A Geometric Approach to Risk Management

A Financial Mathematics Presentation

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Table of contents

Financial Mathematics

Financial Mathematics is the application of mathematical methods to financial problems. It draws many tools from probability, statistics, stochastic processes, and economic theory. Topics include:

- Derivative securities valuation
- Portfolio structuring
- Risk management
- Scenario simulation

Risk Management and Risk Measure

- **Risk Management** is the process of identifying and analysing uncertainties in investment decisions.
- After the market collapse of 1929, risk management has been a core attention of financial institutions and private investors.
- Since the financial crisis of 2008, there has been a renewed interest in the methods and techniques of risk management.

Risk Measure

- **Risk measures** try to quantify the risk associated with financial portfolios.
- If we let Ω be the possible outcomes for an asset and $\mathcal{A}$ be the investor's actions in creating a portfolio then a risk measure ρ will be defined as $\rho : \mathcal{A} \to \mathbb{R}$.
- **Risk measures** are used to determine the amount of asset(s) should a financial institution or an investor keep in case of market irregularities.
- There are different ways to measure financial risks. The most used risk measure is called **Value at Risk**.

Value at Risk (VaR)

Value at Risk (VaR) is a risk measure that calculates the loss that will occur with a given probability over a specified period of time[1].

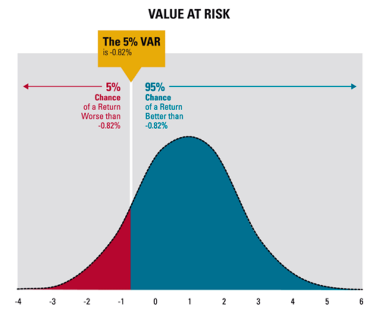

Figure 1: Value at Risk visual[2]

Example of VaR

For example, if a portfolio of stocks has a one-day 5% VaR of $1 million, that means that there is a 0.05 probability that the portfolio will fall in value by more than $1 million over a one-day period.[6]

Remark: To use VaR it is necessary to know the probability distribution of the portfolio value over some period.

Coherent Risk Measures

According to Artzner et. al [4], a minimum requirement for a "good" risk measure is to be coherent. If X and Y are random variables representing the unknown future values of some portfolios, then a risk measure ρ is said to be coherent if it satisfies the following conditions:

1. Monotonicity: If $X \leq Y$ then $\rho(Y) \leq \rho(X)$
2. Translation invariance:
   $\rho(X - m) = \rho(X) - m$, for all $m \in \mathbb{R}$
3. Positive homogeneity
   $\rho(\lambda X) = \lambda \rho(X)$, for all $\lambda > 0$
4. Sub-additivity: $\rho(X + Y) \leq \rho(X) + \rho(Y)$

VaR Criticisms

- VaR is not coherent because it fails the sub-additivity condition.
- Model uncertainties (meaning that the relevant probability distributions are not fully known) arising from VaR may lead into an unreliable risk measure, giving false sense of security to investors and traders.
- VaR tends to underestimate risks.

Objectives

- Construct a risk measure with nice properties, in particular it should be coherent.
- The new proposal should be robust so that uncertainties on probability distributions and parameters do not make the risk measure too unreliable.
- The new risk measure will be primarily based on some geometric structure rather than exclusively using some (not very well known) probabilistic structure.

Our Approach

- Consider an asset whose value changes over time. Let Ω be the set of possible future scenario prices for this asset with a time frame of [0,T].
- We denote $\mathcal{A}$ a set of available portfolio strategies. Assume $\mathcal{A}$ is a linear space.
- We construct a set Ω^*, where $\Omega^* \subset \Omega$
- For $\phi \in \mathcal{A}$ we defined the new risk measure as

$$\rho(\phi) = -\inf_{x \in \Omega^*} V_\phi(x, T)$$

Numerical Simulations i

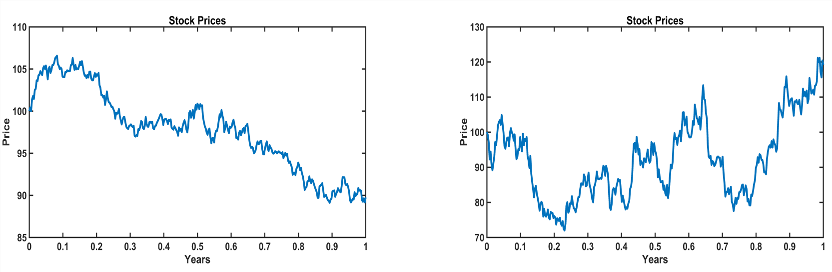

Figure 2: Two possible price trajectories with different volatilities.

Numerical Simulations ii

By construction, the set Ω^* does not include "extreme" trajectories.

For us, extreme trajectories are those where:

- the price itself goes too low at any point, and/or
- the average volatility is too high.

For the construction of the set Ω^* we will take into account model uncertainties.

Numerical Simulations iii

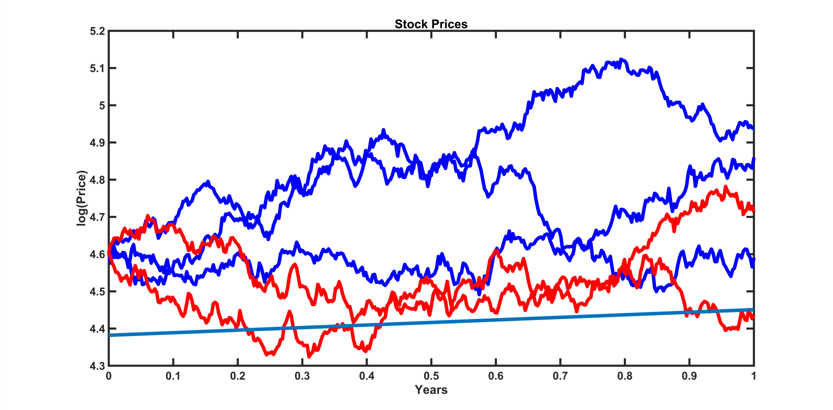

Figure 3: Price trajectories with $E[S_T] - k$,where k is a constant, as threshold

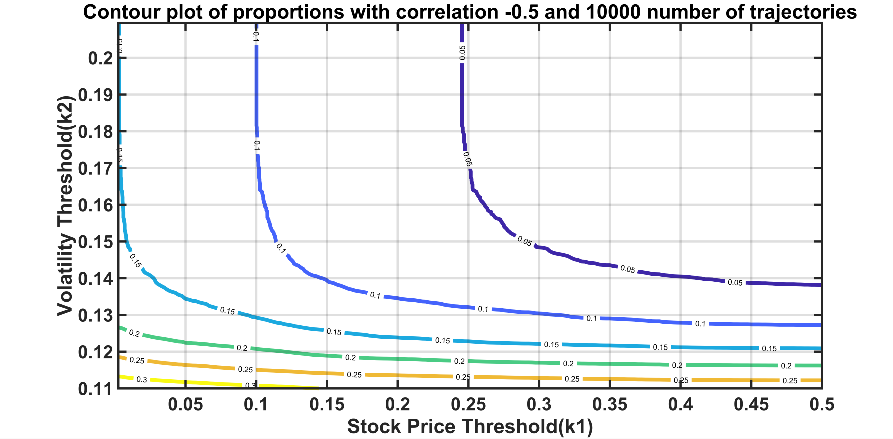

Figure 4: Each plot illustrates α proportions

Numerical Simulations v

We chose the following two important portfolio strategies to show how the method works:

- Short call delta-hedged portfolio (using one stock and a call option)
- Buy-and-hold portfolio (buy stock at $t = 0$ until $t = T$)

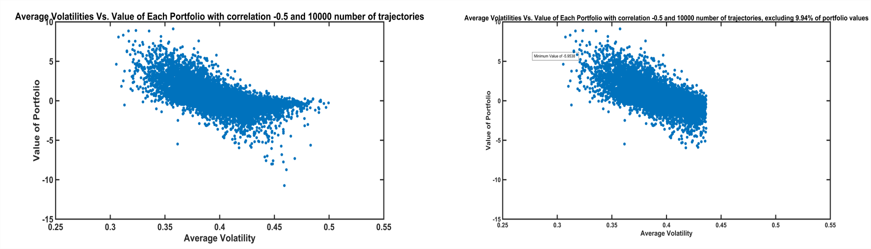

Figure 5: Portfolio values before and after excluding the extreme stock price scenarios ($\alpha = 0.10$)

Results ii

Table 1: Values collected using short call delta-hedged portfolio with correlation of $\rho = -0.5$.

Alpha(α)	Value at Risk	Threshold based Risk Measure
0.01	3.4648	10.7397
0.03	2,2459	10.7397
0.05	1.7409	8.0159
0.07	1.4516	7.5871
0.09	1.2192	5.9538
0.10	1.1305	5.9538

Results iii

Table 2: Values collected using buy-and-hold portfolio with correlation of $\rho = -0.5$.

Alpha(α)	Value at Risk	Threshold based Risk Measure
0.01	68.8245	71.1254
0.03	61.3915	67.5869
0.05	56.3680	65.5239
0.07	52.2180	62.4690
0.09	48.8837	60.6612
0.10	47.6580	60.1613

Economic Interpretations

- This approach is very flexible: it allows us to look for convenient/meaningful metric structures. Some external factors could be used in the construction of Ω^*.
- The new risk measure is coherent. The economic interpretation is that this new risk measure encourages diversification.
- This approach allows us to deal with model uncertainty in a relatively intuitive way.
- The new risk measure is more risk averse (compared to VaR) and puts the investor in a safer investing position.

Future Work

- We only used one asset for our model.
- An investor may have more than one asset in a portfolio.
- The next step will be to create portfolios with multiple assets, that are correlated.
- Find the risk measure for those portfolios.

Thank you!

References

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