

Final project

FFT 8-Points Radix-2

Presented by: Ali Mohamed Mohamed Saied

Agenda

- ▶ Breif about FFT 8-points Radix-2 arch.
- ▶ System modeling.
- ▶ RTL design.
- ▶ Verification.
- ▶ ASIC implementation and results.
- ▶ FPGA implementation and results.

Breif about FFT 8-points Radix-2 arch.

► FFT importance:

- Can be used to obtain the frequency domain view of signals which gives a different point of view and carries information don't present in time domain.
- In modern DSP applications (e.g. DSP IP for OFDM Modulators/Demodulators , Range–Doppler Processing, & Audio codecs with noise reduction).

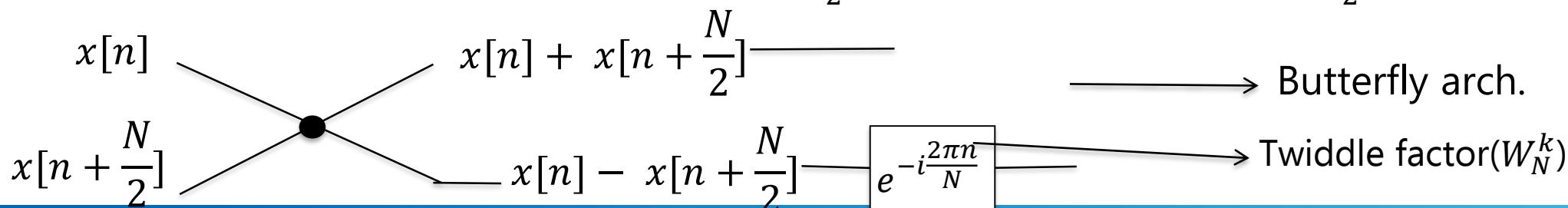
► DFT equation:

$$\tilde{x}[k] = \sum_{n=0}^{N-1} x[n] e^{-i\frac{2\pi kn}{N}}, k = 0, 1, 2, \dots, N-1 \longrightarrow O(N^2)$$

► FFT equation:

$$\tilde{x}[k] = DFT_{N/2}[x_1[n]] + DFT_{N/2}[x_2[n]]e^{-i\frac{2\pi kn}{N}}, k = 0, 1, 2, \dots, N-1. \longrightarrow O(\frac{N}{2} \log N)$$

$$x_1[n] = x[n] + x[n + \frac{N}{2}] \quad \& \quad x_2[n] = x[n] - x[n + \frac{N}{2}]$$

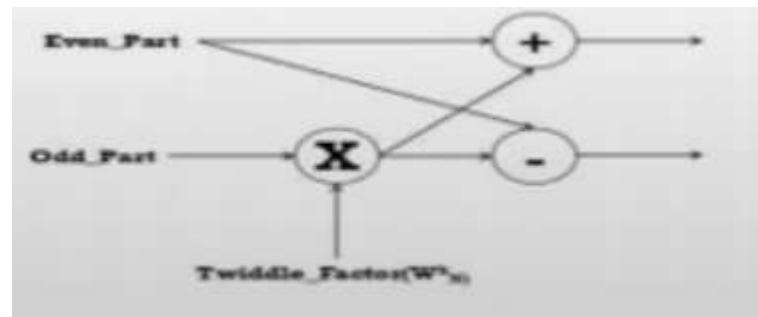


Breif about FFT 8-points Radix-2 arch. cont.

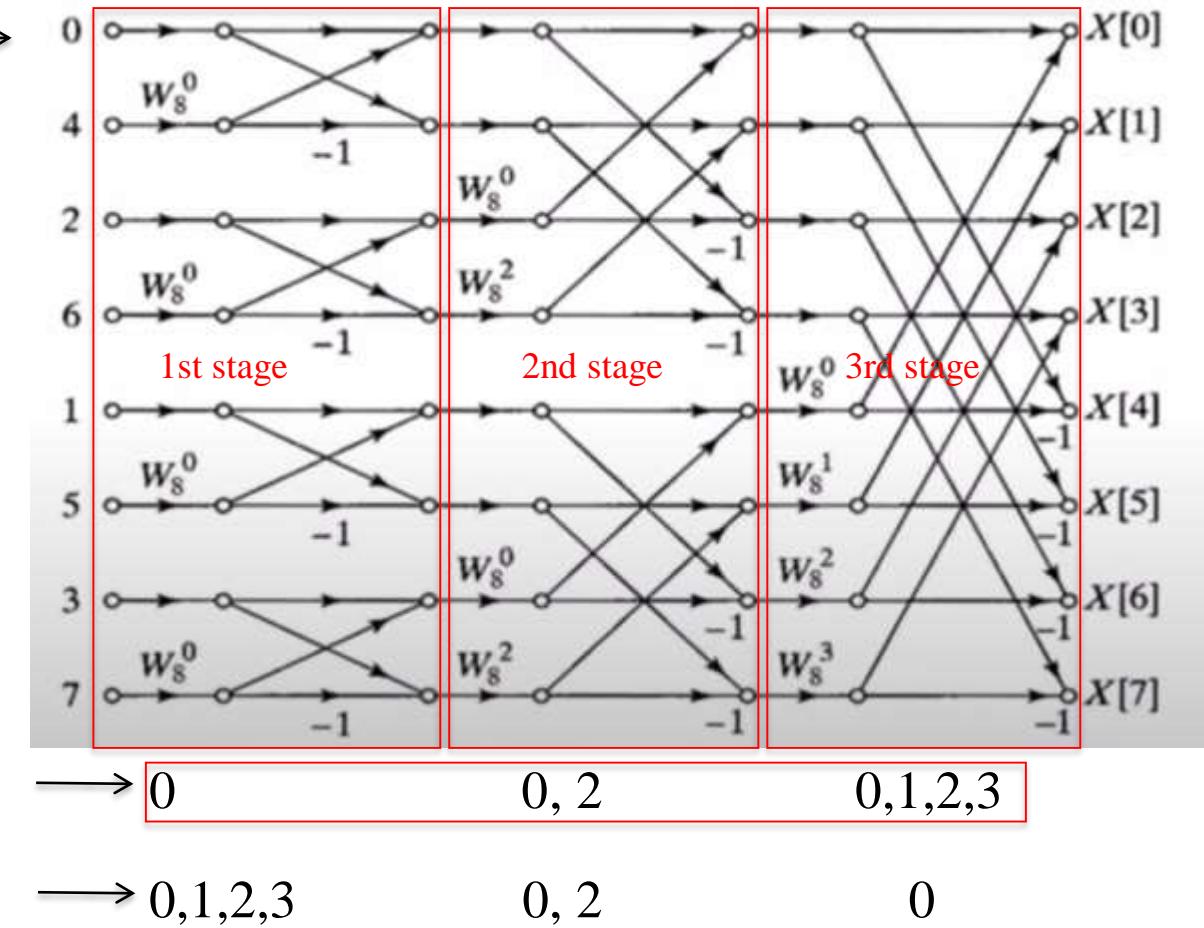
- The used arch. in our RTL can be obtained by expanding the FFT equation for 8-point (8 samples) case and simplify it to be implemented using 2-point blocks. Therefore:

$$\#stages = \log_2 8 = 3 \text{ stages}$$

- Finally, the arch. is: 
- Basic building block is **MAC block**:

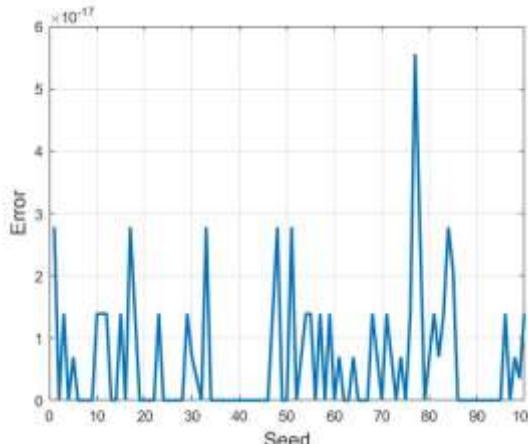


- Required twiddle factors are only: $W_8^0, W_8^1, W_8^2, \text{ and } W_8^3$
- We have two forms:
 - Decimation In Time(DIT):Input is bit-reversed, output is normal order.
 - Decimation In Freq.(DIF):Input is normal order, output is bit-reversed.

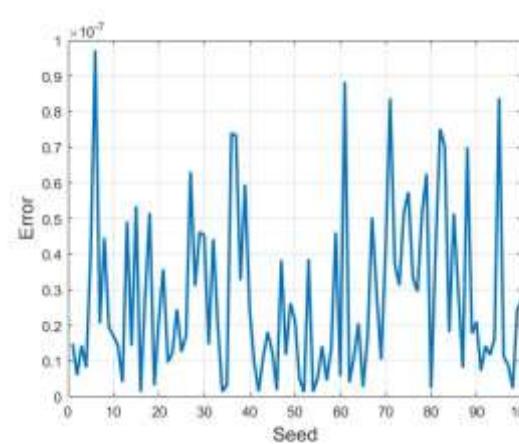


System modeling

- ▶ First write down the chosen architecture exactly similar to signal flow graph. Isolate core algorithm.
- ▶ Run double, single, & fixed-point analysis successively such that in each case calculate the error and do instrumentation. Run is done for **100 seed** and assumed to have **real inputs only**.
- ▶ During the double case run, from its instrumentation results extract the integer bit width required for each signal in the flow graph.
- ▶ With respect to the **twiddle factor**: It's assumed to have **2 integer bits**.
- ▶ Error for double and single runs:
 - double $\sim 10^{-17}$
 - single $\sim 10^{-7}$



double

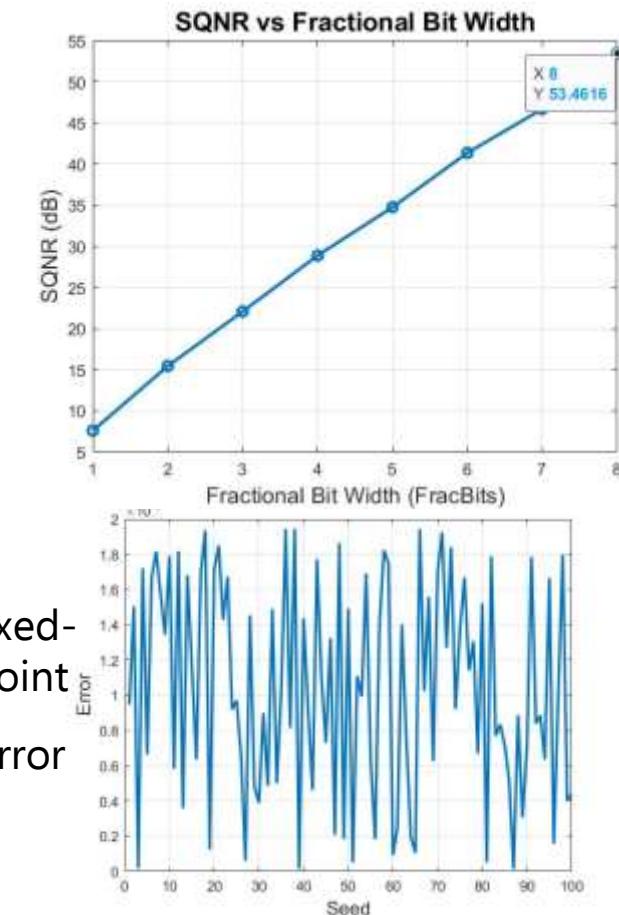
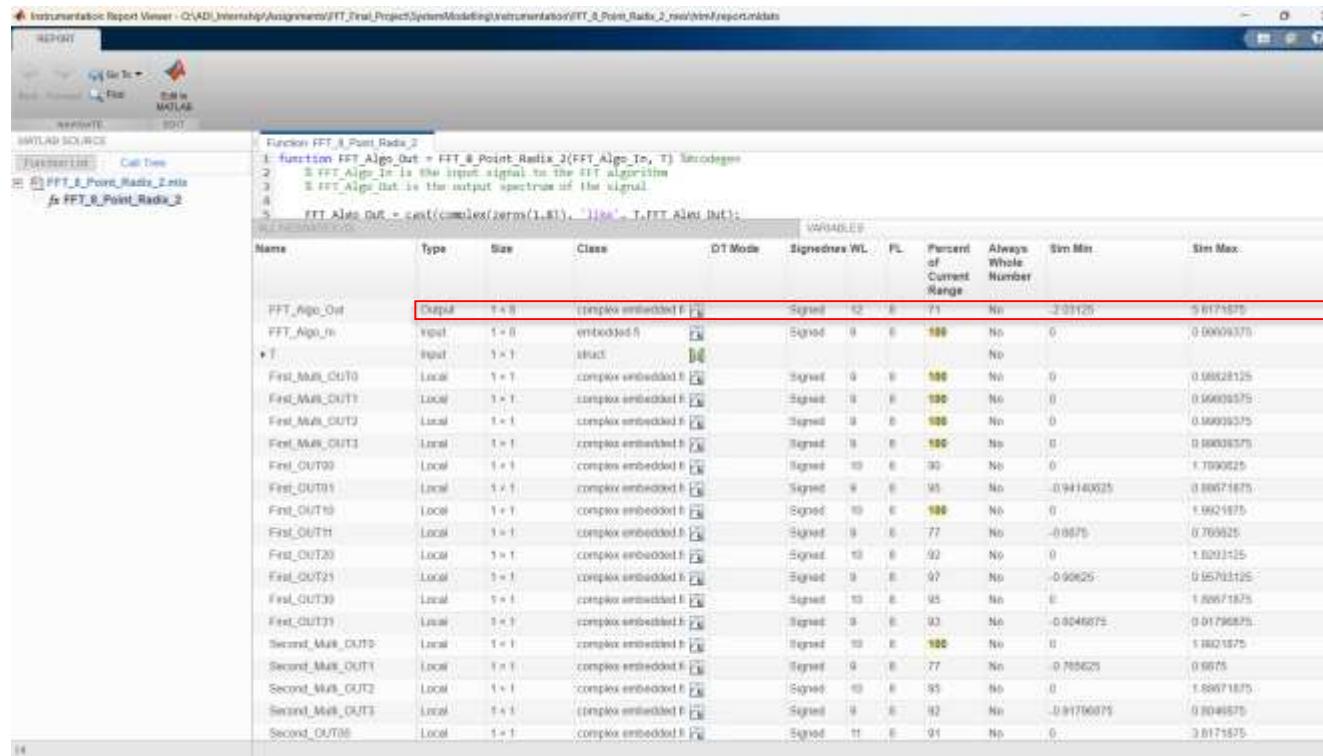


single

signal ID	Integer bit width
First_INxx_*	4
First_Multi_OUTx_*	1
First_OUTx0_*	2
First_OUTx1_*	1
Second_Multi_OUT(0,2)	2
Second_Multi_OUT(1,3)	1
Second_OUT(00,20)	3
Second_OUT(01,21)	2
Second_OUT(10,11,30,31)	1
Third_Multi_OUT0	3
Third_Multi_OUT(1,2,3)	2
Third_OUTxx_*	4

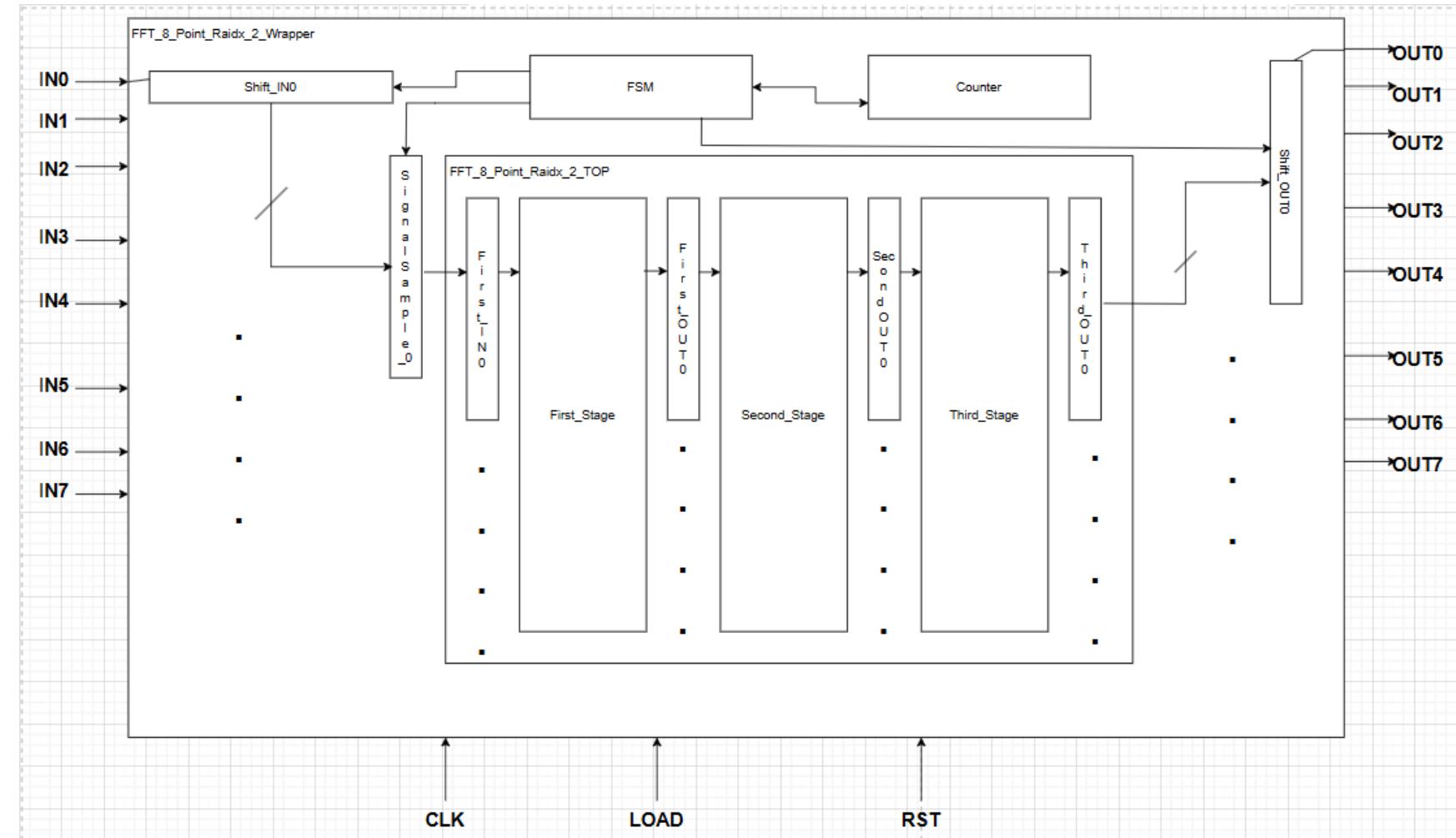
System modeling cont.

- In order to determine the fractional bit width, plot the SQNR vs the fractional bit width:
 - Choose **SQNR = 53.4616 dB @ unified fractional bit width for all signals equals 8 bits.**
 - Therefore the widest signals in the flow graph are the output signals with width = $4 + 8 = 12$ bits.
- Now run the first step in the case of fixed-point after adding the chosen fractional bit width to observe any unwanted difference.
- Error for fixed-point case with 8-bits fractional bit width $\sim 10^{-3}$
- Fixed-point instrumentation results:



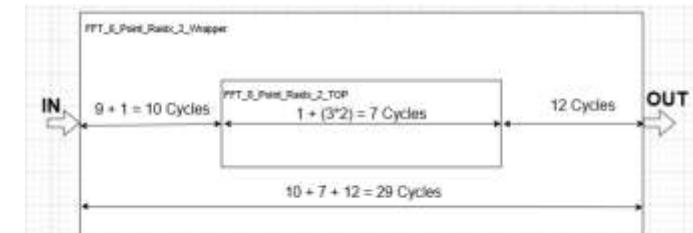
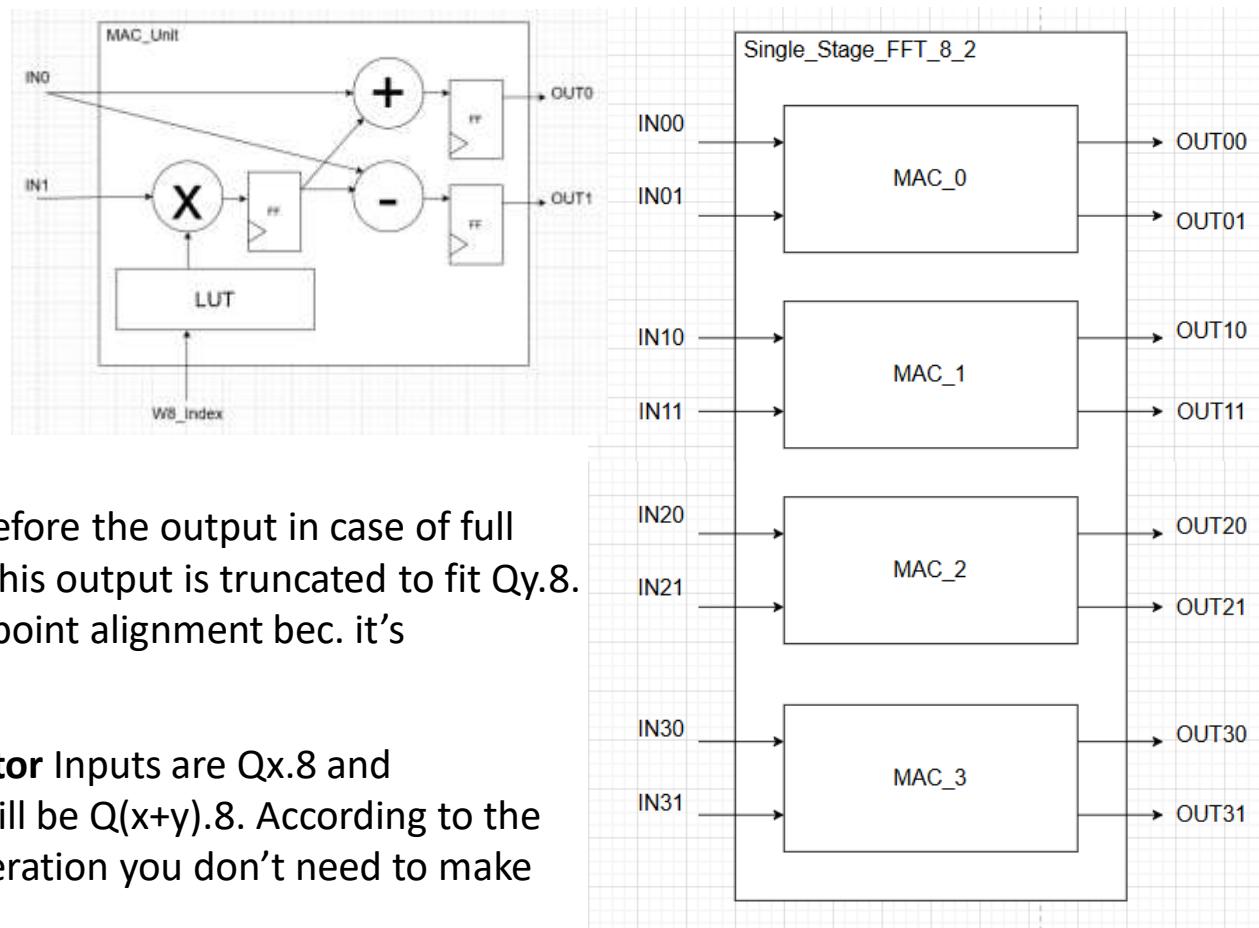
RTL design

- ▶ Design architecture: Serial-IN-Serial-OUT
- ▶ Only the inputs of the first sample bits (9-bits) are shown.
- ▶ Note: Each of the drawn I/P, O/P, & internal signals has to splitted to real signal and imaginary signal (Not drawn).
- ▶ Therefore we will have: $8*2+3 = 19$ I/P Port (bits).
- ▶ And $8*2 = 16$ O/P ports (bits).
- ▶ All inputs are set to be fixed-point representations of notation Qx.8, while outputs are also fixed point representation of notation Q4.8.



RTL design cont.

- ▶ Single stage has 4 MAC blocks:
- ▶ Single stage has the following internal blocks:
 - Single Complex multiplier
 - Single Complex adder.
 - Single Complex subtractor.
 - LUT (Memory) to store twiddle factors. (I should make one LUT for all MACs[Future work]), but the optimizer of the synthesis tool may handle this.
- ▶ Complex multiplier: Inputs are Qx.8 and Q2.10(Twiddle facotr), therefore the output in case of full precision will be Q(x+2).18. According to the system modeling step this output is truncated to fit Qy.8. During operation you don't need to make virtual fractional decimal point alignment bec. it's multiplication.
- ▶ Complex adder and subtractor: **Single module with operation selector** Inputs are Qx.8 and Qy.8(Twiddle facotr), therefore the output in case of full precision will be Q(x+y).8. According to the system modeling step this output is truncated to fit Qz.8. During operation you don't need to make virtual fractional decimal point alignment bec. it's multiplication.
- ▶ **Latency (The number of cycles from the first input sample to the first output sample) = 29 cycles.**



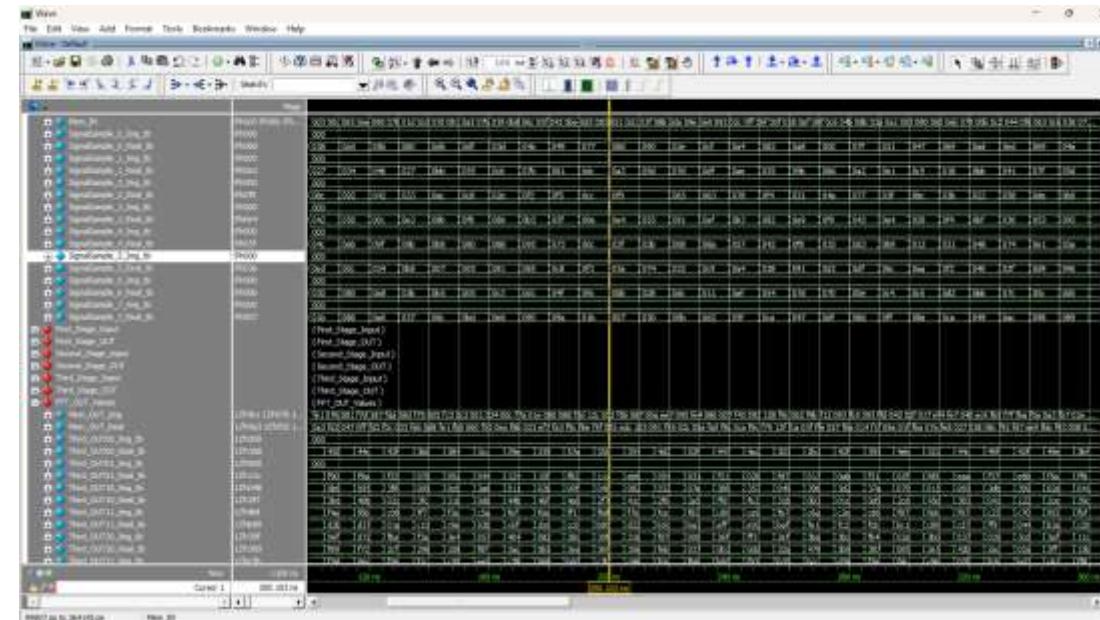
- ▶ Verification based on a normal testbench to verify the system operation and compare between the system's output with the golden model output obtained by Matlab at the system modeling step.
- ▶ Inputs come from the randomization process at matlab are saved as fixed-point Q1.8 in a text file (FFT_Golden_In_bin.txt) and read inside testbench using “readmemb” system function. Also the outputs of the golden model of Matlab **real and imaginary parts**.
- ▶ Error calculation:
 - First = $\text{abs}(\text{FFT_Module_Real_OUT} - \text{FFT_Golden_Real_OUT})$
 - Second = $\text{abs}(\text{FFT_Module_Img_OUT} - \text{FFT_Golden_Img_OUT})$
 - $\text{Error_In_Magnitude} = \sqrt{\text{First}^2 + \text{Second}^2}$ \implies Error in one sample
 - $\text{Single_Seed_Error} = \sum_0^7 \text{Error_In_Magnitude}$ \implies Error in single seed
 - $\text{Error_All_Seeds} = \sum_0^7 \text{Single_Seed_Error}$
 - $\text{Average_Error_All_Seeds} = \text{Error_All_Seeds} / 100$

Verification

► Verification results:

- Waveform: No unknowns
- Transcript:
 - **Average_Error_All_Seeds = 2.966938**
 - For single sample = Average_Error_All_Seeds/8 = 0.371

```
# Average Error for seed no.      97 = 2.543193
# Average Error for seed no.      98 = 1.591838
# Average Error for seed no.      99 = 1.943823
# Total Average Error across simulation = 2.966938
# ** Note: $stop : FFT_8_Point_Radix_2_tb.sv(370)
#   Time: 1100 ns Iteration: 1 Instance: /FFT_8_Point_Radix_2_tb
# Break in Module FFT_8_Point_Radix_2_tb at FFT_8_Point_Radix_2_tb.sv line 370
```



▪ Code coverage report:

► Future work:

Class-based verification environment

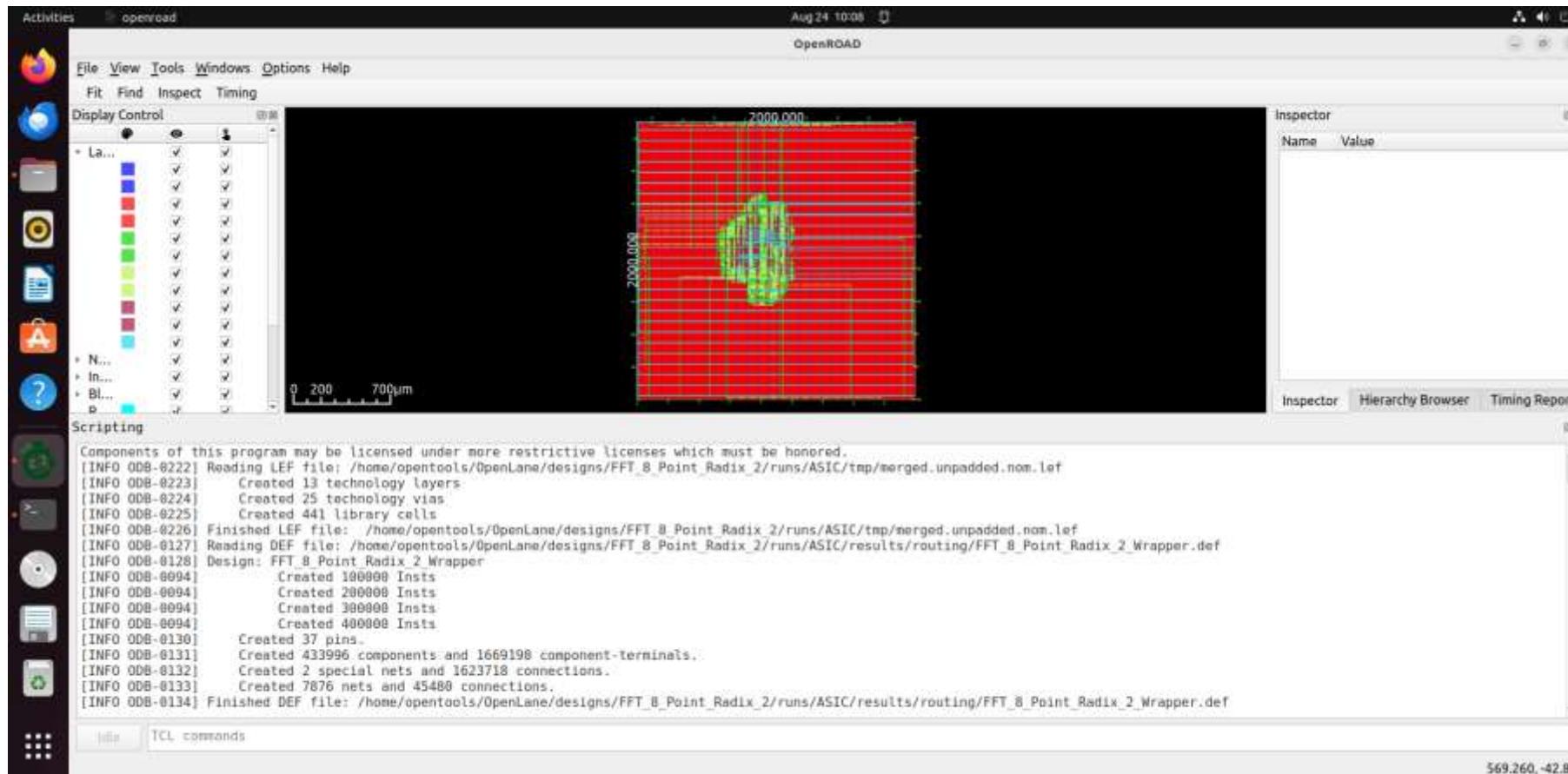
Coverage Report Totals BY INSTANCES: Number of Instances 65						
	Enabled Coverage	Bins	Hits	Misses	Weight	Coverage
3	-----	----	----	-----	-----	-----
4	-----	----	----	-----	-----	-----
5	Branches	156	106	50	1	67.94%
6	Conditions	5	4	1	1	80.00%
7	Statements	666	569	97	1	85.43%
8	Toggles	12556	7068	5488	1	56.29%
9	Total coverage (filtered view):	72.41%				
10						
11						

ASIC implementation

- ▶ OpenLane tool (open-source) is used. Therefore by default the **technology is sky130A PDK and the sky130_fd_sc_hd standard cell library**
- ▶ Max die area is 2mm * 2mm.
- ▶ Constrained clock period is 18ns.
- ▶ Final output die from OpenLane: Post-route utilization = 4% @ placement

=====

Design area 155580 μm^2 4% utilization.



ASIC implementation cont.

- ▶ Worst Negative Slack (WNS):

```
=====
report_worst_slack -max (Setup)
=====
worst slack 11.58
```

- ▶ Minimum achievable clock period = CLOCK_PERIOD - WNS = $18.0 - 11.58 = 6.42$ ns
Therefore the maximum allowed clock = 1 / Minimum achievable clock period = 155.76MHz

- ▶ Hold Slack :

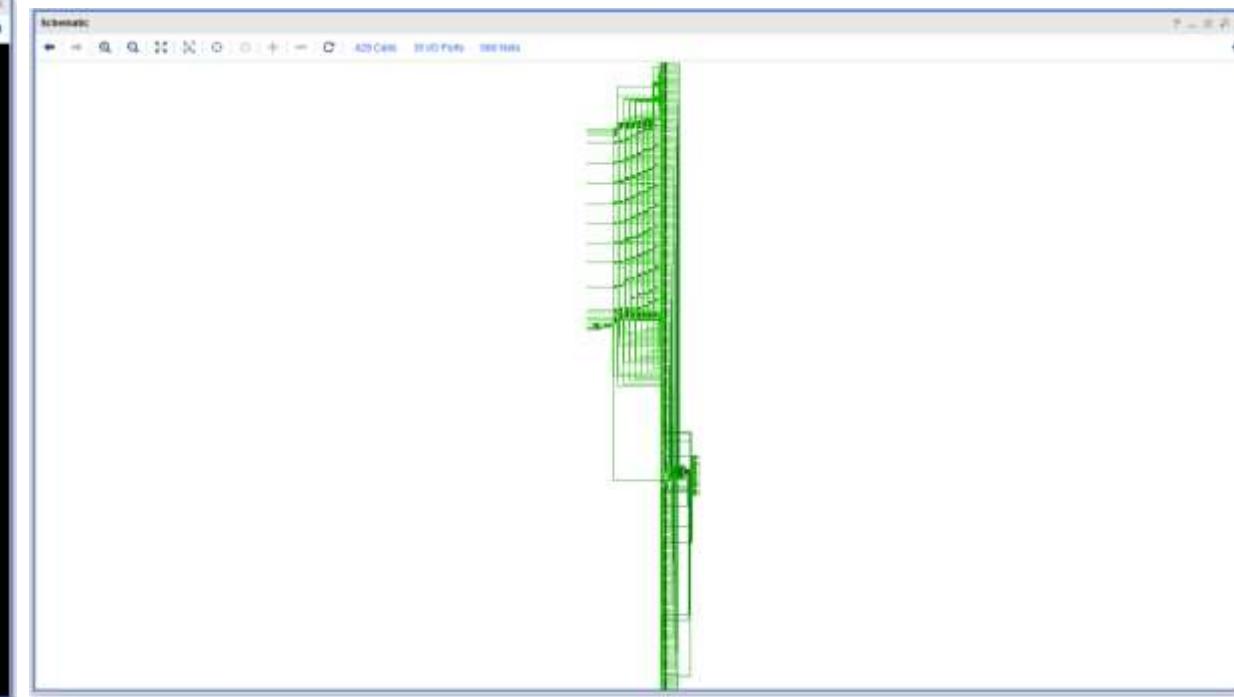
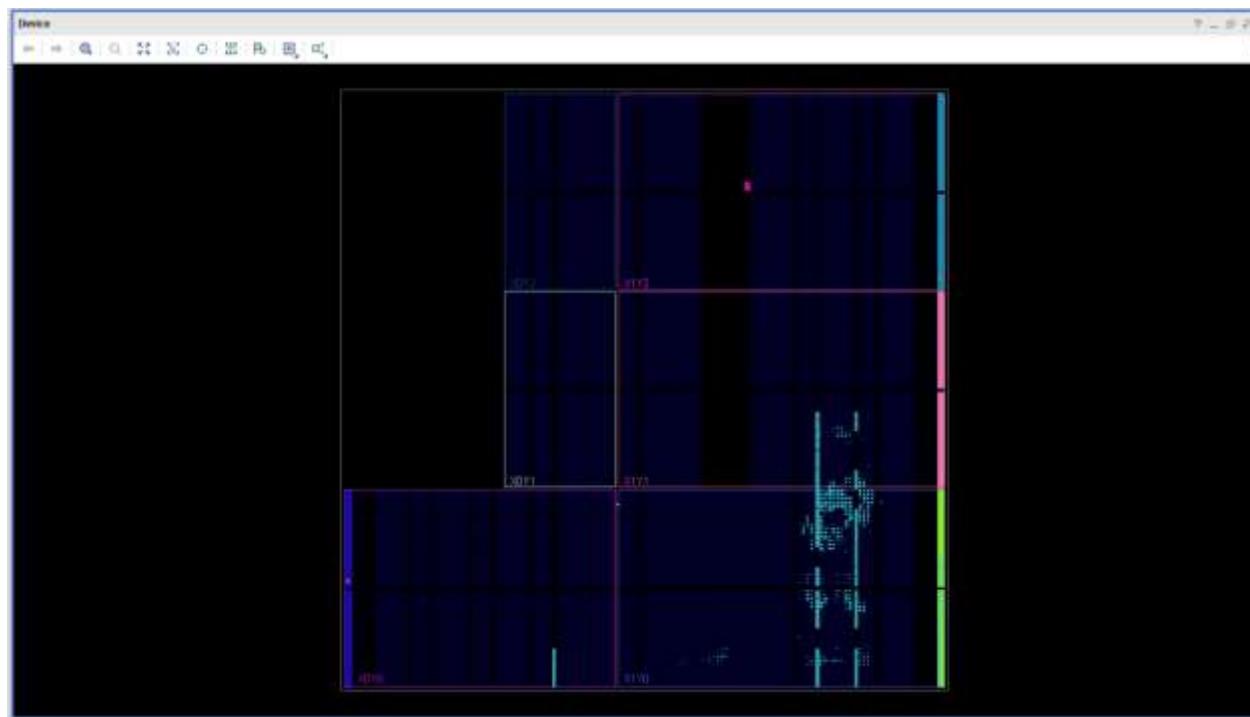
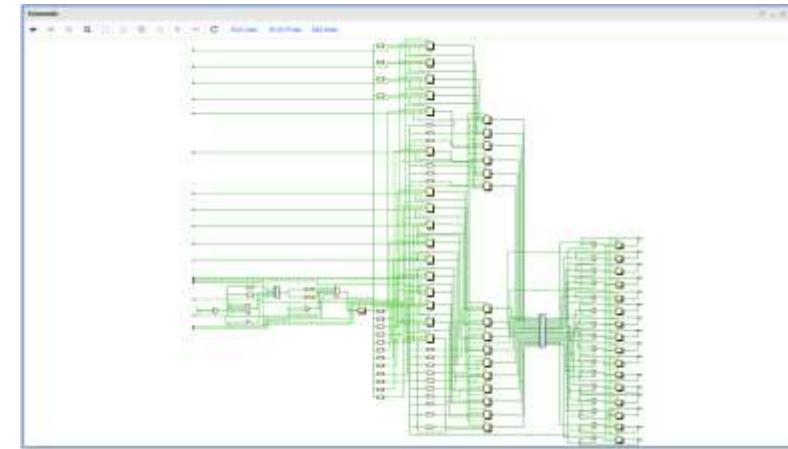
```
=====
report_worst_slack -min (Hold)
=====
worst slack 0.12
worst_slack_end
clock_skew
```

- ▶ Total Negative Slack (TNS):

```
=====
report_tns
=====
tns 0.00
tns_report_end
wns_report
```

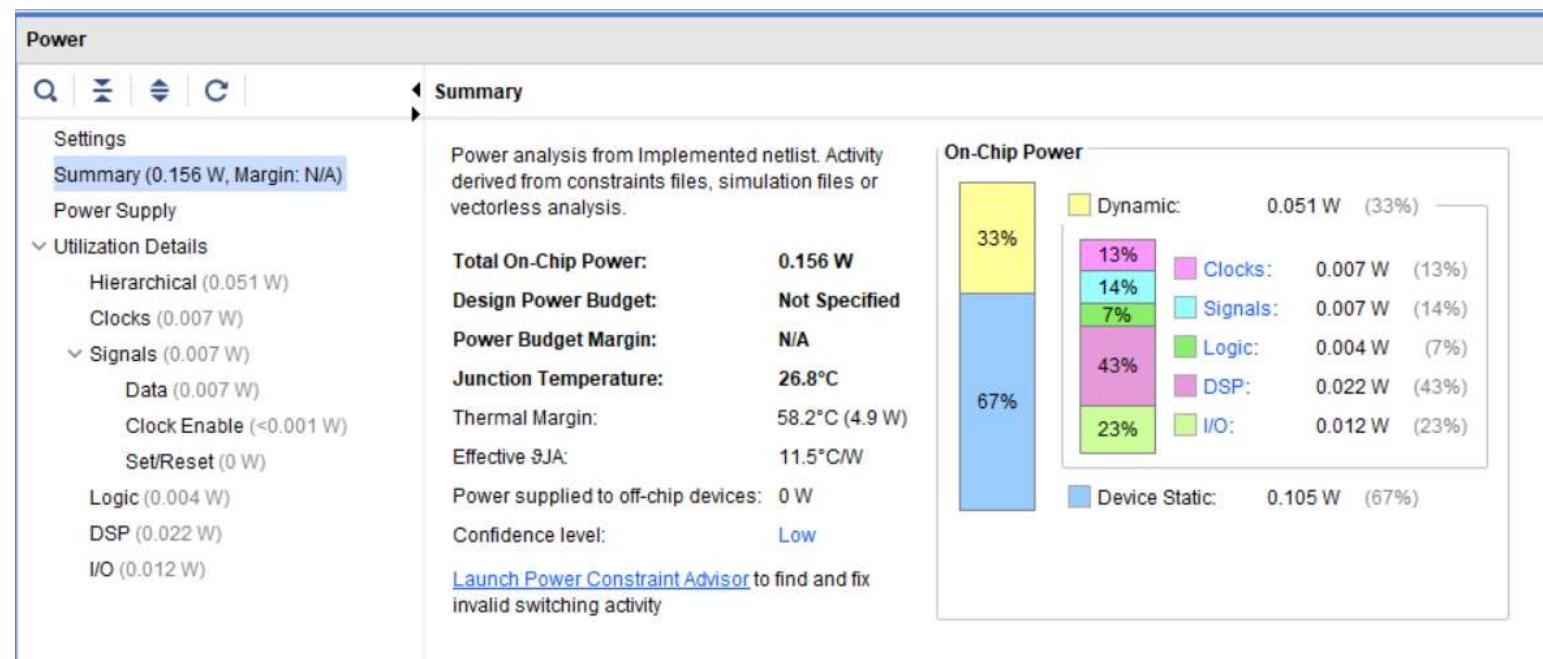
FPGA implementation

- ▶ Vivado tool is used to synthesize the design for FPGA.
- ▶ Implementation is done for the **ZYNQ FPGA** part no.: **xc7z020clg484-2**
- ▶ Constrained clock period is 10ns.
- ▶ Elaborated design:
- ▶ Synthesized design:
- ▶ Implementation:

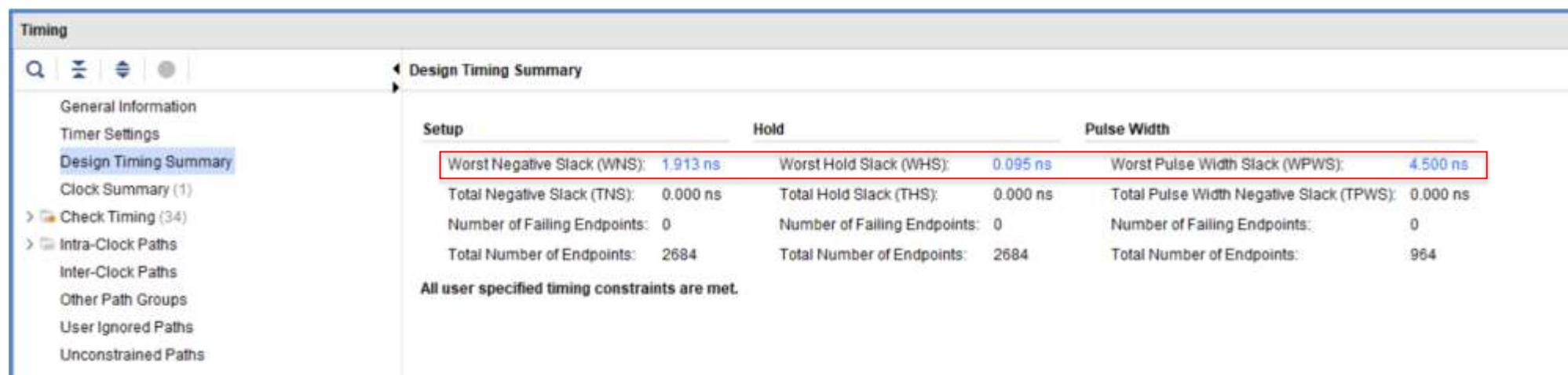


FPGA implementation cont. (reports)

► Power report:



► Timing report:



FPGA implementation cont. (reports)

- ▶ Total cells used report:

Design Runs														
Name	Constraints	Status	WNS	TNS	WHS	THS	TPWS	Total Power	Failed Routes	LUT	FF	BRAMs	URAM	DSP
✓ synth_1	constrs_1	synth_design Complete!								483	939	0.00	0	48
✓ impl_1	constrs_1	route_design Complete!	2.352	0.000	0.104	0.000	0.000	0.156	0	483	939	0.00	0	48

- ▶ Cells used in details report:

Report Cell Usage:		
	Cell	Count
1	BUFG	1
2	CARRY4	116
3	DSP48E1	24
4	DSP48E1_1	24
5	LUT1	2
6	LUT2	32
7	LUT3	511
8	LUT4	3
9	LUT5	2
10	LUT6	4
11	FDCE	939
12	IBUF	19
13	OBUF	16

- ▶ Number of cells used for each instantiation in details report

Report Instance Details	Module	Count
1	BUFG	8881
2	IBUF	241
3	IBUFDS_GTE2	13461
4	IBUFDS_GTE3	4581
5	MAC_0	2261
6	MAC_1	361
7	Calc_Adder	1001
8	Calc_Multiplier	1001
9	Calc_Subtractor	1001
10	MAC_Unit	981
11	Calc_Adder	1001
12	Calc_Multiplier	1001
13	Calc_Subtractor	1001
14	MAC_Unit_0	5421
15	Calc_Adder	1001
16	Calc_Multiplier	1001
17	Calc_Subtractor	1001
18	MAC_Unit_1	5421
19	Calc_Adder	1001
20	Calc_Multiplier	1001
21	Calc_Subtractor	1001
22	MAC_Unit_2	5421
23	Calc_Adder	1001
24	Calc_Multiplier	1001
25	Calc_Subtractor	1001
26	MAC_Unit_3	5421
27	Calc_Adder	1001
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30	MAC_Unit_4	5421
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33	Calc_Subtractor	1001
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37	Calc_Subtractor	1001
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310	MAC_Unit_74	5421
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Thank You!