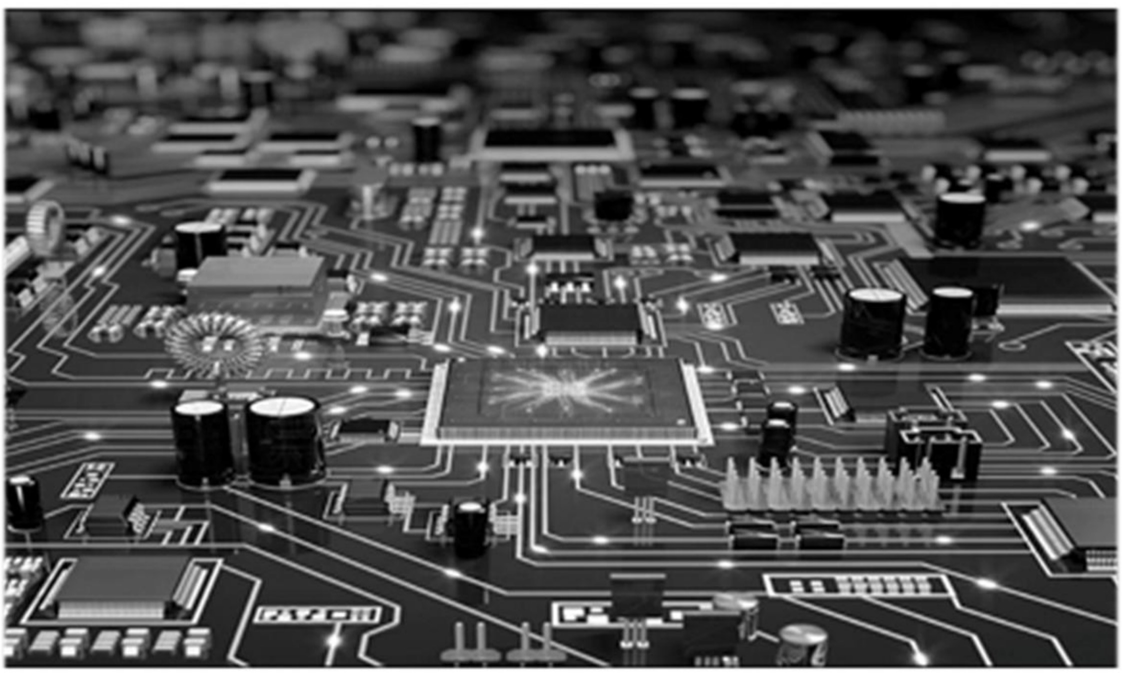
وزارة التعليم العالي

جامعة الشطرة – كلية الهندسة

قسم الحاسوب

Calculus 2



عنوان التقرير: **Matrices and Determinants**

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1. Introduction

Matrices and determinants are fundamental concepts in linear algebra with wide applications in mathematics, physics, engineering, computer science, and economics. A matrix is a rectangular array of numbers arranged in rows and columns, while a determinant is a scalar value that can be computed from a square matrix and provides important properties about the matrix.

This report explores:

- Definition and types of matrices
- Matrix operations (addition, multiplication, transpose)
- Determinants and their properties
- Applications in real-world problems

2. Matrices: Definition and Types

2.1 Definition

A matrix is a structured arrangement of numbers (or functions) in rows and columns. It is denoted as:

$$A = \begin{pmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{pmatrix}$$

where a_{ij} represents the element in the i^{th} row and j^{th} column.

2.2 Types of Matrices

1. **Row Matrix:** A matrix with only one row.

$$\begin{pmatrix} 1 & 2 & 3 \end{pmatrix}$$

2. **Column Matrix:** A matrix with only one column.

$$\begin{pmatrix} 4 \\ 5 \\ 6 \end{pmatrix}$$

3. **Square Matrix:** A matrix where m = n.

$$\begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$$

4. **Diagonal Matrix:**

   A square matrix where non-diagonal elements are zero.

$$\begin{pmatrix} 2 & 0 \\ 0 & 5 \end{pmatrix}$$

5. **Identity Matrix:**

   A diagonal matrix with all diagonal elements equal to 1.

$$I = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

6. **Zero Matrix:** A matrix where all elements are zero.

$$\begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$$

3. Matrix Operations

3.1 Addition and Subtraction

Two matrices A and B can be added or subtracted if they have the same dimensions.

C = A ± B $\rightarrow \boldsymbol{c_{ji}} = \boldsymbol{a_{ij}} \pm \boldsymbol{b_{ij}}$

Example:

$$A = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}, B = \begin{pmatrix} 5 & 6 \\ 7 & 8 \end{pmatrix}$$

$$A + B = \begin{pmatrix} 6 & 8 \\ 10 & 12 \end{pmatrix}$$

3.2 Scalar Multiplication

A matrix can be multiplied by a scalar K:

$$KA = \begin{pmatrix} ka_{11} & ka_{12} \\ ka_{21} & ka_{22} \end{pmatrix}$$

3.3 Matrix Multiplication

For two matrices A (size m × n) and B (size n × p), their product C=AB is defined as:

$$c_{ij} = \sum_{k=1}^{n} a_{ik}\, b_{kj}$$

Example:

$$A = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}, B = \begin{pmatrix} 5 & 6 \\ 7 & 8 \end{pmatrix}$$

$$A\,B = \begin{pmatrix} 1.5 + 2.7 & 1.6 + 2.8 \\ 3.5 + 4.7 & 3.6 + 4.8 \end{pmatrix} = \begin{pmatrix} 19 & 22 \\ 43 & 50 \end{pmatrix}$$

3.4 Transpose of a Matrix

The transpose A^T is obtained by swapping rows and columns:

$$A = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix} \rightarrow A^T = \begin{pmatrix} 1 & 3 \\ 2 & 4 \end{pmatrix}$$

4. Determinants

4.1 Definition

The determinant is a scalar value computed for a **square matrix** and is denoted as det (A) or |A|.

4.2 Determinant of a 2×2 Matrix

For

$$A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}, \det(A) = ad - bc$$

Example:

$$A = \begin{pmatrix} 2 & 3 \\ 1 & 4 \end{pmatrix}, \det(A) = (2)(4) - (3)(1) = 5$$

4.3 Determinant of a 3×3 Matrix

For

$$A = \begin{pmatrix} a & b & c \\ d & e & f \\ g & h & i \end{pmatrix}$$

$$\det(A) = a(ei - fh) - b(di - fg) + c(dh - eg)$$

4.4 Properties of Determinants

1. **Singular Matrix:** If det(A) = 0, the matrix is singular (non-invertible).
2. **Multiplicative Property:** det(AB) = det (A) det (B).
3. **Effect of Row Operations:**
    - Swapping rows changes the sign.
    - Multiplying a row by k multiplies the determinant by k.
    - Adding a multiple of one row to another does not change the determinant.

5. Applications of Matrices and Determinants

5.1 Solving Linear Systems (Cramer's Rule)

For a system AX = B, if det(A) ≠ 0, the solution is:

$$x_i = \frac{\det(A_i)}{\det(A)}$$

where A_i is obtained by replacing the i^{th} column of A with B.

5.2 Computer Graphics & Transformations

Matrices are used for:

- Rotation, Scaling, Translation
- 3D Modeling

5.3 Cryptography

Matrices are used in **encoding** and **decoding** messages.

5.4 Economics (Input-Output Models)

Matrices model economic interactions between industries.

6. Conclusion

Matrices and determinants are powerful tools in mathematics with diverse applications. Understanding their properties and operations is essential for solving complex problems in science and engineering. Further study can include eigenvalues, eigenvectors, and matrix decompositions.

References

- Anton, H., & Rorres, C. (2013). Elementary Linear Algebra. Wiley.
- Strang, G. (2016). Introduction to Linear Algebra. Wellesley-Cambridge Press.