**TABLE 1** Reflections

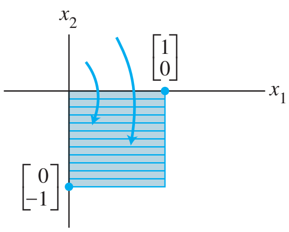
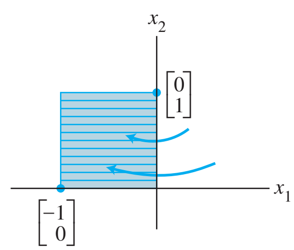
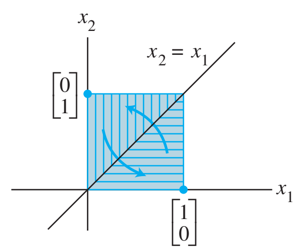
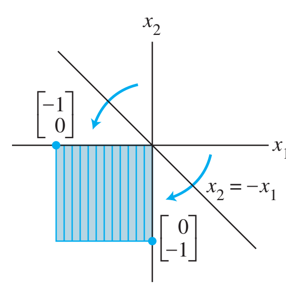
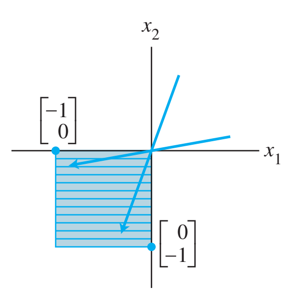
| Transformation | Image of the Unit Square | Standard Matrix |
|---|---|---|
| Reflection through the $x_1$-axis |   | $\begin{bmatrix}1 & 0\\0 & -1\end{bmatrix}$ |
| Reflection through the $x_2$-axis |   | $\begin{bmatrix}-1 & 0\\0 & 1\end{bmatrix}$ |
| Reflection through the line $x_2 = x_1$ |   | $\begin{bmatrix}0 & 1\\1 & 0\end{bmatrix}$ |
| Reflection through the line $x_2 = -x_1$ |   | $\begin{bmatrix}0 & -1\\-1 & 0\end{bmatrix}$ |
| Reflection through the origin |   | $\begin{bmatrix}-1 & 0\\0 & -1\end{bmatrix}$ |

**TABLE 2** Contractions and Expansions

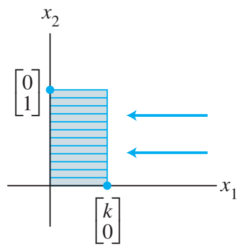
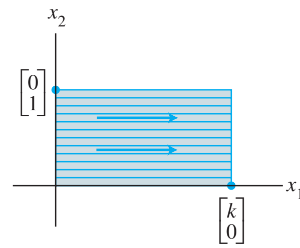
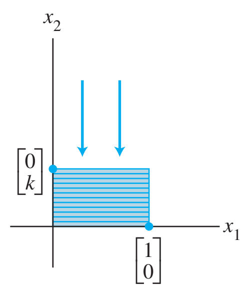
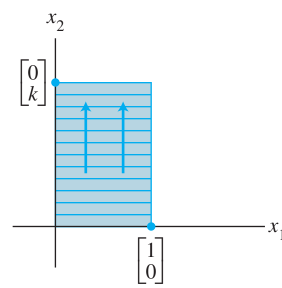
| Transformation | Image of the Unit Square | | Standard Matrix |
|---|---|---|---|
| Horizontal contraction and expansion | <br>$0 < k < 1$ | <br>$k > 1$ | $\begin{bmatrix}k & 0\\0 & 1\end{bmatrix}$ |
| Vertical contraction and expansion | <br>$0 < k < 1$ | <br>$k > 1$ | $\begin{bmatrix}1 & 0\\0 & k\end{bmatrix}$ |

**TABLE 3** Shears

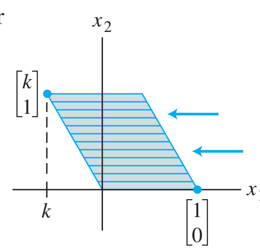
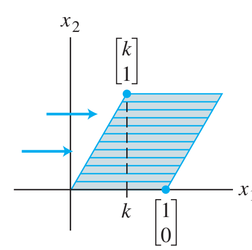
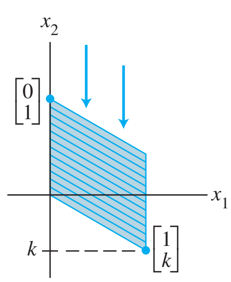
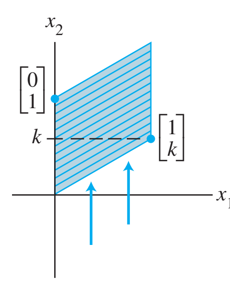
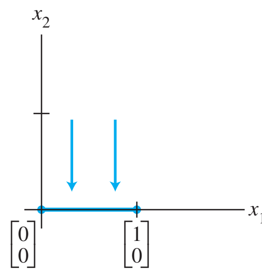
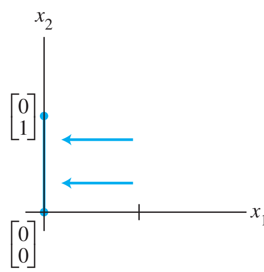
| Transformation | Image of the Unit Square | | Standard Matrix |
|---|---|---|---|
| Horizontal shear | <br>$k < 0$ | <br>$k > 0$ | $\begin{bmatrix}1 & k\\0 & 1\end{bmatrix}$ |
| Vertical shear | <br>$k < 0$ | <br>$k > 0$ | $\begin{bmatrix}1 & 0\\k & 1\end{bmatrix}$ |

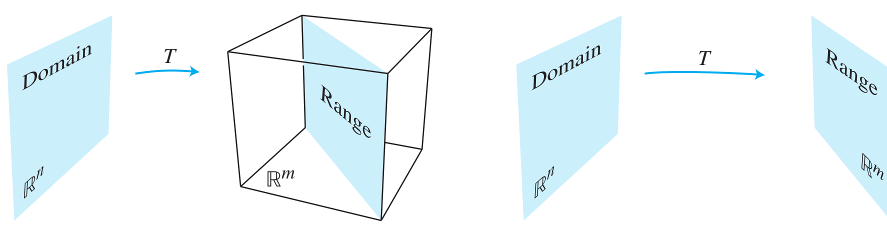
**TABLE 4** Projections

| Transformation | Image of the Unit Square | Standard Matrix |
|---|---|---|
| Projection onto the $x_1$-axis |   | $\begin{bmatrix}1 & 0\\0 & 0\end{bmatrix}$ |
| Projection onto the $x_2$-axis |   | $\begin{bmatrix}0 & 0\\0 & 1\end{bmatrix}$ |

**DEFINITION**

A mapping $T : \mathbb{R}^n \to \mathbb{R}^m$ is said to be **onto** $\mathbb{R}^m$ if each $\mathbf{b}$ in $\mathbb{R}^m$ is the image of *at least one* $\mathbf{x}$ in $\mathbb{R}^n$.

Equivalently, $T$ is onto $\mathbb{R}^m$ when the range of $T$ is all of the codomain $\mathbb{R}^m$. That is, $T$ maps $\mathbb{R}^n$ onto $\mathbb{R}^m$ if, for each $\mathbf{b}$ in the codomain $\mathbb{R}^m$, there exists at least one solution of $T(\mathbf{x}) = \mathbf{b}$. "Does $T$ map $\mathbb{R}^n$ onto $\mathbb{R}^m$?" is an existence question. The mapping $T$ is *not* onto when there is some $\mathbf{b}$ in $\mathbb{R}^m$ for which the equation $T(\mathbf{x}) = \mathbf{b}$ has no solution. See Fig. 3.




$T$ is *not* onto $\mathbb{R}^m$ $\qquad$ $T$ *is* onto $\mathbb{R}^m$

**FIGURE 3** Is the range of $T$ all of $\mathbb{R}^m$?

**DEFINITION**

A mapping $T : \mathbb{R}^n \to \mathbb{R}^m$ is said to be **one-to-one** if each $\mathbf{b}$ in $\mathbb{R}^m$ is the image of *at most one* $\mathbf{x}$ in $\mathbb{R}^n$.