University of Bahrain
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Experiment 4

Laplace transform

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Objective:

1. Introducing the Laplace transformation

Purpose:

The Laplace Transform converts the integral-differential equations of electromechanical system into algebra-based equations that can be easily manipulated. However, evaluating the inverse Laplace transform can be cumbersome. This lab will introduce some tools in MATLAB that can be used to find the inverse Laplace transform.

Procedure

1. Laplace Transform

Laplace transform of a function f(t) is defined as

$$L[f(t)] = F(s) = \int_{0-}^{\infty} f(t)e^{-st}dt$$

where

$$s = \sigma + j\omega$$

Laplace transform of a function f(t) can be obtained with Matlab's function laplace.

Syntax: L =laplace(f)

The usage is demonstrated in the following examples.

Example 1

Find the Laplace transform of

$$f(t) = 5e^{-2t}$$

Matlab performs Laplace transform symbolically. Thus, you need to first define the variable t as a "symbol".

>> syms t

Next, enter the function f(t):

>> f=5*exp(-2*t);

Finally, enter the following command:

>> L=laplace(f)

Matlab yields the following answer:

L =

5/(s+2)

You may want to carry out the transformation by hand (or using Laplace transform table) to verify this result.

Example 2

Find the Laplace transform of:

$$12\frac{d^2y}{dt^2}$$

In Matlab Command Window:

>> laplace(12*diff(sym('y(t)'),2))

Note that the function y(t) is defined as symbol with the imbedded command "sym". The number 2 means we wish to take the second derivative of the function y(t).

Matlab result:

ans =

12*s*(s*laplace(y(t),t,s)-y(0))-12*D(y)(0)

where y(0) is the initial condition.

Example 3

Find the inverse Laplace transform of

$$Y(s) = \frac{1}{s} - \frac{2}{(s+4)} + \frac{1}{(s+5)}$$

In Matlab Command window:

>> ilaplace(1/s-2/(s+4)+1/(s+5))

Matlab result:

ans =

1-2*exp(-4*t)+exp(-5*t)

or

$$y(t) = (1 - 2e^{-4t} + e^{-5t})u(t)$$

which is the solution of the differential equation

$$\frac{d^2y}{dt^2} + 12\frac{dy}{dt} + 32y = 32u(t)$$

As an exercise, you should carryout the Laplace transform of the above differential equation with initial condition y(0) = 0 to arrive to the expression of Y(s) as shown above.

Vector DEN contains the coefficients of the denominator in descending powers of s. The numerator coefficients are returned in matrix NUM with as many rows as there are outputs y. Thus the above answer from MATLAB can be written as:

$$H(s) = \frac{1.6667E3}{s^2 + 33.3s + 1.6667E3}$$

Rational Functions

The rational function $H(s)=B(s)/A(s)$ requires polynomials to describe the numerator and denominator. Therefore we need two polynomial coefficient row vectors for a parameterization. The denominator should be *normalized* in the sense that the leading coefficient should be one. After all, if both $B(s)$ and $A(s)$ are multiplied by the same constant, $H(s)$ will not change. Thus we can force the coefficient of the highest power in the denominator polynomial to be one. For example, the rational function

$$H(s) = \frac{s^2 + 1}{4s^3 + 4s^2 + 2s + 1}$$

has normalized representation b=[.25, 0, .25], and a=[1, 1, .5, .25]. Using the MATLAB 'help' facility, study the MATLAB functions 'residue', 'freqs', 'bode', 'nyquist', and 'rlocus'. We will use only the first two of these, but they all deal with rational $H(s)$ in some way.

Partial Fraction

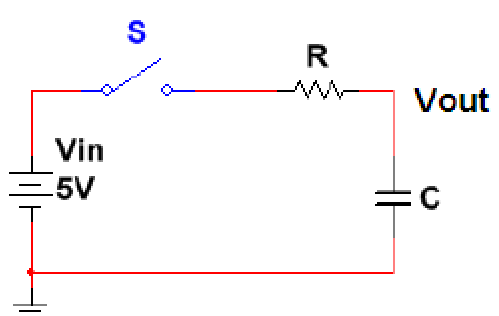
Example

$$H(s) = \frac{s^2+1}{(s+1)(s+2)(s+3)} = \frac{s^2+1}{s^3+6s^2+11s+6}.$$

To do a partial fraction expansion, type

```
»b=[1 0 1]; % B(s)
»a=[1 6 11 6]; % A(s)
»[gamma,alpha,k]=residue(b,a)
gamma =
5.0000
-5.0000
1.0000
alpha =
-3.0000
-2.0000
-1.0000
k = []
```

2- Consider the following RC circuit with a switch



The switch closes at $t = 0$. The output is the capacitor voltage.
Select your own values of R and C. Treat all components as "resistors" and solve for V_B in the Laplace domain.

The voltage divider gives

$$V_{out}(s) = \frac{\frac{1}{sC}}{R + \frac{1}{sC}} V_{in}(s) = \frac{1}{sRC + 1} \cdot \frac{V_{in}}{s} = \frac{V_{in}}{s^2 RC + s}$$

The time-domain solution requires us to find the inverse Laplace transform.

Method 1: Using the MATLAB built-in function residue

Let the denominator be $a = [RC \quad 1 \quad 0]$ and the numerator be $b = V_{in}$

Then the MATLAB function [r, p, K] = residue(b, a) finds the partial fraction expansion:

Note: Here we assume that we have only simple poles. When the order of the numerator b is lower than the order of the denominator a, we always have K = 0.

Method 2: Using the MATLAB's symbolic calculation and function ilaplace

As an exercise, run the following MATLAB script to learn about MATLAB's laplace and ilaplace :

```
syms t          %time variable t
f=2*exp(-t)-2*t*exp(-2*t)-2*exp(-2*t);  %define f(t)
pretty(f)       %looks better
F=laplace(f)   %Laplace transform
pretty(F)       %looks better
F=simplify(F)  %combine partial fractions
fnew=ilaplace(F)  %inverse Laplace transform
pretty(f)         %looks better
```

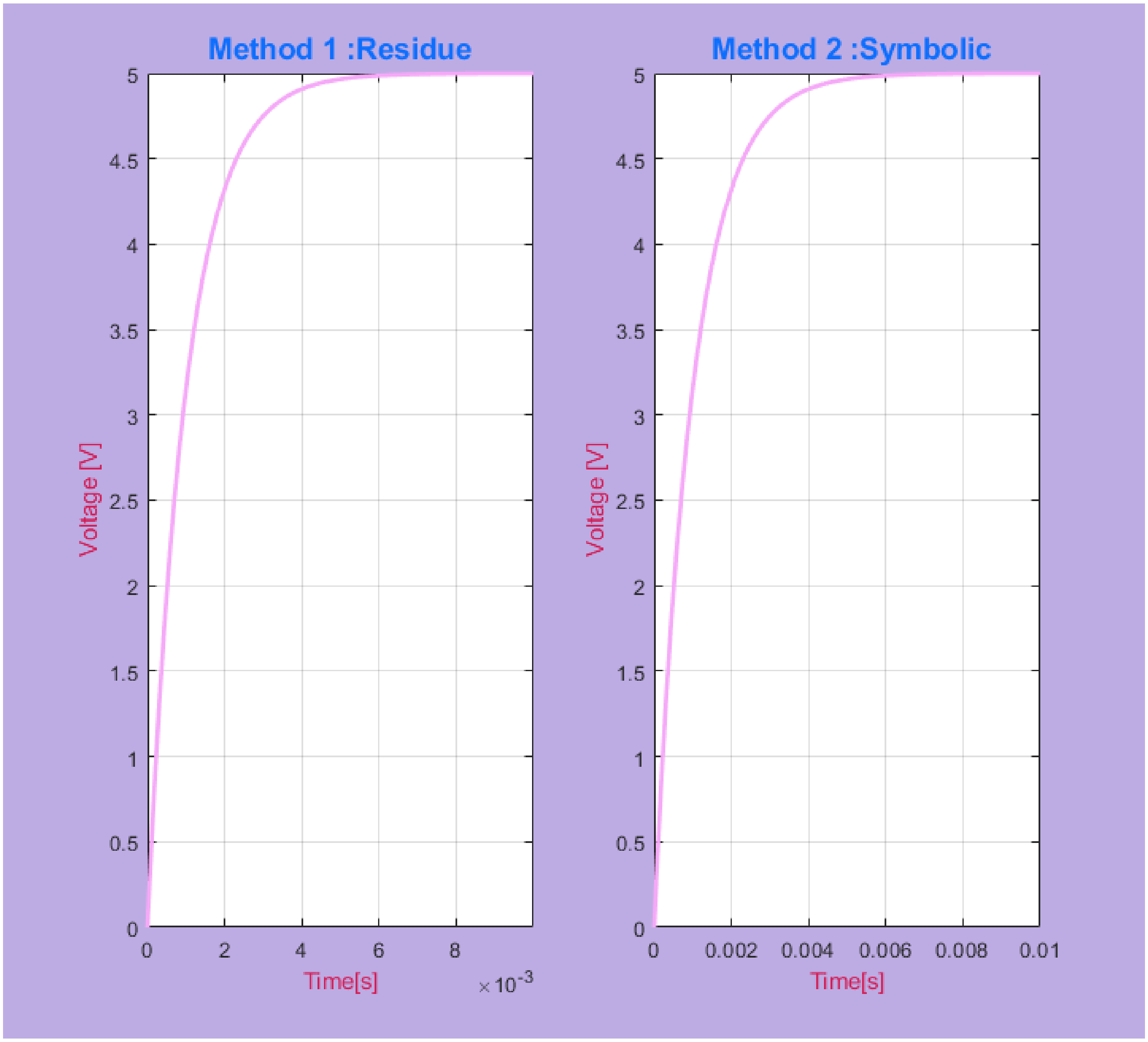
Now you are ready to do the lab using the second method.

i. Defining the symbolic variables to be used (i.e. s)
>> syms s
ii. Writing the Laplace domain function
>> F = b/(R*C*s^2 + s)
iii. Operating on the function
>> f = ilaplace(F)
Now plot $v_B(t)$ to ensure that it depicts a charging capacitor. See the Appendix.

Conclusions:

(1) Did these two methods give you the same mathematical expression for the inverse Laplace transform?
(2) Type (or write) these two time-domain expressions here.
(3) Run a *Multisim* simulation to verify the time-domain $v_B(t)$ is reasonably correct.

```
% Example 1 - Simple Example
% syms t

% f=5*exp(2*t);

% L=laplace(f)


% laplace(12*diff(sym('y'),2))

clc;%reset the workspace command line
clear all; %clear all the variables
close all; %close all the plots
%%====Please use your own value for R and C===================
R = 10000; %10kohm
C = 0.1*10^(-6); %0.1 uF
%%===========================================================
vin = 5; %input amplitude=5 V
a = [R*C 1 0]; %denominator
b = vin; %numerator
%%=========For Plotting==================
set(gca,'fontsize',18,'FontWeight','bold','FontName','Times New Roman');

f14 = figure("Name", 'Signals');
set(f14, 'color', '#BDACE4');
%%======================================
% Method 1: Residue
display('Method1: Residue');
[r, p, K] = residue (b, a)
t=0:0.0001:0.01;
VB=r(1)*exp(p(1)*t)+r(2)*exp(p(2)*t);
subplot(1,2,1)
plot(t, VB, 'color', '#F5A9F7', 'LineWidth', 2)
xlabel('Time[s]', 'color', '#D21D55');
ylabel('Voltage [V]', 'color', '#D21D55');

title('{\bf Method 1 :Residue }', 'color', ...
    '#0d6efd','FontSize', ...
     14,'FontName' ...
    ,'TimeNewRoman'); grid on


% Method 2: Symbolic
display('Method2: Symbolic');
syms s
F = b/(a(1)*s^2+a(2)*s)
f = ilaplace(F)
subplot(1,2,2)
fplot(f, [0, 0.01], 'color', '#F5A9F7', 'LineWidth', 2);%ezplot plots function f over the specified range
xlabel('Time[s]', 'color', '#D21D55');
ylabel('Voltage [V]', 'color', '#D21D55');

title('{\bf Method 2 :Symbolic}', 'color', ...
    '#0d6efd','FontSize', ...
     14,'FontName' ...
    ,'TimeNewRoman'); grid on
```


Method 1 :Residue
Method 2 :Symbolic
Voltage [V]
Time[s]
$\times 10^{-3}$
Voltage [V]
Time[s]

1- Use “laplace” to find the Laplace transforms

A)

$g(t) = 5te^{-5t}u_s(t)$

```
La =

5/(s + 5)
```

```
syms t
f=5*exp(-5*t);
La=laplace(f)
```

$g(t) = (t\sin(2t) + e^{-2t})u_s(t)$

```
L2 =

1/(s + 2) + (4*s)/(s^2 + 4)^2
```

```
f2 = (t*sin(2*t) +exp(-2*t))

L2=laplace(f2)
```

$g(t) = (\sin 2t * \cos 2t\,)u_s(t$

```
f3 = sin(2*t)*cos(2*t) ;
L3=laplace(f3)
```

```
l3 =

2/(s^2 + 16)
```

Find the inverse Laplace transform of

A) $G(s) = \frac{1}{s*(s+2)*(s+3)}$

```
f =(1/s*(s+2)*(s+3));
La=ilaplace(f,s);
```

```
La =

5*dirac(t) + dirac(1, t) + 6
```

B) $G(s) = \frac{10}{(s+1)^2*(s+3)}$

```
fb= (10/(s+1)^2*(s+3));
Lb=ilaplace(fb)
```

```
Lb =

10*exp(-t) + 20*t*exp(-t)
```

C) $G(s) = \frac{2*(s+1)}{2*(s^2+s+2)}$

```
fc = (2*(s+1)/2*(s^2+s+2));
Lc=ilaplace(fc)
```

```
Lc =

2*dirac(t) + 3*dirac(1, t) + 2*dirac(2, t) + dirac(3, t)
```

D) $Y(s)=\frac{1}{s} - \frac{2}{s+4} + \frac{1}{s+5}$

```
fd=(1/s +2/s+4 +1/s+5);
Ld=ilaplace(fd)
```

```
Ld =

9*dirac(t) + 4
```

Conclusion:

In this lab I learned a lot of things which are

- How to use Laplace Transformation in MATLAB
- How to use inverse Laplace Transformation in MATLAB