Probability and Statistics
MATH 121
Assignment – 10 marks

KING FAISAL UNIVERSITY
COLLEGE OF SCIENCE
DEPARTMENT OF MATHEMATICS & STATISTICS

Deadline for submitting the assignment is: 13-12- 2023.

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Answer the following questions:

Q.1. Marks [5]

Consider the following joint probability mass function.

x	y	$f_{XY}(x, y)$
−1.0	−2	1/8
−0.5	−1	1/4
0.5	1	1/2
1.0	2	1/8

Determine the following
  (a) Marginal probability distribution of X.
  (b) Marginal probability distribution of Y.
  (c) The correlation coefficient between X and Y.

x	$f(x)$	$x \cdot f(x)$	$x^2 \cdot f(x)$
-1	1/8	-1/8	1/8
-0.5	1/4	-1/8	1/16
0.5	1/2	1/4	1/8
1	1/8	1/8	1/8

$x = \frac{1}{8}$ $E(x^2) = \frac{7}{16}$

(a)

$$\sigma_x^2 = E(x^2) - \mu^2 = \frac{7}{16} - \left(\frac{1}{8}\right)^2 = \sigma^2 = \frac{27}{64} = 0.4218$$

$$\sigma_x = 0.6495$$

y	$f(y)$	$y \cdot f(y)$	$y^2 \cdot f(y)$
-2	1/8	-1/4	1/2
-1	1/4	-1/4	1/4
1	1/2	1/2	1/2
2	1/8	1/4	1/2

$\mu_2 = \frac{1}{4}$ $E(y^2) = \frac{7}{4}$

(b)

$$\sigma_y^2 = E(y^2) - \mu_y^2 = \frac{7}{4} - \left(\frac{1}{4}\right)^2 = \frac{27}{16}$$

$$\sigma_y = 1.299$$

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$$\rho = \frac{\sigma_{xy}}{\sigma_x \cdot \sigma_y}$$

$$\sigma_{xy} = E(x,y) - E(x) \cdot E(y)$$

$$E(x,y) = -1 \cdot (2) \cdot \frac{1}{8} + (-0.5) \cdot (-1) \cdot \frac{1}{4} + 0.5 \cdot 1 \cdot \frac{1}{2} + 1 \cdot 2 \cdot \frac{1}{8} = 0.875$$

$$\sigma_{xy} = 0.875 - \frac{1}{8} \cdot \frac{1}{4} = 0.8437$$

$$\rho = \frac{\sigma_{xy}}{\sigma_x \cdot \sigma_y} = \frac{0.8737}{0.6495 \cdot 1.299} = 1$$

$$P = 1 \quad \text{So It is perfect and discrete}$$

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Q.2. Marks [5]

Summary of quantities are as follows
$n = 20$

$\sum y_i = 12.75,$
$\sum x_i = 1478, \quad \sum x_i^2 = 143215.8, \quad and \quad \sum x_i y_i = 1083.67$

Assume that the two variables are related according to the simple linear regression model.
- (a) Calculate the least squares estimates of the slope and intercept.
- (b) Use the equation of the fitted line to predict y when x= 85
- (c) Suppose that the observed value of y at x =90 is y=70. Calculate the value of the corresponding residual.

(a) $B_2 = \dfrac{\sum xy - (\sum x \cdot \sum y)/n}{\sum x^2 - (\sum x)^2/n} = \dfrac{1083.67(1478 \cdot 12.75)/20}{143215.8 - (1478)^2/20} = 0.00416$

$B_0 = \bar{y} - B_1 \bar{x} = \dfrac{\sum y}{n} - B_1 \cdot \dfrac{\sum x}{n} = \dfrac{12.75}{20} - 0.00716 \cdot \dfrac{1478}{20} = 0.330076$

$\hat{y} = B_0 + B_2 x = 0.330076 + 0.00716 x$

(b) $\hat{y} = B_0 + B_1 x = 0.330076 + 0.00416 \cdot 85 = 0.683676$

(c) $\hat{y} = 0.330076 + 0.00416 \cdot 90 = 0.704476$

$e = y - \hat{y} = 70 - 0.704476 = 69.295524$