HACETTEPE UNIVERSITY
ENGINEERING FACULTY
ELECTRICAL AND ELECTRONICS ENGINEERING PROGRAM

2023-2024
SPRING SEMESTER

ELE708
NUMERICAL METHODS IN ELECTRICAL ENGINEERING

HW9

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1) Exercises

1.a) 9.2

9.2-) Write each of the following ODEs an equivalent first-order system of ODEs.

a-) Van der Pol equation:

$$y'' = y' \cdot (1-y^2) - y$$

a-) 1. $y_1' = y_2$

2. $y_2' = y_2 \cdot (1-y_1^2) - y_1$

$$\rightarrow \quad y' = \begin{bmatrix} y_1' \\ y_2' \end{bmatrix} = \begin{bmatrix} y_2 \\ y_2 \cdot (1-y_1^2) - y_1 \end{bmatrix}$$

b-) Blasius equation

$$y''' = -y \cdot y''$$

b-) if: $y = y_1$

1. $y_1' = y_2$

2. $y_2' = y_3$

3. $y_3' = -y_1 \cdot y_3$

$$\Rightarrow y' = \begin{bmatrix} y_1' \\ y_2' \\ y_3' \end{bmatrix} = \begin{bmatrix} y_2 \\ y_3 \\ -y_1 \cdot y_3 \end{bmatrix}$$

c-) Newton's Second Law of Motion for two-body problem

$$y_1'' = -G.M. y_1 / (y_1^2 + y_2^2)^{3/2}$$

$$y_2'' = -G.M. y_2 / (y_1^2 + y_2^2)^{3/2}$$

c-) if $y_1 = y$

$y_1 = y$
$y_2 = y'$
$y_3 = y''$

1. $y_1' = y_2$

2. $y_2' = (y_1'') = -G.M. y_1 / (y_1^2 + y_3^2)^{3/2}$

3. $y_3' = y_4$

4. $y_4' = (-G.M. y_3 . / (y_1^2 + y_3^2)^{3/2}$

1.b) 9.5

9.5-) With an initial value of $y_0 = 1$ at $t_0 = 0$ and a time step of $h = 1$, compute the approximate solution value y_1 at time $t_1 = 1$ for the ODE $y' = -y$ using each of the following methods.

a-) Euler's method

a-) In Euler's method;

$$y_{k+1} = y_k + h_k \cdot f(t_k, y_k)$$

$$y_1 = y_0 + h_0 \cdot \underbrace{f(t_0, y_0)}_{\to -y}$$

$$= 1 + 1 \cdot (-1)$$

$$= 0$$

b-) Backward Euler's Method

b-) In BE;

$$y_{k+1} = y_k + h_k \cdot f(t_{k+1}, y_{k+1})$$

$$y_1 = y_0 + h_0 \cdot f(t_1, y_1)$$

$$y_1 = 1 + 1 \cdot (-y_1)$$

$$2y_1 = 1$$

$$y_1 = 0{,}5$$

2) Computer Problems

2.a) 9.1

a) **Use a library routine to solve the LotkaVolterra model of predator-prey population dynamics given in Example 9.4, integrating from t = 0 to t = 25. Use the parameter values α1 = 1, β1 = 0.1, α2 = 0.5, β2 = 0.02, and initial populations y1(0) = 100 and y2(0) = 10. Plot each of the two populations as a function of time, and on a separate graph plot the trajectory of the point (y1(t), y2(t)) in the plane as a function of time. The latter is sometimes called a “phase portrait.” Give a physical interpretation of the behavior you observe. Try other initial populations and observe the results using the same type of graphs. Can you find nonzero initial populations such that either of the populations eventually becomes extinct? Can you find nonzero initial populations that never change? (Hint: You can find such a stationary point without solving the differential equation.)**

b) **Repeat part a, but this time use the Leslie-Gower model**

$$
\begin{aligned}
y_1' &= y_1(\alpha_1 - \beta_1 y_2), \\
y_2' &= y_2(\alpha_2 - \beta_2 y_2 / y_1).
\end{aligned}
$$

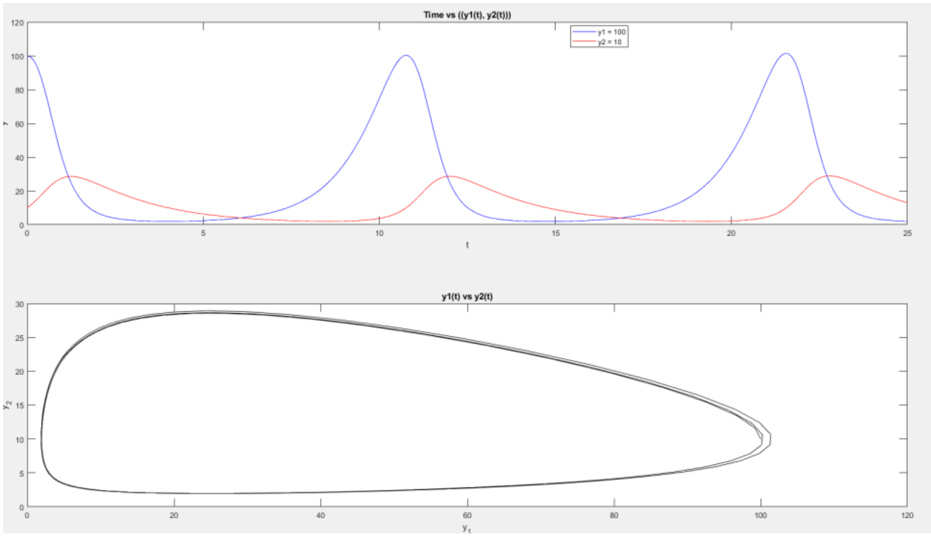
Use the same parameter values except take β2 = 10. How does the behavior of the solutions differ between the two models?

Give a physical interpretation of the behavior you observe. Try other initial populations and observe the results using the same type of graphs.

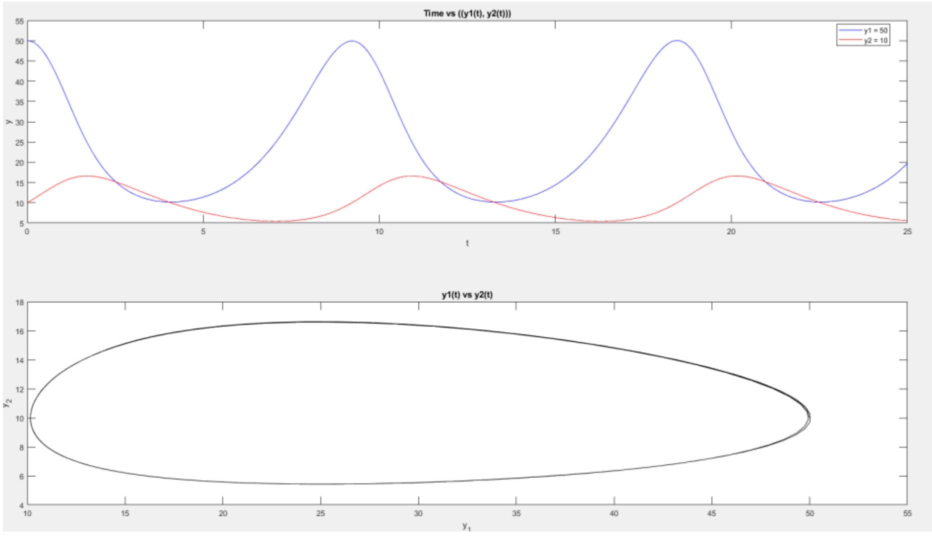
When the equations are examined, we can see that y1 is decreases as y2 increased and y2 increases when y1 is increased. So y1 is food and y2 is the hunter. In other word, Increasing livestock also increases the number of predator animal but increased predator number decreases the livestock number.

a) Using the LotkaVolterra model

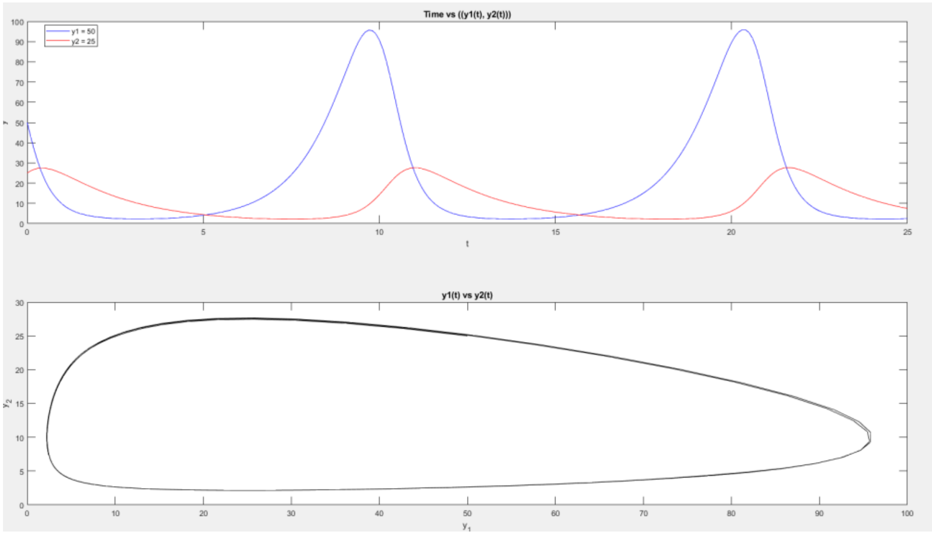
Y1 = 100, Y2 = 10



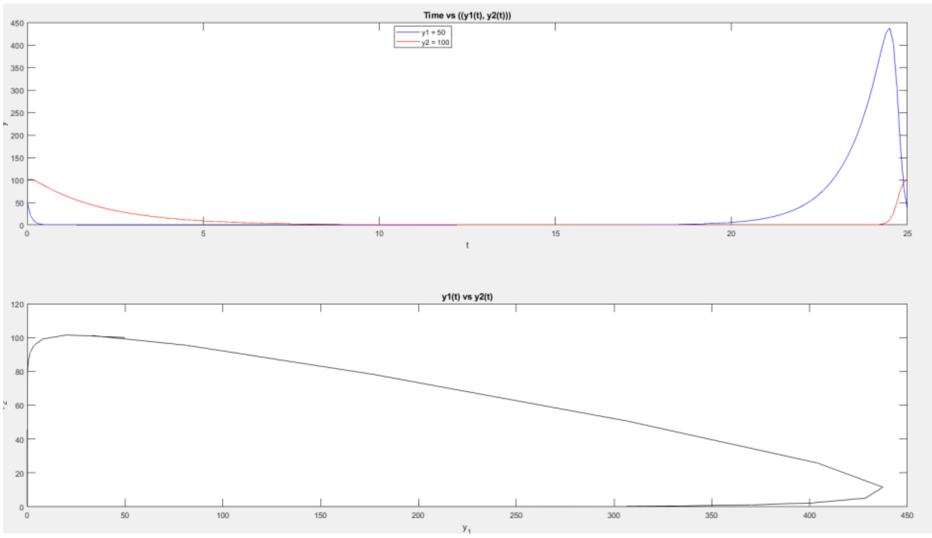
Y1 = 50, Y2 = 10



Y1 = 50, Y2 = 25



Y1 = 50, Y2 = 100



Can you find nonzero initial populations such that either of the populations eventually becomes extinct?

No, the equations are connected to each other. One can not be zero, so that it will always osilacatte between a range of numbers. As we can see from the phase diagrams, the ratio between y1 and y2 draw tight to together.

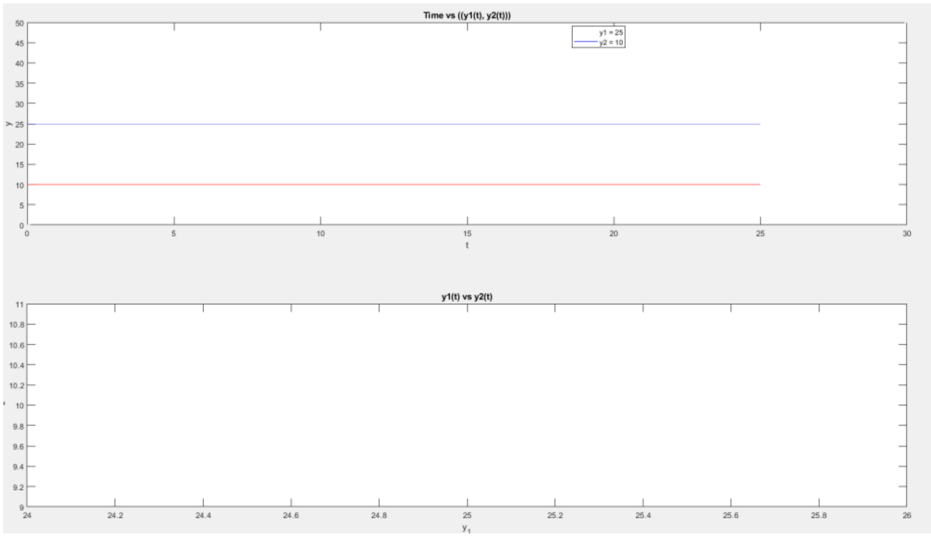
Can you find nonzero initial populations that never change?

Yes, making the derivative equal to zero means that slop will never change. So if we equal the equations to zero.

$$y_1 = \frac{\alpha_2}{\beta_2} = \frac{0.5}{0.02} = 25$$

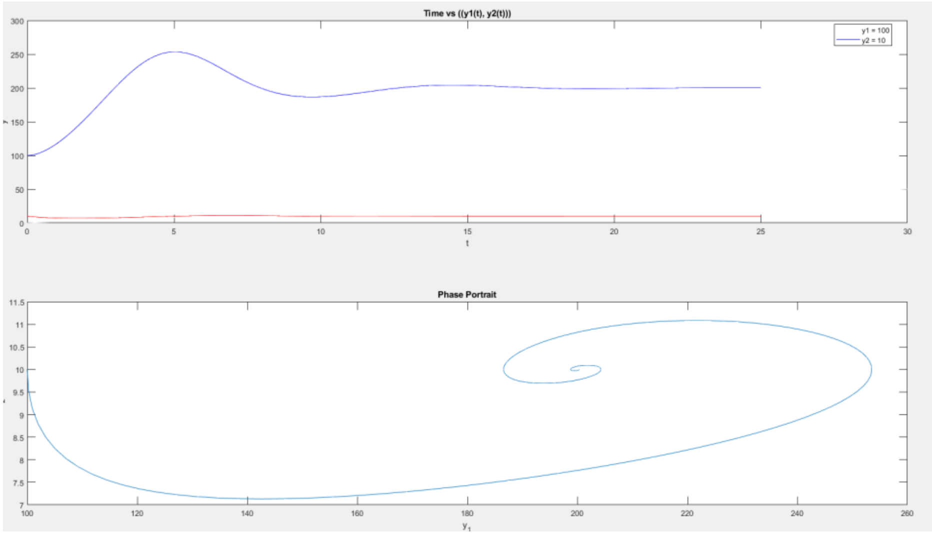
$$y_2 = \frac{\alpha_1}{\beta_1} = \frac{1}{0.1} = 10$$

Y1 = 25, Y2 = 10

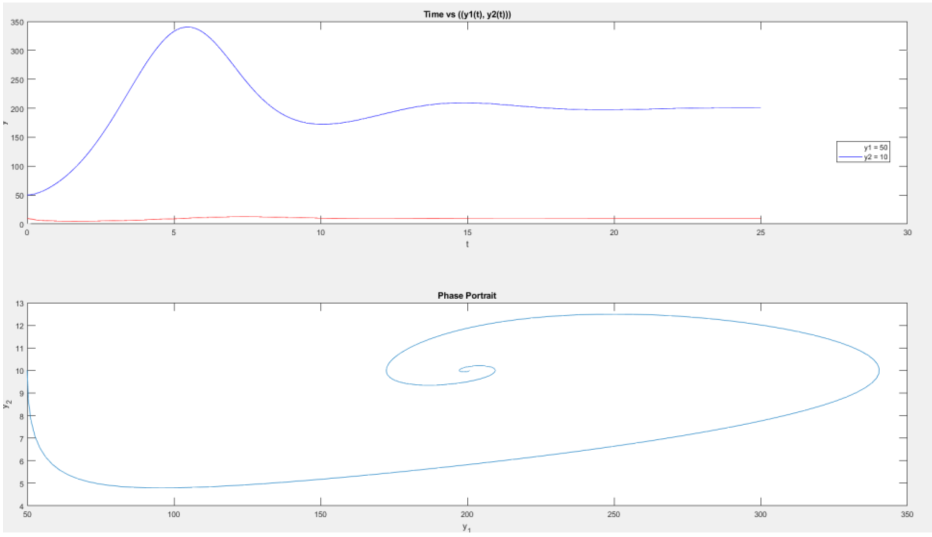


b) Using the Leslie Gower model

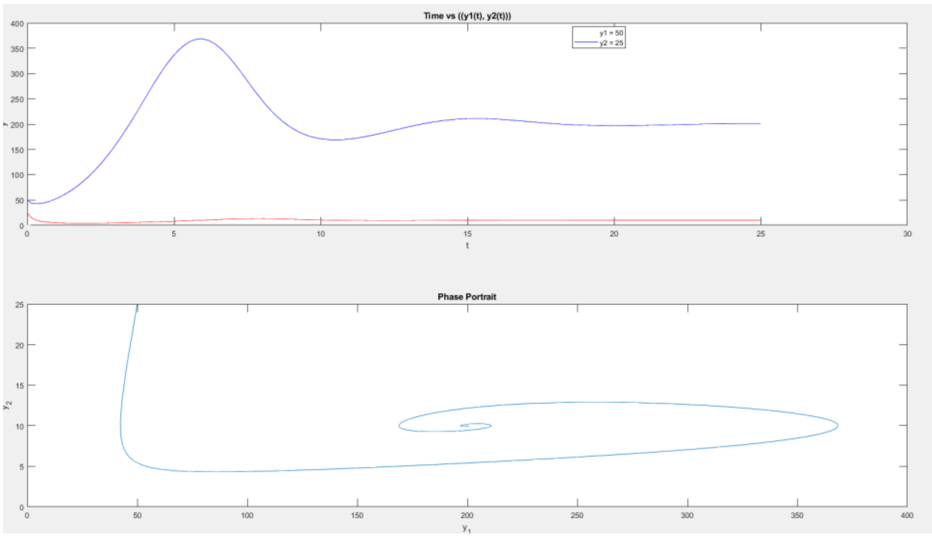
Y1 = 100, Y2 = 10



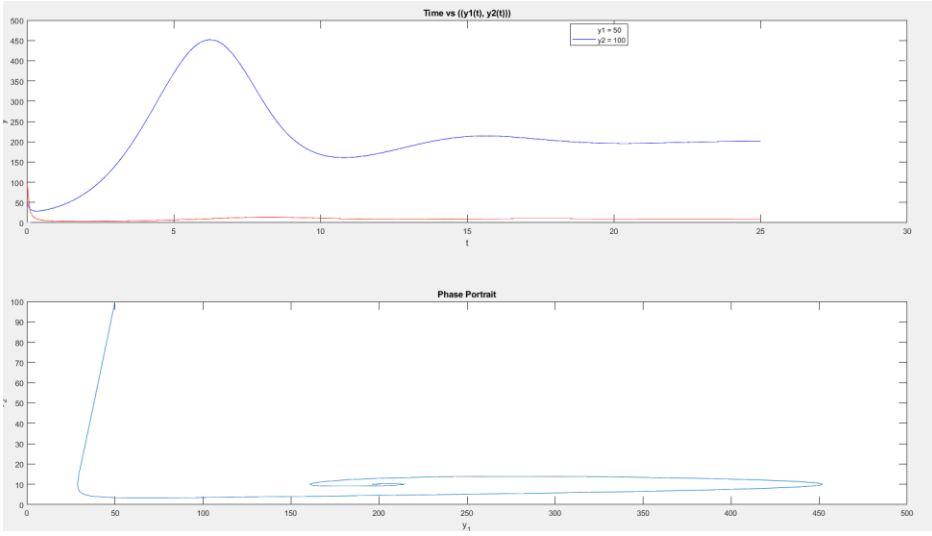
Y1 = 50, Y2 = 10



Y1 = 50, Y2 = 25



Y1 = 50, Y2 = 100



Can you find nonzero initial populations such that either of the populations eventually becomes extinct?

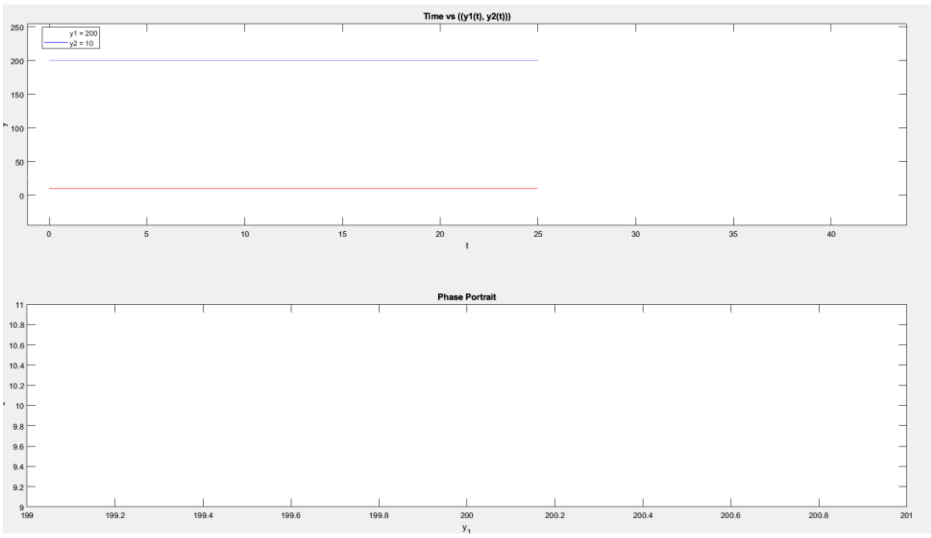
No, the equations are connected to each other. One can not be zero, so that it will always osilacatte between a range of numbers. As we can see from the image above, the values of y1 and y2 go to stabilize position in time.

Can you find nonzero initial populations that never change?

Yes, making the derivative equal to zero means that slop will never change. So if we equal the equations to zero.

$$y_1 = \frac{\alpha_1.\beta_2}{\beta_1.\alpha_2} = \frac{10.1}{0.1.0.5} = 200$$

$$y_2 = \frac{\alpha_1}{\beta_1} = \frac{1}{0.1} = 10$$



How does the behavior of the solutions differ between the two models?

The difference in the two models is that the Lotka-Volterra model draws a loop in its phase portrait. It means it cycles between the numbers or in other word it finds a periodic solution.

The Leslie-Gower model in the other hand, always converge to stabilize solution when its phase portrait examined. All the starting points tend to go that stable point.

2.b) 9.5

The following system of ODEs, formulated by Lorenz, represents a crude model of atmospheric circulation:

$$\begin{aligned} y_1' &= \sigma(y_2 - y_1), \\ y_2' &= ry_1 - y_2 - y_1y_3, \\ y_3' &= y_1y_2 - by_3. \end{aligned}$$

Taking σ = 10, b = 8/3, r = 28, and initial values y1(0) = y3(0) = 0 and y2(0) = 1, integrate this ODE from t = 0 to t = 100. Plot each of y1, y2, and y3 as a function of t, and also plot each of the trajectories (y1(t), y2(t)), (y1(t), y3(t)), and (y2(t), y3(t)) as a function of t, each on a separate plot. Try perturbing the initial values by a tiny amount and see how much difference this makes in the final value of y(100).

Initial values : 0 1 0

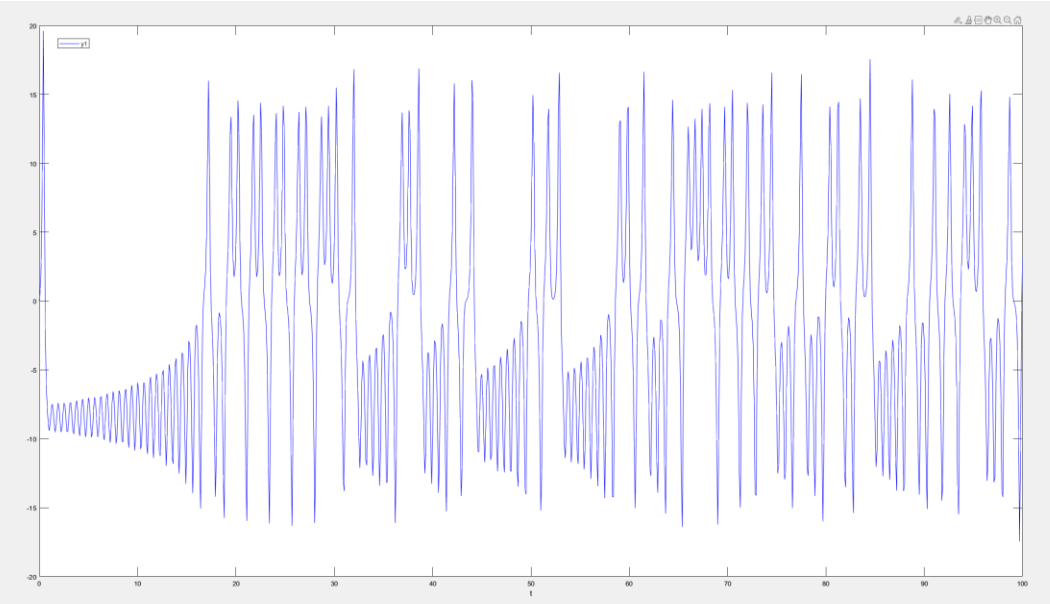
Results at t = 100:

y1 = 3.2323

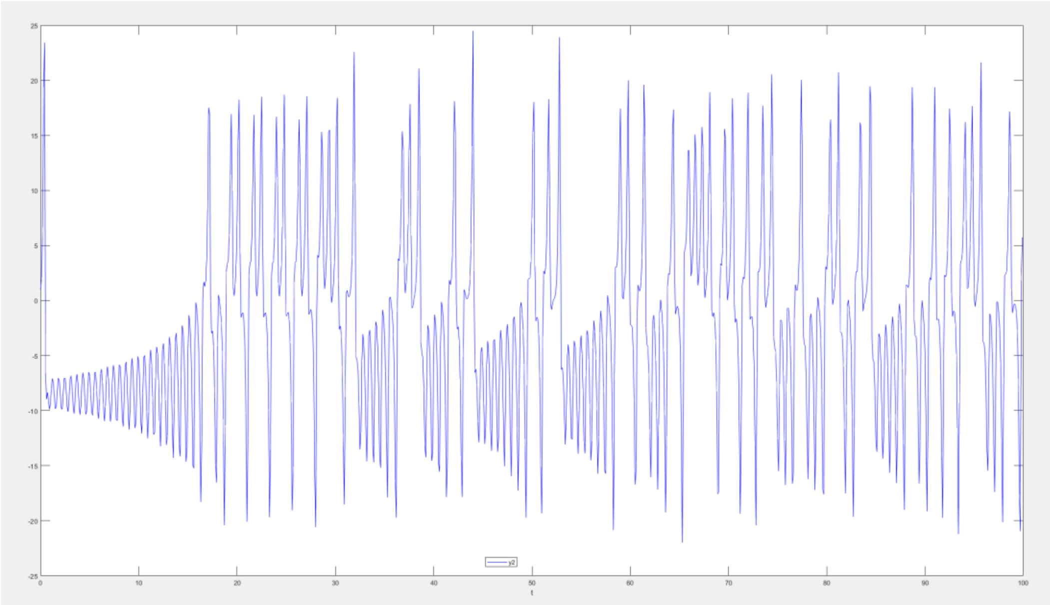
y2 = 5.6847

y3 = 23.1373

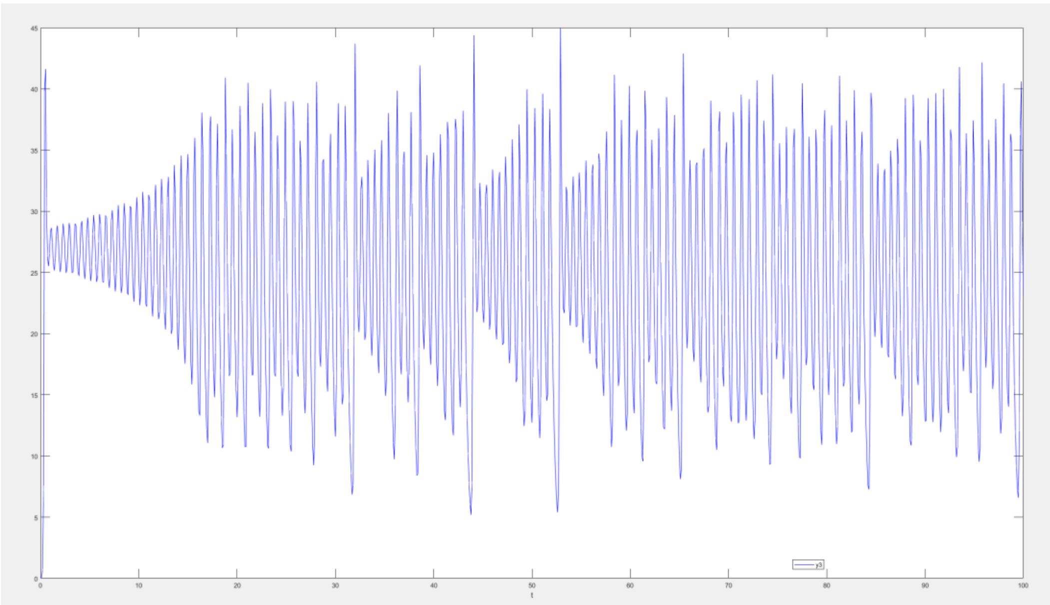
a) Plot of y1



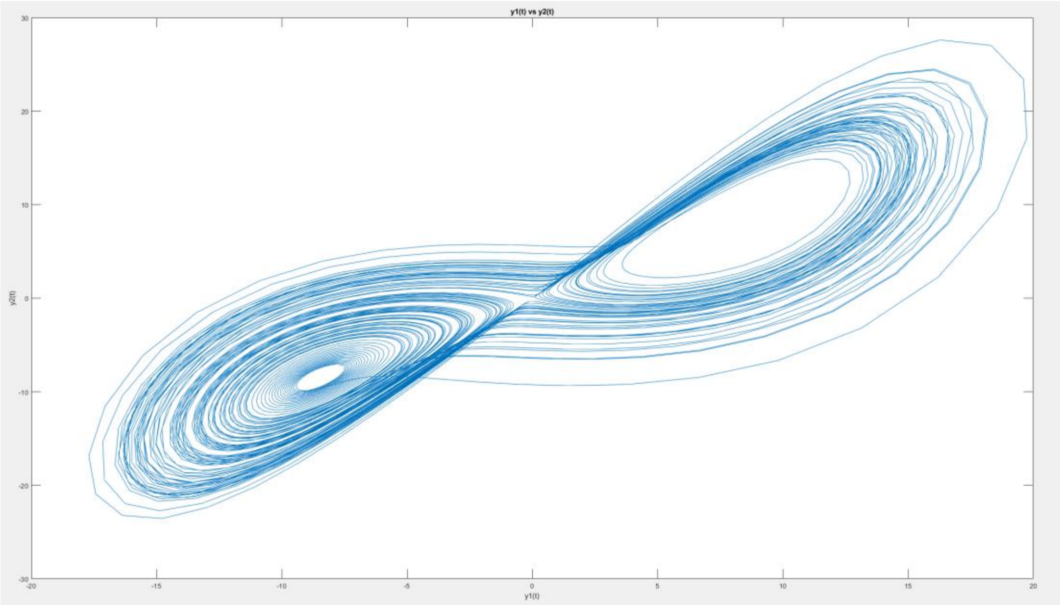
b) Plot of y2



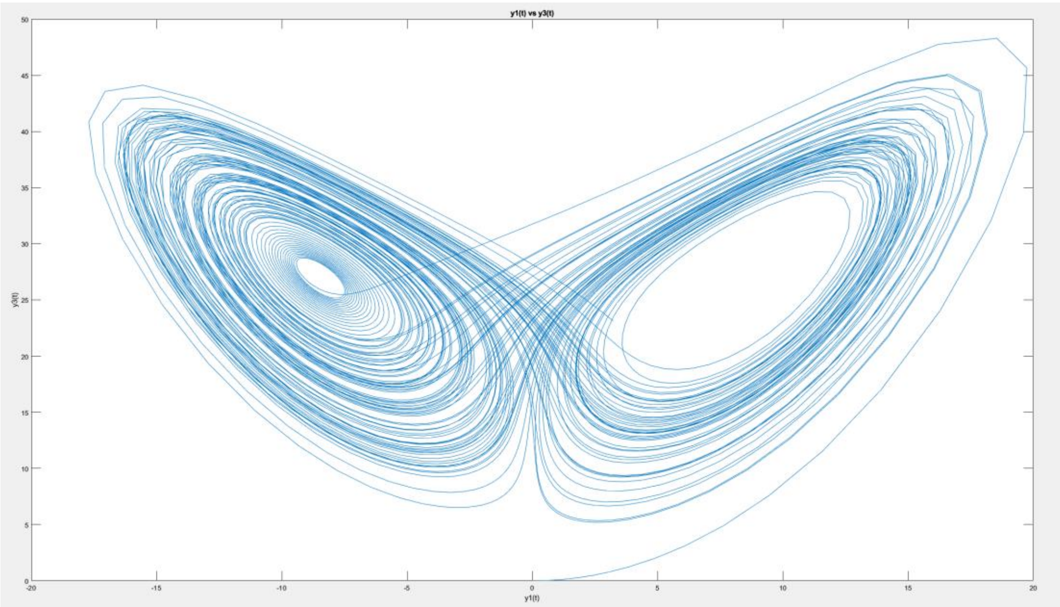
c) Plot of y3



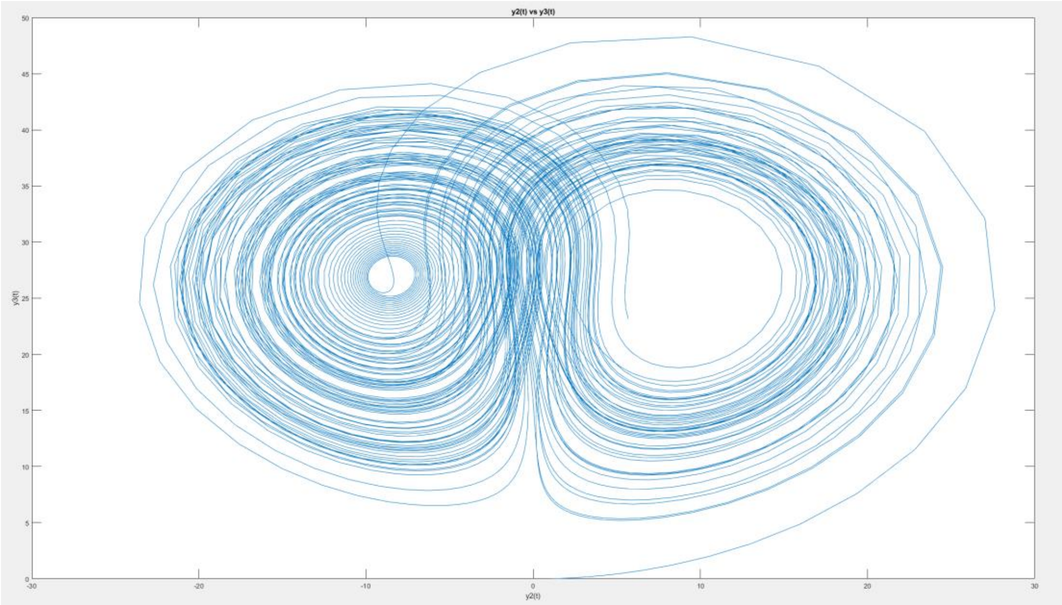
d) Plot of y1 vs y2



e) Plot of y1 vs y3



f) Plot of y2 vs y3



g) The perturbation when initial values changed tiny amount.

We can see that the equation is ill-conditioned, therefore, sensitive to changes.

The eigenvalues of the Jacobian matrix are greater then zero ($\lambda > 0$) so the ODE is not stable.

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Initial values : 0 1 0
Results at t = 100:
y1 = 4.1794
y2 = 7.1405
y3 = 13.1821


Initial values : 1.000000e-02 1 0
Perturbed solution at t = 100:
y = -0.0879299      1.98574      22.7139


Initial values : 0 9.900000e-01 0
Perturbed solution at t = 100:
y = 4.15977      7.69722      10.0964


Initial values : 1.000000e-02 9.900000e-01 -1.000000e-02
Perturbed solution at t = 100:
y = -5.25885     -8.30759       16.159
>>
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