UPPER TAIL TEST

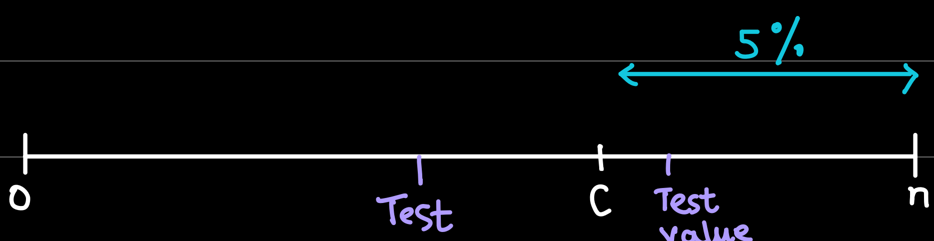
H_0 = ordinary Scenario   70% accuracy. $P = 0.7$

H_1 = Claim   Improve $P > 0.7$

I play 20 shot out of which 18 are accurate.

$n=20$ Binomial $X = 18$ (Test value)

Significance level = α %   5% mostly. (given).



(H₀ is accepted)
(H₁ is rejected)

Claim accept
(H_1 is accepted)
(H_0 is rejected)

Ordinary   Claim.

$P(X \geq \text{Test value}) < 5\%$

(H_1 accept) (H_0 reject)

$P(X \geq \text{Test value}) > 5\%$

(H_0 accept) (H_1 reject)

LOWER TAIL TEST

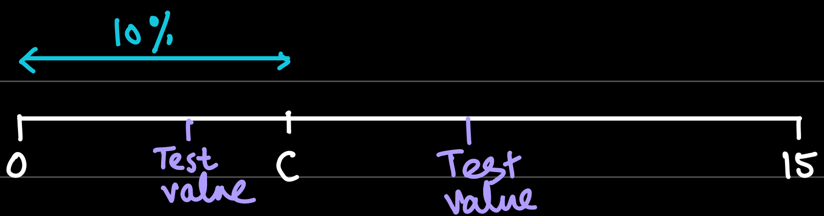
Q: P(Red cars) in an area is 0.2. Ali claims that the red cars have reduced recently. Out of 15 cars 2 were red. Use significance level of 10%.

Binomial $n=15$

Test value

$H_0 = \quad p = 0.2$

$H_1 = \quad p < 0.2 \quad$ (Lower Tail test)



(H₁ is accepted) (H₀ is rejected)

(H₁ is reject) (H₀ is accept)

$P(X \leq \text{Test value}) < 10\%$

(H_1 accept) (H_0 reject)

(claim accepted)

$P(x \leq \text{Test value}) > 10\%$

(H_0 accept) (H_1 reject)

TWO TAILED TEST

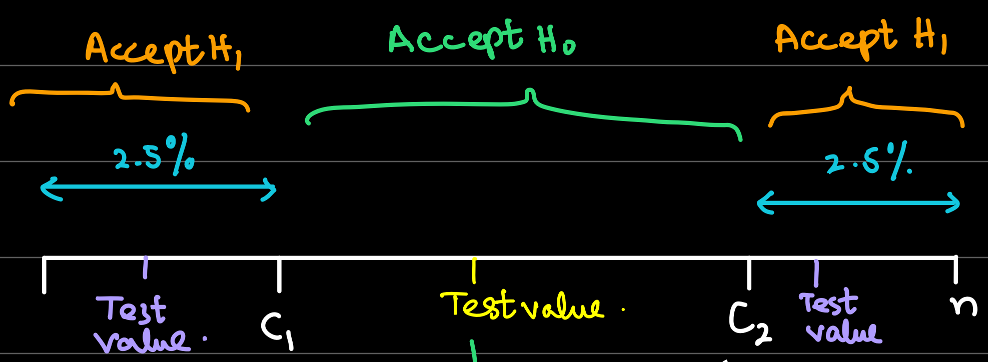
CHANGE

$\neq$

Q: Mean length of nails is 8 cm. It is claimed that this is now changed. In a sample of 8 nails the mean is found to be 7.1. Carry out a 5% significance test.

$H_0: \mu = 8$

$H_1: \mu \neq 8$



$P(X \leq \text{Test value}) < 2.5$
(H₁ accept) (H₀ reject)

H_0 accept
H_1 reject.

$P(X \geq \text{Test value}) < 2.5\%$
(H₁ accept) (H₀ reject)

STEPS TO CARRY OUT HYPOTHESIS

1- Define variables and distributions.

eg $X \sim B(20, p)$

2- State the Pypothesis

eg: $H_0 : \quad p = 0.25$ (ordinary)

$H_1 : \quad p > 0.25$ (claim)

3. State distribution according to Ho (ordinary)

eg: $X \sim B(20, 0.25)$

↓

H_0 ki value of p.

4. Level of significance and type of test

$\alpha \% =$ Level of significance

$H_1 :$ $>$ (upper tail test)

$<$ (Lower tail test)

$\neq$ (Two tail test)

5. State the rejected region / critical region.

6- Check probability with test value.

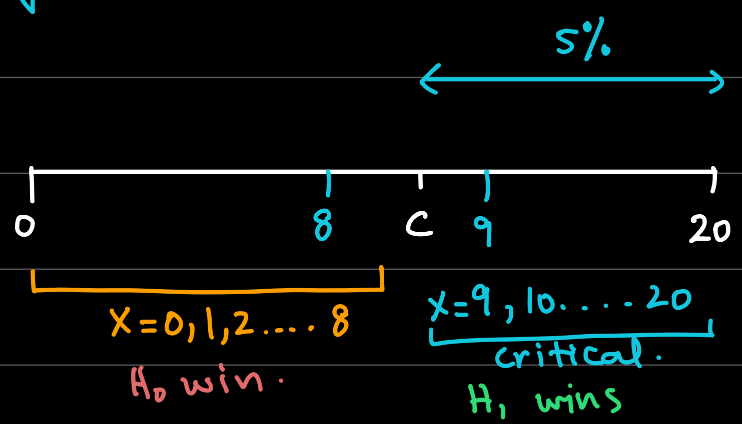
7- Make your decision.

DISCRETE (BINOMIAL, POISSON)	CONTINUOUS (NORMAL)

DISCRETE (BINOMIAL, POISSON)

Trial and error

eg:

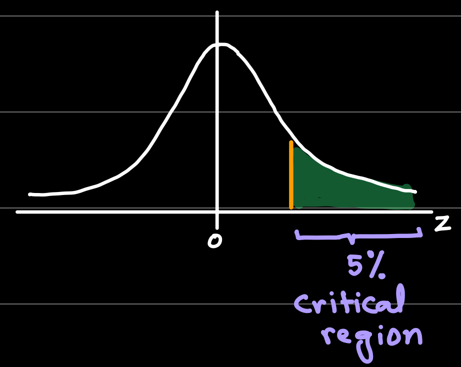


if $P(X \geq 8) = 10\%$

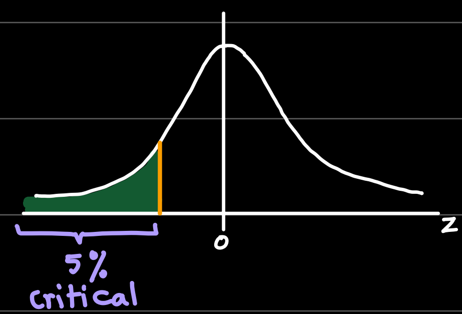
$P(X \geq 9) = 4\%$

CONTINUOUS (NORMAL)

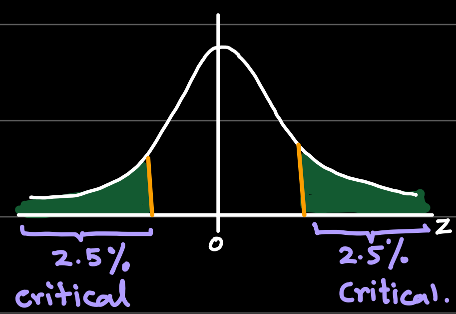
upper tail test 5%



Lower tail test 5%



Two Tailed Test 5%



TYPES OF QUESTIONS

POPULATIONS

1- Binomial	$X \sim B(n, p)$ $P(X=r) = {}^nC_r\, p^r q^{n-r}$
2- Normal Approximation to a Binomial Distribution.	Conditions : $np > 5$, $nq > 5$ 1- $\mu = np$ 2- $\sigma = \sqrt{npq}$ 3. Correction of continuity.
3- Poisson Distribution	$X \sim P_o(\lambda)$ $P(X=r) = \frac{e^{-\lambda}\lambda^r}{r!}$
4- Poisson approximation of a Binomial Distribution	Condition : $n > 50$, $p < 0.1$ 1- $\mu = np = \lambda$ 2- $Var = \lambda$ No correction of continuity.
5- Normal Approximation of a poisson Distribution.	Condition $\lambda > 15$ 1- $\mu = \lambda$ 2- $\sigma = \sqrt{\lambda}$ 3- Correction of continuity.

ERRORS

Factors : 1- Biased Sample data.

2- Probability model not correct.

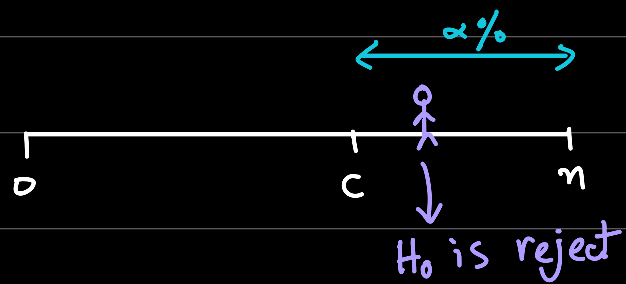
3- $\alpha\%$ significance level is not appropriate.

Type I True H_0 is rejected

Roll a dice and six land.

$H_0: \; p = \frac{1}{6}$ (Dice is fair)

$H_1: \; p > \frac{1}{6}$ (Dice is biased)

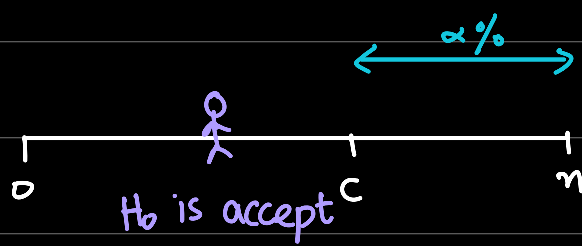


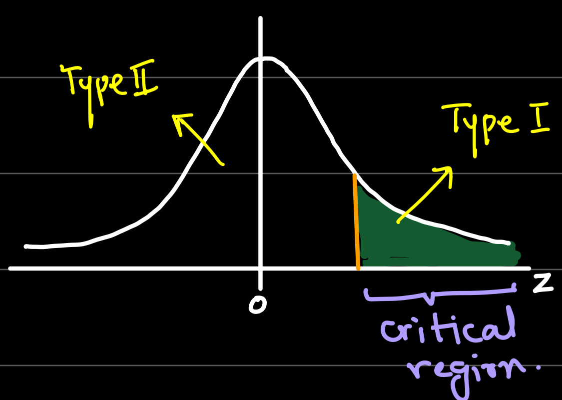
Type II False H_0 is accepted.

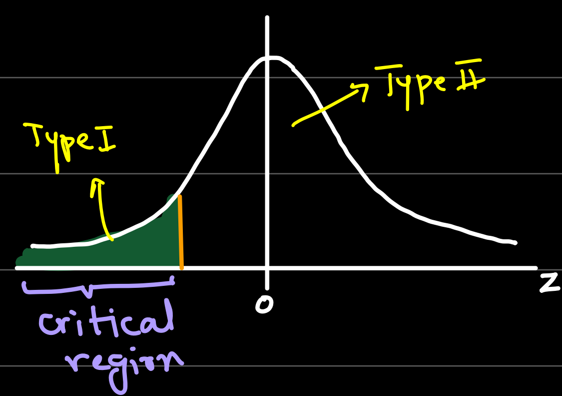
Roll a dice and six land.

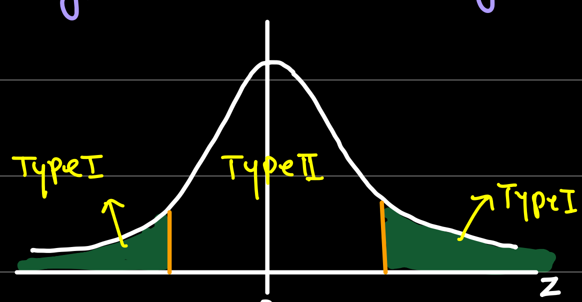
$H_0: \; p = \frac{1}{6}$ (Dice is fair)

$H_1: \; p > \frac{1}{6}$ (Dice is biased)









	we accepted H_0	we rejected H_0
In actual H_0 was True	Correct Decision (No error)	wrong Decision (Type I)
In actual H_0 was False	Wrong Decision (Type II)	Correct decision (No error)

Q. Before attending a basketball course, a player found that 60% of his shots made a score. After attending the course the player claimed he had improved. In his next game he tried 12 shots and scored in 10 of them. Assuming shots to be independent, test this claim at the 10% significance level. [5]

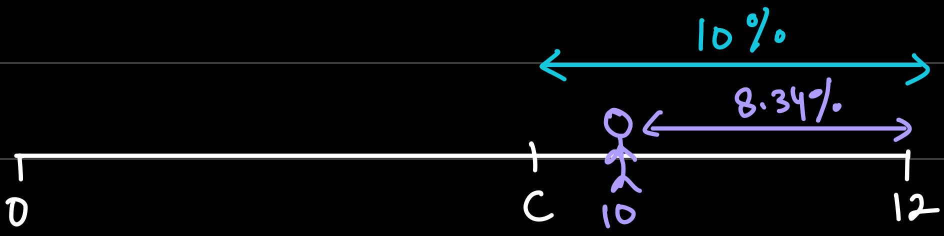
Test value = 10

$\alpha = 10\%$

$\underbrace{n = 12, \ p = 0.6, \ q = 0.4}_{\text{Binomial.}}$

$H_0 : \quad p = 0.6$

$H_1 : \quad p > 0.6 \qquad$ (upper Tail Test)



$P(X \geq 10) = P(10, 11, 12)$

$= {}^{12}C_{10}(0.6)^{10}(0.4)^{2} + {}^{12}C_{11}(0.6)^{11}(0.4)^{1} + {}^{12}C_{12}(0.6)^{12}(0.4)^{0}$

$= 0.0834 = 8.34\%$

10 lies in critical region.

H_0 is rejected.

Claim is accepted that he has improved.

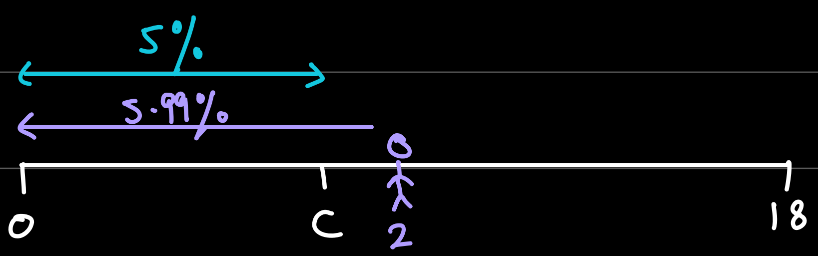
Isaac claims that 30% of cars in his town are red. His friend Hardip thinks that the proportion is less than 30%. The boys decided to test Isaac's claim at the 5% significance level and found that 2 cars out of a random sample of 18 were red. Carry out the hypothesis test and state your conclusion. [5]

H_0 : $p = 0.3$ (Issac)

H_1 : $p < 0.3$ (Hardip)

$n = 18$, $p = 0.3$, $q = 0.7$

Test value = 2.



$$P(X \leq 2) = P(2, 1, 0)$$

$$= {}^{18}C_2 (0.3)^2 (0.7)^{16} + {}^{18}C_1 (0.3)^1 (0.7)^{17} + {}^{18}C_0 (0.3)^0 (0.7)^{18}$$

$$= 0.0599$$

$$= 5.99\%$$

Since 2 is not in critical region.

H_0 is accepted and Issac's claim is valid.

It is claimed that a certain 6-sided die is biased so that it is more likely to show a six than if it was fair. In order to test this claim at the 10% significance level, the die is thrown 10 times and the number of sixes is noted.

(i) Given that the die shows a six on 3 of the 10 throws, carry out the test. [5]

On another occasion the same test is carried out again.

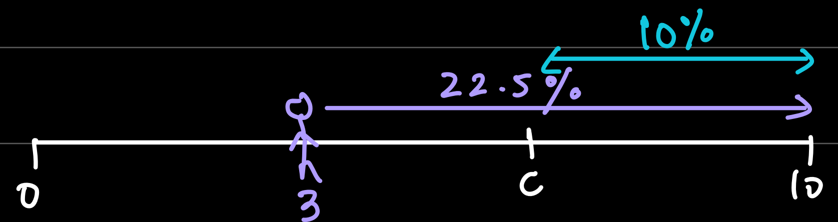
(ii) Find the probability of a Type I error. [3]

(iii) Explain what is meant by a Type II error in this context. [1]

$H_0 : \quad p = \frac{1}{6}$ Binomial: $n = 10,\ p = \frac{1}{6},\ q = \frac{5}{6}$

$H_1 : \quad p > \frac{1}{6}$

Test = 3



$P(X \geq 3) = P(3, 4, 5, \ldots 10)$

$= 1 - P(0, 1, 2)$

$= 1 - \left[{}^{10}C_0 \left(\frac{1}{6}\right)^0 \left(\frac{5}{6}\right)^{10} - {}^{10}C_1 \left(\frac{1}{6}\right)^1 \left(\frac{5}{6}\right)^9 - {}^{10}C_2 \left(\frac{1}{10}\right)^2 \left(\frac{5}{6}\right)^8 \right]$

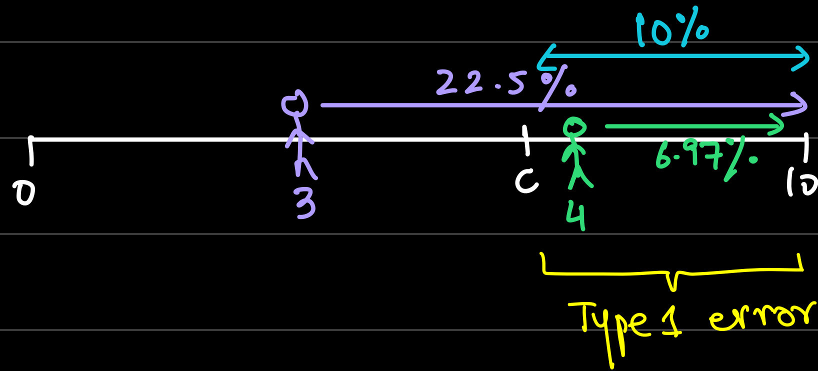
$= 0.2248$

$= 22.5\%$

3 is not in critical region

H_0 is accepted. The claim that dice is biased is rejected.

(ii)



$$P(X \geq 4) = 1 - P(0, 1, 2, 3)$$

$$= 1 - \left[{}^{10}C_0 \left(\frac{1}{6}\right)^0 \left(\frac{5}{6}\right)^{10} + {}^{10}C_1 \left(\frac{1}{6}\right)^1 \left(\frac{5}{6}\right)^9 + {}^{10}C_2 \left(\frac{1}{6}\right)^2 \left(\frac{5}{6}\right)^8 + {}^{10}C_3 \left(\frac{1}{6}\right)^3 \left(\frac{5}{6}\right)^7 \right]$$

$$= 0.0697$$

$$= 6.97\%$$

$P(X \geq 3) = 22.5\%$ (Not in critical region)

$P(X \geq 4) = 6.97\%$ (in critical region.

Type 1 error : $P(X \geq 4) = 0.0697$.

(iii) **(iii)** Explain what is meant by a Type II error in this context. **[1]**

It would be an error if we said dice is fair when it was not.