CMB lensing and Minkowski Functionals

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based on work in progress

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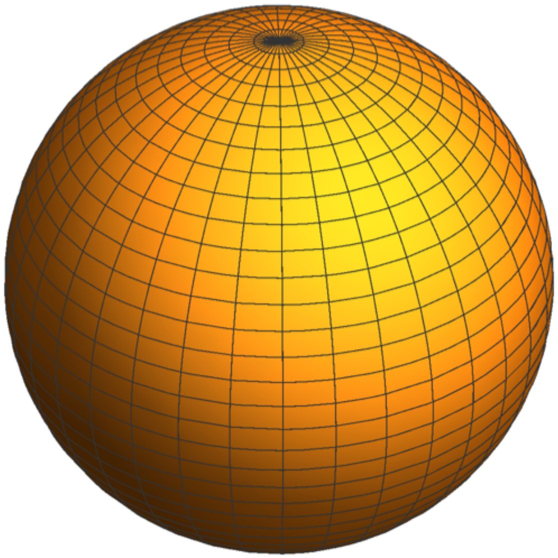
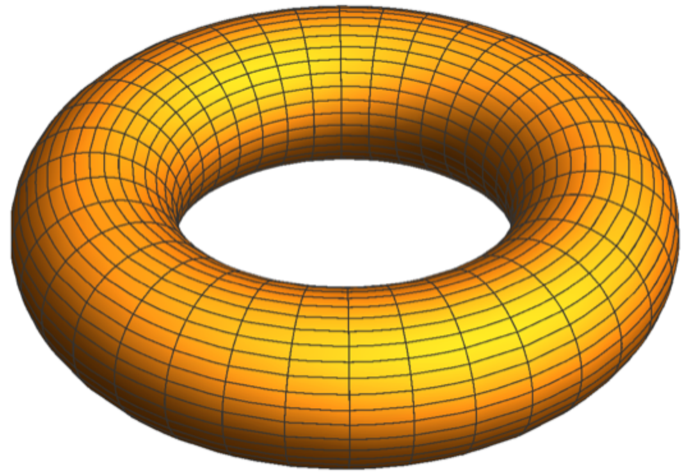
Topology





no holes

one hole

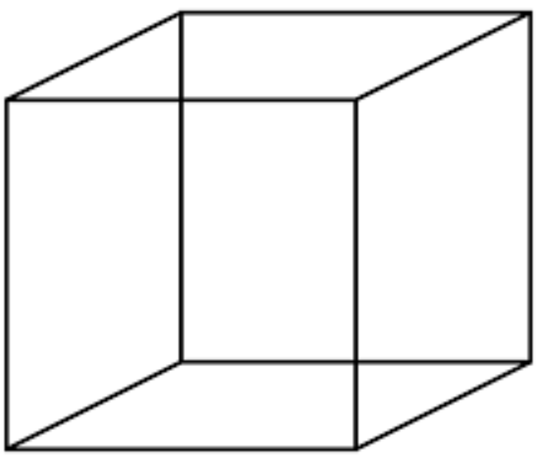
Invariant under smooth differentiable deformations
(diffeomorphisms)

Euler's formula for convex polyhedra

$$V + F - E = 2$$

[Euler (1758)]

vertices faces edges



8	vertices
6	faces
- 12	edges
2	

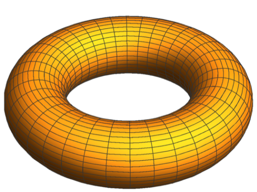
Euler's formula v2

$$V + F - E = \chi$$

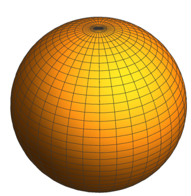
vertices faces edges

Euler characteristic

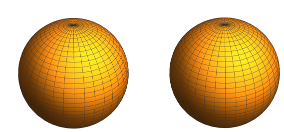
$$\boxed{\chi = 2(\#\text{ of disconnected parts}) - (\#\text{ of holes})}$$



$\chi = 0$



$\chi = 2$



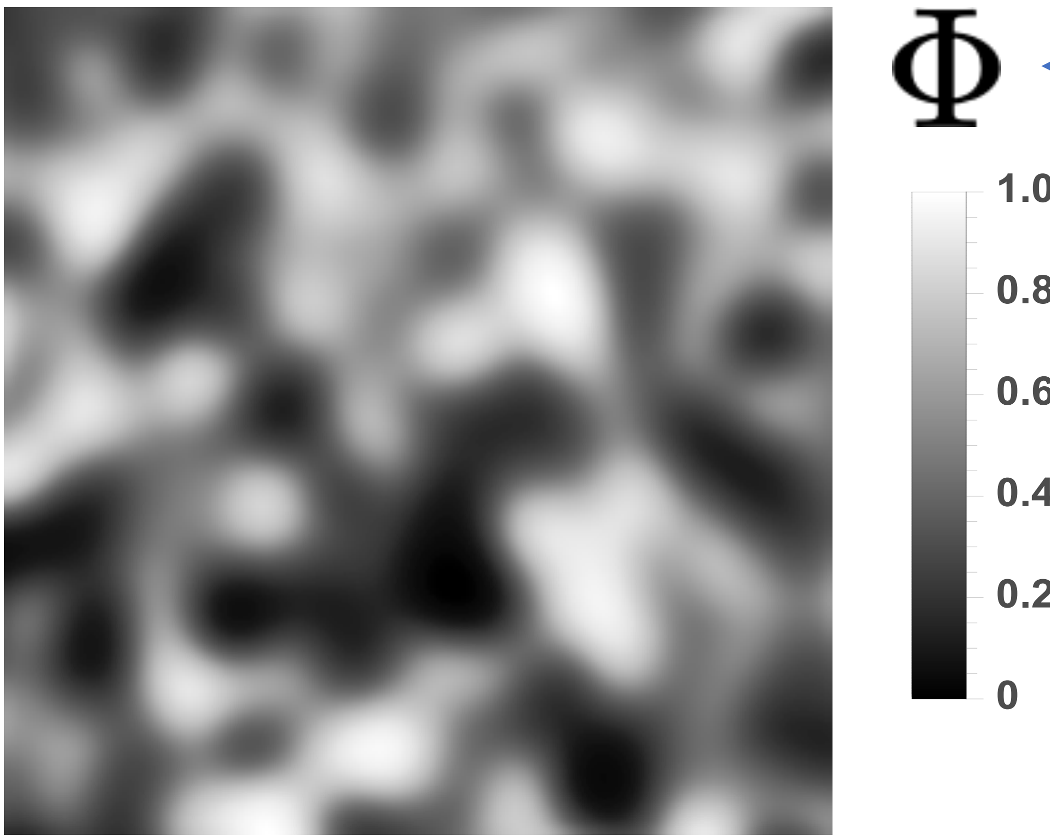
$\chi = 4$

Topology and Morphology

Minkowski Functionals (MFs)

d	1	2	3
V_0	length	area	volume
V_1	χ	circumference	surface area
V_2	–	χ	total mean curvature
V_3	–	–	χ

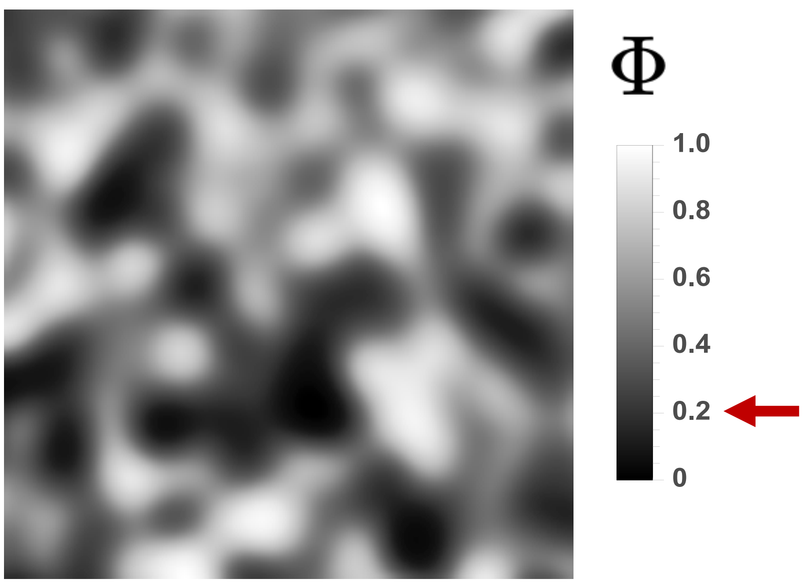
Morphological analysis of 2d maps



Some scalar quantity, e.g., temperature, density, potential, etc.

Generating excursion sets from a map

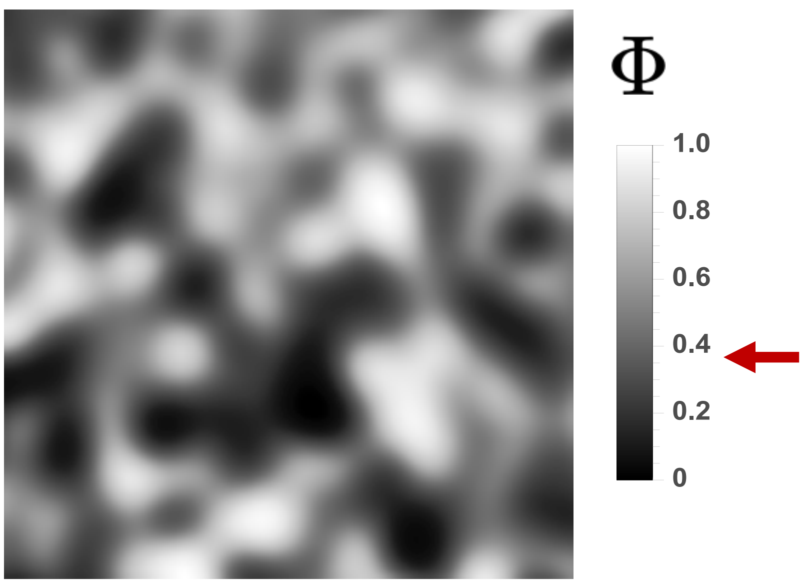
- Define a threshold value ν
- $\Phi > \nu \longrightarrow$ white $\quad \Phi < \nu \longrightarrow$ black

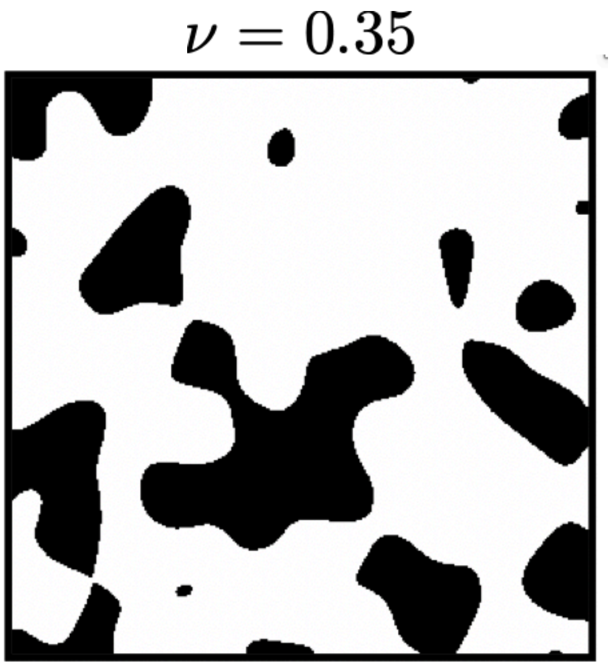


$\nu = 0.2$

Generating excursion sets from a map

- Define a threshold value ν
- $\Phi > \nu \longrightarrow$ white $\quad \Phi < \nu \longrightarrow$ black





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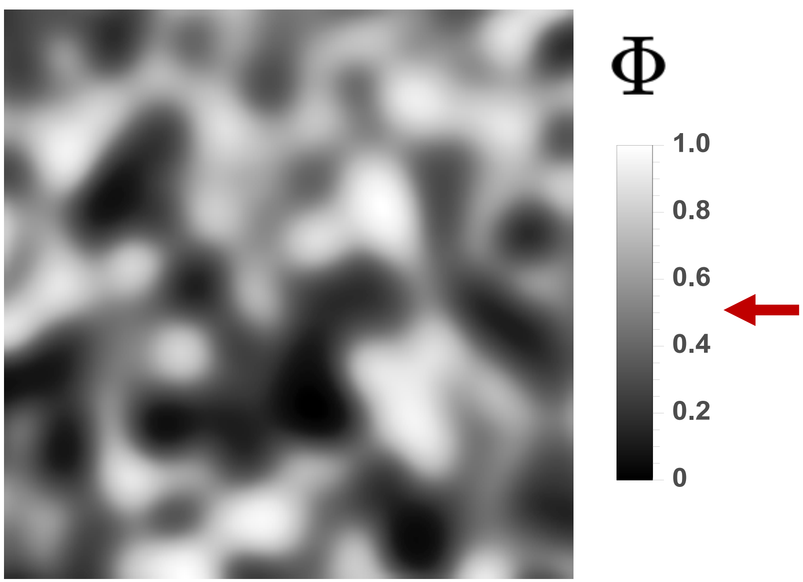
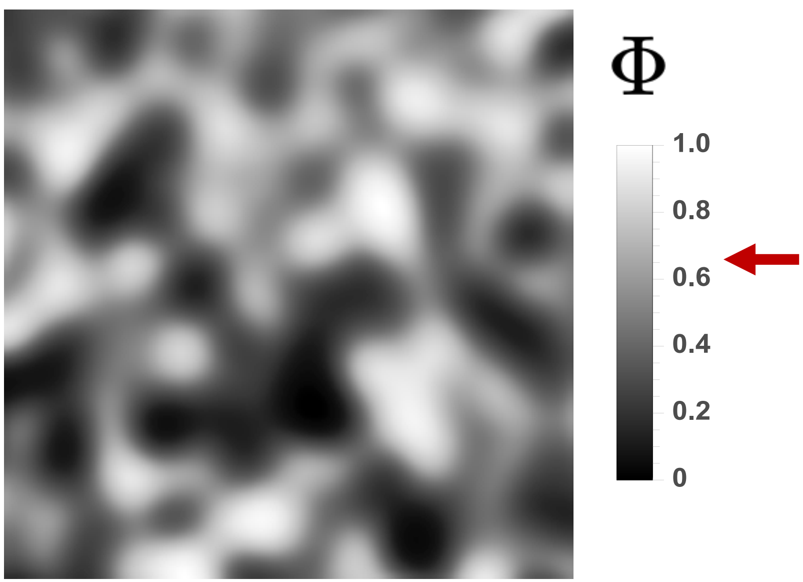


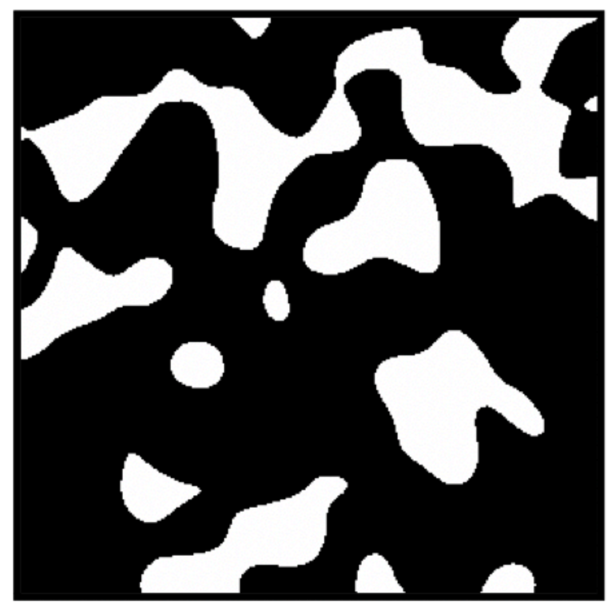
$\nu = 0.5$

Generating excursion sets from a map

- Define a threshold value ν
- $\Phi > \nu \longrightarrow$ white $\qquad \Phi < \nu \longrightarrow$ black



$\nu = 0.65$



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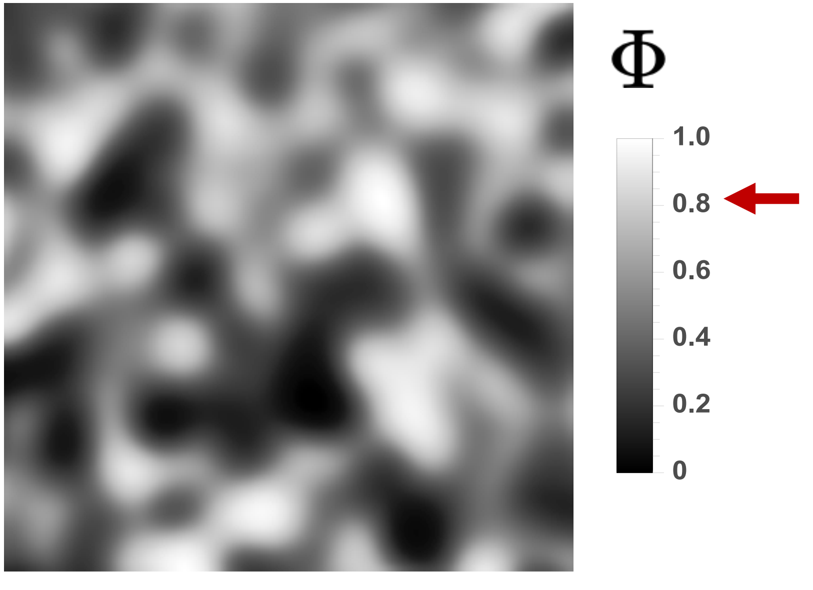
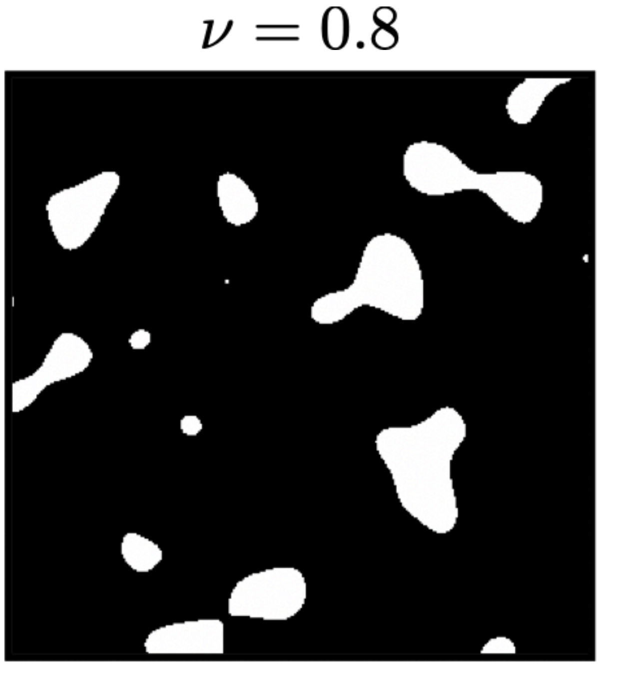
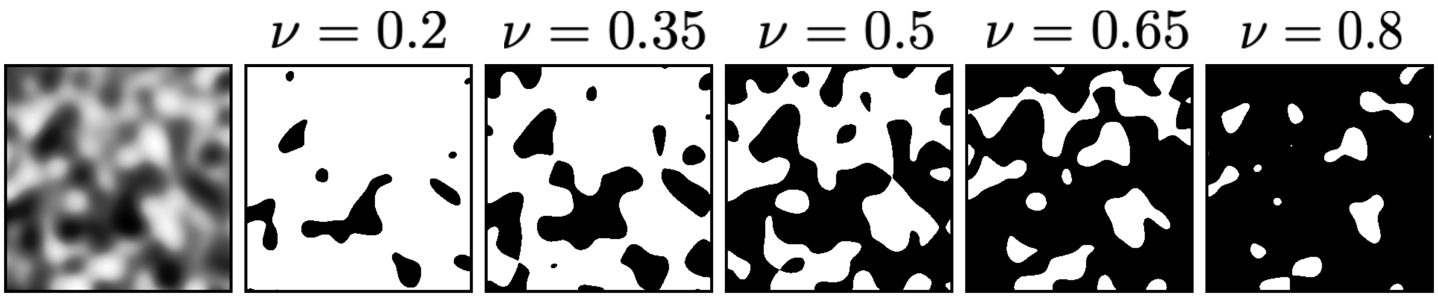
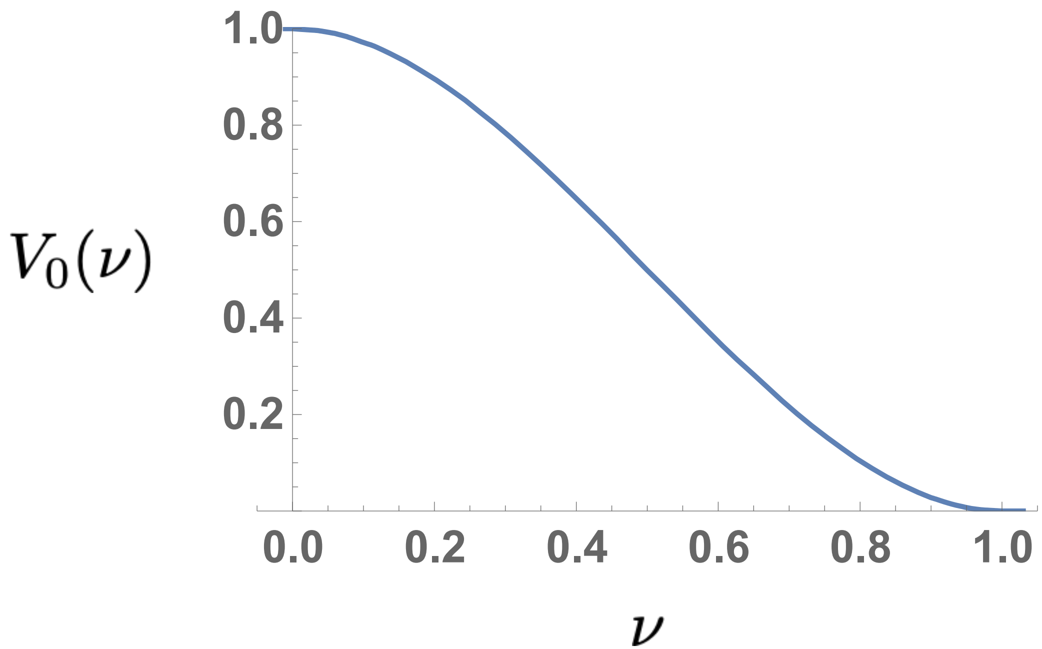




V_0: area

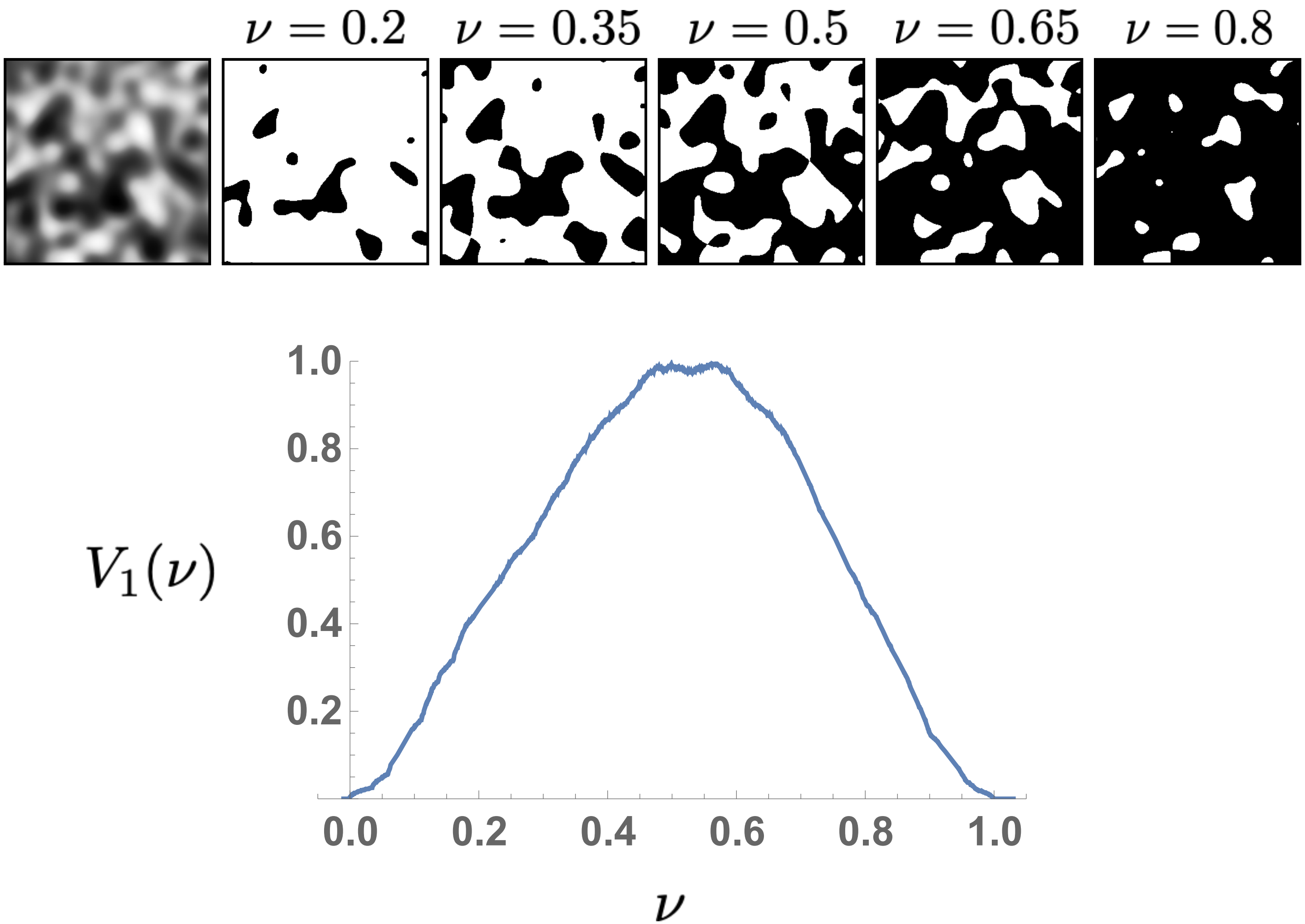


$\nu = 0.2$
$\nu = 0.35$
$\nu = 0.5$
$\nu = 0.65$
$\nu = 0.8$

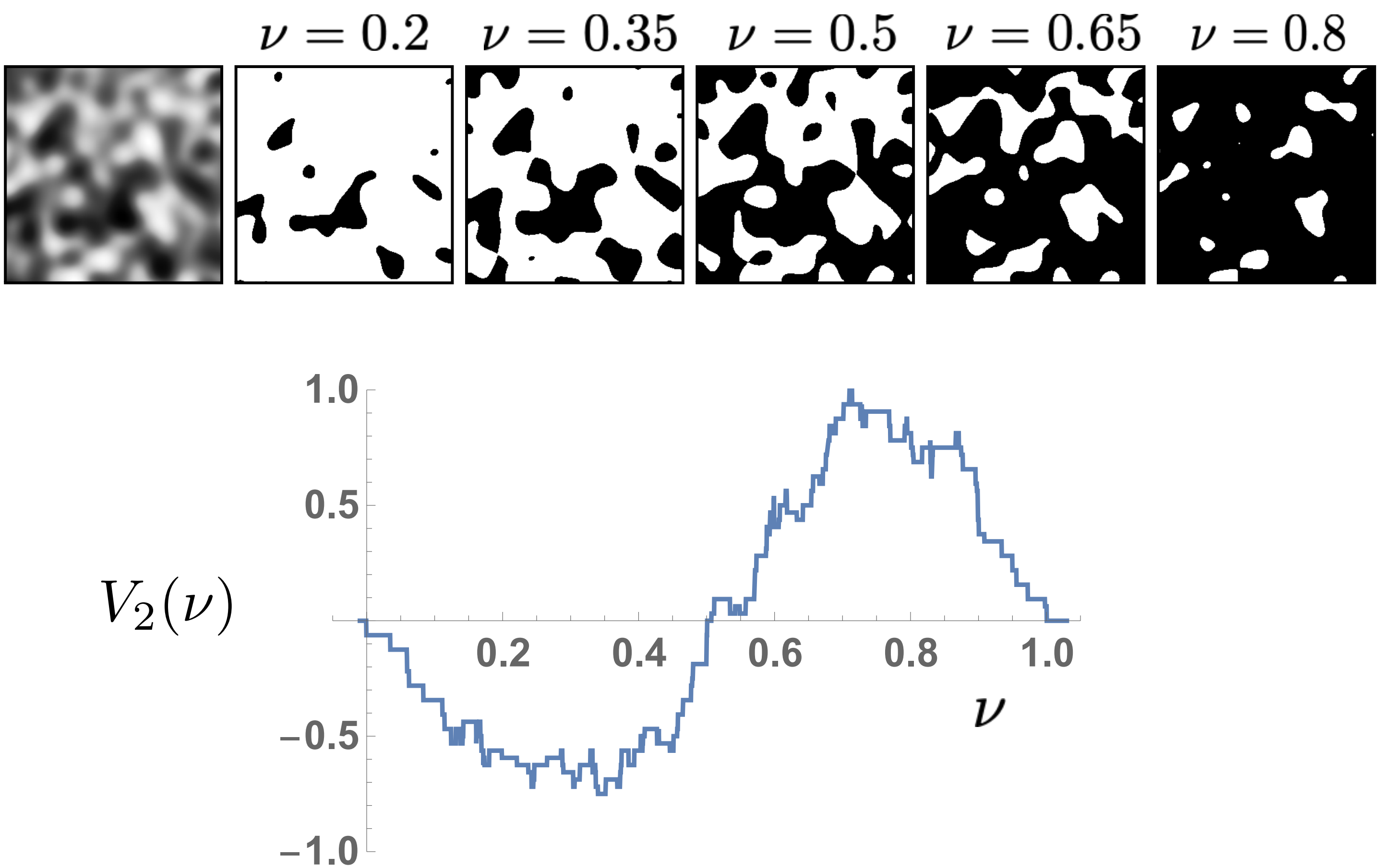


$V_0(\nu)$
1.0
0.8
0.6
0.4
0.2
0.0
0.2
0.4
0.6
0.8
1.0
ν

V_1: length of perimeter

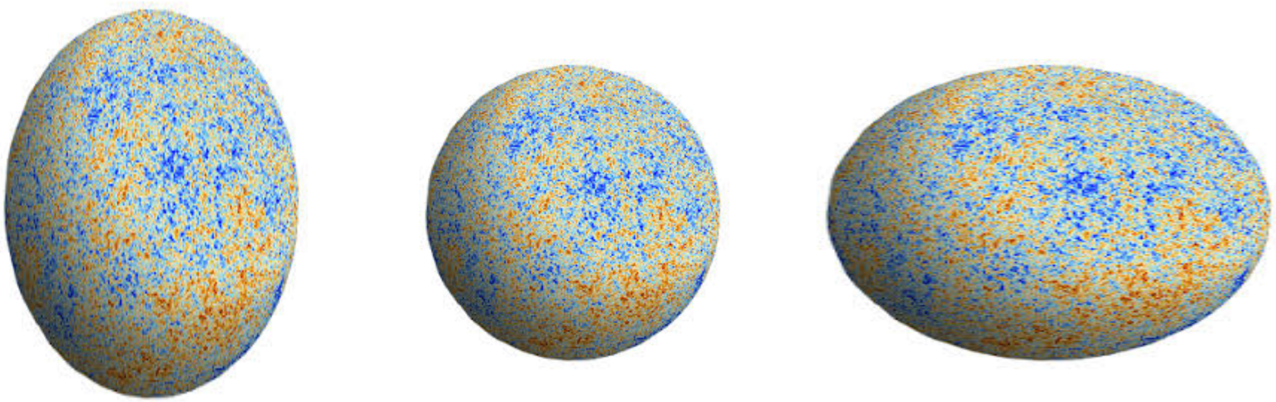


V_2: Euler characteristic



Why might MFs be useful?

- Relatively straightforward to evaluate
- Very robust with respect to systematics
  (χ is invariant under diffeomorphisms!)



- Sensitive to all higher order correlations

Why might MFs be useful?

Gaussian random fields

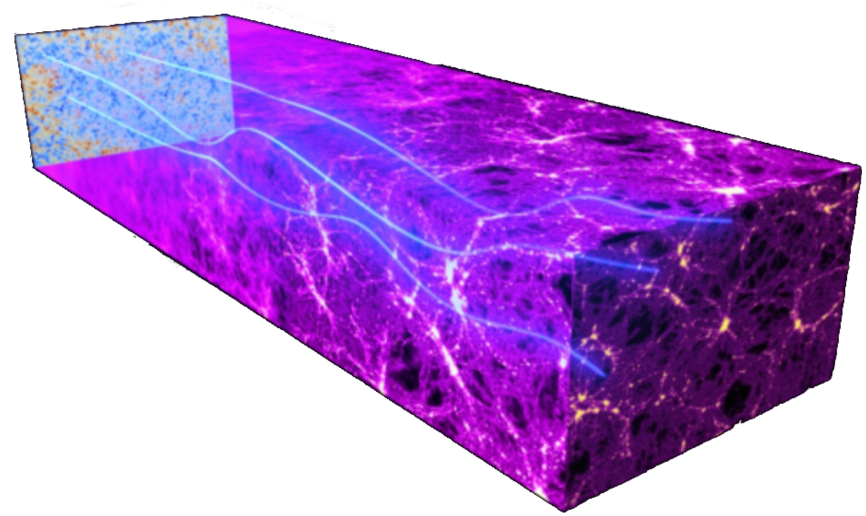
- all statistical information is contained in the power spectrum
- MFs can still provide consistency check

non-Gaussian random fields

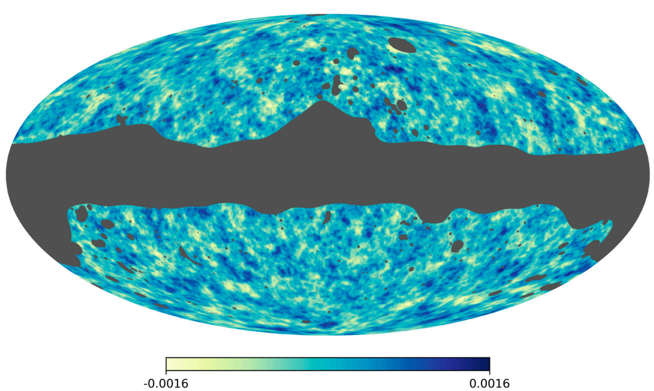
- MFs contribute information beyond the power spectrum
- much easier to evaluate than higher order spectra

Why (CMB) Lensing?

- Lensing potential is (mildly) non-Gaussian
  - General relativity is non-linear theory
  - lens is non-Gaussian
  - possibly primordial non-Gaussianity
- Isotropic source distribution
  - isotropic noise



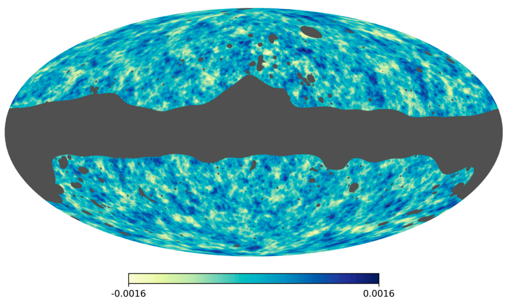
CMB lensing potential

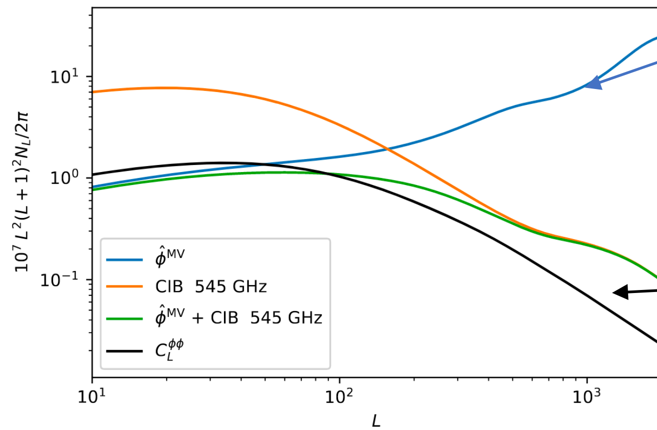


[*Planck* (2018)]

Why (CMB) Lensing?

Currently: lots of noise



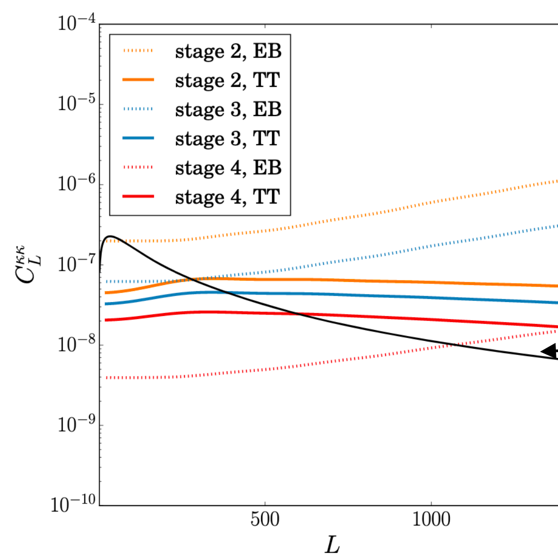


noise

signal

[*Planck* (2018)]

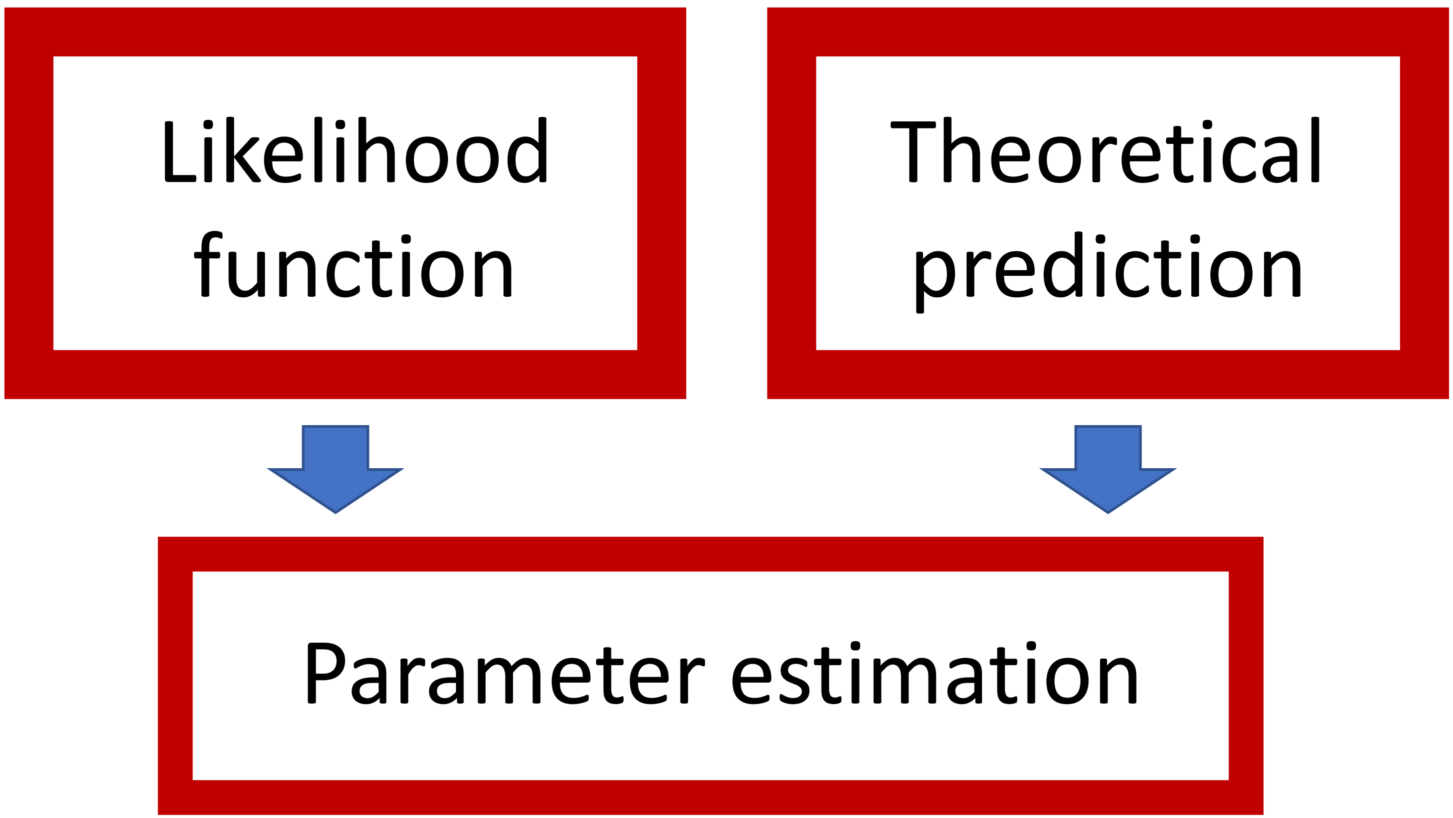
Soon: much better quality lensing maps



signal

[*CMB-S4* (2018)]

CMB lensing MFs as observable

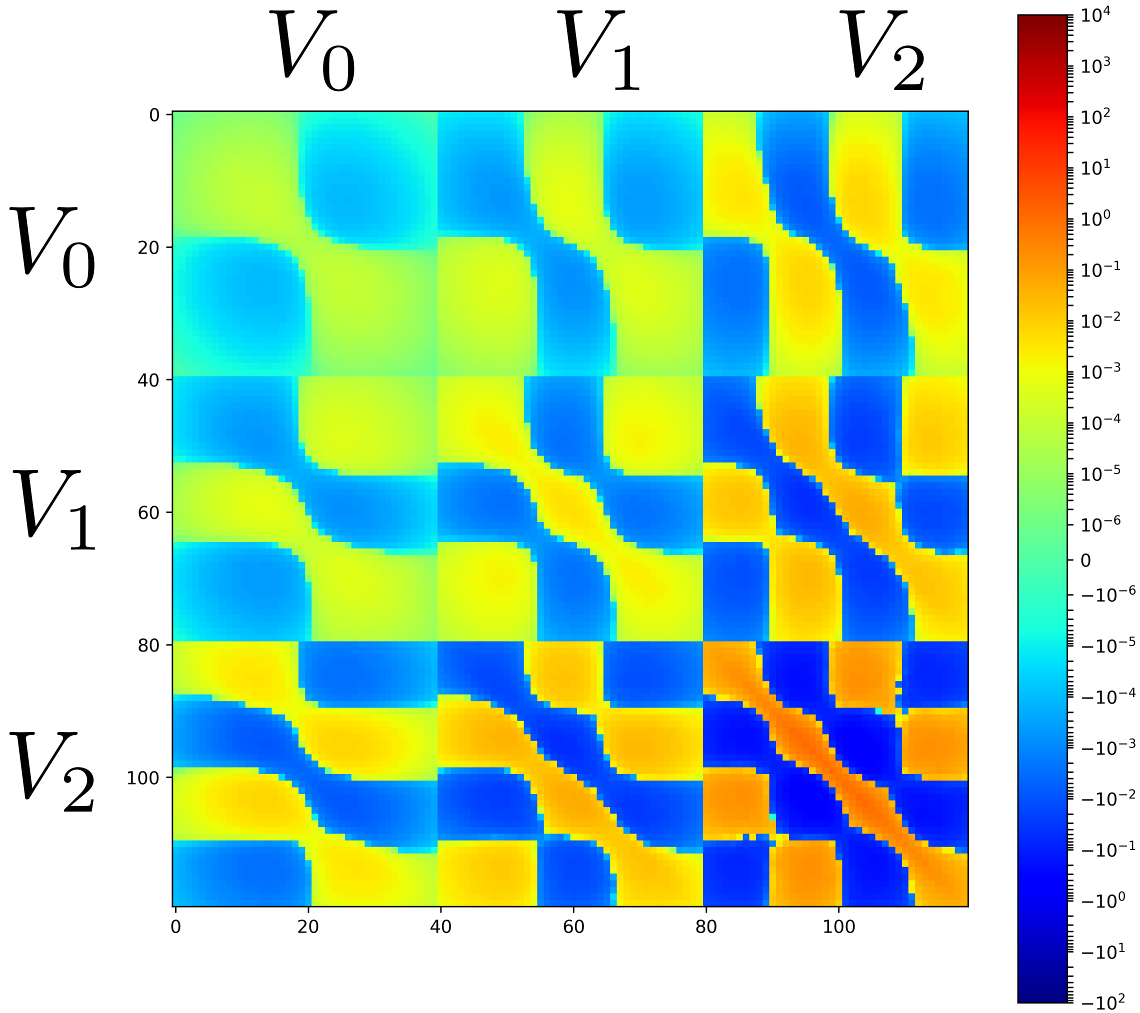


CMB lensing MF likelihood

$$\mu_{ij} \equiv (\underbrace{V_i^{\mathrm{obs}}(\nu_j)}_{\text{observed value}} - \underbrace{V_i^{\mathrm{th}}(\nu_j, \underbrace{\mathbf{\Theta}}_{\text{cosmological parameters}})}_{\text{theory prediction}})$$

$$-2\ln\mathcal{L}(\mathrm{data}|\mathbf{\Theta}) = \underbrace{\sum_{\mathrm{i,j,k,l}}}_{\text{sum over MFs and bins}} \mu_{\mathrm{ij}} \, (\underbrace{\mathrm{Cov}^{-1}}_{\text{covariance matrix}})_{\mathrm{ijkl}} \, \mu_{\mathrm{kl}}$$

MF covariance matrix



[Kang, JH (in progress)]

Theoretical prediction for Gaussian random fields

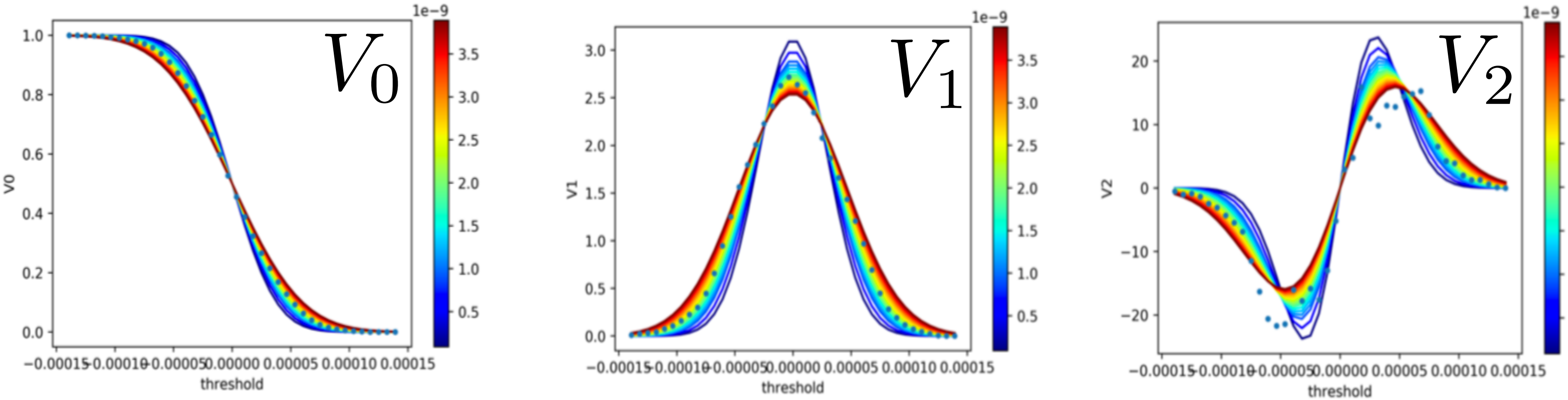
$$V_i^{\mathrm{G}}(\nu) \propto \left(\frac{\sigma_1}{\sqrt{2}\sigma}\right)^i e^{-\nu^2/2\sigma^2} H_{i-1}(\nu/\sigma)$$

Hermite polynomials

- Depend on second moments of the field (i.e., the variance) and the field gradient
- These can be expressed in terms of the angular power spectrum

[Tomita (1986)]

Theoretical prediction for Gaussian random fields



- Evaluate Gaussian prediction for MFs via angular power spectrum (`CAMB/Class`)

Beyond Gaussianity: perturbative approach

- Expand in σ around Gaussian result

$$V_i = V_i^G + V_i^{(1)}\,\sigma + V_i^{(2)}\,\sigma^2 + \ldots$$

third moments of the field (skewness) and derivatives
$\rightarrow$ bispectrum of Φ

fourth moments of the field (kurtosis) and derivatives
$\rightarrow$ trispectrum of Φ

[Matsubara (2003,2010)]

Beyond Gaussianity: non-perturbative approach

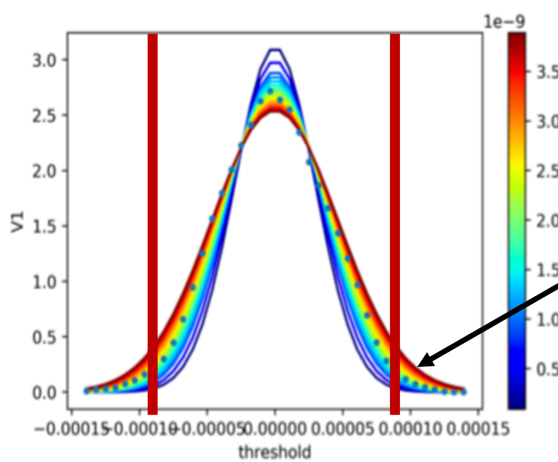
- *N*-body simulations + ray-tracing
- Not easy to implement for parameter estimation (need emulator)
- Has been applied to forecast LSST lensing MF constraints with promising results

[Marques+ (2018)]

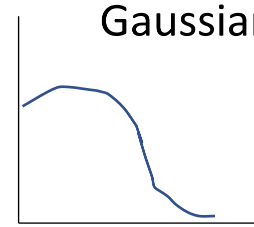
Proof of concept: Gaussian approximation

- *Planck*-like fiducial data, assuming
  - *Planck* best-fit cosmology
  - *Planck* noise properties
  - f_{sky} fudge factor to rescale covariance
  - tails cut off generously *(too generously?)*

$$-2.5\ \sigma < \nu < 2.5\ \sigma$$



likelihood not quite Gaussian in tails



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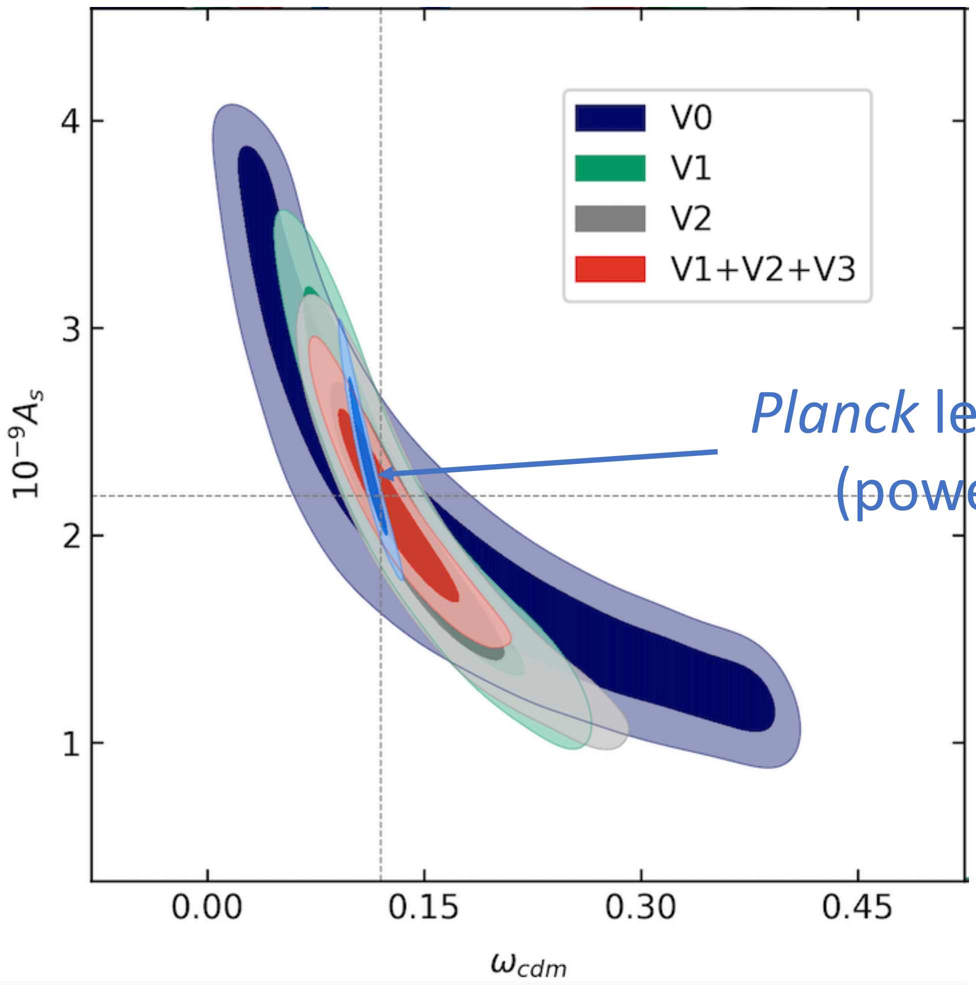
$$-2.5\ \sigma < \nu < 2.5\ \sigma$$

- analytical prediction calculated from power spectrum

Preliminary results

primordial
power spectrum
amplitude



Planck lensing likelihood
(power spectrum)

matter density

[Kang, JH (in progress)]

To be done

- Proper treatment of mask
- Generalize to non-Gaussian case
  - analytically to second order?
  - *N*-body simulations/ray tracing?
- Apply to real *Planck* lensing map
- Forecast for future experiments

Conclusions

- Minkowski functionals describe topological and morphological properties of maps
- For non-Gaussian maps, MFs can add information that is missed by the power spectrum
- CMB lensing potential is a promising target for the application of MFs
- Watch this space!