DREXEL UNIVERSITY
Department of Mechanical Engineering & Mechanics
Applied Engineering Analytical & Numerical Methods I

MEM 592 - Winter 2022-2023

HOMEWORK #3: Due Monday, 03/06/2022 at 1:59PM

1. [25 points] (MATLAB or Python Programming)

Consider the motion of a pendulum mass m, length L in a constant gravitational field of strength g. The equation of motion for the angle the pendulum makes with the vertical, θ, is:

$$\ddot{\theta} = -\frac{g}{L}\sin(\theta)$$

For small angles, the motion of the pendulum is given by

$$\ddot{\theta} = -\frac{g}{L}\theta$$

(a) Assuming the acceleration due to gravity is $g = 9.81\ m/s^2$, the length of the pendulum is $L = 0.6m$, and the pendulum starts from rest at an angle of $\theta = 10°$, solve the linearized equations for $0 \leq t \leq 6$, with the Explicit Euler and 2nd-Order Runge-Kutta methods as start-up schemes, and h=0.1, 0.2, and 0.5 as time steps, using the: (i) Leapfrog Method, and; (ii) Second-Order Adams Bashforth Method. Plot the exact result along with the numerical solutions. This should result in 4 plots, with each showing the exact and numerical results for all step sizes for a given combination of a single main method and start-up scheme. (b) The linearized damped equation of motion is:

$$\ddot{\theta} + c\dot{\theta} + \frac{g}{L}\theta = 0$$

With $c = 4s^{-1}$, repeat part A using the linearized damped equation of motion. (c) Discuss how the stability of your linearized damped computations compare with the stability of those obtained in part A. (d) Do your results change significantly using different start-up schemes?

2. [25 points] (Hand Calculation)

Find the exact solution of the boundary value problem shown below:

$$y''x = 9yx - 3x^2 \quad for \quad 0 < x < 1 \quad with \quad y_0 = 0 \quad and \quad y_1 = 1$$

3. [25 points] (MATLAB or Python Programming)

A conical rod fabricated from stainless steel is immersed in air at a temperature T_a=0. The cross-sectional area of the rod is circular but varies with x. The large end is located at x = 0 and is held at a temperature T_A = 5. The small end is located at x = L = 2 and is held at temperature T_B = 4.

Conservation of energy can be used to develop a heat balance in the rod and at any point in the body. Assuming that the body is not insulated and is at steady state, its temperature satisfies the ordinary differential equation:

$$\frac{d^2T}{dx^2} + a(x)\frac{dT}{dx} + b(x)T = f(x)$$

where a(x), b(x) and c(x) are functions of the cross-sectional area, thermal properties of the material and heat sources and sinks in the body. Let's assume that:

$$a(x) = -\frac{x+3}{x+1}$$

$$b(x) = \frac{x+3}{(x+1)^2}$$

$$f(x) = 2(x+1) + 3b(x)$$

(a) Re-write the 2nd order ODE into a system of first order ODE's, including boundary conditions. (b) Use the shooting method to solve the boundary value problem. Use the 4th order Runge-Kutta method to integrate with a step size of h = 0.2. (c) Now considering using the direct method with second order accurate formulas for the derivatives. Set up the linear algebra problem (you are expected to know how to do this by hand, so this is an excellent time to practice) and find the solution using MATLAB or Python. Use the same step size as part b. (d) Plot both solutions on one graph with a straight line for the shooting method and circles for the direct method.

4. [25 points] (MATLAB or Python Programming)

Consider again the boundary value problem you solved exactly (problem 1):

$$y''x = 9yx - 3x^2 \quad for \quad 0 < x < 1 \quad with \quad y_0 = 0 \quad and \quad y_1 = 1$$

(a) Write the formula of the Second-Order Central Difference approximation for y(x). (b) Plot your numerical results along with the exact solution you found in problem 1 (using h=0.1).

NOTE: All problems that require the use of numerical methods and programming in MATLAB or Python should be solved using scripted elements that you have created yourself. Using shortcut methods for any numerical method may result in a zero or a deduction of points. If you are not sure whether or not you are allowed to use a certain MATLAB or Python function, contact the TA.