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Plotting $X_{In}^{b}(t)$ For $Ga_{0.8}In_{0.2}As$/GaAs

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- From Eq-20 we have:

$$X_A^b(t) = -X_A^s(t) + X_A^s(0) + X_A^b(0) + \Phi_A t$$

- We replace $X_A^b(t)$ by Eq-20 into Eq-17 then we will have:

$$\frac{dX_B^s(t)}{dt} = \Phi_B + P_{A/B}^{b\to s} X_B^b(t) X_A^s(t) - P_{A/B}^{s\to b} X_B^s(t)\left[-X_A^s(t) + X_A^s(0) + X_A^b(0) + \Phi_A t\right]$$

$$= \Phi_B + P_{A/B}^{b\to s} X_B^b(t) X_A^s(t) + P_{A/B}^{s\to b} X_B^s(t) X_A^s(t) - P_{A/B}^{s\to b} X_B^s(t) X_A^s(0) - P_{A/B}^{s\to b} X_B^s(t) X_A^b(0)$$

$$- P_{A/B}^{s\to b} X_B^s(t) \Phi_A t$$

- From Eq-18 we have:

$$X_B^b(t) = -X_B^s(t) + X_B^s(0) + X_B^b(0) + \Phi_B t$$

Plotting $X_{In}^{b}(t)$ For $Ga_{0.8}In_{0.2}As/GaAs$

- We replace $X_B^b(t)$ by Eq-18 into $\frac{dX_B^s(t)}{dt}$ then we will have:

$$\frac{dX_B^s(t)}{dt}$$

$$= \Phi_B + P_{\frac{A}{B}}^{b\to s} X_A^s(t)\left[-X_B^s(t) + X_B^s(0) + X_B^b(0) + \Phi_B t\right] + P_{A/B}^{s\to b} X_B^s(t) X_A^s(t)$$
$$- P_{A/B}^{s\to b} X_B^s(t) X_A^s(0) - P_{A/B}^{s\to b} X_B^s(t) X_A^b(0) - P_{A/B}^{s\to b} X_B^s(t) \Phi_A t$$

$$= \Phi_B - P_{\frac{A}{B}}^{b\to s} X_A^s(t) X_B^s(t) + P_{\frac{A}{B}}^{b\to s} X_A^s(t) X_B^s(0) + P_{\frac{A}{B}}^{b\to s} X_A^s(t) X_B^b(0) + P_{\frac{A}{B}}^{b\to s} X_A^s(t) \Phi_B t$$
$$+ P_{A/B}^{s\to b} X_B^s(t) X_A^s(t) - P_{A/B}^{s\to b} X_B^s(t) X_A^s(0) - P_{A/B}^{s\to b} X_B^s(t) X_A^b(0) - P_{A/B}^{s\to b} X_B^s(t) \Phi_A t$$

$$= \Phi_B - P_{\frac{A}{B}}^{b\to s} X_B^s(t) X_A^s(t) + P_{\frac{A}{B}}^{b\to s} X_B^s(0) X_A^s(t) + P_{\frac{A}{B}}^{b\to s} X_B^b(0) X_A^s(t) + P_{\frac{A}{B}}^{b\to s} \Phi_B t X_A^s(t)$$
$$+ P_{A/B}^{s\to b} X_B^s(t) X_A^s(t) - P_{A/B}^{s\to b} X_B^s(t) X_A^s(0) - P_{A/B}^{s\to b} X_B^s(t) X_A^b(0) - P_{A/B}^{s\to b} X_B^s(t) \Phi_A t$$

Plotting $X_{In}^{b}(t)$ For $Ga_{0.8}In_{0.2}As/GaAs$

- From Eq-19 we have:

$$X_A^s(t) = -X_B^s(t) + X_A^s(0) + X_B^s(0) + (\Phi_A + \Phi_B)t$$

- Therefore by replacing $X_A^s(t)$ from Eq-19 into $\frac{dX_B^s(t)}{dt}$ we will have:

$$\frac{dX_B^s(t)}{dt}$$

$$= \Phi_B - P_{\frac{A}{B}}^{b\to s} X_B^s(t) X_A^s(t) + P_{\frac{A}{B}}^{b\to s} X_B^s(0) X_A^s(t) + P_{\frac{A}{B}}^{b\to s} X_B^b(0) X_A^s(t) + P_{\frac{A}{B}}^{b\to s} \Phi_B t X_A^s(t) + P_{A/B}^{s\to b} X_B^s(t) X_A^s(t)$$

$$- P_{A/B}^{s\to b} X_B^s(t) X_A^s(0) - P_{A/B}^{s\to b} X_B^s(t) X_A^b(0) - P_{A/B}^{s\to b} X_B^s(t) \Phi_A t$$

$$= \Phi_B - P_{\frac{A}{B}}^{b\to s} X_B^s(t)[-X_B^s(t) + X_A^s(0) + X_B^s(0) + (\Phi_A + \Phi_B)t] + P_{\frac{A}{B}}^{b\to s} X_B^s(0)[-X_B^s(t) + X_A^s(0) + X_B^s(0) + (\Phi_A + \Phi_B)t]$$

$$+ P_{\frac{A}{B}}^{b\to s} X_B^b(0)[-X_B^s(t) + X_A^s(0) + X_B^s(0) + (\Phi_A + \Phi_B)t] + P_{\frac{A}{B}}^{b\to s} \Phi_B t[-X_B^s(t) + X_A^s(0) + X_B^s(0) + (\Phi_A + \Phi_B)t]$$

$$+ P_{A/B}^{s\to b} X_B^s(t)[-X_B^s(t) + X_A^s(0) + X_B^s(0) + (\Phi_A + \Phi_B)t] - P_{A/B}^{s\to b} X_B^s(t) X_A^s(0) - P_{A/B}^{s\to b} X_B^s(t) X_A^b(0) - P_{A/B}^{s\to b} X_B^s(t) \Phi_A t$$

Plotting $X_{In}^{b}(t)$ For $Ga_{0.8}In_{0.2}As/GaAs$

$$= \Phi_B + P_{\frac{A}{B}}^{b\to s} X_B^s(t) X_B^s(t) - P_{\frac{A}{B}}^{b\to s} X_B^s(t) X_A^s(0) - P_{\frac{A}{B}}^{b\to s} X_B^s(t) X_B^s(0) - P_{\frac{A}{B}}^{b\to s} X_B^s(t)(\Phi_A + \Phi_B)t - P_{\frac{A}{B}}^{b\to s} X_B^s(0) X_B^s(t)$$

$$+ P_{\frac{A}{B}}^{b\to s} X_B^s(0) X_A^s(0) + P_{\frac{A}{B}}^{b\to s} X_B^s(0) X_B^s(0) + P_{\frac{A}{B}}^{b\to s} X_B^s(0)(\Phi_A + \Phi_B)t - P_{\frac{A}{B}}^{b\to s} X_B^b(0) X_B^s(t) + P_{\frac{A}{B}}^{b\to s} X_B^b(0) X_A^s(0)$$

$$+ P_{\frac{A}{B}}^{b\to s} X_B^b(0) X_B^s(0) + P_{\frac{A}{B}}^{b\to s} X_B^b(0)(\Phi_A + \Phi_B)t - P_{\frac{A}{B}}^{b\to s} \Phi_B t X_B^s(t) + P_{\frac{A}{B}}^{b\to s} \Phi_B t X_A^s(0) + P_{\frac{A}{B}}^{b\to s} \Phi_B t X_B^s(0)$$

$$+ P_{\frac{A}{B}}^{b\to s} \Phi_B t(\Phi_A + \Phi_B)t - P_{\frac{A}{B}}^{s\to b} X_B^s(t) X_B^s(t) + P_{A/B}^{s\to b} X_B^s(t) X_A^s(0) + P_{A/B}^{s\to b} X_B^s(t) X_B^s(0) + P_{A/B}^{s\to b} X_B^s(t)(\Phi_A + \Phi_B)t$$

$$- P_{A/B}^{s\to b} X_B^s(t) X_A^s(0) - P_{A/B}^{s\to b} X_B^s(t) X_A^b(0) - P_{A/B}^{s\to b} X_B^s(t) \Phi_A t$$

$$= \Phi_B + P_{\frac{A}{B}}^{b\to s} [X_B^s(t)]^2 - P_{\frac{A}{B}}^{b\to s} X_B^s(t) X_A^s(0) - P_{\frac{A}{B}}^{b\to s} X_B^s(t) X_B^s(0) - P_{\frac{A}{B}}^{b\to s} X_B^s(t)(\Phi_A + \Phi_B)t - P_{\frac{A}{B}}^{b\to s} X_B^s(0) X_B^s(t)$$

$$+ P_{\frac{A}{B}}^{b\to s} X_B^s(0) X_A^s(0) + P_{\frac{A}{B}}^{b\to s} X_B^s(0) X_B^s(0) + P_{\frac{A}{B}}^{b\to s} X_B^s(0)(\Phi_A + \Phi_B)t - P_{\frac{A}{B}}^{b\to s} X_B^b(0) X_B^s(t) + P_{\frac{A}{B}}^{b\to s} X_B^b(0) X_A^s(0)$$

$$+ P_{\frac{A}{B}}^{b\to s} X_B^b(0) X_B^s(0) + P_{\frac{A}{B}}^{b\to s} X_B^b(0)(\Phi_A + \Phi_B)t - P_{\frac{A}{B}}^{b\to s} X_B^s(t) \Phi_B t + P_{\frac{A}{B}}^{b\to s} X_A^s(0) \Phi_B t + P_{\frac{A}{B}}^{b\to s} X_B^s(0) \Phi_B t$$

$$+ P_{\frac{A}{B}}^{b\to s} \Phi_B(\Phi_A + \Phi_B)t^2 - P_{\frac{A}{B}}^{s\to b} [X_B^s(t)]^2 + P_{A/B}^{s\to b} X_B^s(t) X_A^s(0) + P_{A/B}^{s\to b} X_B^s(t) X_B^s(0) + P_{A/B}^{s\to b} X_B^s(t)(\Phi_A + \Phi_B)t$$

$$- P_{A/B}^{s\to b} X_B^s(t) X_A^s(0) - P_{A/B}^{s\to b} X_B^s(t) X_A^b(0) - P_{A/B}^{s\to b} X_B^s(t) \Phi_A t$$

Plotting $X_{In}^{b}(t)$ For $Ga_{0.8}In_{0.2}As/GaAs$

$$= P_{\frac{A}{B}}^{b\to s}[X_B^s(t)]^2 - P_{\frac{A}{B}}^{s\to b}[X_B^s(t)]^2 - P_{\frac{A}{B}}^{b\to s}X_B^s(t)X_A^s(0) - P_{\frac{A}{B}}^{b\to s}X_B^s(t)X_B^s(0) - P_{\frac{A}{B}}^{b\to s}X_B^s(t)(\Phi_A + \Phi_B)t - P_{\frac{A}{B}}^{b\to s}X_B^s(t)X_B^s(0)$$

$$+ P_{\frac{A}{B}}^{b\to s}X_B^s(0)X_A^s(0) + P_{\frac{A}{B}}^{b\to s}X_B^s(0)X_B^s(0) + P_{\frac{A}{B}}^{b\to s}X_B^s(0)(\Phi_A + \Phi_B)t - P_{\frac{A}{B}}^{b\to s}X_B^s(t)X_B^b(0) + P_{\frac{A}{B}}^{b\to s}X_B^b(0)X_A^s(0)$$

$$+ P_{\frac{A}{B}}^{b\to s}X_B^b(0)X_B^s(0) + P_{\frac{A}{B}}^{b\to s}X_B^b(0)(\Phi_A + \Phi_B)t - P_{\frac{A}{B}}^{b\to s}X_B^s(t)\Phi_B t + P_{\frac{A}{B}}^{b\to s}X_A^s(0)\Phi_B t + P_{\frac{A}{B}}^{b\to s}X_B^s(0)\Phi_B t$$

$$+ P_{\frac{A}{B}}^{b\to s}\Phi_B(\Phi_A + \Phi_B)t^2 + P_{\frac{A}{B}}^{s\to b}X_B^s(t)X_A^s(0) + P_{\frac{A}{B}}^{s\to b}X_B^s(t)X_B^s(0) + P_{\frac{A}{B}}^{s\to b}X_B^s(t)(\Phi_A + \Phi_B)t - P_{\frac{A}{B}}^{s\to b}X_B^s(t)X_A^s(0)$$

$$- P_{\frac{A}{B}}^{s\to b}X_B^s(t)X_A^b(0) - P_{\frac{A}{B}}^{s\to b}X_B^s(t)\Phi_A t + \Phi_B$$

$$= P_{\frac{A}{B}}^{b\to s}[X_B^s(t)]^2 - P_{\frac{A}{B}}^{s\to b}[X_B^s(t)]^2 - P_{\frac{A}{B}}^{b\to s}X_B^s(t)X_A^s(0) - P_{\frac{A}{B}}^{b\to s}X_B^s(t)X_B^s(0) - P_{\frac{A}{B}}^{b\to s}X_B^s(t)(\Phi_A + \Phi_B)t - P_{\frac{A}{B}}^{b\to s}X_B^s(t)X_B^s(0)$$

$$+ P_{\frac{A}{B}}^{b\to s}X_B^s(0)(\Phi_A + \Phi_B)t - P_{\frac{A}{B}}^{b\to s}X_B^s(t)X_B^b(0) + P_{\frac{A}{B}}^{b\to s}X_B^b(0)(\Phi_A + \Phi_B)t - P_{\frac{A}{B}}^{b\to s}X_B^s(t)\Phi_B t + P_{\frac{A}{B}}^{b\to s}X_A^s(0)\Phi_B t$$

$$+ P_{\frac{A}{B}}^{b\to s}X_B^s(0)\Phi_B t + P_{\frac{A}{B}}^{b\to s}\Phi_B(\Phi_A + \Phi_B)t^2 + P_{\frac{A}{B}}^{s\to b}X_B^s(t)X_A^s(0) + P_{\frac{A}{B}}^{s\to b}X_B^s(t)X_B^s(0) + P_{\frac{A}{B}}^{s\to b}X_B^s(t)(\Phi_A + \Phi_B)t$$

$$- P_{\frac{A}{B}}^{s\to b}X_B^s(t)X_A^s(0) - P_{\frac{A}{B}}^{s\to b}X_B^s(t)X_A^b(0) - P_{\frac{A}{B}}^{s\to b}X_B^s(t)\Phi_A t + P_{\frac{A}{B}}^{b\to s}X_B^s(0)X_A^s(0)$$

$$+ P_{\frac{A}{B}}^{b\to s}X_B^s(0)X_B^s(0) + P_{\frac{A}{B}}^{b\to s}X_B^b(0)X_A^s(0) + P_{\frac{A}{B}}^{b\to s}X_B^b(0)X_B^s(0) + \Phi_B$$

Plotting $X_{In}^{b}(t)$ For $Ga_{0.8}In_{0.2}As/GaAs$

- We define:

$$M \equiv P_{\frac{A}{B}}^{b\to s} X_B^s(0) X_A^s(0) + P_{\frac{A}{B}}^{b\to s} X_B^s(0) X_B^s(0) + P_{\frac{A}{B}}^{b\to s} X_B^b(0) X_A^s(0) + P_{\frac{A}{B}}^{b\to s} X_B^b(0) X_B^s(0) + \Phi_B$$

- Then we will have:

$$\frac{dX_B^s(t)}{dt}$$

$$= P_{\frac{A}{B}}^{b\to s} [X_B^s(t)]^2 - P_{\frac{A}{B}}^{s\to b} [X_B^s(t)]^2 - P_{\frac{A}{B}}^{b\to s} X_B^s(t) X_A^s(0) - P_{\frac{A}{B}}^{b\to s} X_B^s(t) X_B^s(0) - P_{\frac{A}{B}}^{b\to s} X_B^s(t)(\Phi_A + \Phi_B)t$$

$$- P_{\frac{A}{B}}^{b\to s} X_B^s(t) X_B^s(0) + P_{\frac{A}{B}}^{b\to s} X_B^s(0)(\Phi_A + \Phi_B)t - P_{\frac{A}{B}}^{b\to s} X_B^s(t) X_B^b(0) + P_{\frac{A}{B}}^{b\to s} X_B^b(0)(\Phi_A + \Phi_B)t$$

$$- P_{\frac{A}{B}}^{b\to s} X_B^s(t)\Phi_B t + P_{\frac{A}{B}}^{b\to s} X_A^s(0)\Phi_B t + P_{\frac{A}{B}}^{b\to s} X_B^s(0)\Phi_B t + P_{\frac{A}{B}}^{b\to s} \Phi_B(\Phi_A + \Phi_B)t^2 + P_{\frac{A}{B}}^{s\to b} X_B^s(t) X_A^s(0)$$

$$+ P_{\frac{A}{B}}^{s\to b} X_B^s(t) X_B^s(0) + P_{\frac{A}{B}}^{s\to b} X_B^s(t)(\Phi_A + \Phi_B)t - P_{\frac{A}{B}}^{s\to b} X_B^s(t) X_A^s(0) - P_{\frac{A}{B}}^{s\to b} X_B^s(t) X_A^b(0) - P_{\frac{A}{B}}^{s\to b} X_B^s(t)\Phi_A t$$

$$+ M$$

Plotting $X_{In}^{b}(t)$ For $Ga_{0.8}In_{0.2}As/GaAs$

$$= \left(P_{\frac{A}{B}}^{b\to s} - P_{\frac{A}{B}}^{s\to b}\right)[X_B^s(t)]^2 + P_{\frac{A}{B}}^{s\to b} X_B^s(t) X_A^s(0) + P_{\frac{A}{B}}^{s\to b} X_B^s(t) X_B^s(0) - P_{\frac{A}{B}}^{s\to b} X_B^s(t) X_A^s(0) - P_{\frac{A}{B}}^{s\to b} X_B^s(t) X_A^b(0)$$

$$- P_{\frac{A}{B}}^{b\to s} X_B^s(t) X_A^s(0) - P_{\frac{A}{B}}^{b\to s} X_B^s(t) X_B^s(0) - P_{\frac{A}{B}}^{b\to s} X_B^s(t)(\Phi_A + \Phi_B)t - P_{\frac{A}{B}}^{b\to s} X_B^s(t) X_B^s(0) - P_{\frac{A}{B}}^{b\to s} X_B^s(t) X_B^b(0)$$

$$+ P_{\frac{A}{B}}^{b\to s} X_B^s(0)(\Phi_A + \Phi_B)t + P_{\frac{A}{B}}^{b\to s} X_B^b(0)(\Phi_A + \Phi_B)t - P_{\frac{A}{B}}^{b\to s} X_B^s(t)\Phi_B t + P_{\frac{A}{B}}^{b\to s} X_A^s(0)\Phi_B t + P_{\frac{A}{B}}^{b\to s} X_B^s(0)\Phi_B t$$

$$+ P_{\frac{A}{B}}^{s\to b} X_B^s(t)(\Phi_A + \Phi_B)t - P_{\frac{A}{B}}^{s\to b} X_B^s(t)\Phi_A t + P_{\frac{A}{B}}^{b\to s}\Phi_B(\Phi_A + \Phi_B)t^2 + M$$

$$= \left(P_{\frac{A}{B}}^{b\to s} - P_{\frac{A}{B}}^{s\to b}\right)[X_B^s(t)]^2$$

$$+ \left[P_{\frac{A}{B}}^{s\to b} X_A^s(0) + P_{\frac{A}{B}}^{s\to b} X_B^s(0) - P_{\frac{A}{B}}^{s\to b} X_A^s(0) - P_{\frac{A}{B}}^{s\to b} X_A^b(0) - P_{\frac{A}{B}}^{b\to s} X_A^s(0) - P_{\frac{A}{B}}^{b\to s} X_B^s(0) - P_{\frac{A}{B}}^{b\to s} X_B^s(0)\right.$$

$$\left.- P_{\frac{A}{B}}^{b\to s} X_B^b(0)\right] X_B^s(t) + P_{\frac{A}{B}}^{s\to b} X_B^s(t)(\Phi_A + \Phi_B)t - P_{\frac{A}{B}}^{b\to s} X_B^s(t)(\Phi_A + \Phi_B)t - P_{\frac{A}{B}}^{b\to s} X_B^s(t)\Phi_B t - P_{\frac{A}{B}}^{s\to b} X_B^s(t)\Phi_A t$$

$$+ P_{\frac{A}{B}}^{b\to s} X_B^s(0)(\Phi_A + \Phi_B)t + P_{\frac{A}{B}}^{b\to s} X_B^b(0)(\Phi_A + \Phi_B)t + P_{\frac{A}{B}}^{b\to s} X_A^s(0)\Phi_B t + P_{\frac{A}{B}}^{b\to s} X_B^s(0)\Phi_B t + P_{\frac{A}{B}}^{b\to s}\Phi_B(\Phi_A + \Phi_B)t^2$$

$$+ M$$

Plotting $X_{In}^{b}(t)$ For $Ga_{0.8}In_{0.2}As/GaAs$

$$= \left(P_{\frac{A}{B}}^{b\to s} - P_{\frac{A}{B}}^{s\to b}\right)[X_B^s(t)]^2$$

$$+ \Big[P_{\frac{A}{B}}^{s\to b} X_A^s(0) + P_{\frac{A}{B}}^{s\to b} X_B^s(0) - P_{\frac{A}{B}}^{s\to b} X_A^s(0) - P_{\frac{A}{B}}^{s\to b} X_A^b(0) - P_{\frac{A}{B}}^{b\to s} X_A^s(0) - P_{\frac{A}{B}}^{b\to s} X_B^s(0) - P_{\frac{A}{B}}^{b\to s} X_B^s(0)$$

$$- P_{\frac{A}{B}}^{b\to s} X_B^b(0)\Big] X_B^s(t) + \Big[P_{\frac{A}{B}}^{s\to b}(\Phi_A + \Phi_B) - P_{\frac{A}{B}}^{b\to s}(\Phi_A + \Phi_B) - P_{\frac{A}{B}}^{b\to s}\Phi_B - P_{\frac{A}{B}}^{s\to b}\Phi_A\Big] X_B^s(t)t$$

$$+ P_{\frac{A}{B}}^{b\to s} X_B^s(0)(\Phi_A + \Phi_B)t + P_{\frac{A}{B}}^{b\to s} X_B^b(0)(\Phi_A + \Phi_B)t + P_{\frac{A}{B}}^{b\to s} X_A^s(0)\Phi_B t + P_{\frac{A}{B}}^{b\to s} X_B^s(0)\Phi_B t + P_{\frac{A}{B}}^{b\to s}\Phi_B(\Phi_A + \Phi_B)t^2$$

$$+ M$$

$$= \left(P_{\frac{A}{B}}^{b\to s} - P_{\frac{A}{B}}^{s\to b}\right)[X_B^s(t)]^2$$

$$+ \Big[P_{\frac{A}{B}}^{s\to b} X_A^s(0) + P_{\frac{A}{B}}^{s\to b} X_B^s(0) - P_{\frac{A}{B}}^{s\to b} X_A^s(0) - P_{\frac{A}{B}}^{s\to b} X_A^b(0) - P_{\frac{A}{B}}^{b\to s} X_A^s(0) - P_{\frac{A}{B}}^{b\to s} X_B^s(0) - P_{\frac{A}{B}}^{b\to s} X_B^s(0)$$

$$- P_{\frac{A}{B}}^{b\to s} X_B^b(0)\Big] X_B^s(t) + \Big[P_{\frac{A}{B}}^{s\to b}(\Phi_A + \Phi_B) - P_{\frac{A}{B}}^{b\to s}(\Phi_A + \Phi_B) - P_{\frac{A}{B}}^{b\to s}\Phi_B - P_{\frac{A}{B}}^{s\to b}\Phi_A\Big] X_B^s(t)t$$

$$+ \Big[P_{\frac{A}{B}}^{b\to s} X_B^s(0)(\Phi_A + \Phi_B) + P_{\frac{A}{B}}^{b\to s} X_B^b(0)(\Phi_A + \Phi_B) + P_{\frac{A}{B}}^{b\to s} X_A^s(0)\Phi_B + P_{\frac{A}{B}}^{b\to s} X_B^s(0)\Phi_B\Big] t + P_{\frac{A}{B}}^{b\to s}\Phi_B(\Phi_A + \Phi_B)t^2$$

$$+ M$$

Plotting $X_{In}^{b}(t)$ For $Ga_{0.8}In_{0.2}As/GaAs$

$$= \left(P_{\frac{A}{B}}^{b\to s} - P_{\frac{A}{B}}^{s\to b}\right)[X_B^s(t)]^2$$

$$+ \left[P_{\frac{A}{B}}^{s\to b} X_A^s(0) + P_{\frac{A}{B}}^{s\to b} X_B^s(0) - P_{\frac{A}{B}}^{s\to b} X_A^s(0) - P_{\frac{A}{B}}^{s\to b} X_A^b(0) - P_{\frac{A}{B}}^{b\to s} X_A^s(0) - P_{\frac{A}{B}}^{b\to s} X_B^s(0) - P_{\frac{A}{B}}^{b\to s} X_B^s(0)\right.$$

$$\left. - P_{\frac{A}{B}}^{b\to s} X_B^b(0)\right] X_B^s(t) + \left[P_{\frac{A}{B}}^{s\to b}(\Phi_A + \Phi_B) - P_{\frac{A}{B}}^{b\to s}(\Phi_A + \Phi_B) - P_{\frac{A}{B}}^{b\to s}\Phi_B - P_{\frac{A}{B}}^{s\to b}\Phi_A\right] X_B^s(t)t$$

$$+ \left[P_{\frac{A}{B}}^{b\to s} X_B^s(0)(\Phi_A + \Phi_B) + P_{\frac{A}{B}}^{b\to s} X_B^b(0)(\Phi_A + \Phi_B) + P_{\frac{A}{B}}^{b\to s} X_A^s(0)\Phi_B + P_{\frac{A}{B}}^{b\to s} X_B^s(0)\Phi_B\right] t$$

$$+ P_{\frac{A}{B}}^{b\to s}\Phi_B(\Phi_A + \Phi_B)t^2 + M$$

$$= \left(P_{\frac{A}{B}}^{b\to s} - P_{\frac{A}{B}}^{s\to b}\right)[X_B^s(t)]^2$$

$$+ \left[P_{\frac{A}{B}}^{s\to b} X_B^s(0) - P_{\frac{A}{B}}^{s\to b} X_A^b(0) - P_{\frac{A}{B}}^{b\to s} X_A^s(0) - 2P_{\frac{A}{B}}^{b\to s} X_B^s(0) - P_{\frac{A}{B}}^{b\to s} X_B^b(0)\right] X_B^s(t)$$

$$+ \left[P_{\frac{A}{B}}^{s\to b}\Phi_B - P_{\frac{A}{B}}^{b\to s}(\Phi_A + 2\Phi_B)\right] X_B^s(t)t$$

$$+ \left[P_{\frac{A}{B}}^{b\to s} X_B^s(0)(\Phi_A + 2\Phi_B) + P_{\frac{A}{B}}^{b\to s} X_B^b(0)(\Phi_A + \Phi_B) + P_{\frac{A}{B}}^{b\to s} X_A^s(0)\Phi_B\right] t + P_{\frac{A}{B}}^{b\to s}\Phi_B(\Phi_A + \Phi_B)t^2 + M$$

Plotting $X_{In}^{b}(t)$ For $Ga_{0.8}In_{0.2}As/GaAs$

- We define:

$$Z \equiv P_{\frac{A}{B}}^{b\to s} - P_{\frac{A}{B}}^{s\to b}$$

$$L \equiv P_{\frac{A}{B}}^{s\to b} X_B^s(0) - P_{\frac{A}{B}}^{s\to b} X_A^b(0) - P_{\frac{A}{B}}^{b\to s} X_A^s(0) - 2P_{\frac{A}{B}}^{b\to s} X_B^s(0) - P_{\frac{A}{B}}^{b\to s} X_B^b(0)$$

$$W \equiv P_{\frac{A}{B}}^{s\to b} \Phi_B - P_{\frac{A}{B}}^{b\to s}(\Phi_A + 2\Phi_B)$$

$$Y \equiv P_{\frac{A}{B}}^{b\to s} X_B^s(0)(\Phi_A + 2\Phi_B) + P_{\frac{A}{B}}^{b\to s} X_B^b(0)(\Phi_A + \Phi_B) + P_{\frac{A}{B}}^{b\to s} X_A^s(0)\Phi_B$$

$$T \equiv P_{\frac{A}{B}}^{b\to s} \Phi_B(\Phi_A + \Phi_B)$$

$$\frac{dX_B^s(t)}{dt} = Z[X_B^s(t)]^2 + LX_B^s(t) + WX_B^s(t)t + Yt + Tt^2 + M$$

- In the above equation, “Z”, “L”, “W”, “Y”, “T”, and “M” are constant and we have their values.

Plotting $X_{In}^{b}(t)$ For $Ga_{0.8}In_{0.2}As/GaAs$

- In order to find these constants, based on APL_66_52, we have:

$$v_{A/B}^{b\to s} = 10^{13}\ s^{-1}$$

$$E_{\frac{A}{B}}^{b\to s} = 1.8\ eV$$

- Since we have In segregation then E_s for In is positive and based on APL_66_52 we have:

$$E_s = 0.2\ eV$$

$$E_s + E_{\frac{A}{B}}^{b\to s} = E_{\frac{A}{B}}^{s\to b} = 2.0\ eV$$

- **T:** Growth temperature is T=500^{o}C or 773.15 K.
- k_B=1.3806485X10^{-23} J/K =8.617333262X10^{-5} eV/K.

Plotting $X_{In}^{b}(t)$ For $Ga_{0.8}In_{0.2}As/GaAs$

- Based on the definition of $X_B^s(n-1)$ is the same as $X_B^s(0)$ and "B" is representative of "In" atoms and "A" is the representative of "Ga" atoms.
- $\boldsymbol{X_B^s(0)}$: The $X_B^s(0)$ is zero for the first and all upcoming MLs because whatever was left from the previous ML is the bulk material for the next ML.
- $\boldsymbol{X_B^b(0)}$: For $X_B^b(0)$ or $X_{In}^b(0)$ is the percentage of "In" atoms in the bulk from previous ML. For the first ML this is zero however for next MLs its value comes from $\boldsymbol{X_B^s(t)}$**.**
- $\boldsymbol{X_A^s(0)}$: Since surface material from the previous layer is the surface material for the upcoming layers, $\boldsymbol{X_A^s(0)}$ is zero for all layers.
- $\boldsymbol{X_A^b(0)}$: This is the surface percentage of Ga from previous layer.

Plotting $X_{In}^{b}(t)$ For $Ga_{0.8}In_{0.2}As/GaAs$

- If we compare the paper and our note, we have:

$$P_1 \rightarrow P_{\frac{A}{B}}^{b\rightarrow s}$$

$$P_1 \rightarrow P_{\frac{A}{B}}^{s\rightarrow b}$$

$$X_{In\,s}(t) = X_B^s(t)$$

$$X_{In\,b}(t) = X_B^b(t)$$

$$X_{Ga\,s}(t) = X_A^s(t)$$

$$X_{Ga\,b}(t) = X_A^b(t)$$

Plotting $X_{In}^{b}(t)$ For $Ga_{0.8}In_{0.2}As/GaAs$

- After we start the flux, t≠0, then since we have a growth rate of 1 ML/s that means for growing 1 ML of $Ga_{0.8}In_{0.2}As$ per second we need to have “Ga” growth rate of 0.8 ML/s and “In” growth rate of 0.2 ML/s. Therefore:

$$\Phi_A = \Phi_{Ga} = 0.8$$

$$\Phi_B = \Phi_{In} = 0.2$$

Mathematica Programing

```
kB = 8.617333262*10^-5 (*eVK-1*);
Tempe = 773.15 (*K which is 500 C*);
E1 = 1.8(*eV*);
E2 = 2.0 (*eV*);

XsIn0 = 0; (*The surface from the previous ML is our bulk0 on this ML therefore, XsIn0=0*)
XsGa0 = 0;(*The surface from the previous ML is our bulk0 on this ML therefore, XsGa0=0*)
XbIn0 = 0.1886450487493853`;(*The surface from the previous ML is our bulk0 on this ML*)
XbGa0 = 0.8113549512506146`;(*The surface from the previous ML is our bulk0 on this ML*)
fGa = 0.8 (*ML/s*);
fIn = 0.2 (*ML/s*);

PbsAB = (10^13) Exp[-(E1)/(kB * Tempe )];
PsbAB = (10^13) Exp[-(E2)/(kB * Tempe )];
Z = PbsAB - PsbAB;
L = PsbAB * XsIn0 - PsbAB *XbGa0 - PbsAB * XsGa0 - 2 PbsAB * XsIn0 - PbsAB*XbIn0;
W = PsbAB *fIn - PbsAB * (fGa + 2*fIn);
Y = PbsAB * XsIn0 * (fGa + 2 fIn) + PbsAB *XbIn0* (fGa + fIn) + PbsAB * XsGa0 * fIn;
T = PbsAB * fIn*(fGa + fIn);
M = PbsAB*XsIn0 * XsGa0 + PbsAB *XsIn0 * XsIn0 + PbsAB *XbIn0 * XsGa0 + PbsAB *XbIn0 * XsIn0 + fIn;
Clear[t] (*Changig t as a constant to a variable*);
```

Mathematica Programing

- The previous Mathematica program is for the first ML.
- Note based on the APL_66_52 paper, this is after the first second of the deposition; therefore, we already have our first ML of GaInAs on the surface.

Plotting $X_{In}^{b}(t)$ For $Ga_{0.8}In_{0.2}As/GaAs$

- From Mathematical programing we will have:

$X_{In}^{s}(t)$ = -(0.0569424 (**C[1]** (-6.75507 E^(-1.13383 t - 9.23996 t^2) HermiteH[-2.23464, -0.358887 + 2.73565 t] + E^(-1.13383 t - 9.23996 t^2) (-1.13383 - 18.4799 t) HermiteH[-1.23464, -0.358887 + 2.73565 t]) + E^(-1.13383 t - 9.23996 t^2) (-1.13383 - 18.4799 t) Hypergeometric1F1[0.617318, 1/2, (-0.358887 + 2.73565 t)^2] + 6.75507 E^(-1.13383 t - 9.23996 t^2) (-0.358887 + 2.73565 t) Hypergeometric1F1[1.61732, 3/2, (-0.358887 + 2.73565 t)^2]))/(E^(-1.13383 t - 9.23996 t^2) **C[1]** HermiteH[-1.23464, -0.358887 + 2.73565 t] + E^(-1.13383 t - 9.23996 t^2) Hypergeometric1F1[0.617318, 1/2, (-0.358887 + 2.73565 t)^2]) Eq-21

- The above answer has a constant, “C[1]”, and a variable, “t”.
- The “C[1]” constant can be determined from boundary condition. Based on the $X_{In}^{s}(0)$ boundary condition at t=0 we can find “C[1]”.

Plotting $X_{In}^{b}(t)$ For $Ga_{0.8}In_{0.2}As/GaAs$

- In Eq-21, we put t=0, then we will get:

$$X_{In}^{s}(0) = -\frac{0.0569424454611059(-4.1119523879687065-7.480764625024727\boldsymbol{C}[\mathbf{1}])}{1.1705758337776504+1.343826525956692\boldsymbol{C}[\mathbf{1}]} \qquad \text{Eq-22}$$

Or

$$C[1] = -0.54967$$

- Now we can put the “C[1]” value from Eq-23 into Eq-21 and get the final form of $X_{In}^{s}(t)$.

Plotting $X_{In}^{b}(t)$ For $Ga_{0.8}In_{0.2}As/GaAs$

$X_{In}^{s}(t)$ = -(0.05694244546111059` (**-0.5496700663744135`** (-6.755073433330022` E^(-1.1338303443619282` t - 9.239962590038246` t^2) HermiteH[-2.23463588135501`, -0.35888670848026455` + 2.7356541047212826` t] + E^(-1.1338303443619282` t -9.239962590038246` t^2) (-1.1338303443619282` - 18.479925180076492` t) HermiteH[-1.2346358813550098`, -0.35888670848026455` + 2.7356541047212826` t]) + E^(-1.1338303443619282` t - 9.239962590038246` t^2) (-1.1338303443619282` - 18.479925180076492` t) Hypergeometric1F1[0.6173179406775049`, 1/2, (-0.35888670848026455` + 2.7356541047212826` t)^2] + 6.755073433330022` E^(-1.1338303443619282` t - 9.239962590038246` t^2) (-0.35888670848026455` + 2.7356541047212826` t) Hypergeometric1F1[1.617317940677505`, 3/2, (-0.35888670848026455` + 2.7356541047212826` t)^2]))/(E^(-1.1338303443619282` t -9.239962590038246` t^2) *(**-0.5496700663744135`**) \HermiteH[-1.2346358813550098`, -0.35888670848026455` + 2.7356541047212826` t] + E^(-1.1338303443619282` t - 9.239962590038246` t^2)

Hypergeometric1F1[0.6173179406775049`, 1/ 2, (-0.35888670848026455` + 2.7356541047212826` t)^2]) Eq-24

- Therefore, from Eq-24 we can calculate $X_{In}^{s}(t)$ which is "In" concentration at the surface after time "t".
- Now we need to know how much is the $X_{In}^{s}(t)$ at t=1.
- We put t=1 in Eq-24 and we get:

$$X_{In}^{s}(1) = 0.359434$$

This means that after passing 2 seconds or second ML, the "In" concentration at the surface is about 0.359%.

Plotting $X_{In}^{b}(t)$ For $Ga_{0.8}In_{0.2}As/GaAs$

- This surface “In” concentration will change after we have more layers.
- What does not change is the $X_{In}^{b}(1)$ which is “In” concentration in the bulk after passing 2 seconds (t=1).
- $X_{In}^{b}(t)$ is what was drawn in Fig 2 (a) in APL_66_52 article.
- In order we calculate $X_{In}^{b}(t)$ we use Eq-18:

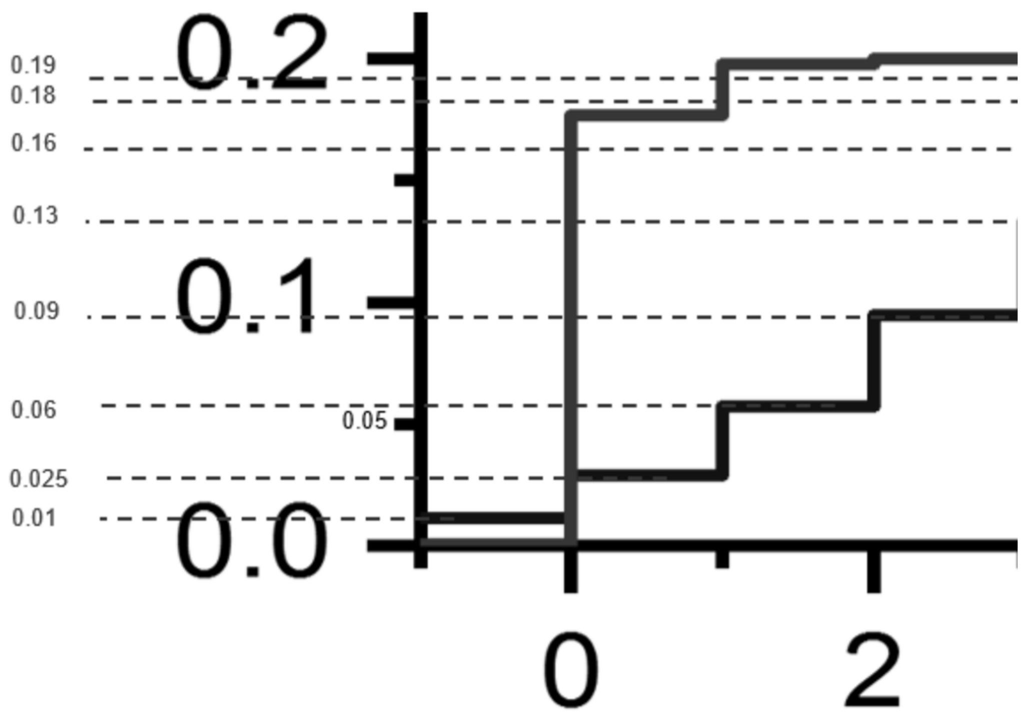
$$X_{B}^{b}(t) = -X_{B}^{s}(t) + X_{B}^{s}(0) + X_{B}^{b}(0) + \Phi_{B} t$$

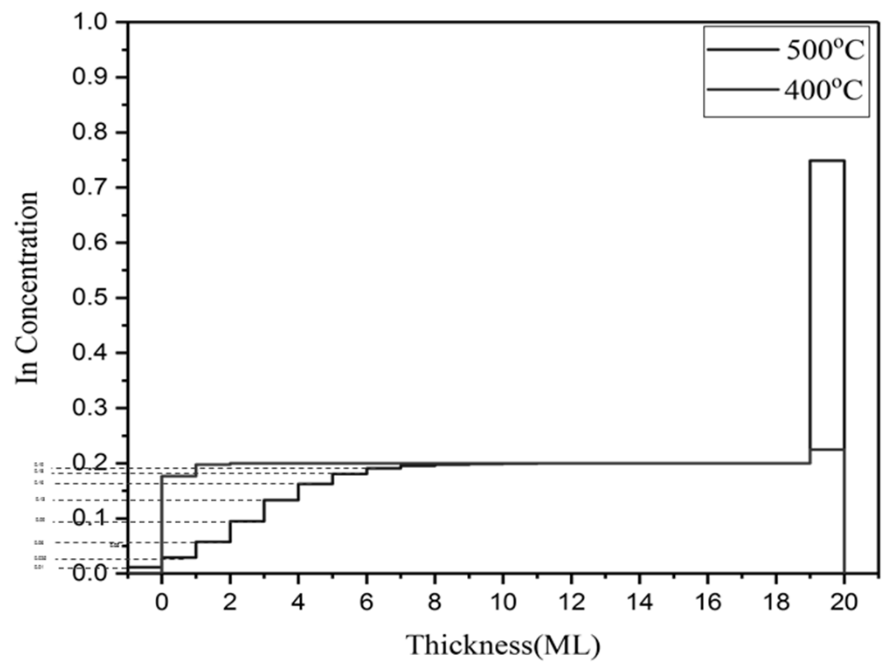
$$X_{In}^{b}(t) = -X_{In}^{s}(t) + X_{In}^{s}(0) + X_{In}^{b}(0) + \Phi_{In} t$$

Plotting $X_{In}^{b}(t)$ For $Ga_{0.8}In_{0.2}As/GaAs$

$$X_{In}^{b}(1) = 0.0292111$$

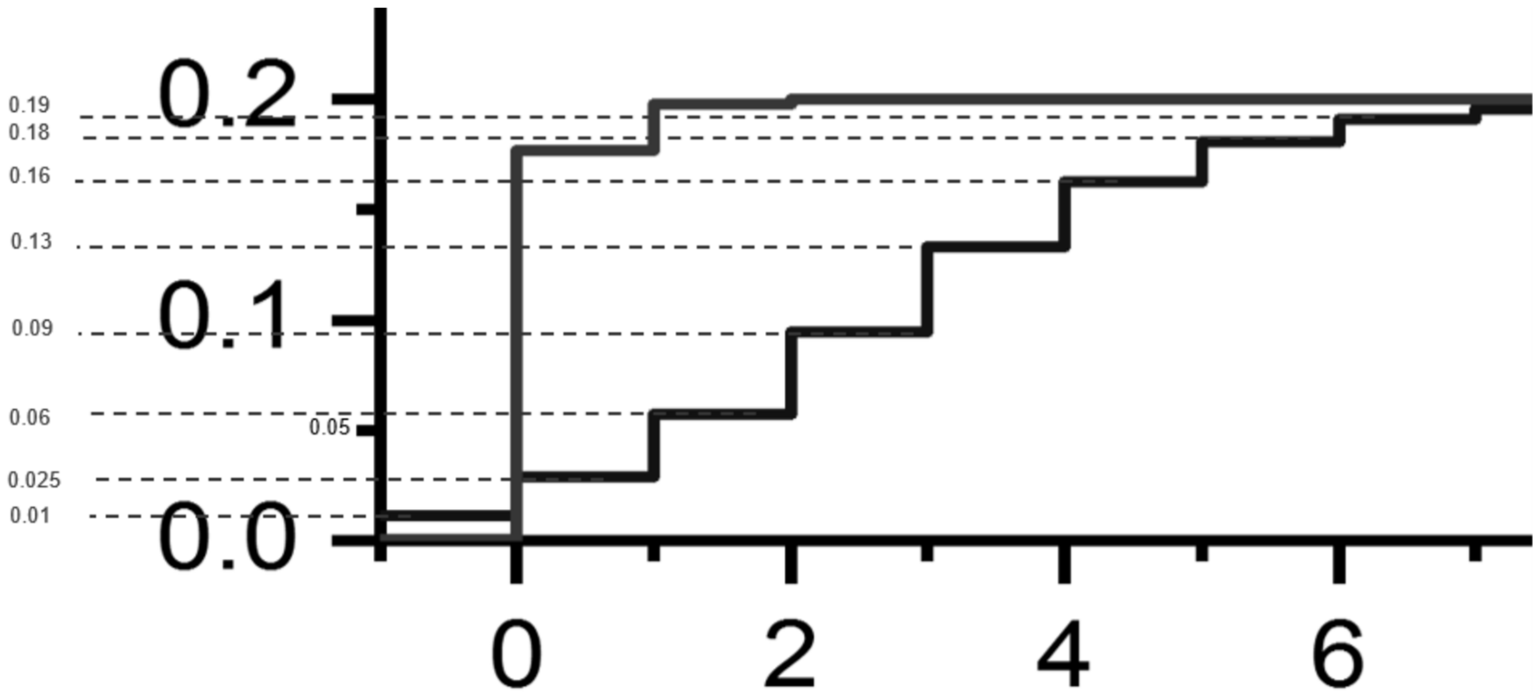
- Therefore, in Fig 2(a) after passing 2 seconds or at t=1 the $X_{In}^{b}(1)$ or "In" concentration in the bulk is about 0.029%.





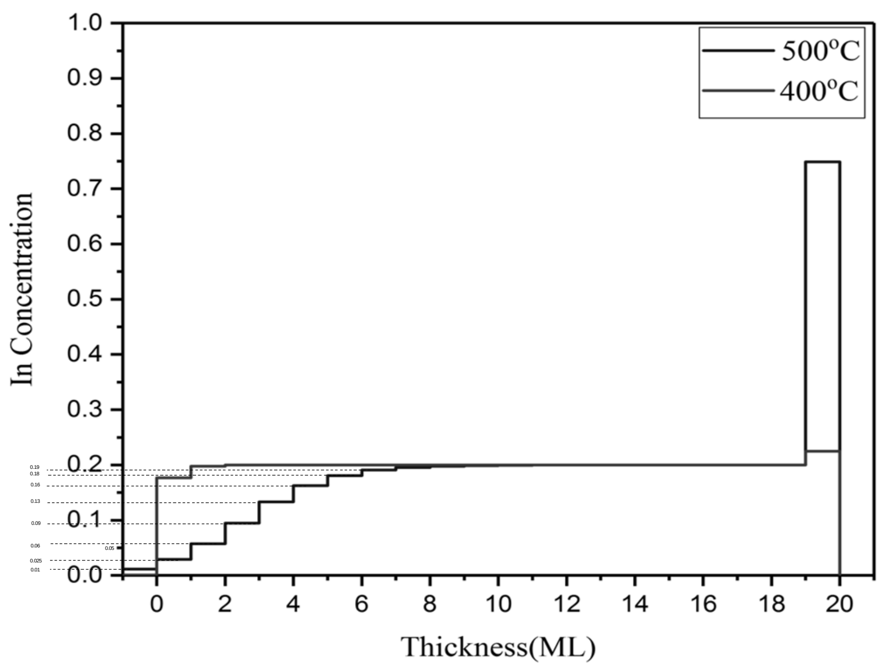
Verification of Simulated Atoms Profiles Due to The Segregation

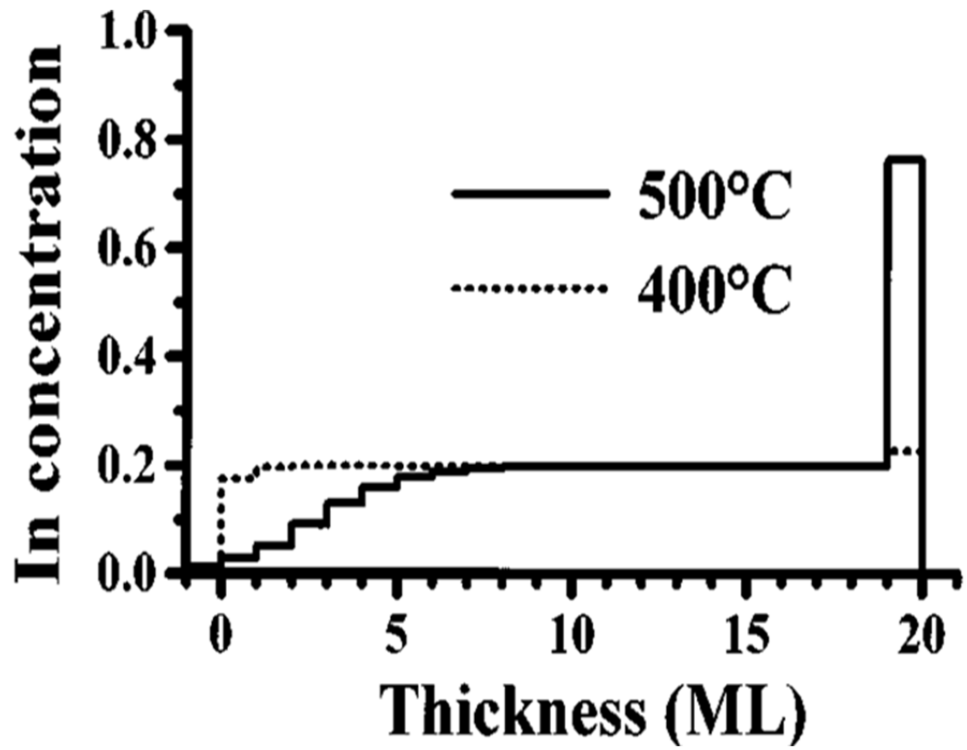
ML	0.00	1.00	2.00	3.00	4.00	5.00	6.00	7.00	8.00	9.00	10.00
t (s)	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
XsIn0	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
XsGa0	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
XbIn0	0.00	0.19	0.36	0.50	0.61	0.68	0.72	0.74	0.75	0.75	0.75
XbGa0	1.00	0.81	0.64	0.50	0.39	0.32	0.28	0.26	0.25	0.25	0.25
XsInt	0.19	0.36	0.50	0.61	0.68	0.72	0.74	0.75	0.75	0.75	0.75
XsGat	0.81	0.64	0.50	0.39	0.32	0.28	0.26	0.25	0.25	0.25	0.25
XbInt	0.01	0.03	0.06	0.09	0.13	0.16	0.18	0.19	0.20	0.20	0.20
XbGat	0.99	0.97	0.94	0.91	0.87	0.84	0.82	0.81	0.80	0.80	0.80



Verification of Simulated Atoms Profiles Due to The Segregation

ML	0.00	1.00	2.00	3.00	4.00	5.00	6.00	7.00	8.00	9.00	10.00
t (s)	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
XsIn0	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
XsGa0	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
XbIn0	0.00	0.19	0.36	0.50	0.61	0.68	0.72	0.74	0.75	0.75	0.75
XbGa0	1.00	0.81	0.64	0.50	0.39	0.32	0.28	0.26	0.25	0.25	0.25
XsInt	0.19	0.36	0.50	0.61	0.68	0.72	0.74	0.75	0.75	0.75	0.75
XsGat	0.81	0.64	0.50	0.39	0.32	0.28	0.26	0.25	0.25	0.25	0.25
XbInt	0.01	0.03	0.06	0.09	0.13	0.16	0.18	0.19	0.20	0.20	0.20
XbGat	0.99	0.97	0.94	0.91	0.87	0.84	0.82	0.81	0.80	0.80	0.80





Verification of Simulated Atoms Profiles Due to The Segregation

- When we considering "Ga" segregation, based on KM, barrier energy of atoms and growth conditions such as growth temperature and growth rate, can alter the segregation length which is the maximum "Ga" segregation length.
- For example, "In" segregation length in AlSb/InSb system has been reported to be 15 ML at growth temperature of 520^{o}C with a growth rate of 0.5 ML/s.