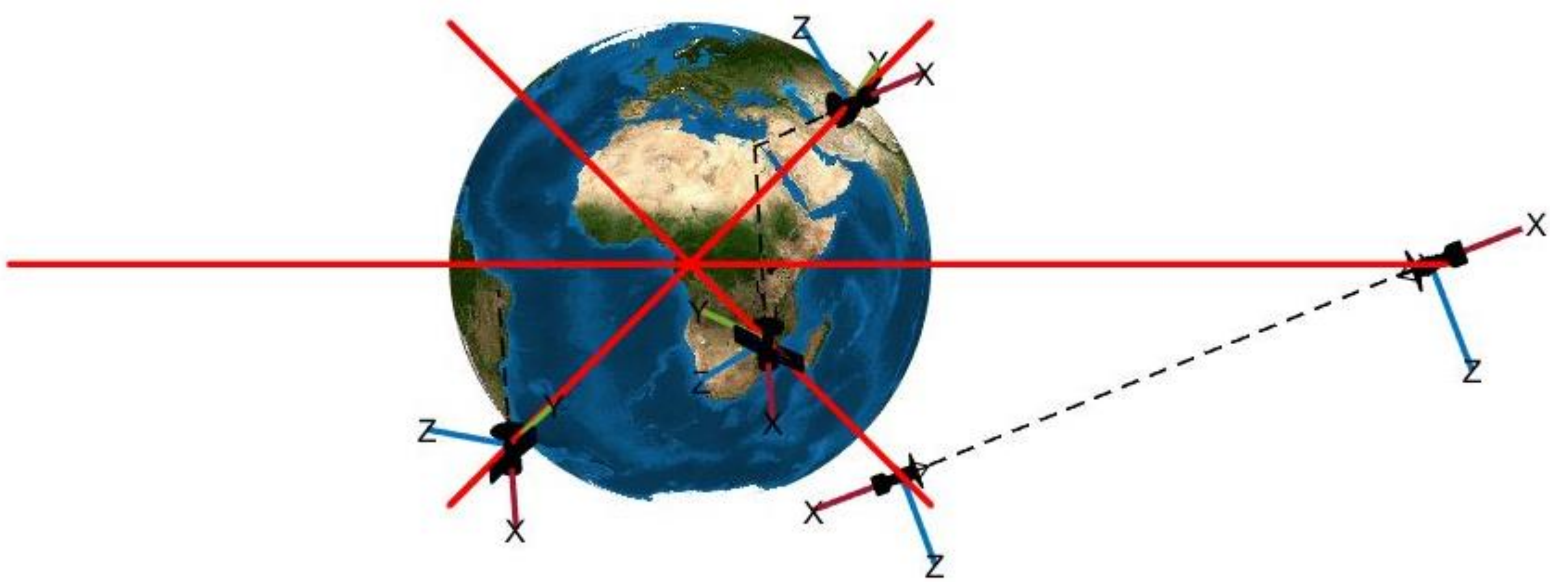
Satelllite Pointing using Nonlinear Control



Ali Ahmed Almorshdy

Without any kind of control, a satellite's attitude can behave in a way that is quite unpredictable and probably risky. Many external factors that satellites encounter while in orbit, such as atmospheric drag, magnetic fields, and gravitational perturbations from the Moon and Sun, can cause them to spin or tumble. Depending on how the satellite's mass is distributed, in this document I will show you how we can apply a nonlinear control input to force the spacecraft to point to a specific point like (center of the earth – ground station on earth – another spacecraft).

Steps:

We can simplify main steps for all cases to be:

1. **Define epoch in term of Juliandate, you can find it at [2].**
2. **Define your inertial frame (ECI for our case).**
3. **Define the Spacecraft body frame (like $\hat{X}$ axis to antenna $\hat{Y}$ to Solar Panel $\hat{Z}$ Right-handed).**
4. **at epoch find:**
    1. **the spacecraft position vector ($\hat{R}$) in inertial frame.**
    2. **the position vector of the point in inertial frame to which you want the spacecraft to point ($\hat{P}$).**
    3. **Find the pointing vector $\hat{r} = \hat{P} - \hat{R}$.**
    4. **Define another Vector [$\hat{H}$] to determine the orthonormal Matrix, it could be vector of inertial frame like $[1\ 0\ 0]$, Velocity Vector,**
5. **find the Matrix ([RM]) which is sets of orthonormal right-hand sets of unit vectors it can be**

   Like:

   $\widehat{\imath 1} = \frac{\hat{r}}{norm(\hat{r})}$, $\widehat{\imath 2} = \frac{cross(\hat{r},\hat{H})}{norm(cross(\hat{r},\hat{H}))}$, $\widehat{\imath 3}$ **Right-handed.**

6. **find The Rational Matrix (C) from Body Frame to [RM] like -$\hat{X}$ pint to $\hat{r}$ or $\hat{Y}$ point to $\hat{r}$**

7. **Find [RefN] = [C]*[RM].**

8. **at [1] $\frac{N_d}{dt}[\text{RefN}] = -skew(w_{refn})[\text{RefN}]$, where w_{refn} is the angular velocity of our frame.**

   And given that $\frac{N_d}{dt}[\text{RefN}] = \frac{\text{RefN(t+dt)}-\text{RefN(t)}}{dt}$ you can find the w_{refn}.

9. **RefN and w_{refn} is set to be seen by the inertial frame, now we need to transfer it to body**

   Frame given the [BN] matrix and w_{BN} like [1] it:

   [BR] =[BN]*[RefN]' , $B_{W_{refn}}$**=[BN]*** w_{ref} , $w_{Bref} = w_{BN} - B_{W_{refn}}$

10. **convert [BR] attitude to Modified Rodrigues Parameters σ_{BR},[1].**

11. **like [1] the simple Nonlinear Control input could be:**

    $u = -K\,\sigma_{BR} - P\,w_{Bref} - L$ **, Where L is the external torque**

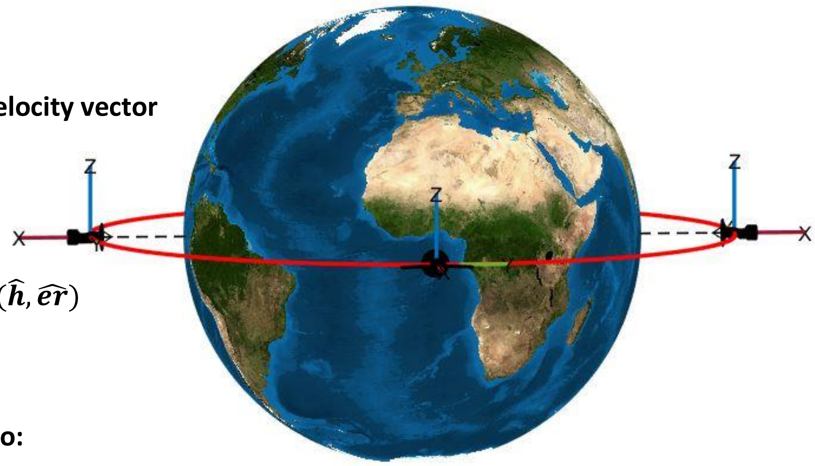
Example (1) antenna point to center of the earth:

- **After finding the position vector $\widehat{R}$, we don't have to calculate**

$\widehat{P}$ *as it equal to zero here* **so $\hat{r} = \widehat{R}$.**

- **Another vector here may be $\hat{v}$ which is the velocity vector**

In inertial frame.



- **The matrix [RM],**

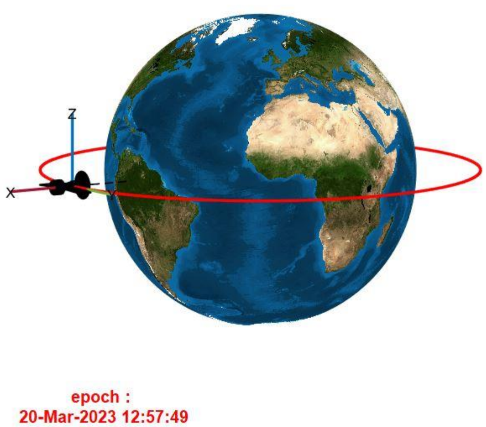
$$\widehat{er} = \frac{\hat{r}}{norm(\hat{r})} \ , \widehat{h} = \frac{cross(\hat{r},\hat{v})}{norm(cross(\hat{r},\hat{v}))} \ , \widehat{th} = cross(\widehat{h}, \widehat{er})$$

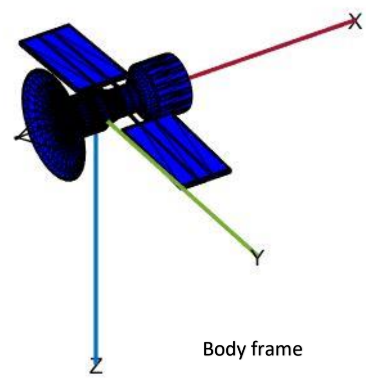
As Right-handed [RM]= $[\widehat{er}, \widehat{th}, \widehat{h}]'$.

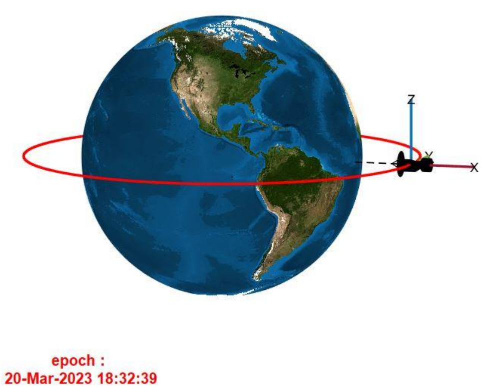
- **Our body frame axis is practically coaligned so:**

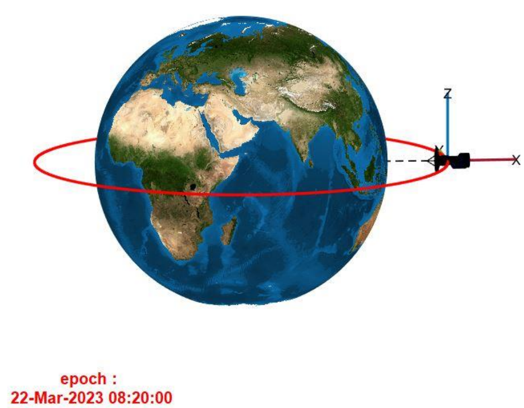
$C = [\mathbf{1\ 0\ 0; 0\ 1\ 0; 0\ 0\ 0}]$.

- $[\mathbf{RefN}] = [\mathbf{C}] * [\mathbf{RM}]$.
- **Now from 8 you can find w_{refn} , but note that our chosen frame [RM] is the Orbit Frame, where $w_O = [\ 0\ 0\ \ sqrt(\frac{mu}{r^3})\ ]'$. from here w_{refn} could be [RM]* w_O.**
- **Now you can easily apply steps 9→11**









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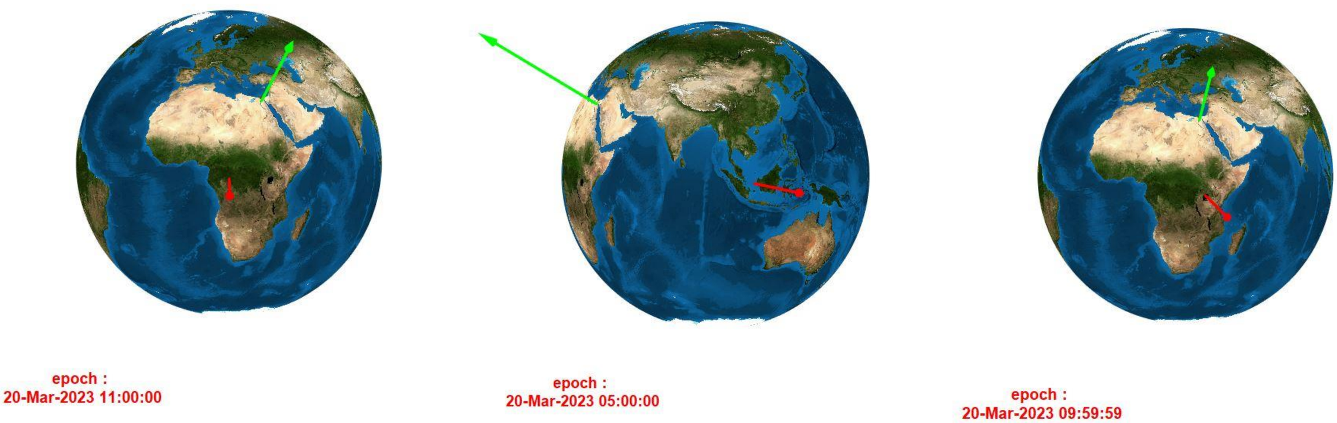
Example (2) antenna point to ground station on the earth.

Let's take Beni Suef university (Egypt) at $29°$ N and $31.2°\ E$ as an example

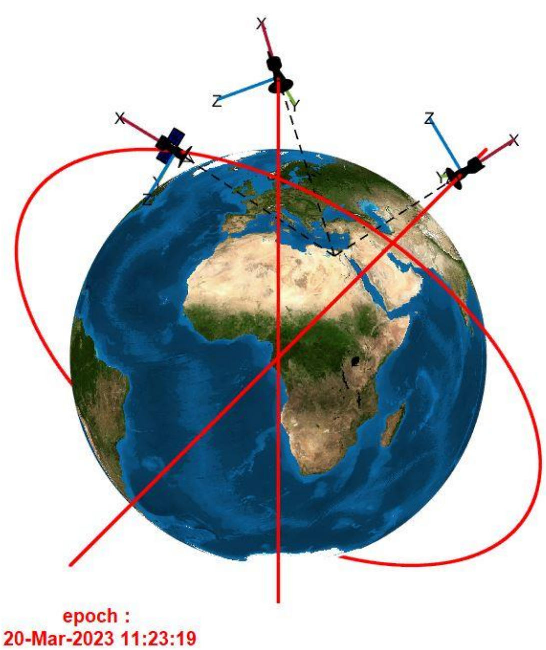
- **To find $\widehat{P}$ at epoch, we have to calculate (Local Sidereal time) in term of degree, you can find it at [2], from which we can calculate the angle between X vector in inertial frame and $\widehat{P}$ given epoch as juliandate format as [2].**
  So, we have to update the longitude in inertial frame to be equal to LST as the location is moving in ECI frame, and Transfer the latitude and new the longitude to Cartesian coordinate like:
  $\widehat{\boldsymbol{P}} = \boldsymbol{6378} * [(\boldsymbol{cosd}(\boldsymbol{long}) * \boldsymbol{cosd}(\boldsymbol{lat}))', (\boldsymbol{sind}(\boldsymbol{long}) * \boldsymbol{cosd}(\boldsymbol{lat}))', \boldsymbol{sind}(\boldsymbol{lat})'];$



Red line here is X vector of ECI frame and the green one refers to location fixed to earth moving

Now we can calculate $\hat{\boldsymbol{r}} = \widehat{\boldsymbol{P}} - \widehat{\boldsymbol{R}}$, and go like we do in previous Example.

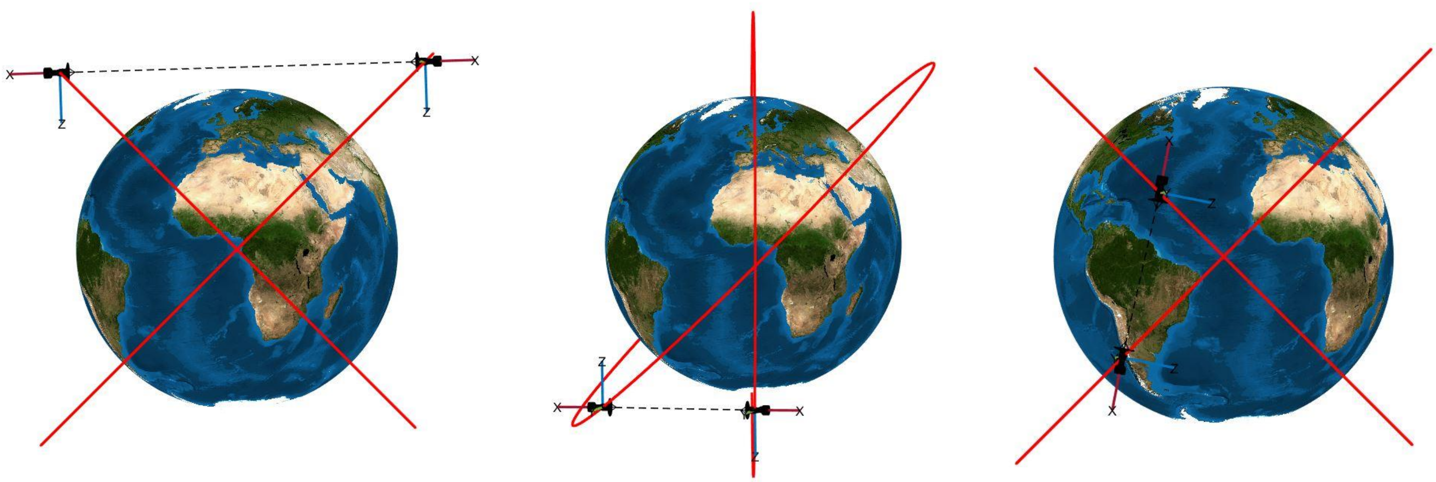
You can also apply this algorithm to constellation of satellites:



Example (3) Two Satellites antenna point to each other.

- **Find the position vector of both satellites.**
- $\hat{\boldsymbol{r}} = \widehat{\boldsymbol{R2}} - \widehat{\boldsymbol{R1}}$
- **Another vector here may be $n3\ like\ [\mathbf{0}, \mathbf{0}, \mathbf{1}]$.**
- **As the pointing Vector is opposite direction with X body frame so $\boldsymbol{C} = [-\mathbf{1}, \mathbf{0}, \mathbf{0}; \mathbf{0}, \mathbf{1}, \mathbf{0}; \mathbf{0}, \mathbf{0}, -\mathbf{1}]$**

Now you can use the same algorithm to each satellite.



REFERENCES

[1] Schaub, H., & Junkins, J. L. (2018). Analytical Mechanics of Space Systems. Reston, VA: AIAA.

[2] Curtis, H. D. (2020). Orbital Mechanics for Engineering Students. Oxford: Butterworth-Heinemann.