

Heap

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Creation of Max Heap and Heap Sort Time Complexity

For the creation of a max heap using Heapify the time complexity is $O(n)$. This is because you go through each element in the list ensuring it is a max heap. This time complexity holds for both the average and the worst case of the algorithm. This is because in the worst case you still have to traverse through n elements. For the creation of a max heap using one by one it will be $O(n \log n)$. If we insert elements one by one into an initially empty heap to create a max-heap, we need to perform n insertions. Each insertion may require traversing the height of the heap, which is $\log n$ in the worst case. Therefore, the time complexity for creating a max-heap using one-by-one insertion is $O(n \log n)$ in the worst case. The time complexity for the average case is still $O(n \log n)$. In the average case we still have to traverse the height of the heap which is $O(\log n)$ and we still have to insert n elements which is $O(n)$. Therefore making the average case $O(n \log n)$. The heap sort algorithm will have to make a max heap and then sort it. To make a max heap is $O(n)$ using heapify. In the Heap Sort algorithm, after creating the max-heap, we repeatedly extract the maximum element from the heap and then re-heapify the remaining elements. Both operations take $O(\log n)$ time. Since we perform these operations n times (once for each element), the overall time complexity for the sorting step is $O(n \log n)$ in both the average and worst cases.