A Geometric Approach to Financial Risk Management

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"Don't cross a river if it is four feet deep on AVERAGE." Nassim Taleb

Value at Risk

Value at Risk is a risk measure that calculates the loss that will occur with a given probability over a specified period of time[5].
Value at risk is the most commonly used risk measure since it was developed in 1990s. In recent years, specially after crisis of 2008, there has been many criticism of this risk measure.

According to "Coherent Measures of Risk" by Artzner[3], a minimum requirement for a "good" risk measure is to be coherent. The main criticism is that VaR is not coherent because it fails sub-additivity condition which states if X and Y are two portfolio values and ρ calculates the risk, then

$$\rho(X+Y) \le \rho(X) + \rho(Y) \tag{1}$$

Another criticism of VaR is the probabilistic structure which may lead into an unreliable risk measure which gives a false sense of security to the traders.
David Einhorn in Grant's Spring investment Conference states that value at risk "is like an air bag that works all the time, except when you have a car accident."[4]

Objectives

1- Construct a risk measure with nice properties, in particular that is coherent. This will encourage diversification unlike VaR.

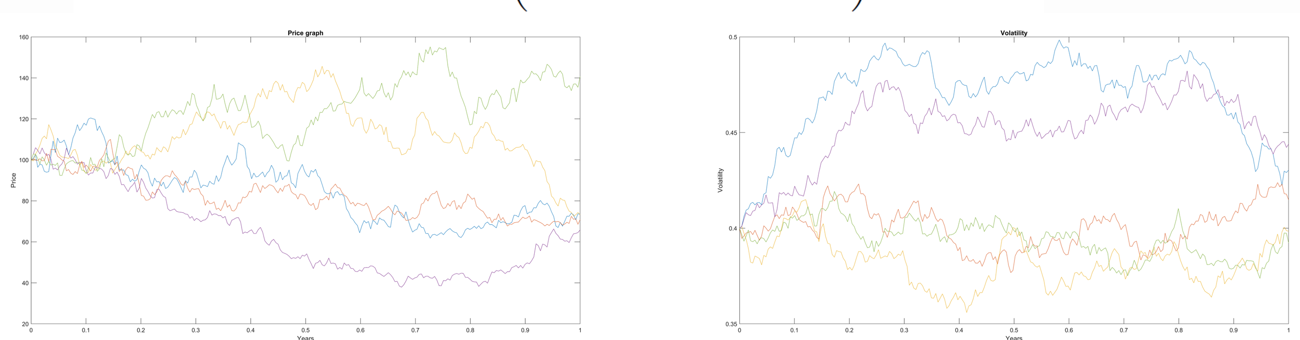
2- The new proposal should be robust so that uncertainties on probability distributions and parameters do not make the risk measure to be unreliable.

3- The new risk measure would be constructed based on some geometric structure instead of probabilistic structure.

Procedure

1- Familiarize and simulate stochastic volatility using Cox–Ingersoll–Ross process, correlated stock price. Figure 1.

$$\begin{aligned} dS_t &= \mu S_t dt + \sigma(Y_t) S_t dW_t^1;\ S_0 > 0 \\ dY_t &= m(t,Y_t)dt + v(t,Y_t)\left(\rho dW_t^1 + \sqrt{1-\rho^2}dW_t^2\right);\ Y_0 \text{ given.} \end{aligned} \tag{2}$$



equation 2- Correlated prices and volatility. CIR process was used to create stochastic volatility.
figure 1 - Stock prices Vs. Time. Left
Stochastic Volatility Vs. Time.Right

3- Create two thresholds, one for stock sample size, and one for the average volatility sample size. Figure 2.

figure 2 - log(Stock Prices) Vs. Time and expected value as threshold for stock prices.
Red trajectories indicate that these prices fall below an expected value

4- We compute the α proportion of extreme trajectories (they exceed the thresholds). Figure 3.

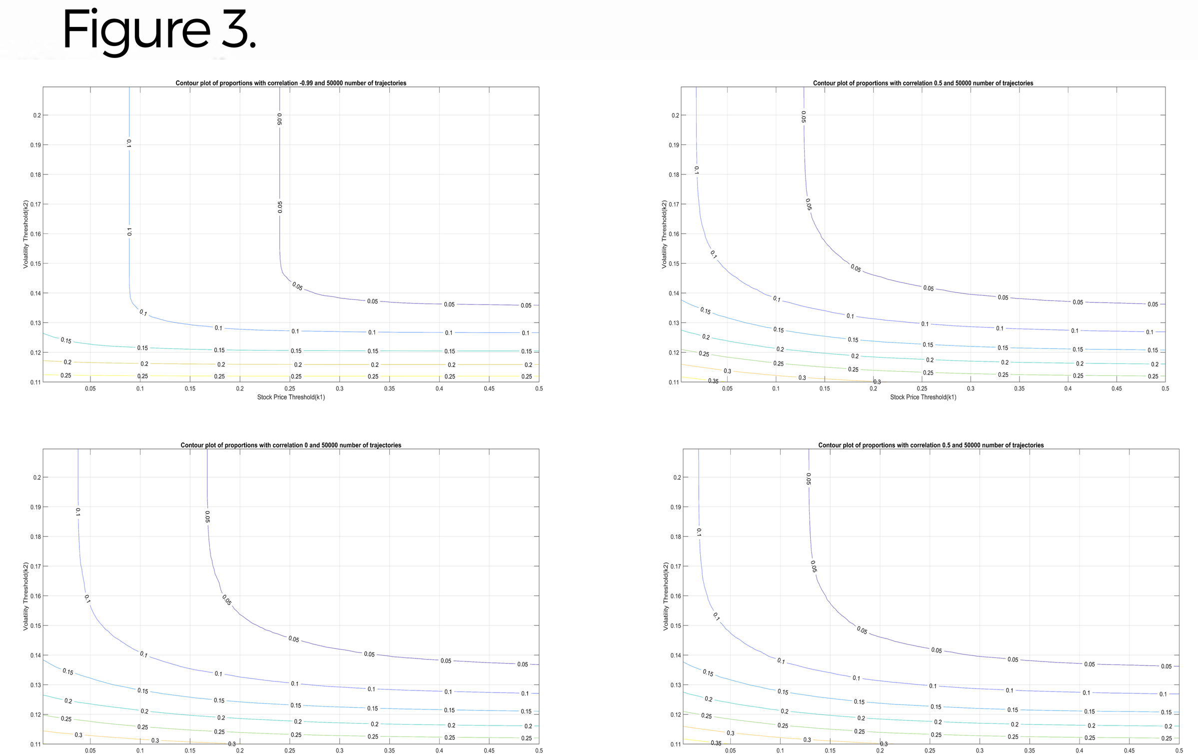


figure 3 - Each plot illustrates α proportions (on each graph) changing according to change in each threshold. Different correlation between stock prices and volatilities result into different α proportions.

Theoretical Framework

Let Ω be a concrete set of possible future scenarios for some underlying asset within [0,T], and Φ be a set of available portfolio strategies for the investor. Assuming Φ is a linear space, then $V_\phi(x,T)$ is the value of portfolio that investor creates[1].

Then excluding the α proportion of trajectories will result into sample Ω', where Ω'⊂Ω. Then in [1] a risk functional can be defined as:

$$\rho(\phi) = -\inf_{x\in\Omega'} V_\phi(x,T) \tag{3}$$

Results

Alpha (α)	Value at Risk	Minimum Value Risk Measure
0.01	3.8265	13.0387
0.03	2.4758	13.0387
0.05	1.9150	12.3826
0.07	1.5673	10.6149
0.10	1.2279	9.20260

Table 1. Comparing value at risk and geometric approach risk measure

Alpha (α)	Value at Risk	Minimum Value Risk Measure
0.01	73.0599	76.3864
0.03	64.1628	73.1104
0.05	58.9840	73.1104
0.07	55.0804	73.1104
0.10	49.9962	70.1258

Table 2. Comparing value at risk and geometric approach risk measure

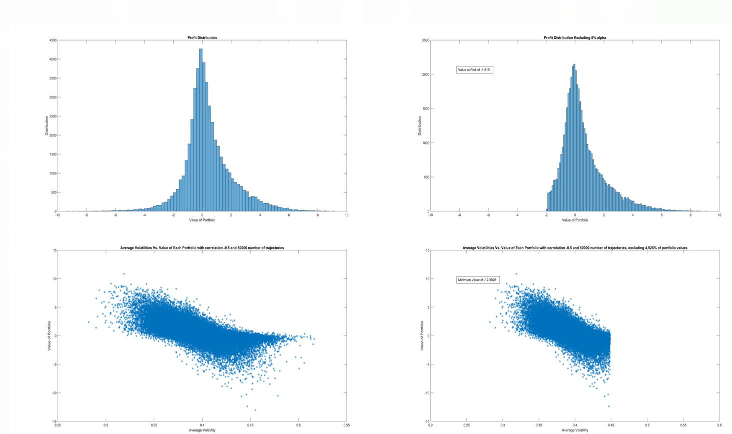


figure 4 - comparing VaR and geometric approach for portfolio value

Table 1 is data to compare VaR and the new geometric approach risk measure values. Delta hedging was used to find portfolio values in this simulation.

In Table 2, a simple buy and selling of an asset strategy is used for portfolio values. The data stays consistent with different ways of creating a portfolio.

Figure 4 shows the difference between the results of two risk measures. VaR discards portfolio values after a certain amount, yet the geometric approach picks the minimum value.

Conclusions

- The new risk measure is coherent and uses geometric structure. The economic interpretation is that this new risk measure encourages diversification.
- This approach is very flexible: it allows us to look for convenient/meaningful metric structures. Some external factors could be used in the construction of Ω'. This makes the risk measure to robust and reliable.
- The new risk measure is more risk averse and puts the investor in a safer investing position.

References and Acknowledgements

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