

Final project

FFT 8-Points Radix-2

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- ▶ Breif about FFT 8-points Radix-2 arch.
- ▶ System modeling.
- ▶ RTL design.
- ▶ Verification.
- ▶ ASIC implementation and results.
- ▶ FPGA implementation and results.

Breif about FFT 8-points Radix-2 arch.

► FFT importance:

- Can be used to obtain the frequency domain view of signals which gives a different point of view and carries information don't present in time domain.
- In modern DSP applications (e.g. DSP IP for OFDM Modulators/Demodulators , Range-Doppler Processing, & Audio codecs with noise reduction).

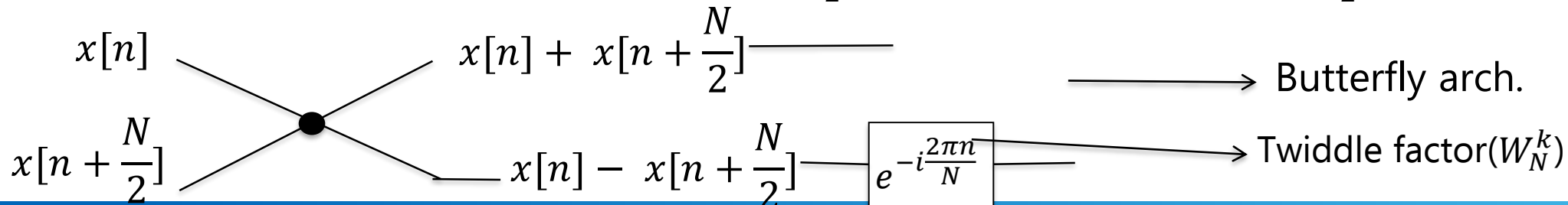
► DFT equation:

$$\tilde{x}[k] = \sum_{n=0}^{N-1} x[n] e^{-i\frac{2\pi kn}{N}}, k = 0, 1, 2, \dots, N-1 \longrightarrow O(N^2)$$

► FFT equation:

$$\tilde{x}[k] = DFT_{N/2}[x_1[n]] + DFT_{N/2}[x_2[n]]e^{-i\frac{2\pi n}{N}}, k = 0, 1, 2, \dots, N-1. \longrightarrow O\left(\frac{N}{2} \log N\right)$$

$$x_1[n] = x[n] + x\left[n + \frac{N}{2}\right] \text{ \& } x_2[n] = x[n] - x\left[n + \frac{N}{2}\right]$$



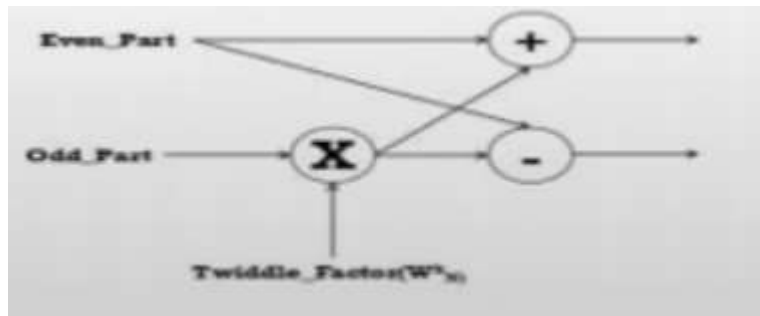
Breif about FFT 8-points Radix-2 arch. cont.

- ▶ The used arch. in our RTL can be obtained by expanding the FFT equation for 8-point (8 samples) case and simplify it to be implemented using 2-point blocks. Therefore:

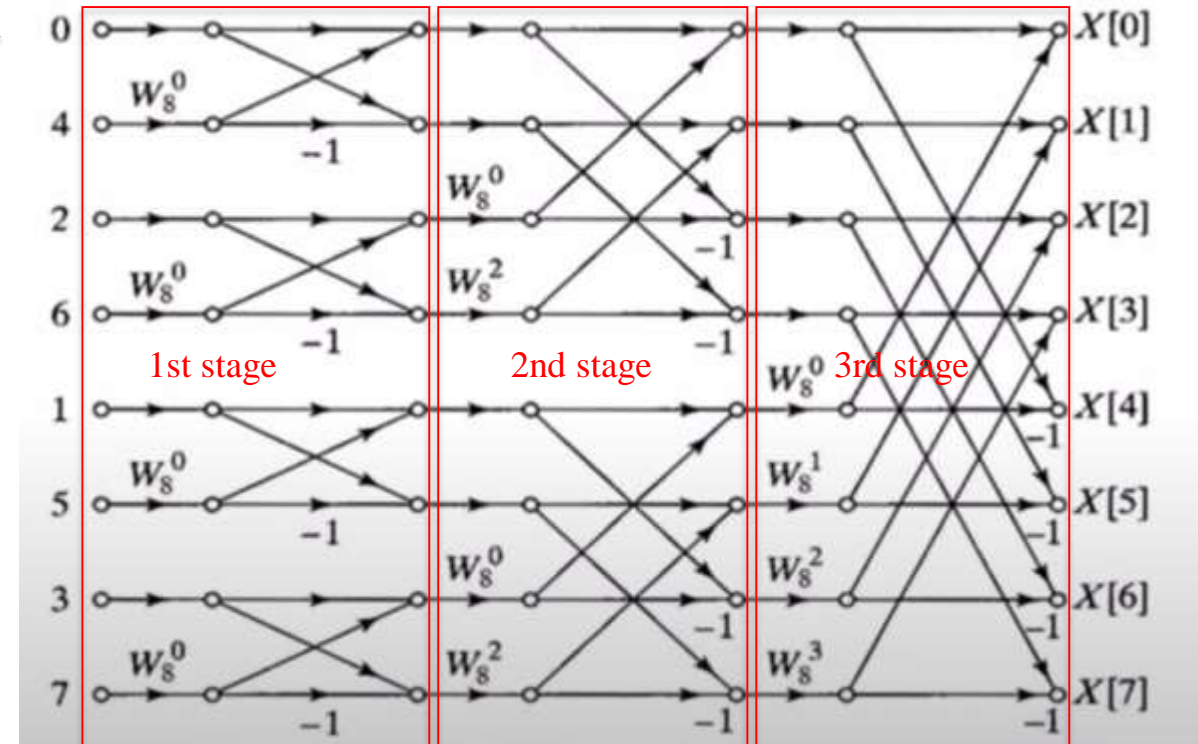
$$\#stages = \log_2 8 = 3 \text{ stages}$$

- ▶ Finally, the arch. is: 

- ▶ Basic building block is **MAC block**:



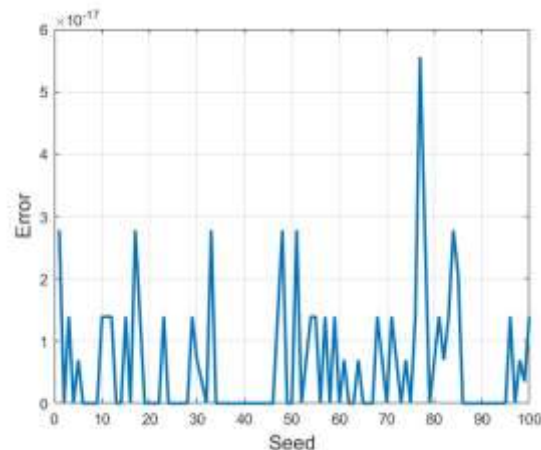
- ▶ Required twiddle factors are only: W_8^0, W_8^1, W_8^2 , and W_8^3
- ▶ We have two forms:
 - Decimation In Time(DIT):Input is bit-reversed, output is normal order.
 - Decimation In Freq.(DIF):Input is normal order, output is bit-reversed.



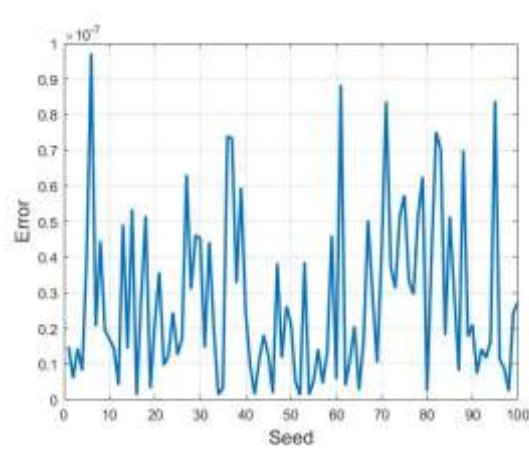
→ 0 0, 2 0,1,2,3

→ 0,1,2,3 0, 2 0

- ▶ First write down the chosen architecture exactly similar to signal flow graph. Isolate core algorithm.
- ▶ Run double, single, & fixed-point analysis successively such that in each case calculate the error and do instrumentation. Run is done for **100 seed** and assumed to have **real inputs only**.
- ▶ During the double case run, from its instrumentation results extract the integer bit width required for each signal in the flow graph.
- ▶ With respect to the **twiddle factor**: It's assumed to have **2 integer bits**.
- ▶ Error for double and single runs:
 - double $\sim 10^{-17}$
 - single $\sim 10^{-7}$



double



single

signal ID	Integer bit width
First_INxx_*	4
First_Multi_OUTx_*	1
First_OUTx0_*	2
First_OUTx1_*	1
Second_Multi_OUT(0,2)	2
Second_Multi_OUT(1,3)	1
Second_OUT(00,20)	3
Second_OUT(01,21)	2
Second_OUT(10,11,30,31)	1
Third_Multi_OUT0	3
Third_Multi_OUT(1,2,3)	2
Third_OUTxx_*	4

- ▶ In order to determine the fractional bit width, plot the SQNR vs the fractional bit width:
 - Choose **SQNR = 53.4616 dB @ unified fractional bit width for all signals equals 8 bits.**
 - Therefore the widest signals in the flow graph are the output signals with width = 4 + 8 = 12 bits.
- ▶ Now run the first step in the case of fixed-point after adding the chosen fractional bit width to observe any unwanted difference.
- ▶ Error for fixed-point case with 8-bits fractional bit width $\sim 10^{-3}$
- ▶ Fixed-point instrumentation results:

Instrumentation Report Viewer - C:\ADI\Internship\Assignment\FFT_Final_Project\SystemModeling\instrumentation\FFT_8_Point_Radix_2_new\bin\report\index.html

REPORT

FUNCTION LIST

Call Tree

FUNCTION LIST

FFT_8_Point_Radix_2.m

fft_8_point_radix_2

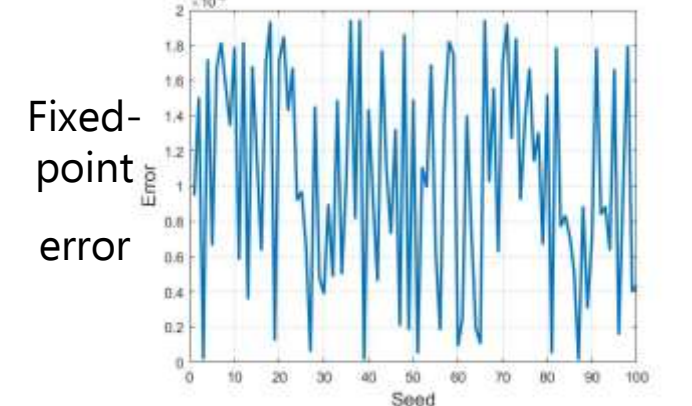
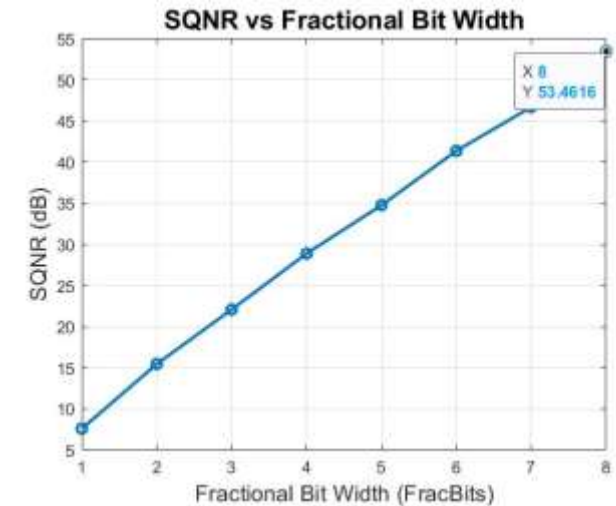
```

1 function FFT_8_Point_Radix_2
2   % FFT_8_Point_Radix_2(FFT_Algo_In, T) %decodes
3   % FFT_Algo_In is the input signal to the FFT algorithm
4   % FFT_Algo_Out is the output spectrum of the signal
5   FFT_Algo_Out = cast((complex(zeros(1,81), 'like', T, FFT_Algo_In)));

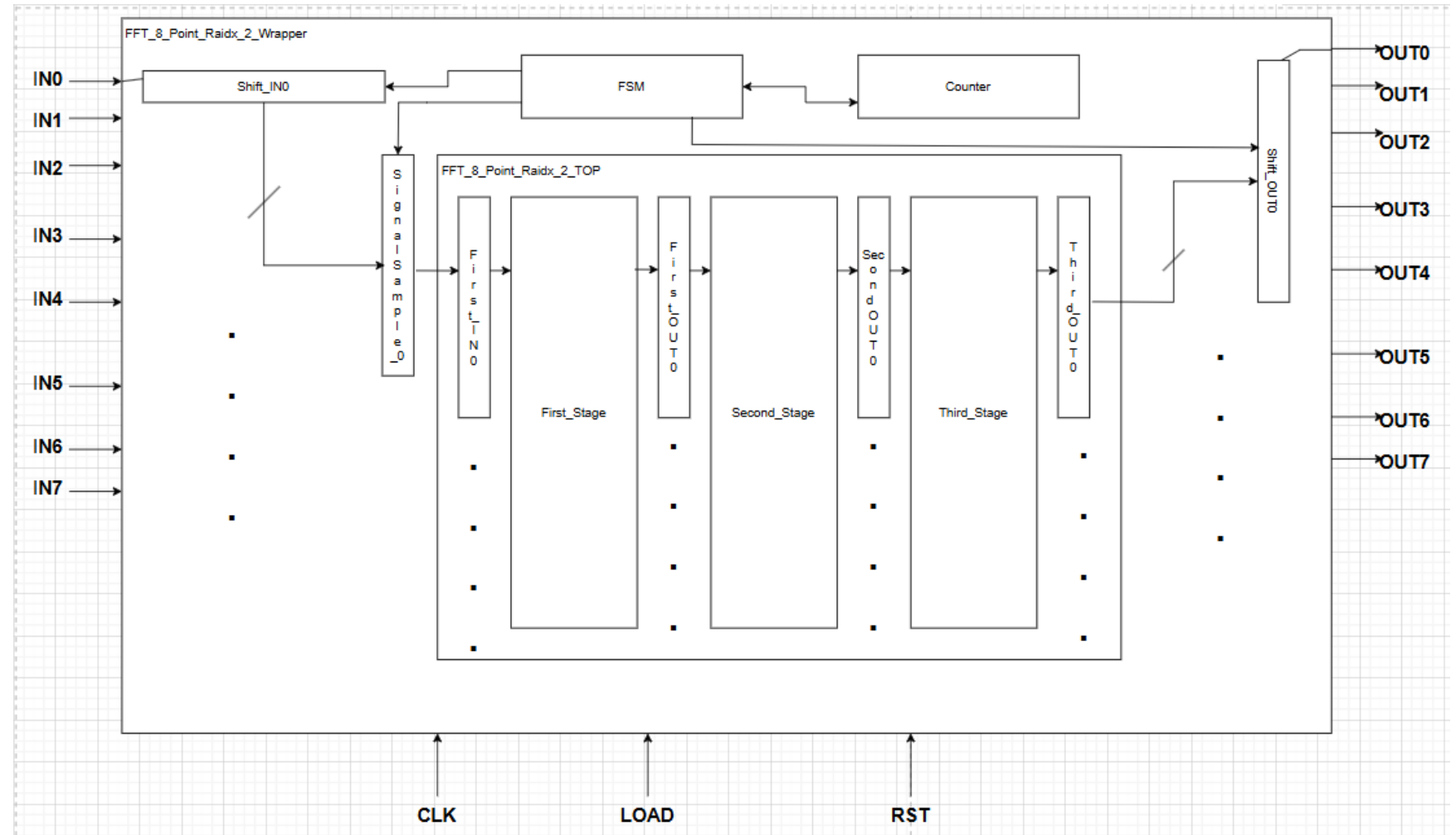
```

VARIABLES

Name	Type	Size	Class	DT Mode	Signedness	WL	PL	Percent of Current Range	Always Whole Number	Sim Min	Sim Max
FFT_Algo_Out	Output	1 x 8	complex embedded f	Fixed	Signed	32	8	71	No	-2.01125	3.8171875
FFT_Algo_In	Input	1 x 8	embedded f	Fixed	Signed	8	8	100	No	0	0.00009375
T	Input	1 x 1	struct	Fixed	Signed	8	8	100	No	0	0.00009375
First_Mat_Out0	Local	1 x 1	complex embedded f	Fixed	Signed	8	8	100	No	0	0.00028125
First_Mat_Out1	Local	1 x 1	complex embedded f	Fixed	Signed	8	8	100	No	0	0.00039375
First_Mat_Out2	Local	1 x 1	complex embedded f	Fixed	Signed	8	8	100	No	0	0.00039375
First_Mat_Out3	Local	1 x 1	complex embedded f	Fixed	Signed	8	8	100	No	0	0.00039375
First_Out00	Local	1 x 1	complex embedded f	Fixed	Signed	10	8	90	No	0	1.7090625
First_Out01	Local	1 x 1	complex embedded f	Fixed	Signed	8	8	95	No	-0.94140625	0.88671875
First_Out10	Local	1 x 1	complex embedded f	Fixed	Signed	10	8	100	No	0	1.8821875
First_Out11	Local	1 x 1	complex embedded f	Fixed	Signed	8	8	77	No	-0.0875	0.7090625
First_Out20	Local	1 x 1	complex embedded f	Fixed	Signed	10	8	92	No	0	1.8203125
First_Out21	Local	1 x 1	complex embedded f	Fixed	Signed	8	8	97	No	-0.90625	0.95703125
First_Out30	Local	1 x 1	complex embedded f	Fixed	Signed	10	8	95	No	0	1.88671875
First_Out31	Local	1 x 1	complex embedded f	Fixed	Signed	8	8	93	No	-0.9046875	0.91796875
Second_Mat_Out0	Local	1 x 1	complex embedded f	Fixed	Signed	10	8	100	No	0	1.8821875
Second_Mat_Out1	Local	1 x 1	complex embedded f	Fixed	Signed	8	8	77	No	-0.7090625	0.9075
Second_Mat_Out2	Local	1 x 1	complex embedded f	Fixed	Signed	10	8	95	No	0	1.88671875
Second_Mat_Out3	Local	1 x 1	complex embedded f	Fixed	Signed	8	8	92	No	-0.91796875	0.9046875
Second_Out00	Local	1 x 1	complex embedded f	Fixed	Signed	11	8	91	No	0	3.8171875

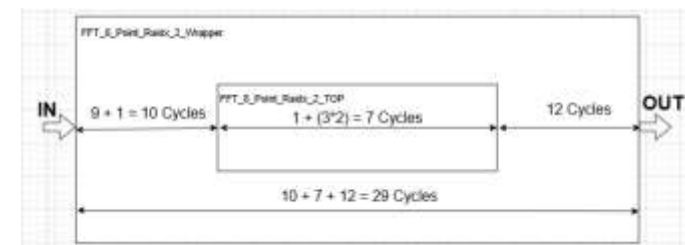
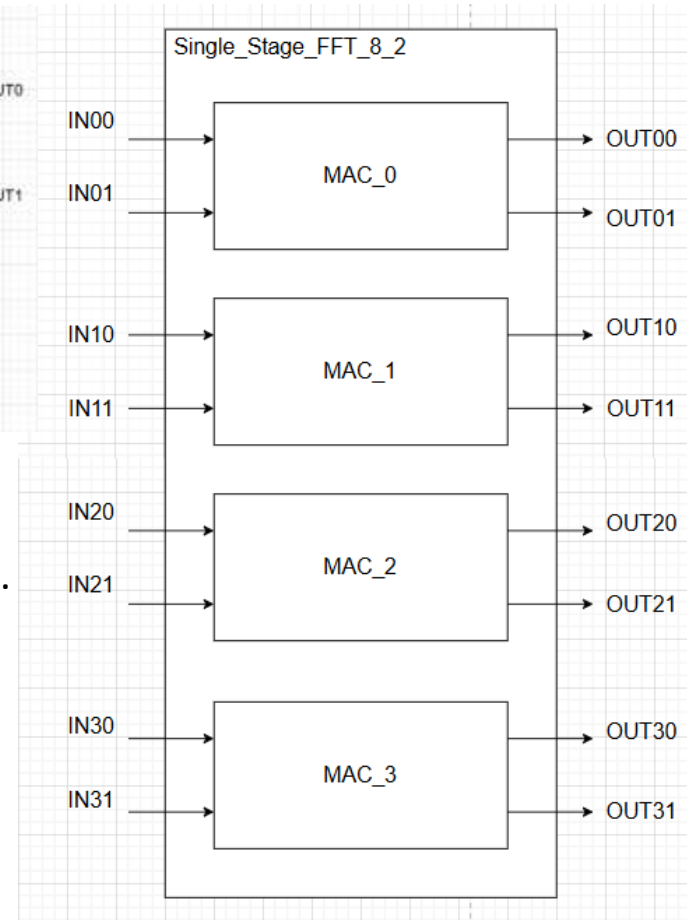
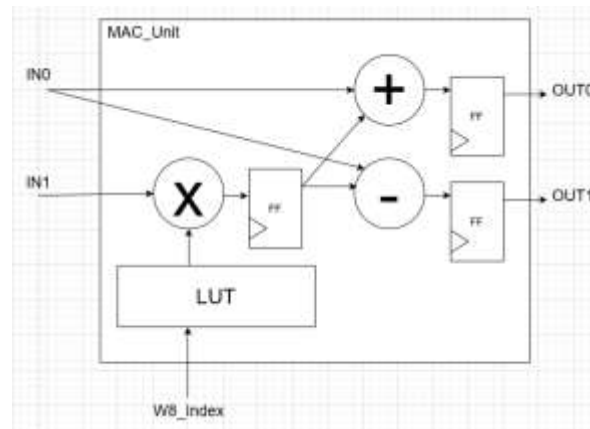


- ▶ Design architecture: Serial-IN-Serial-OUT
- ▶ Only the inputs of the first sample bits (9-bits) are shown.
- ▶ Note: Each of the drawn I/P, O/P, & internal signals has to be split to real signal and imaginary signal (Not drawn).
- ▶ Therefore we will have: $8 \times 2 + 3 = 19$ I/P Port (bits).
- ▶ And $8 \times 2 = 16$ O/P ports (bits).
- ▶ All inputs are set to be fixed-point representations of notation Qx.8, while outputs are also fixed point representation of notation Q4.8.



RTL design cont.

- ▶ Single stage has 4 MAC blocks:
- ▶ Single stage has the following internal blocks:
 - Single Complex multiplier
 - Single Complex adder.
 - Single Complex subtractor.
 - LUT (Memory) to store twiddle factors. (I should make one LUT for all MACs[Future work]), but the optimizer of the synthesis tool may handle this.
- ▶ Complex multiplier: Inputs are $Q_{x.8}$ and $Q_{2.10}$ (Twiddle factor), therefore the output in case of full precision will be $Q_{(x+2).18}$. According to the system modeling step this output is truncated to fit $Q_{y.8}$. During operation you don't need to make virtual fractional decimal point alignment bec. it's multiplication.
- ▶ Complex adder and subtractor: **Single module with operation selector** Inputs are $Q_{x.8}$ and $Q_{y.8}$ (Twiddle factor), therefore the output in case of full precision will be $Q_{(x+y).8}$. According to the system modeling step this output is truncated to fit $Q_{z.8}$. During operation you don't need to make virtual fractional decimal point alignment bec. it's multiplication.
- ▶ **Latency** (The number of cycles from the first input sample to the first output sample) = 29 cycles.

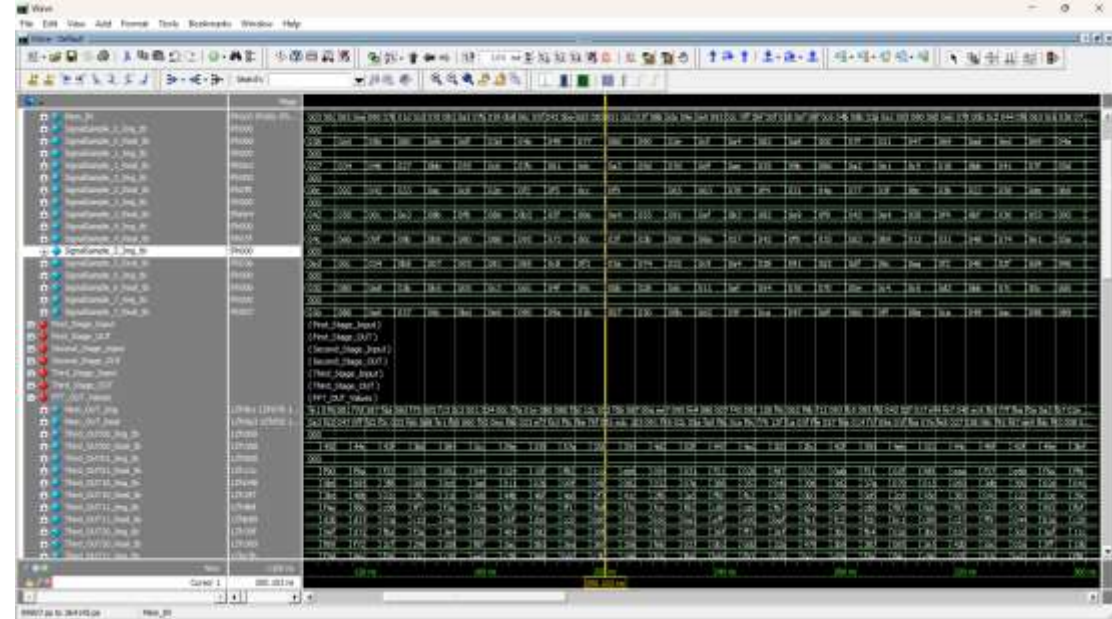


- ▶ Verification based on a normal testbench to verify the system operation and compare between the system's output with the golden model output obtained by Matlab at the system modeling step.
- ▶ Inputs come from the randomization process at matlab are saved as fixed-point Q1.8 in a text file (FFT_Golden_In_bin.txt) and read inside testbench using "readmemb" system function. Also the outputs of the golden model of Matlab **real and imaginary parts**.
- ▶ Error calculation:
 - $\text{First} = \text{abs}(\text{FFT_Module_Real_OUT} - \text{FFT_Golden_Real_OUT})$
 - $\text{Second} = \text{abs}(\text{FFT_Module_Img_OUT} - \text{FFT_Golden_Img_OUT})$
 - $\text{Error_In_Magnitude} = \text{sqrt}(\text{First} * \text{First} + \text{Second} * \text{Second}) \implies$ Error in one sample
 - $\text{Single_Seed_Error} = \sum_0^7 \text{Error_In_Magnitude} \implies$ Error in single seed
 - $\text{Error_All_Seeds} = \sum_0^7 \text{Single_Seed_Error}$
 - $\text{Average_Error_All_Seeds} = \text{Error_All_Seeds} / 100$

► Verification results:

- Waveform: **No unknowns**
- Transcript:
 - Average_Error_All_Seeds = 2.966938**
 - For single sample = Average_Error_All_Seeds/8 = 0.371**

```
# Average Error for seed no.      97 = 2.543193
# Average Error for seed no.      98 = 1.591838
# Average Error for seed no.      99 = 1.943823
# Total Average Error across simulation = 2.966938
# ** Note: $stop : FFT_8_Point_Radix_2_tb.sv(370)
# Time: 1100 ns Iteration: 1 Instance: /FFT_8_Point_Radix_2_tb
# Break in Module FFT_8_Point_Radix_2_tb at FFT_8_Point_Radix_2_tb.sv line 370
```



■ Code coverage report:

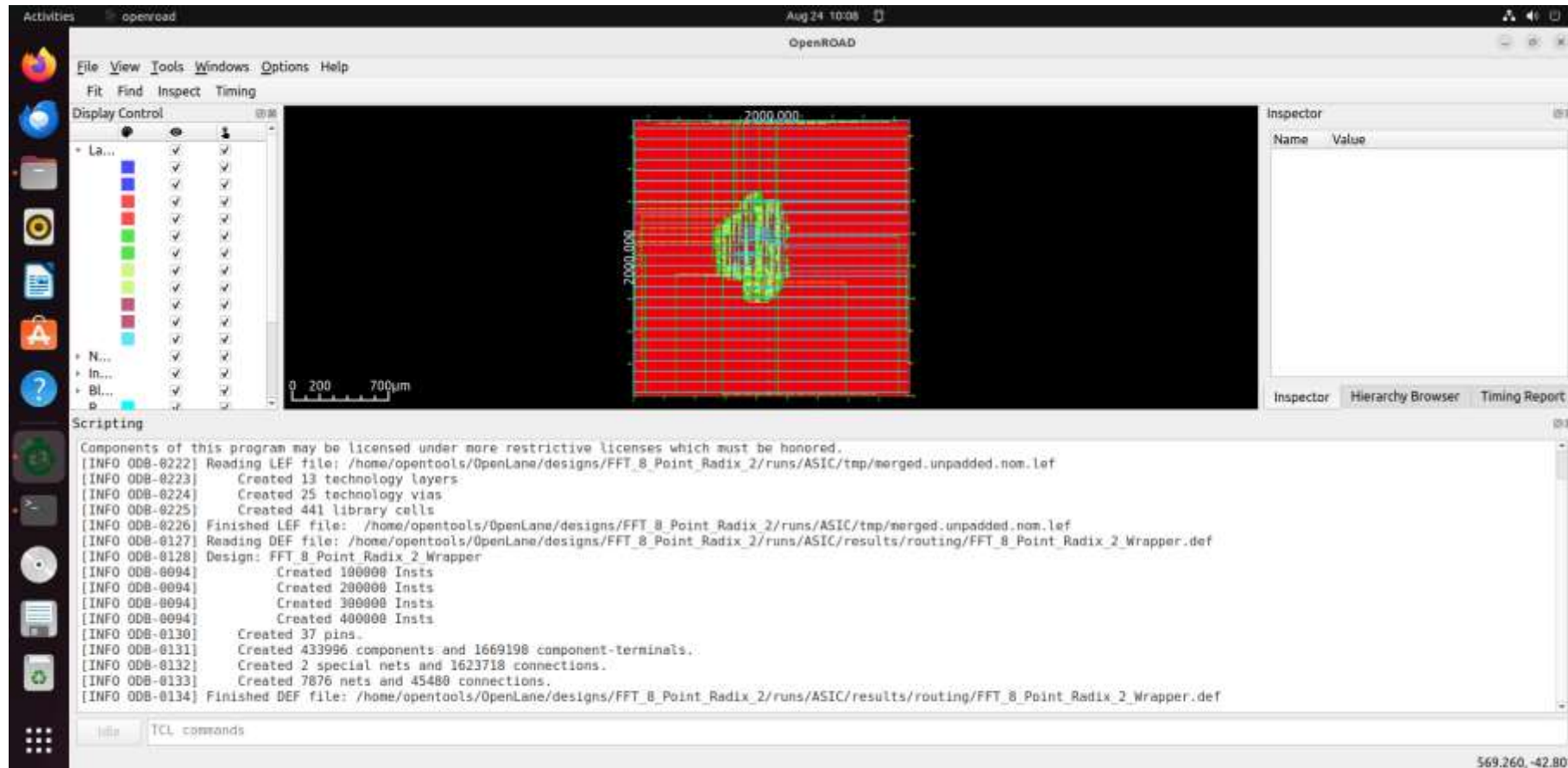
► Future work:

Class-based verification environment

1	Coverage Report Totals BY INSTANCES: Number of Instances 65					
2						
3	Enabled Coverage	Bins	Hits	Misses	Weight	Coverage
4	-----	----	----	-----	-----	-----
5	Branches	156	106	50	1	67.94%
6	Conditions	5	4	1	1	80.00%
7	Statements	666	569	97	1	85.43%
8	Toggles	12556	7068	5488	1	56.29%
9	Total coverage (filtered view): 72.41%					
10						
11						

- ▶ OpenLane tool (open-source) is used. Therefore by default the technology is sky130A PDK and the sky130_fd_sc_hd standard cell library
- ▶ Max die area is 2mm * 2mm.
- ▶ Constrained clock period is 18ns.
- ▶ Final output die from OpenLane: Post-route utilization = 4% @ placement

Design area 155580 u² 4% utilization.



► Worst Negative Slack (WNS):

```
=====
report_worst_slack -max (Setup)
=====
worst_slack 11.58
```

► Minimum achievable clock period = $\text{CLOCK_PERIOD} - \text{WNS} = 18.0 - 11.58 = 6.42 \text{ ns}$

Therefore the maximum allowed clock = $1 / \text{Minimum achievable clock period} = 155.76\text{MHz}$

► Hold Slack :

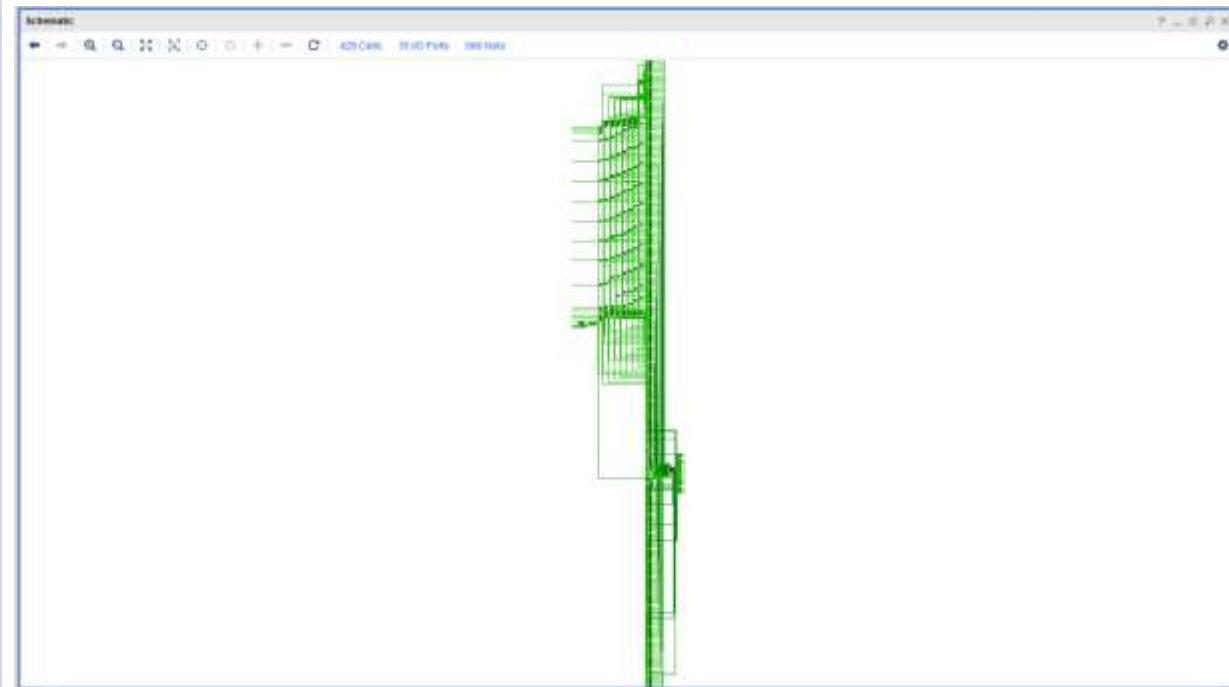
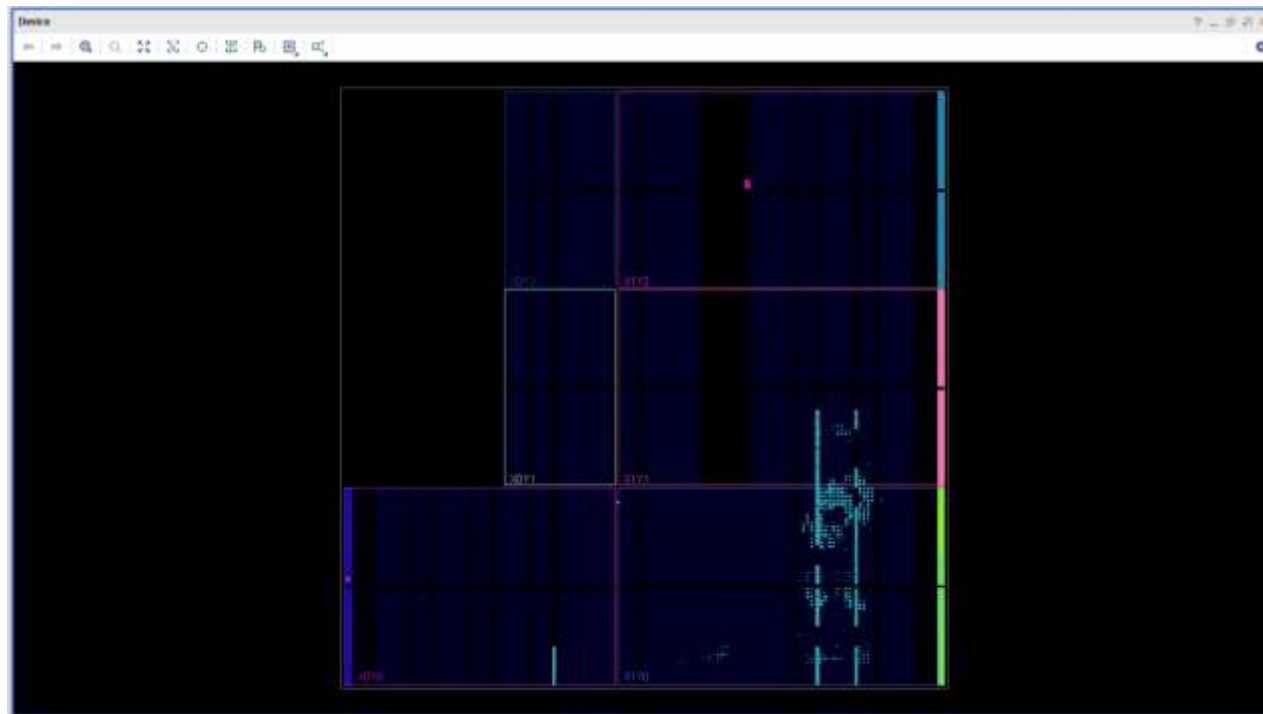
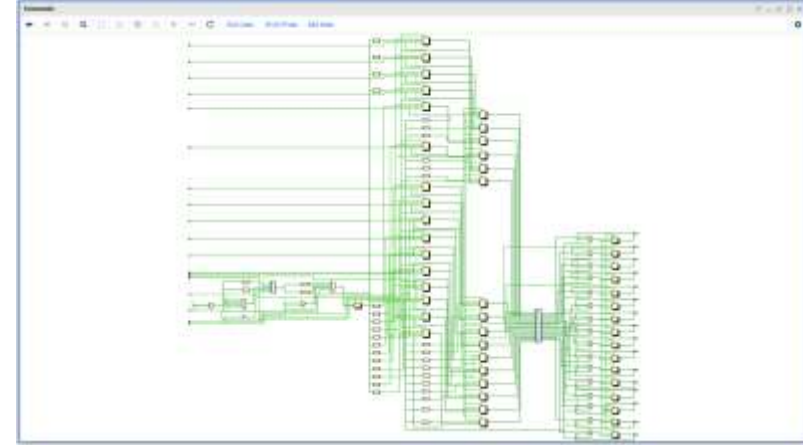
```
=====
report_worst_slack -min (Hold)
=====
worst_slack 0.12
worst_slack_end
clock_skew
```

► Total Negative Slack (TNS):

```
=====
report_tns
=====
tns 0.00
tns_report_end
wns_report
```

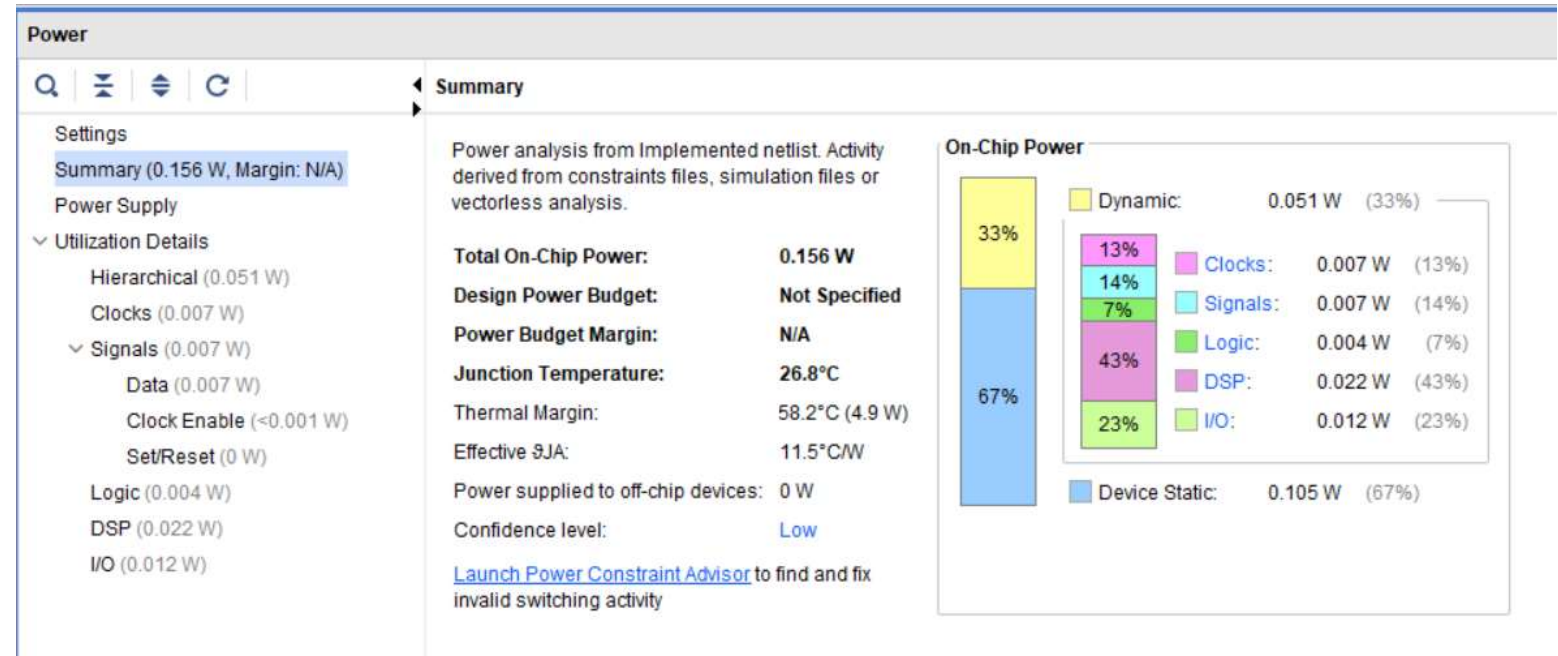
FPGA implementation

- ▶ Vivado tool is used to synthesize the design for FPGA.
- ▶ Implementation is done for the **ZYNQ FPGA** part no.: **xc7z020clg484-2**
- ▶ Constrained clock period is 10ns.
- ▶ Elaborated design:
- ▶ Synthesized design:
- ▶ Implementation:



FPGA implementation cont. (reports)

► Power report:



► Timing report:



Timing

Design Timing Summary

Setup	Hold	Pulse Width
Worst Negative Slack (WNS): 1.913 ns	Worst Hold Slack (WHS): 0.095 ns	Worst Pulse Width Slack (WPWS): 4.500 ns
Total Negative Slack (TNS): 0.000 ns	Total Hold Slack (THS): 0.000 ns	Total Pulse Width Negative Slack (TPWS): 0.000 ns
Number of Failing Endpoints: 0	Number of Failing Endpoints: 0	Number of Failing Endpoints: 0
Total Number of Endpoints: 2684	Total Number of Endpoints: 2684	Total Number of Endpoints: 964

All user specified timing constraints are met.

General Information

Timer Settings

Design Timing Summary

Clock Summary (1)

Check Timing (34)

Intra-Clock Paths

Inter-Clock Paths

Other Path Groups

User Ignored Paths

Unconstrained Paths

FPGA implementation cont. (reports)

- Total cells used report:

Design Runs															
        															
Name	Constraints	Status	WNS	TNS	WHS	THS	TPWS	Total Power	Failed Routes	LUT	FF	BRAMs	URAM	DSP	
✓ synth_1	constrs_1	synth_design Complete!								483	939	0.00	0	48	
✓ impl_1	constrs_1	route_design Complete!	2.352	0.000	0.104	0.000	0.000	0.156	0	483	939	0.00	0	48	

- Cells used in details report:

Report Cell Usage:		
	Cell	Count
1	BUFG	1
2	CARRY4	116
3	DSP48E1	24
4	DSP48E1_1	24
5	LUT1	2
6	LUT2	32
7	LUT3	511
8	LUT4	3
9	LUT5	2
10	LUT6	4
11	FDCE	939
12	IBUF	19
13	OBUF	16

- Number of cells used for each instantiation in details report

Report Instance Areas:		
Instance	Module	Cells
1	fft_module	1
2	fft_0	1
3	fft_0	1
4	fft_0	1
5	fft_0	1
6	fft_0	1
7	fft_0	1
8	fft_0	1
9	fft_0	1
10	fft_0	1
11	fft_0	1
12	fft_0	1
13	fft_0	1
14	fft_0	1
15	fft_0	1
16	fft_0	1
17	fft_0	1
18	fft_0	1
19	fft_0	1
20	fft_0	1
21	fft_0	1
22	fft_0	1
23	fft_0	1
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95	fft_0	1
96	fft_0	1
97	fft_0	1
98	fft_0	1
99	fft_0	1
100	fft_0	1

Thank You!