In the Name of God

University of Tehran

Electrical and Computer Engineering faculty

Signals and Systems, Fall 95

Computer Assignment #3

Due Date: Tuesday, 30 Azar 1395, 11:55 PM

Important: Read two PDF files about Fourier Transform.

Problem 1

1. Compute and plot the Fourier transform of the following continuous-time ($X(j\omega)$) and Discrete-time ($X(e^{j\omega})$) signals. In case of complex transform, plot the magnitude, angle, real part and imaginary part of the $X(j\omega)$ or $X(e^{j\omega})$.
   - ❖ Try to compute and plot yourself and compare with MATLAB results.

   1.a) $x(t) = rect\left(\frac{t+1}{2}\right) + rect\left(\frac{t-1}{2}\right)$; plot over interval $-10 \le \omega \le 10$

   1.b) $x(t) = 10\ tri(\frac{t-4}{20})$; plot over interval $-5 \le \omega \le 5$

   1.c) $x[n] = \cos[\frac{\pi n}{3}]$; $0 \le n \le 10$, plot over interval $-3\pi \le \omega \le 3\pi$

   1.d) $x[n] = (u[n+4] - u[n-5]) * \cos(\frac{2\pi n}{6})$; $0 \le n \le 10$, plot over interval $-3\pi \le \omega \le 3\pi$

2. Compute and plot the inverse Fourier transform of the following continuous-time signals.

   2.a) $X(j\omega) = \frac{2j\omega}{1+j\omega}$

   2.b) $X(\Omega) = 20\ tri(8\omega)$

   2.c) $X(\Omega) = \frac{\pi\delta(\omega)}{(2+j\omega)(5+j\omega)}$

Problem 2

1. Using the multiplication–convolution duality of the CTFT, find an expression for $y(t)$, that does not use the convolution operator $*$ and graph $y(t)$.

   1.a) $y(t) = e^{-t}u(t) * e^{-t}\cos(2\pi t)u(t)$

   1.b) $y(t) = sinc(t) * sinc^2(2\pi t)$

2. Suppose that $y(t) = xe^{-x}u(x)$. Compute the convolution between the signals $y_1(t) = dy(t)/dt$ and $y_2(t) = \int_{-\infty}^{t} dr$.
3. Compute the energy of the following signals

   3.a) $x(t) = e^{-0.1t}\cos(t)\,u(t)$

   3.b) $x(t) = 2e^{-t}u(t)$

Problem 3

Autocorrelation and Cross-Correlation. The autocorrelation function $R_x(\tau)$of $x(t)$ is defined as

$$R_x(\tau) = \int_{-\infty}^{\infty} x(t) * x(t-\tau)dt = \int_{-\infty}^{\infty} x(t+\tau) * x(t)dt$$

The energy of a signal $x(t)$ is equal to its autocorrelation function $R_x(\tau)$ evaluated at $\tau = 0$

$$R_x(0) = E_x$$

and the maximum value of $R_x(\tau)$ is at $\tau = 0$.

The Fourier transform of the autocorrelation function of an energy signal is equal to the energy spectral density of the signal.

$$F\{R_x(\tau)\} = |X(j\omega)|^2$$

In MATLAB, the autocorrelation function of a signal is computed by using the command

'xcorr'. The syntax is $R = xcorr(x) * step$ where $step$ is the time step used in the definition of the signal $x(t)$. If the length of x is M, the outcome of command *'xcorr'* (here is the vector R) is of length $2M-1$. Hence, R must be plotted in the double time interval from that of $x(t)$. More specifically, if $x(t)$ is defined in the time interval $[-T, T]$. The command *'xcorr'* is used in a similar way to the command '*conv*'.

1. Compute and plot the autocorrelation function of the signal $x(t) = e^{-3t}u(t)$

   1.a) Using the definition of autocorrelation function ('*int*' command)

   1.b) Using the *'xcorr'* command and compare your answer with previous part.
2. Consider the signal $x(t) = e^{-3t}u(t)$. Compute and plot the Fourier transform of its autocorrelation function as well as the energy spectral density of $x(t)$.