Problem Statement

The problem is formulated as the P-Median model, which makes the location and allocation decisions to **minimize the total system cost**, which is both the total travel time of evacuees finding and not finding shelters.

Evacuees follow a trial-and-error process to find the nearest shelter. They go from affected zones to the nearest shelters. People stay at the shelter if it can accommodate them unless they go to the nearest shelter from their current position. They follow this behavior until they can find a shelter with capacity.

Assumptions

1. Nodes within a predefined vicinity of zones hit by hurricane are considered as a 'affected zones', which are unsafe for opening shelters.
2. Each shelter has a predefined capacity for the total number of people accommodated in them.
3. It may not be possible to accommodate all the people in a given scenario.
4. P shelters can be opened in any given scenario.
5. People who cannot find shelter are assumed to receive service from

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Notations

$j \epsilon J$: Index of candidate nodes for opening shelters

$i \epsilon I$: Index of network zones

$N_i \subseteq J$: Sorted neighboring candidate nodes of node i which are sorted based on people's trial-and-error behavior

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Parameters

1. P: Numbers of shelters to be opened or located
2. c_j: Capacity of each shelter
3. h_i: Number of evacuees from node i seeking shelters
4. n_j: Binary parameters that show if shelter at site j is safe to be opened
5. t_{ij}: Travel time between node i and node $j \in N_i$, Travel time are computed following people's trial-and-error behavior
6. γ: Assumed travel time for the people not accommodate in shelters

Decision Variables

1. X_j a binary decsion variable to be one if a shelter is located at site j and zero otherwise.
2. $Y_{i,j}$ a non-negative decision variable to indicate the number of evacuees from origin i that use a shelter at site j
3. Z_i a non-negative decision variable to indicate the number of evacuees from origin i who seek shelters but cannot be accommodated.

Formulation

$$\min \quad \gamma \sum_i Z_i + \sum_i \sum_j t_{i,j} Y_{i,j}$$

$$\text{s.t.} \quad \sum_j X_j = P$$

$$X_j \leq n_j \qquad \forall j \in J$$

$$\sum_i Y_{i,j} \leq c_j X_j \qquad \forall j \in J$$

$$\sum_j Y_{i,j} + Z_i = h_i \qquad \forall j \in N_i$$

Note that we can add the following constraints. However, it is redundant.

$y_{i,ju} \geq y_{i,ju+1} \quad \forall i \in I, \ \forall j_u \in N_i$

Proof:

$t_{i,u+1} = t_{i,u} + \lambda, \ \lambda > 0$

$t_{i,u} \leq t_{i,u+1}$

λ: travel time between node u and node $u+1$

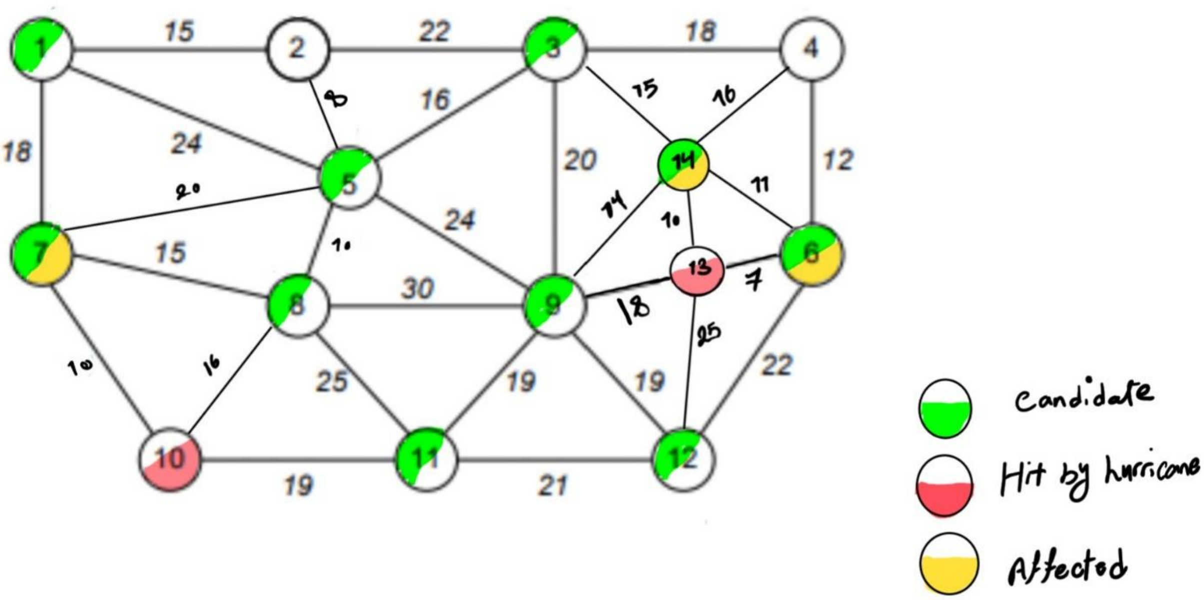
Since the objective function try to minimize the evacuees' travel time, it atomically ensures the constraint.

Illustration on N_i and t_{ij}

We assume that people follow a trial-and-error approach to find shelters. In other words, evacuees travel between shelters in a greedy manner to find the first shelter with capacity. This behavior is modeled in N_i set for each node of the network. N_i is a sorted list of neighboring shelters for people at the node i that shows how people move from one shelter to the other. In fact, N_i is the sorted list of destination with the shortest travel time in a greedy manner

Furthermore, the parameter t_{ij} ,travel time between node i and node $j \in N_i$, are calculated based on the aforementioned people's behavior.

In the following example we show how N_i and t_{ij} are constructed in a toy network.



Affected zones within 10 minutes travel time from nodes hit by hurricane: {14,6,7}

$N_{13} = \{9, 11, 12\}$

$N_{10} = \{8, 5, 3, 9,11, 12\}$

$t_{13,9} = 18$

$t_{13,11} = 18 + 19 = 37$

$t_{13,12} = 37 + 21 = 58$

$$t_{10,8} = 16$$

$$t_{10,5} = 16 + 10 = 26$$

$$t_{10,9} = 26 + 30 = 56$$

$$t_{10,11} = 56 + 19 = 75$$

$$t_{10,12} = 75 + 21 = 96$$