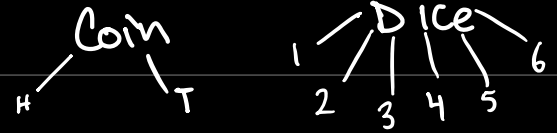
Binomial Distribution

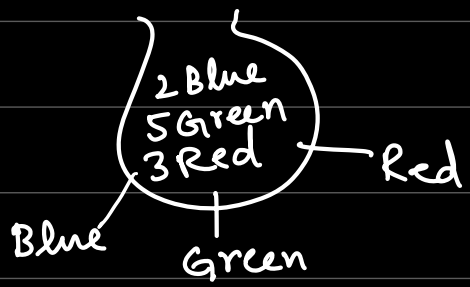
(without replacement cannot be Binomial ever)

Conditions

1- Repeated experiment (n)

2- Discrete (fixed) outcomes





Probability.
1- Conditional Prob.
2- Binomial
3. Geometric
4. Normal.

3- Success and failure

(p) $q = 1 - p$

4. n, p, and q are constants.

$$P(X = r) = {}^{n}C_{r}\, p^{r} q^{n-r}$$

Success event

desired value of success

P1

${}^{n}C_{r}\, a^{n-r} b^{r}$

Be careful.

Q. A fair dice is thrown 20 times.
X denotes the random variable for number of times that dice 1, 2, 3, 4, 5, 6 lands on a multiple of 3 (Success (X)). Find the probability that: $p = \frac{2}{6} = \frac{1}{3}$, $q = \frac{2}{3}$, $n = 20$

(a) Dice lands on multiple of 3 exactly twice.
(Success (X))

$$P(X=2) = {}^{20}C_2 \left(\frac{1}{3}\right)^2 \left(\frac{2}{3}\right)^{18} = \boxed{}$$

This means that we threw dice 20 times, out of which two times it landed on multiple of 3 and 18 times it did not land on a multiple of 3.

(b) Dice lands on multiple of 3 at least 18 times.
(X $\geqslant$ 18)

$$P(X \geqslant 18) = P(X=18) \text{ or } P(X=19) \text{ or } P(X=20)$$

$$= {}^{20}C_{18}\left(\frac{1}{3}\right)^{18}\left(\frac{2}{3}\right)^2 + {}^{20}C_{19}\left(\frac{1}{3}\right)^{19}\left(\frac{2}{3}\right)^1 + {}^{20}C_{20}\left(\frac{1}{3}\right)^{20}\left(\frac{2}{3}\right)^0$$

$$= \boxed{}$$

(c) Dice lands on multiple of 3 at most twice.
(X $\leqslant$ 2)

$$P(X \leqslant 2) = P(X=2) + P(X=1) + P(X=0)$$

X
0
1
2
3

$X \leqslant 2$

$= {}^{20}C_2 \left(\frac{1}{3}\right)^2 \left(\frac{2}{3}\right)^{18} + {}^{20}C_1 \left(\frac{1}{3}\right)^1 \left(\frac{2}{3}\right)^{19} + {}^{20}C_0 \left(\frac{1}{3}\right)^0 \left(\frac{2}{3}\right)^{20}$

$= \square$

(d) Dice lands on multiple of 3 at least once.

$X \geq 1$

$P(X \geqslant 1) = P(X=1) + P(X=2) + P(X=3) + \cdots\cdots P(X=20)$

$P(X \geqslant 1) = 1 - P(X=0)$

$= 1 - {}^{20}C_0 \left(\frac{1}{3}\right)^0 \left(\frac{2}{3}\right)^{20}$

$= \square$

X
0
1
2
3
⋮
18
19
20

$X \geqslant 1$

(e) Dice lands on a multiple of 3 at least twice.

$X \geqslant 2$

$P(X \geqslant 2) = P(X=2) + P(X=3) \cdots\cdots P(X=20)$

$P(X \geqslant 2) = 1 - P(X=0) - P(X=1)$

$= 1 - {}^{20}C_0 \left(\frac{1}{3}\right)^0 \left(\frac{2}{3}\right)^{20} - {}^{20}C_1 \left(\frac{1}{3}\right)^1 \left(\frac{2}{3}\right)^{19}$

X
0
1
2
3
⋮
18
19
20

$X \geqslant 2$

ADVANCED

THIS QUESTION IS ALL ABOUT YOUR GRADE 7 ENGLISH TEACHER!

Q. A Fair dice is thrown n times.
X denotes the random variable for number of times that dice lands on a multiple of 3.

Success (X)

$$p = \frac{1}{3}, \quad q = \frac{2}{3}$$

(i) Find smallest value of n for which the probability that dice lands on a multiple of three at least once is at least 0.95.

	Dice lands on multiple of 3	at least once	is at least 0.95
Probability	X	≥ 1	≥ 0.95

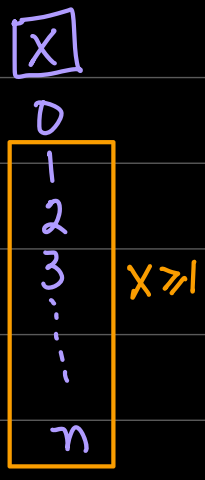
$P(X \geq 1) \geq 0.95$

$1 - P(X = 0) \geq 0.95$

$1 - {}^{n}C_{0}\left(\frac{1}{3}\right)^{0}\left(\frac{2}{3}\right)^{n} \geq 0.95$

$1 - (1)(1)\left(\frac{2}{3}\right)^{n} \geq 0.95$

$1 - \left(\frac{2}{3}\right)^{n} \geq 0.95$



${}^{5}C_{0} = 1$	${}^{5}C_{1} = 5$	${}^{5}C_{5} = 1$
${}^{4}C_{0} = 1$	${}^{4}C_{1} = 4$	${}^{4}C_{4} = 1$
${}^{n}C_{0} = 1$	${}^{n}C_{1} = n$	${}^{n}C_{n} = 1$

$$-\left(\frac{2}{3}\right)^n \geq 0.95 - 1$$

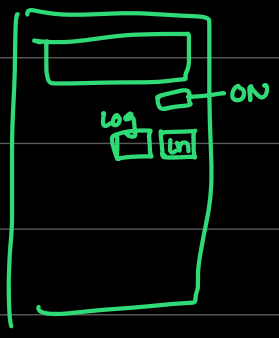
$$-\left(\frac{2}{3}\right)^n \geq -0.05$$

Divide both sides with -1 and inequality sign flips.

$$\left(\frac{2}{3}\right)^n \leq 0.05$$

Introduce Ln on both sides

$$\ln\left(\frac{2}{3}\right)^n \leq \ln 0.05$$



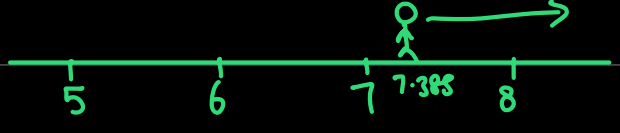
$$n \ln\left(\frac{2}{3}\right) \leq \ln(0.05)$$

NOW FIRST CALCULATE BOTH Ln values before making n subject

$$n(-0.4055) \leq -2.99573$$

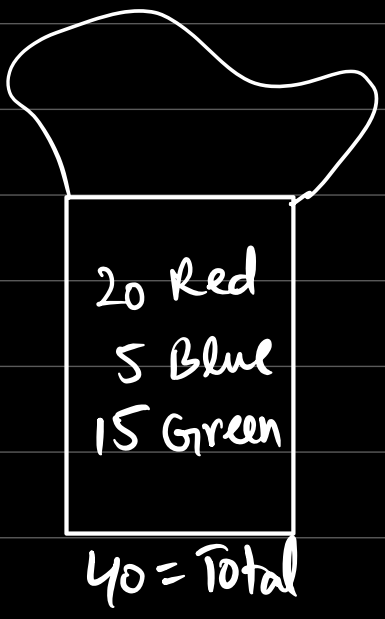
$$n \geq \frac{-2.99573}{-0.4055}$$

$$n \geq 7.388$$



Smallest value of $n = 8$

(i) Find Largest value of n for which the probability that dice lands on a multiple of three at least once is less than 0.10.



Three sweets are taken out of this bag without replacement.
X denotes the number of Green Sweets.

Conditions:

✓ 1) Repeat
✓ 2) Discrete
✓ 3) Sucess (Green)
✗ 4) n, P, q must be constant

V.V.IMP Without replacement Cannot be BINOMIAL.

parents: At least 3 A*

you : At most 3 B