$\Delta y_t = y_C - y_{C_0} - Rwheel\,.\,\varphi$$

$$\Delta y_t = (k_3 - Rwheel\,.\,k_2)(z_u - z_s)$$

$$\Delta y_t = k_4(z_u - z_s) \quad (22)$$

Δy_t is the distance by which the tire has moved form its initial position on the y-axis.

$$\Delta z_t = (z_u - z_r) \quad (23)$$

Δz_t is the distance by which the tire has moved away or into the road, and z_r is the road vertical displacement.

After representing all points and goal variables as functions of generalized coordinates, Dynamics of the system will be introduced to let the system move.

C. Dynamics

The dynamic model of a system studies the relation between forces and torques applied to the system and the motion they cause. It is necessary to study the system dynamics to get the response of the system variables to the input.

Newton second law is the most common way in developing the dynamic model of a mechanical system, but sometimes it may be so complicated to imply in terms of existence of motion constraints and multi-body system. In this case, other approaches may be more general and simpler, e.g., Hamilton's Principle and Lagrangian dynamics. Hamilton's principle states the following:

> *Of all possible paths along which a dynamical system may move from one point to another within a specified time interval (consistent with any constraints), the actual path followed is that which minimizes the time integral of the difference between the kinetic and potential energies.[6]*

Hamilton's Principle itself represents one of the *least action* principles. In fact, the difference of the kinetic and potential energies is due to the action made on or by the system. Furthermore, the integral of the difference represents the sum of action within a specific time interval. So, Hamilton's principles minimize that integral, which means there is an extremum of the integral *functional*. This can be translated mathematically to the following equation,

$$\delta \int_{t1}^{t2} T - V \, dt = 0$$

$$\delta \int_{t1}^{t2} L \, dt = 0 \quad (22)$$

Where T is the kinetic energy, V is the potential energy, L is called the *Lagrangian* or *Lagrange function*, and δ represent variation. Depending on the calculus of variation,[6]

$$\frac{\partial L}{\partial y_i} - \frac{d}{dt}\frac{\partial L}{\partial \dot{y}_i} = 0 \quad (23)$$

Where y_i is the i-th generalized coordinate, and $\dot{y}_i$ is its time derivative. The last equations are called Lagrange-Euler equations, they describe the system where there are no dissipative objects and forces, e.g., strut damping and tire damping. These dissipative forces can be added as Rayleigh's dissipation function F which describes the dissipative energies and get into Lagrange equations as the following,[1]

$$\frac{d}{dt}\frac{\partial L}{\partial \dot{y}_i} - \frac{\partial L}{\partial y_i} + \frac{\partial F}{\partial \dot{y}_i} = 0 \quad (24)$$

Eq. (23) will be used to develop the dynamical model of McPherson system after determining the Lagrangian function and Rayleigh's dissipative function.

$$T = \frac{1}{2} m_s \dot{z_s}^2 + \frac{1}{2} m_u (\dot{z_u}^2 + \dot{y_c}^2) + \frac{1}{2} I_\Delta \dot{\varphi}^2 \quad (24)$$

$$V = \frac{1}{2} k_s \Delta s^2 + \frac{1}{2} k_{t1} (z_u - z_r)^2 + \frac{1}{2} k_{t2} \Delta {y_t}^2 \quad (25)$$

$$F = \frac{1}{2} b_s \Delta\dot{s}^2 + \frac{1}{2} b_{t1} (\dot{z_u} - \dot{z_r})^2 + \frac{1}{2} b_{t2} \Delta \dot{y_t}^2 \quad (26)$$

$$\frac{d}{dt}\frac{\partial (T-V)}{\partial \dot{z_s}} - \frac{\partial (T-V)}{\partial z_s} + \frac{\partial F}{\partial \dot{z_s}} = 0 \quad (27)$$

$$\frac{d}{dt}\frac{\partial (T-V)}{\partial \dot{z_u}} - \frac{\partial (T-V)}{\partial z_u} + \frac{\partial F}{\partial \dot{z_u}} = 0 \quad (28)$$

Kinetic energies are due the vertical movement of sprung mass, vertical, lateral, and rotational movement of unsprung mass. Potential energies are due the extension of the strut, and vertical and lateral extension of the tire. Dissipation energies are due the damping of the strut, and vertical and lateral damping of the tire.

Substituting from kinematics equations to have the energies as functions of generalized coordinates.,

$$T = \frac{1}{2} m_s \dot{z_s}^2 + \frac{1}{2} m_u (\dot{z_u}^2 + {k_3}^2 (\dot{z_u} - \dot{z_s})^2) + \frac{1}{2} I_\Delta {k_2}^2 (\dot{z_u} - \dot{z_s})^2$$

$$V = \frac{1}{2}k_s\left(\sqrt{2k_1\left(y_{D_0}z_{B_0} - z_{D_0}y_{B_0}\right)(z_u - z_s) + {L_{02}}^2} - L_{02}\right)^2 + \frac{1}{2}k_{t1}(z_u - z_r)^2 + \frac{1}{2}k_{t2}{k_4}^2(z_u - z_s)^2$$

$$F = \frac{1}{2}b_s\frac{{k_1}^2\left(y_{D_0}z_{B_0} - z_{B_0}y_{B_0}\right)^2(\dot{z}_u - \dot{z}_s)^2}{2k_1\left(y_{D_0}z_{B_0} - z_{D_0}y_{B_0}\right)(z_u - z_s) + {L_{02}}^2} + \frac{1}{2}b_{t1}(\dot{z}_u - \dot{z}_r)^2 + \frac{1}{2}b_{t2}{k_4}^2(\dot{z}_u - \dot{z}_s)^2$$

Applying Eqs. (27) and (28) will result in the following dynamic system equations.

For coordinate z_s,

$$m_s\ddot{z}_s - m_u{k_3}^2(\ddot{z}_u - \ddot{z}_s) - I_\Delta{k_2}^2(\ddot{z}_u - \ddot{z}_s) - k_sf_s - k_{t_2}{k_4}^2(z_u - z_s) - b_sf_b - b_{t_2}{k_4}^2(\dot{z}_u - \dot{z}_s) = 0 \quad (29)$$

For coordinate z_u,

$$m_u\ddot{z}_u + m_u{k_3}^2(\ddot{z}_u - \ddot{z}_s) + I_\Delta{k_2}^2(\ddot{z}_u - \ddot{z}_s) + k_sf_s + k_{t_1}(z_u - z_r) + k_{t_2}{k_4}^2(z_u - z_s) + b_sf_b + b_{t_1}(\dot{z}_u - \dot{z}_r) + b_{t_2}{k_4}^2(\dot{z}_u - \dot{z}_s) = 0 \quad (30)$$

where,

$$f_s = k_1k_5\frac{\sqrt{2k_1k_5(z_u - z_s) + {L_{02}}^2} - L_{02}}{\sqrt{2k_1k_5(z_u - z_s) + {L_{02}}^2}} \quad (31)$$

$$f_b = {k_1}^2{k_5}^2\frac{(\dot{z}_u - \dot{z}_s)}{2k_1k_5(z_u - z_s) + {L_{02}}^2} \quad (32)$$

and $k_5 = y_{D_0}z_{B_0} - z_{D_0}y_{B_0}$.

Eqs. (29) and (30) represent the dynamical model of McPherson suspension system. It can be noticed that the model is nonlinear due to the strut extension and damping. These equations will be solved numerically using Runge-Kuta 4th order in the next section.

V. Simulation and analysis

The dynamical model of McPherson system is described by two nonlinear ODEs, each is of second order. To use Runge-Kuta for higher order ODEs, equations must be reconstructed to have four first order ODEs. This is a mathematical manipulation process by isolating the terms $\ddot{z}_s$ and $\ddot{z}_u$ then substituting each one of them in the other's equation. The final result will be as,

A. $\dot{z}_u = U$
B. $\dot{z}_s = S$
C. $\dot{S} = {k_{6s}}^{-1}\left(k_sf_s + k_{t_2}{k_4}^2(z_u - z_s) + b_sf_b + b_{t_2}{k_4}^2(U - S)\right)(1 - k_{7s}) - k_{7s}(k_{t_1}(z_u - z_r) + b_{t_1}(U - \dot{z}_r))$
D. $\dot{U} = {k_{6u}}^{-1}\left(k_sf_s + k_{t_2}{k_4}^2(z_u - z_s) + b_sf_b + b_{t_2}{k_4}^2(U - S)\right)(k_{7u} - 1) - (k_{t_1}(z_u - z_r) + b_{t_1}(U - \dot{z}_r))$

where,

$$k_{6s} = m_s + m_uk_3 + I_\Delta{k_2}^2 - \frac{(m_uk_3 + I_\Delta{k_2}^2)^2}{m_u + m_uk_3 + I_\Delta{k_2}^2}$$

$$k_{6u} = m_u + m_uk_3 + I_\Delta{k_2}^2 - \frac{(m_uk_3 + I_\Delta{k_2}^2)^2}{m_s + m_uk_3 + I_\Delta{k_2}^2}$$

$$k_{7s} = \frac{m_uk_3 + I_\Delta{k_2}^2}{m_u + m_uk_3 + I_\Delta{k_2}^2}$$

$$k_{7u} = \frac{m_uk_3 + I_\Delta{k_2}^2}{m_s + m_uk_3 + I_\Delta{k_2}^2}$$

Parameters and initial positions were taken from [3], then labeled below.

Parameter	***Value***
Chassis Mass Kg	453
Wheel Mass Kg	71
Strut Spring Constant N/m	17658
Strut damper Constant $N.s/m$	1950
Wheel Radius m	0.29
Wheel Moment of inertia $Kg.m^2$	0.021
Tire Deflection Constant k_{t1} N/m	183887
Tire Lateral Deflection Const. k_{t2} N/m	50, 000
Tire Damping Constant b_{t1} N/m	2500
Tire Lateral Damping Const. b_{t2} $N.s/m$	2500

	Y coordinates	***Z coordinates***
A	0	0
B	0.2490	−0.0608
C	0.3721	0.0275
D	0.1074	0.5825

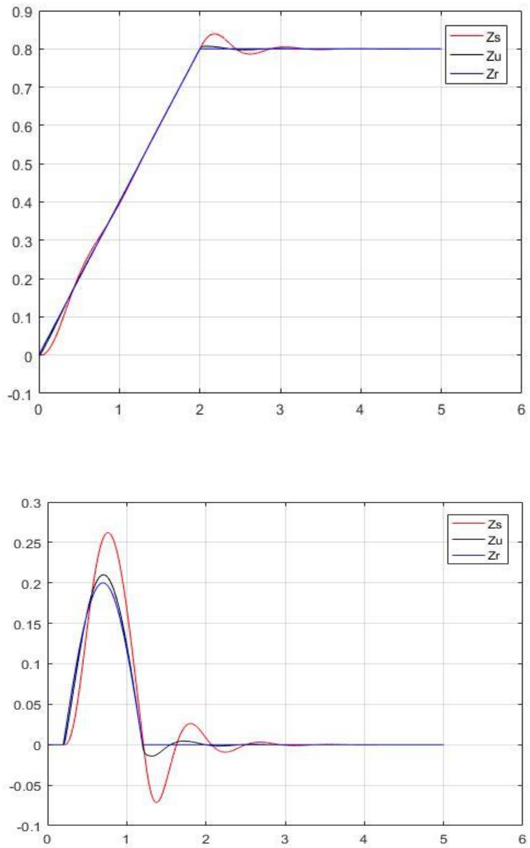

Figure 3 Simulation Results

Figure 3 represents the results of simulating McPherson system with two inputs. The first one is a semi-ramp input plotted in blue. The input road profile increases with a slope of 0.4 to reach its final value of 0.8m in 2 seconds. It can be noticed that z_u is almost similar to z_r, this is due to the high spring constant of the wheel. By looking at the curves, it can be read that a person sitting inside this car will feel a bounce right after the end of the ramp with an overshoot of about 0.039.

The second input is a half sinus wave starting at 0.1 second with 0.5 Hz frequency and 0.2m Amplitude. This input profile is the very similar to bumps on the roads. It can be noticed that a rider overshoots the height by about 5 cm.

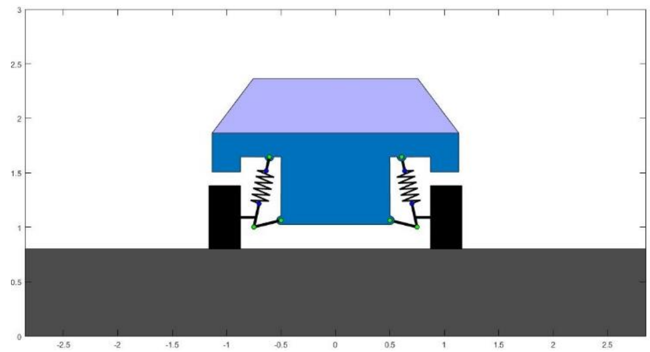

Figure 4 Animation of a semi-car with McPherson suspensions

VI. Code

The simulation is conducted using MATLAB coding. The code includes the initialization of the system, numerically solving the dynamic model using Runge-Kuta, plotting the generalized variables and input function to the time variable, and drawing the animation of the semi-car.

VII. Conclusion

In this report, the McPherson strut is investigated and simulated using MATLAB. First, study assumptions are set to simplify the work. Second, the kinematics equations are derived. Third, the Dynamic equations are derived using Lagrangian dynamics. The resulting equations are simulated on 2 input signals and animated using MATLAB.

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