

# Final Project

## STAT 4110 Statistical Simulation

### Winter 2026 Semester

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## Overview

This project is designed to familiarize students with real-world applications of statistical simulation, stochastic modelling, and analytical reasoning. The available options for the final project may involve developing a computational model, implementing simulation techniques covered in class, and using simulation-based or analytical methods to study system behaviour. Students are free to use any dataset or programming language of their choice, unless otherwise specified.

Choose **one** of the following three project directions:

1. Use your modelling skills to model and implement **one of the following three** real world problems:
  - Forest Fire Simulation – Poisson and Markov Chain Simulation
  - Numerical Option Pricing: Crank–Nicolson PDE vs. Monte Carlo Simulation
  - Bayesian Neural Networks using Markov Chain Monte Carlo
  - Your very own problem and modelling. Some examples are: Queueing Problems, Molecule Collisions in a Reactor, Prey/Predator using Poisson Point Estimates, 1D and 2D Ising Model.
2. Replicating the full design and implementation of **one** published paper related to our course material
3. Literature review of **5–15 papers** on a topic related to the course

Projects will be completed in teams of three members. Each team will be responsible for defining the scope, data sources, and motivation of their work.

## Deliverables

For any of the project choices, the deliverables are as follows:

1. **Introductory Summary** (2 pages max, 5/30 points): Includes a brief description of the chosen project, references and papers that the group will use, and the roles of each student in the group.

2. **Summary Report** (5 pages max, 20/30 points): A detailed report summarizing the approach, implementation, results, and discussion. Include any supporting figures, tables, or appendices as needed. You are also responsible to attach a ZIP file containing all code used in the project. If you are using Git (GitHub or GitLab), please add me as the owner, so I can review your code.
3. **Anonymous Peer Review** (5/30 Points): A peer review questionnaire with 5 grading questions and one paragraph question describing your peers contribution to the project.

**Disclaimer:** The use of LLMs, including GPT, is permitted for coding assistance only. Any use of such tools must be explicitly acknowledged in the report.

# Project Choice 1: Modelling Option

## Purpose

The purpose of this project choice is to give you hands-on experience in developing and analysing statistical/mathematical models for real-world problems. By working on a modelling-based project, you will learn how to formalize assumptions, implement computational methods, and interpret model outcomes. This choice emphasizes on using the material learnt throughout the course and understanding the behaviour of complex systems under uncertainty

## Learning Objectives

- Design and implement a computational model for a selected application.
- Apply simulation or numerical methods to approximate solutions when analytic solutions are impossible.
- Compare model predictions with alternative approaches or benchmarks.
- Communicate methodology, results, and insights effectively through written reports.

## Scope

You are expected to choose **one of the four** modelling-based problems:

- Forest Fire Simulation – Poisson and Markov Chain Simulation
- Numerical Option Pricing: Crank–Nicolson PDE vs. Monte Carlo Simulation
- Bayesian Neural Networks using Markov Chain Monte Carlo
- Your very own problem and modelling. Some examples are: Queueing Problems, Molecule Collisions in a Reactor, Prey/Predator using Poisson Point Estimates, 1D and 2D Ising Model.

As a team, you are responsible for selecting appropriate datasets, defining the project scope, and documenting their methodology and results.

# Option 1: Forest Fire Simulation – Poisson and Markov Chain Simulation

Forest fires are a significant environmental hazard that can have devastating ecological and economic impacts. Fires can be initiated naturally, such as through lightning, or by human activity. The spread of a fire through a forest depends on multiple factors including tree density, forest topology, weather conditions, and seasonal variations. The dynamics of forest fires are complex but by understanding these dynamics, starting with simple models and then making them more complex and complex, allows fire management teams to better predict high-risk regions, estimate time-to-burn, and plan effective response strategies.

Spatial heterogeneity in forests is a key factor influencing fire behaviour. Trees are often not distributed uniformly; regions near water sources or fertile soil may have higher tree density, creating areas more susceptible to fire spread. Simulation models that incorporate stochastic spatial distributions and probabilistic fire spread can help quantify the risk in these heterogeneous environments.

In this scenario, you are hired by Natural Resources Canada to provide insights and a model that provides actionable insights for forest fire risk management. You will simulate forest fires in a 2D square forest of size  $100 \times 100$  kilometres squared. You will need to implement Monte Carlo simulations to estimate expected outcomes, such as the number of trees burnt and the time required to burn the forest under different seasonal conditions. Note that Trees are distributed using an inhomogeneous Poisson point process (IPP), creating higher density near the forest center. Furthermore fires can ignite at a tree and spread to neighbouring trees probabilistically. Fire spread is modelled as a Markov chain, with tree states transitioning from unburnt to burnt based on neighbourhood interactions.

Your model implementation needs to be accompanied by a report that will show both the tree density and fire risk across months, providing actionable insights for forest management.

## Objectives

In this problem you will:

1. Model a spatially heterogeneous forest using an **inhomogeneous Poisson point process**.
2. Simulate fire spread using a **Markov chain** approach.
3. Implement **Monte Carlo simulations** to estimate:
  - Expected number of burnt trees per month
  - Expected time-to-burn for the entire forest
4. Visualize the forest, fire spread, and fire risk heatmaps.
5. Analyze seasonal variation in fire risk and time-to-burn.

## Forest Model

The forest is modeled as a 2D square with coordinates  $[0, 100] \times [0, 100]$ . Tree locations are generated using an inhomogeneous Poisson point process with intensity function:

$$\lambda(x, y) = \Lambda_{\max} \exp \left( - \frac{(x - 50)^2 + (y - 50)^2}{2\sigma^2} \right)$$

where  $\Lambda_{\max}$  is the maximum expected tree density (trees per unit area) and  $\sigma$  controls how concentrated trees are near the forest center.

## Fire Spread Dynamics

Each tree can be in one of two states:

- Unburnt (0)
- Burnt (1)

Fire spreads probabilistically to neighboring trees within a radius  $r_{\text{fire}}$ . For a tree with  $k$  burning neighbors, the probability of ignition in one timestep is:

$$P(\text{ignite}) = 1 - (1 - p_{\text{fire}})^k$$

where  $p_{\text{fire}}$  is the monthly fire spread probability. Each timestep represents one minute. The fire spreads until no new trees are ignited or a maximum number of timesteps is reached.

## Seasonal Variation

Fire risk depends on the month, with lower probabilities in winter and higher probabilities in summer. Use the following monthly probabilities:

$$p_{\text{fire}} = [0.10, 0.12, 0.15, 0.20, 0.25, 0.35, 0.40, 0.38, 0.30, 0.20, 0.15, 0.12]$$

representing January through December.

## Monte Carlo Simulation

To account for stochastic variation in tree positions and fire spread:

- Run  $N_{\text{MC}}$  independent simulations per month.
- Record the number of burnt trees and total timesteps for each trial.
- Compute the expected number of burnt trees and expected time-to-burn for each month.

## Tasks and Suggested Parameters

### 1. Forest Initialization

- Generate tree locations using IPP.
- Visualize tree locations and create a heatmap of tree density.

### 2. Single Fire Simulation

- Ignite a random tree (or the tree closest to the forest center).
- Simulate fire spread using Markov chain dynamics.

- Record the number of burnt trees and timesteps.
- Visualize fire spread on the forest map.

### 3. Monte Carlo Analysis

- Run multiple trials per month.
- Estimate expected burnt trees and average burn time.
- Plot histograms for number of burnt trees.

### 4. Seasonal Fire Risk Visualization

- Create risk heatmaps by combining tree density and monthly  $p_{\text{fire}}$ .
- Compare risk patterns between winter and summer months.

### 5. Analysis and Discussion

- Identify months with the highest fire risk.
- Determine which areas are consistently at higher risk.
- Discuss implications for forest management and fire response.

Parameter	Value	Notes
Forest size	100 units	2D square forest
Maximum tree intensity ( $\Lambda_{\text{max}}$ )	0.05	High density near center
Fire radius ( $r_{\text{fire}}$ )	5 units	Neighbor search radius
Max timesteps	120	Each timestep = 1 minute (2 hours)
Monte Carlo trials ( $N_{\text{MC}}$ )	500–1000	Per month
Random seed	42	For reproducibility

Table 1: Suggested parameters for the forest fire simulation.

### Optional Tasks

- Introduce multiple ignition points.
- Simulate heterogeneous terrain (e.g., dry vs wet areas).
- Compute probability that more than 50% of the forest burns per month.
- Explore the effect of varying tree density on fire risk.

## Option 2: Black–Scholes Model, Monte Carlo Pricing, and Crank–Nicolson Finite Differences

### 1. What is the Black–Scholes model and why use it?

The Black–Scholes model is a continuous-time stochastic model for the evolution of a financial asset price  $S_t$ . It assumes that the asset price follows a geometric Brownian motion (GBM):

$$dS_t = \mu S_t dt + \sigma S_t dW_t$$

where

- $\mu$  is the instantaneous expected return (drift),
- $\sigma > 0$  is the volatility (instantaneous standard deviation),
- $W_t$  is a standard Brownian motion.

The model is used because (i) it is analytically tractable in many cases, (ii) under risk-neutral valuation it leads to a linear parabolic PDE for derivative prices, and (iii) it captures multiplicative stochastic growth and log-normal marginal distributions for  $S_t$ . The Black–Scholes framework underpins much of modern quantitative finance and provides a baseline for comparing numerical methods (PDE solvers, Monte Carlo, lattice models, etc.).

### 2. Risk-neutral pricing and derivation of the Black–Scholes PDE

Consider a derivative security with payoff at maturity  $T$  given by  $\Phi(S_T)$ . The *no-arbitrage* (risk-neutral) pricing formula states that the arbitrage-free price at time  $t$  is the discounted risk-neutral expectation:

$$V(S_t, t) = e^{-r(T-t)} \mathbb{E}^{\mathbb{Q}}[\Phi(S_T) \mid S_t]$$

where  $r$  is the risk-free rate and  $\mathbb{Q}$  denotes the risk-neutral measure. Under the risk-neutral measure the asset dynamics become

$$dS_t = rS_t dt + \sigma S_t dW_t^{\mathbb{Q}}$$

Applying Itô's formula to  $V(S_t, t)$  gives

$$dV = \left( \frac{\partial V}{\partial t} + rS \frac{\partial V}{\partial S} + \frac{1}{2} \sigma^2 S^2 \frac{\partial^2 V}{\partial S^2} \right) dt + \sigma S \frac{\partial V}{\partial S} dW_t^{\mathbb{Q}}$$

Under risk-neutral valuation, the discounted derivative price must be a martingale, which leads to the Black–Scholes backward PDE:

$$\boxed{\frac{\partial V}{\partial t} + \frac{1}{2} \sigma^2 S^2 \frac{\partial^2 V}{\partial S^2} + rS \frac{\partial V}{\partial S} - rV = 0}$$

Terminal condition:  $V(S, T) = \Phi(S)$ .

### 3. Probabilistic (Monte Carlo) representation

The derivative price can be approximated by the risk-neutral expectation:

$$V(S_0, 0) = e^{-rT} \mathbb{E}^{\mathbb{Q}}[\Phi(S_T) \mid S_0]$$

Under GBM,

$$S_T = S_0 \exp \left( \left( r - \frac{1}{2} \sigma^2 \right) T + \sigma \sqrt{T} Z \right), \quad Z \sim \mathcal{N}(0, 1)$$

### 3.1 Euler–Maruyama discretization

Discretize the interval  $[0, T]$  into  $N$  steps  $\Delta t = T/N$ . The Euler–Maruyama scheme:

$$S_{n+1} = S_n + rS_n\Delta t + \sigma S_n\sqrt{\Delta t}Z_n \quad Z_n \sim N(0, 1)$$

or the log-Euler form:

$$S_{n+1} = S_n \exp \left( (r - \frac{1}{2}\sigma^2)\Delta t + \sigma\sqrt{\Delta t} Z_n \right)$$

### 3.2 Monte Carlo pricing of European options

Simulate  $M$  paths  $S_T^{(m)}$  and compute:

$$V_0 \approx e^{-rT} \frac{1}{M} \sum_{m=1}^M \Phi(S_T^{(m)})$$

## 4. Numerical solution of the PDE: Crank–Nicolson finite differences

Finite-difference methods discretize the PDE on a grid in asset price  $S$  and time  $t$ . Crank–Nicolson (CN) is a widely used implicit scheme that is second-order accurate in both space and time and is unconditionally stable (for the linear parabolic PDEs we consider).

Below we derive the CN discretization and obtain the tridiagonal linear system that must be solved at each time step.

### 4.1 Domain and grid

Truncate the infinite spatial domain  $S \in (0, \infty)$  to a large finite domain  $S \in [S_{\min}, S_{\max}]$ . A common choice is  $S_{\min} = 0$  and  $S_{\max} = LS_0$  with  $L = 3$  or  $4$ , chosen so the option value is insensitive to further increases in  $S_{\max}$ .

Define a uniform spatial grid with  $M + 1$  nodes:

$$S_i = S_{\min} + i\Delta S \quad \Delta S = \frac{S_{\max} - S_{\min}}{M} \quad i = 0, 1, \dots, M$$

Define a temporal grid stepping backward from  $T$  to  $0$  with  $N$  time-steps:

$$t^n = n\Delta t \quad \Delta t = \frac{T}{N} \quad n = 0, 1, \dots, N$$

We will denote the numerical approximation to  $V(S_i, t^n)$  by  $V_i^n$ . (Because the PDE is typically integrated backward in time from  $t = T$  to  $t = 0$ , many authors index time reversely; here we take  $n = 0$  for time  $t = 0$  and note the CN update moves from known  $V^n$  to  $V^{n+1}$ .)

### 4.2 Spatial derivatives approximation

Approximate first and second derivatives by centered finite differences:

$$\left. \frac{\partial V}{\partial S} \right|_{S_i t^n} \approx \frac{V_{i+1}^n - V_{i-1}^n}{2\Delta S} \quad \left. \frac{\partial^2 V}{\partial S^2} \right|_{S_i t^n} \approx \frac{V_{i+1}^n - 2V_i^n + V_{i-1}^n}{(\Delta S)^2}$$



### 4.3 Semi-discrete operator

Plugging these into the PDE, at grid node  $(S_i, t^n)$  the differential operator

$$\mathcal{L}V := \frac{1}{2}\sigma^2 S^2 V_{SS} + rSV_S - rV$$

is approximated by

$$(\mathcal{L}V)_i^n \approx \frac{1}{2}\sigma^2 S_i^2 \frac{V_{i+1}^n - 2V_i^n + V_{i-1}^n}{(\Delta S)^2} + rS_i \frac{V_{i+1}^n - V_{i-1}^n}{2\Delta S} - rV_i^n$$

It is convenient to rewrite this as a linear combination

$$(\mathcal{L}V)_i^n \approx \alpha_i^n V_{i-1}^n + \beta_i^n V_i^n + \gamma_i^n V_{i+1}^n$$

with coefficients (dropping the explicit time index for the coefficients since they depend only on  $S_i$ ):

$$\alpha_i := \frac{1}{2}\sigma^2 S_i^2 \frac{1}{(\Delta S)^2} - \frac{rS_i}{2\Delta S}$$

$$\beta_i := -\frac{1}{2}\sigma^2 S_i^2 \frac{2}{(\Delta S)^2} - r$$

$$\gamma_i := \frac{1}{2}\sigma^2 S_i^2 \frac{1}{(\Delta S)^2} + \frac{rS_i}{2\Delta S}$$

Thus

$$(\mathcal{L}V)_i^n \approx \alpha_i V_{i-1}^n + \beta_i V_i^n + \gamma_i V_{i+1}^n$$

### 4.4 Crank–Nicolson time discretization

Crank–Nicolson averages the spatial operator at times  $t^n$  and  $t^{n+1}$ . The time derivative is approximated by

$$\frac{V_i^{n+1} - V_i^n}{\Delta t} \approx \frac{1}{2}((\mathcal{L}V)_i^{n+1} + (\mathcal{L}V)_i^n).$$

Rearrange to obtain the CN update equation:

$$V_i^{n+1} - \frac{\Delta t}{2}(\mathcal{L}V)_i^{n+1} = V_i^n + \frac{\Delta t}{2}(\mathcal{L}V)_i^n$$

Substituting the finite-difference representation of  $\mathcal{L}$  yields a linear relation for interior nodes  $i = 1, \dots, M - 1$ :

$$V_i^{n+1} - \frac{\Delta t}{2}(\alpha_i V_{i-1}^{n+1} + \beta_i V_i^{n+1} + \gamma_i V_{i+1}^{n+1}) = V_i^n + \frac{\Delta t}{2}(\alpha_i V_{i-1}^n + \beta_i V_i^n + \gamma_i V_{i+1}^n)$$

Collect terms for  $V_{i-1}^{n+1}$ ,  $V_i^{n+1}$ ,  $V_{i+1}^{n+1}$  on the left-hand side and known  $V^n$  terms on the right-hand side. Define coefficients:

$$A_i := -\frac{\Delta t}{2}\alpha_i$$

$$B_i := 1 - \frac{\Delta t}{2}\beta_i \quad (\text{left-hand side coefficients})$$

$$C_i := -\frac{\Delta t}{2}\gamma_i$$

and

$$\begin{aligned}\tilde{A}_i &:= \frac{\Delta t}{2} \alpha_i \\ \tilde{B}_i &:= 1 + \frac{\Delta t}{2} \beta_i \quad (\text{right-hand side coefficients}) \\ \tilde{C}_i &:= \frac{\Delta t}{2} \gamma_i\end{aligned}$$

Then the CN equation becomes, for each interior  $i$ ,

$$A_i V_{i-1}^{n+1} + B_i V_i^{n+1} + C_i V_{i+1}^{n+1} = \tilde{A}_i V_{i-1}^n + \tilde{B}_i V_i^n + \tilde{C}_i V_{i+1}^n$$

#### 4.5 Matrix form

Stack the interior unknowns into a vector

$$\mathbf{V}^n = \begin{bmatrix} V_1^n \\ V_2^n \\ \vdots \\ V_{M-1}^n \end{bmatrix}$$

For each time-step the system can be written compactly as

$$\boxed{A \mathbf{V}^{n+1} = B \mathbf{V}^n + \mathbf{d}^n}$$

where  $A$  and  $B$  are  $(M-1) \times (M-1)$  tridiagonal matrices with entries given by the  $A_i, B_i, C_i$  and  $\tilde{A}_i, \tilde{B}_i, \tilde{C}_i$  coefficients, and  $\mathbf{d}^n$  is a vector that collects boundary contributions. Explicitly,  $A$  has the form

$$A = \begin{bmatrix} B_1 & C_1 & 0 & \cdots & 0 \\ A_2 & B_2 & C_2 & \ddots & \vdots \\ 0 & \ddots & \ddots & \ddots & 0 \\ \vdots & \ddots & A_{M-2} & B_{M-2} & C_{M-2} \\ 0 & \cdots & 0 & A_{M-1} & B_{M-1} \end{bmatrix}$$

and  $B$  is similarly

$$B = \begin{bmatrix} \tilde{B}_1 & \tilde{C}_1 & 0 & \cdots & 0 \\ \tilde{A}_2 & \tilde{B}_2 & \tilde{C}_2 & \ddots & \vdots \\ 0 & \ddots & \ddots & \ddots & 0 \\ \vdots & \ddots & \tilde{A}_{M-2} & \tilde{B}_{M-2} & \tilde{C}_{M-2} \\ 0 & \cdots & 0 & \tilde{A}_{M-1} & \tilde{B}_{M-1} \end{bmatrix}$$

The boundary vector  $\mathbf{d}^n$  accounts for known values  $V_0^n$  and  $V_M^n$  appearing in the finite-difference stencils for the first and last interior nodes. For example, the first equation (for  $i = 1$ ) includes terms proportional to  $V_0^n$  and  $V_0^{n+1}$ ; move these to the right-hand side to obtain the appropriate entry in  $\mathbf{d}^n$ .

**Boundary conditions.** Reasonable boundary conditions for European call options:

$$\begin{aligned} V(S_{\min} = 0, t) &= 0 && \text{(call is worthless at zero)} \\ V(S_{\max}, t) &\approx S_{\max} - Ke^{-r(T-t)} \end{aligned}$$

Use these values to compute contributions to  $\mathbf{d}^n$ . If one uses  $S_{\min} = 0$  then  $V_0^n = 0$  simplifies the first equation. For the last equation, known  $V_M^n$  provides a known term on the right-hand side.

## 5. Implementation notes and practical considerations

**Defining the option and model.** Choose a standard option (e.g., European call or put) and clearly define all parameters ( $S_0$ ,  $K$ ,  $r$ ,  $\sigma$ ,  $T$ , dividends). For European options, the analytical Black–Scholes solution can serve as a reference.

### Accuracy comparison.

- **Monte Carlo (MC):** Simulate  $N$  paths of the underlying asset, compute discounted payoff averages, and measure absolute or relative error against the analytical solution. Optionally, report standard error of the estimator.
- **Crank–Nicolson (CN):** Construct a time–price grid ( $\Delta t$ ,  $\Delta S$ ) and solve the PDE. Compute the error relative to the analytical or highly accurate numerical solution.

Compare errors for both methods and observe convergence under grid refinement (CN) or increasing number of paths (MC).

### Efficiency comparison.

- **Monte Carlo:** Runtime scales linearly with the number of simulated paths. Error decreases as  $1/\sqrt{N}$ . Record CPU time for each  $N$ .
- **Crank–Nicolson:** Runtime scales with  $M \times N$ , where  $M$  is the number of time steps and  $N$  the number of spatial grid points. Finer grids improve accuracy faster than MC for the same runtime in low-dimensional problems.

Generate work–precision plots (error vs CPU time) to visualize efficiency trade-offs.

### Practical considerations.

- MC is more suitable for high-dimensional or path-dependent options; variance reduction techniques improve efficiency.

## 6. Tasks

You are asked to implement both Monte Carlo and Crank–Nicolson methods for European call options and perform a comparative study.

**1. Constants table (suggested values):**

Parameter	Value
Initial asset price $S_0$	100
Strike $K$	110 (high K values for variance reduction)
Time to maturity $T$	1 year
Volatility $\sigma$	0.2, 0.4
Risk-free rate $r$	0.01, 0.05
Number of time steps $N$	100
Number of Monte Carlo paths $M$	50,000

**2. Experiments:**

- Compute option prices for all combinations of  $\sigma$  and  $r$ .
- Compare Monte Carlo estimated prices with Crank–Nicolson numerical PDE prices.
- Measure CPU time and compare efficiency of Monte Carlo vs CN.
- Use variance reduction techniques for the MC estimator. Compare efficiency with new estimators.
- Plot option price vs volatility, option price vs interest rate.

# Option 3: Bayesian Neural Networks via MCMC

## 1. Background and Motivation

Traditional neural networks optimize weights to single point estimates, usually via gradient descent. This approach does not quantify uncertainty, which is important when:

- making decisions in risk-sensitive applications (medicine, finance, engineering)
- detecting anomalies or rare events
- forecasting with small datasets
- requiring confidence intervals or credible predictions

Bayesian neural networks (BNNs) treat the network weights as random variables with prior distributions. The posterior distribution of the weights given observed data allows us to capture uncertainty:

$$p(w \mid X, y) = \frac{p(y \mid X, w) p(w)}{p(X, y)},$$

where  $w$  are the weights,  $X$  the inputs,  $y$  the observed outputs,  $p(w)$  the prior, and  $p(y \mid X, w)$  the likelihood.

Because the posterior is usually intractable, we approximate it using Markov Chain Monte Carlo (MCMC) methods such as:

- Metropolis–Hastings
- Gibbs sampling
- Hamiltonian Monte Carlo (HMC)

## 2. Model Structure

Consider a simple feedforward neural network with input  $x$ , one hidden layer of  $H$  neurons, and output  $y$ . Denote the network function as  $f(x; w)$ .

**Priors:** Assign independent Gaussian priors to weights and biases:

$$w \sim \mathcal{N}(0, \sigma_w^2), \quad b \sim \mathcal{N}(0, \sigma_b^2).$$

**Likelihood (for regression):** Assume Gaussian noise on outputs:

$$y \mid x, w \sim \mathcal{N}(f(x; w), \sigma^2).$$

**Posterior:** The posterior distribution of the weights is proportional to:

$$p(w \mid X, y) \propto p(y \mid X, w)p(w).$$

**Posterior predictive distribution:** For a new input  $x_{\text{new}}$ , approximate the predictive distribution by averaging over posterior samples:

$$\hat{y}(x_{\text{new}}) \approx \frac{1}{S} \sum_{s=1}^S f(x_{\text{new}}; w^{(s)}),$$

where  $w^{(1)}, \dots, w^{(S)}$  are sampled from the posterior using MCMC.

### 3. Datasets

You can use any dataset, but it is recommended to choose **small or simple datasets** so that MCMC sampling completes in reasonable time. Examples include:

- **Tabular / regression:** Boston Housing, Diabetes, or synthetic datasets such as  $y = \sin(x) + \epsilon$ .
- **Time series:** Stock prices (subset), daily sales, or temperature data.
- **Classification / small images:** Iris dataset, or a subset of MNIST (e.g., 1000–5000 images).

The key is to keep the dataset size manageable for a few thousand MCMC iterations.

### 4. Suggested Experiments

- Vary the prior variance  $\sigma_w^2$  and observe the effect on predictive uncertainty.
- Change the noise variance  $\sigma^2$  and compare predictive intervals.
- Compare performance and uncertainty estimates between MCMC BNN and standard NN.
- Optional: Vary the number of hidden neurons or layers and observe convergence of MCMC samples.

### 5. Tasks

1. Implement a Bayesian neural network using MCMC.
2. Generate posterior samples for the weights and biases.
3. Compute posterior predictive distributions for test inputs.
4. Investigate how predictions and uncertainty change when:
  - the prior variance changes
  - the output noise changes
  - the network size changes
5. Compare accuracy and efficiency with a standard neural network.

## Option 2: Replicating the Design and Implementation of a Published Paper

### Background and Motivation

Replication is a core aspect of scientific research. By attempting to reproduce the results of a published study, you gain a deeper understanding of modelling assumptions, computational methods, and analysis techniques. This option allows you to connect course concepts to real-world research and critically evaluate methodological choices.

### Project Task

You will:

- Select a published paper related to stochastic modelling, Monte Carlo and/or Monte Carlo simulation or other topics covered in the course.
- Carefully review the paper, focusing on the problem statement, assumptions, model structure, and computational approach.
- Implement the methods or experiments described in the paper using a programming language of their choice.
- Compare your results with the published results and analyse any discrepancies.
- Document challenges encountered, adaptations made, and lessons learned from the replication process.

### Learning Objectives

- Understand the translation of theoretical models into practical computational methods.
- Critically evaluate published results and methodologies.
- Develop practical skills in programming, numerical methods, and model validation.
- Gain experience in scientific documentation and reproducibility.

## Option 3: Literature Review of 5–15 Papers

### Background and Motivation

A literature review allows you to synthesize knowledge on a particular topic, identify trends, gaps, and open questions, and develop critical evaluation skills. This project option emphasizes scholarly analysis rather than implementation, and is ideal for you interested in exploring broader research directions.

### Project Task

You will:

- Select a coherent topic or research question related to the course (e.g., stochastic simulations, Bayesian inference, neural network modelling, queueing systems).
- Search for and read 5–15 relevant academic papers, preprints, or technical reports.
- Summarize key methods, results, assumptions, and conclusions of each paper.
- Compare approaches, highlight similarities and differences, and identify gaps in the literature.
- Discuss potential future directions or applications suggested by the literature.

### Learning Objectives

- Develop skills in critical reading, synthesis, and scholarly writing.
- Identify research trends and gaps within a focused area.
- Learn to communicate complex ideas clearly in written and oral form.
- Gain familiarity with academic databases, citation practices, and literature evaluation techniques.