

Chapter 1 - Random Number Generation

Transformation of Random Variables. Box Muller Transformation.

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The inverse transform method is an example of how to transform a uniform random variable into a random variable with a different distribution.

This is just one example of a possible transformation of random variables.

There are more complicated transformations of random variables that could be useful in some cases.

In some cases it might be convenient to transform a whole random vector from \mathbb{R}^d into a new random vector from \mathbb{R}^d in order to get random numbers/vectors with a desired distribution.

Perhaps the most noticeable example of this is the **Box-Muller transform** for the generation of normal random variables.

This comes as an application of a very general result from the textbook (**Theorem 1.34**).

Algorithm

1. Generate $\Theta \sim U[0, 2\pi]$ and $U \sim U[0, 1]$ independently
2. Compute $R = \sqrt{-2 \ln(U)}$
3. Compute $(X, Y) = (R \cos \Theta, R \sin \Theta)$
4. Return (X, Y)

The random variables X, Y are independent, standard normal random variables.

Remarks:

- Unfortunately, as we have discussed earlier in the course (see slides corresponding to the Inverse transform method) **we cannot transform a single uniformly distributed random variable into a single normal random variable easily**
- However, with the Box-Muller algorithm **we can transform two independent, uniformly distributed random variables into two independent standard normal random variables.**
- This type of transformations (from \mathbb{R}^d to \mathbb{R}^d) are not very common and the Box-Muller algorithm is a very notable exception.
- Some textbooks also refer to this method as the **polar method**.

Box-Muller Transform Example

Code

R Code

```
1  n <- 1000
2  m <- n/2
3  X <- vector()
4  R <- vector()
5  U <- runif(m, min=0, max =1)
6  Theta <-
7    runif(m, min=0, max = 2*pi)
8
9  for (i in c(1:m)) {
10    R[i] <- sqrt(-2*log(U[i]))
11    X[2*i-1] <- R[i]*cos(Theta[i])
12    X[2*i] <- R[i]*sin(Theta[i])
13  }
14
15  hist(X)
```

Python Code

```
1  import numpy as np
2  import matplotlib.pyplot as plt
3
4  n = 1000
5  m = n // 2
6  X = np.zeros(n)
7  R = np.zeros(m)
8  U = np.random.uniform(0, 1, m)
9  Theta = np.random.uniform(0, 2*np.pi, m)
10
11  for i in range(m):
12    R[i] = np.sqrt(-2 * np.log(U[i]))
13    X[2*i] = R[i] * np.cos(Theta[i])
14    X[2*i+1] = R[i] * np.sin(Theta[i])
15
16  plt.hist(X, bins=30)
17  plt.show()
```