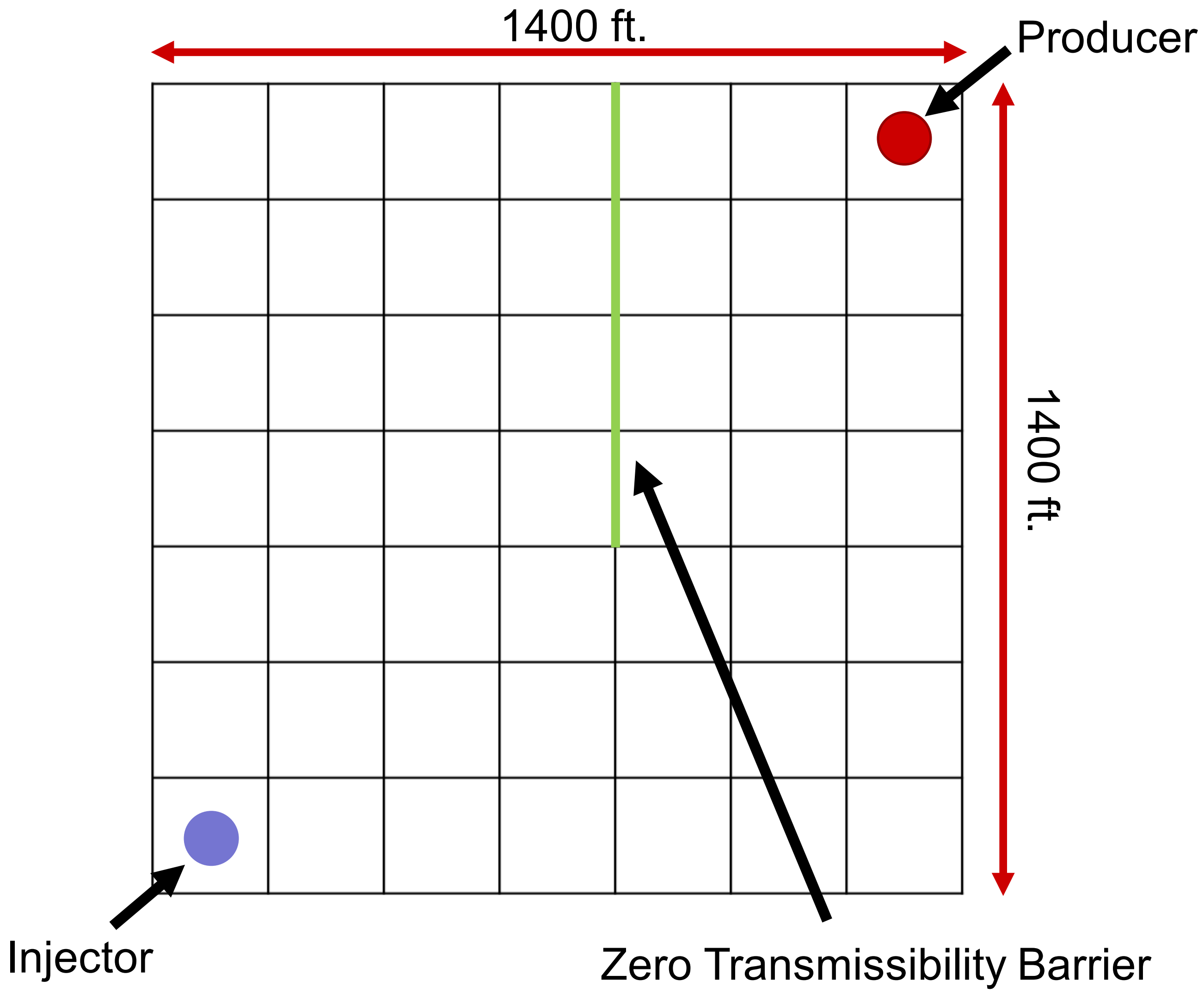
HW7 – 2D Implicit

PETR 5309

Homework Problem

2D Implicit Solver Homework

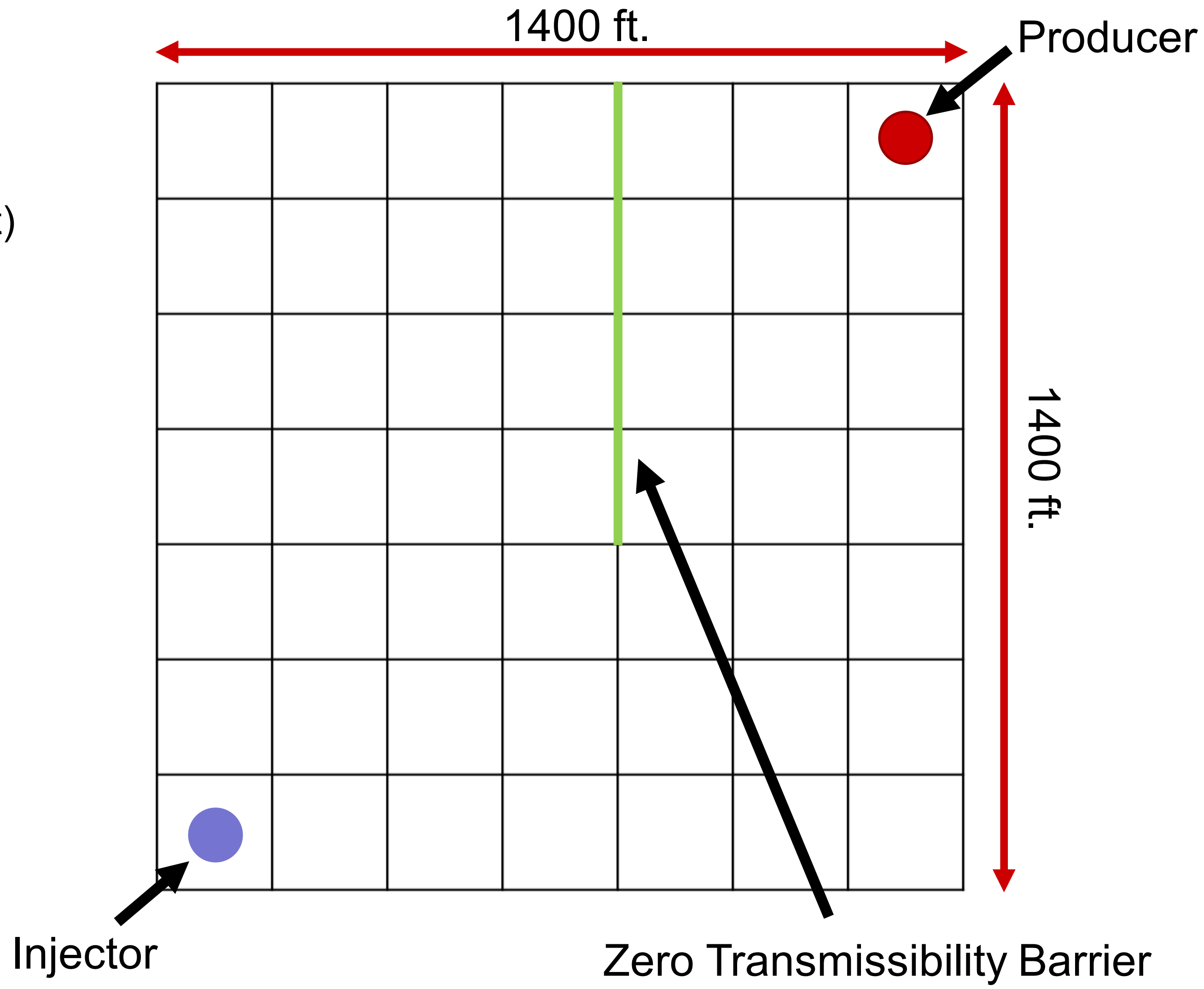
- Create a 7x7 grid reservoir with properties on the following page.
- Within this reservoir is a fault, depicted within the reservoir picture.
- Included are slides from lecture that should help guide you through the problem.
- You may use your Gaussian solver from the previous homework to find the pressure solution at each time step.
  - NOTE: Remember that to define a zero transmissibility barrier, the matrix cell defining the transmissibility between two cells will be zero



Problem Data

Consider

- Homogenous, 2D, Single Phase
- Fluid is flowing into the system with a pressure constrained Injector (Pinlet) and Producer (Poutlet)
- Properties:
  - Pinlet=6000 psi
  - Poutlet=3000 psi
  - Reservoir Pressure: 4500
  - Length=1400 feet
  - Permx=250 md
  - Height(Grid Thickness) =60 feet
  - Width=1400 Feet
  - Viscosity=1 cp
  - FVF= 1 RB/STB
  - Porosity = 18%
- What is the steady state flow rate?



- Upload your code, and a pressure matrix of the field once the field reaches steady state flow.

- Did your code mange to reach the precise inlet and outlet pressures?
- If not why do you think it did not?

Review

Remember: Setup of Implicit Solution

alphac	5.615
betac	1.127
dx	1000
dy	1000
dz	75
visc	10
perm	15
FVF	1
por	0.18
cl	3.50E-06
Pinit	6000
dt	15

$$\boxed{T_{lx} = \left(\beta_c \frac{A_x k_x}{\mu_l B_l \Delta x}\right)}$$

$$\boxed{(VT) = \left(\frac{\alpha_c B_l^0 \Delta t}{V_b \phi c_l}\right)}$$

Areax	75000
Vb	75000000
Perm_D	0.0150
(VT)	1.7825
Tlx	0.1268
(TV)	0.5610

$$\boxed{\left[T_{lx_{i+1/2}} p_{i+1}^{n+1} - \left((TV) + T_{lx_{i+1/2}} + T_{lx_{i-1/2}}\right) p_i^{n+1} + T_{lx_{i-1/2}} p_{i-1}^{n+1}\right] = -\left[(TV)\, p_i^n + q_{lsc_i}\right]}$$

												Pn	Qi
-0.6878	P1(n+1) +	0.1268	P2(n+1)							=	-3365.98	6000.00	0
0.1268	P1(n+1) +	-0.8146	P2(n+1) +	0.1268	P3(n+1)					=	-3365.98	6000.00	0
		0.1268	P2(n+1) +	-0.8146	P3(n+1) +	0.1268	P4(n+1)			=	-3365.98	6000.00	0
				0.1268	P3(n+1) +	-0.8146	P4(n+1) +	0.1268	P5(n+1)	=	-3215.98	6000.00	-150
						0.1268	P4(n+1) +	-0.6878	P5(n+1)	=	-3365.98	6000.00	0

Implicit Setup as a Matrix equation

$$\begin{bmatrix} -0.6878 & 0.1268 & & & \\ 0.1268 & -0.8146 & 0.1268 & & \\ & 0.1268 & -0.8146 & 0.1268 & \\ & 0.0000 & 0.1268 & -0.8146 & 0.1268 \\ & & & 0.1268 & -0.6878 \end{bmatrix} \begin{bmatrix} P1(n+1) \\ P2(n+1) \\ P3(n+1) \\ P4(n+1) \\ P5(n+1) \end{bmatrix} = \begin{bmatrix} -3365.98 \\ -3365.98 \\ -3365.98 \\ -3215.98 \\ -3365.98 \end{bmatrix}$$

From Reference "Basic Applied Reservoir Simulation", Ertekin. Et. Al.

$$\mathrm{T}^n_{lx_{i+½}} p^{n+1}_{i+1} - \left[\left(\frac{V_b \phi c_l}{\alpha_c B^{\circ}_l \Delta t} \right)_i + \mathrm{T}^n_{lx_{i+½}} + \mathrm{T}^n_{lx_{i-½}} \right] p^{n+1}_i + \mathrm{T}^n_{lx_{i-½}}$$

$$\times\, p^{n+1}_{i-1} = - \left[q_{lsc_i} + \left(\frac{V_b \phi c_l}{\alpha_c B^{\circ}_l \Delta t} \right)_i p^n_i \right], \quad \ldots\ldots\ldots \quad (5.105)$$

TABLE 5.6—SYSTEM OF EQUATIONS IN EXAMPLE 5.11

$$\begin{bmatrix} -0.6878 p_1^{n+1} + 0.1268 p_2^{n+1} \\ +0.1268 p_1^{n+1} - 0.8146 p_2^{n+1} + 0.1268 p_3^{n+1} \\ +0.1268 p_2^{n+1} - 0.8146 p_3^{n+1} + 0.1268 p_4^{n+1} \\ +0.1268 p_3^{n+1} - 0.8146 p_4^{n+1} + 0.1268 p_5^{n+1} \\ +0.1268 p_4^{n+1} - 0.6878 p_5^{n+1} \end{bmatrix} = \begin{bmatrix} -3,365.98 \\ -3,365.98 \\ -3,365.98 \\ -3,215.98 \\ -3,365.98 \end{bmatrix}$$

From Reference "Basic Applied Reservoir Simulation", Ertekin. Et. Al.

TABLE 5.6—SYSTEM OF EQUATIONS IN EXAMPLE 5.11

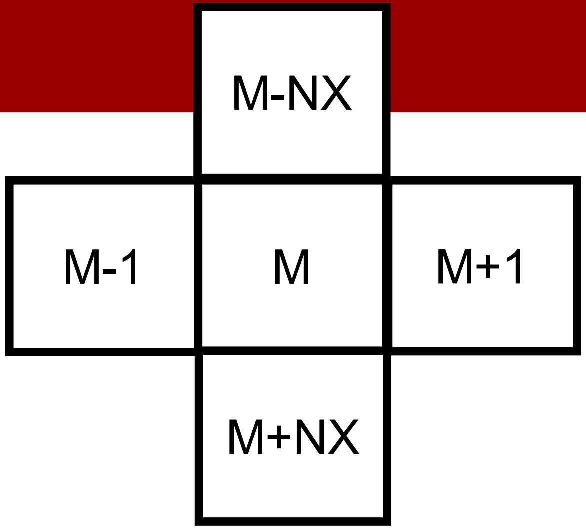
$$\begin{bmatrix} -0.6878p_1^{n+1} + 0.1268p_2^{n+1} \\ +0.1268p_1^{n+1} - 0.8146p_2^{n+1} + 0.1268p_3^{n+1} \\ +0.1268p_2^{n+1} - 0.8146p_3^{n+1} + 0.1268p_4^{n+1} \\ +0.1268p_3^{n+1} - 0.8146p_4^{n+1} + 0.1268p_5^{n+1} \\ +0.1268p_4^{n+1} - 0.6878p_5^{n+1} \end{bmatrix} = \begin{bmatrix} -3,365.98 \\ -3,365.98 \\ -3,365.98 \\ -3,215.98 \\ -3,365.98 \end{bmatrix}$$

$$\begin{bmatrix} -0.6878 & 0.1268 & & & \\ 0.1268 & -0.8146 & 0.1268 & & \\ & 0.1268 & -0.8146 & 0.1268 & \\ & 0.0000 & 0.1268 & -0.8146 & 0.1268 \\ & & & 0.1268 & -0.6878 \end{bmatrix} \begin{bmatrix} P1(n+1) \\ P2(n+1) \\ P3(n+1) \\ P4(n+1) \\ P5(n+1) \end{bmatrix} \begin{matrix} = \\ = \\ = \\ = \\ = \end{matrix} \begin{bmatrix} -3365.98 \\ -3365.98 \\ -3365.98 \\ -3215.98 \\ -3365.98 \end{bmatrix}$$

Solution Process

1. Specify well rates, dt, number of gridblocks, gridblock properties
2. Initialize Pressure
3. Initialize t=0
4. Update t = t + dt
5. If t > tmax go to Step 11
6. Set Pressure_old = Pressure
7. Calculate Matrix Equation Coefficients
8. Calculate RHS using Pressure_old, rates, dt
9. Use Linear Solver to calculate Pressure at current time
10. Go to step 4
11. Stop

2D Implicit Formulation



For Grid Block M = I + (J-1) * NX

$$(T^{n}_{M+1,M}P^{n+1}_{M+1}) + (T^{n}_{M-1,M}P^{n+1}_{M-1}) + (T^{n}_{M+NX,M}P^{n+1}_{M+NX}) + (T^{n}_{M-NX,M}\, P^{n+1}_{M-NX})$$

$$- \left[(T^{n}_{M+1,M} + T^{n}_{M-1,M} + T^{n}_{M+NX,M} + T^{n}_{M-NX,M}) + \left(\frac{Vb\ \emptyset\ c}{\alpha_c\ B\ \Delta t}\right)_M\right] P^{n+1}_{M}$$

$$= \ - \left[Q_M + \left(\frac{Vb\ \emptyset\ c}{\alpha_c\ B\ \Delta t}\right)_M P^{n}_{M} \right)\right]$$

$$\boldsymbol{T_x} = \left(\beta_c \frac{A_x k_x}{\mu_l B_l \Delta x}\right)$$

Initial Pressure Matrix for 5x5 problem well in center

			block	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	
			i	1	2	3	4	5	1	2	3	4	5	1	2	3	4	5	1	2	3	4	5	1	2	3	4	5	RHS
			j	1	1	1	1	1	2	2	2	2	2	3	3	3	3	3	4	4	4	4	4	5	5	5	5	5	
block	i	j																											
1	1	1		-0.534	0.127	0.000	0.000	0.000	0.127	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	-1683.0
2	2	1		0.127	-0.661	0.127	0.000	0.000	0.000	0.127	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	-1683.0
3	3	1		0.000	0.127	-0.661	0.127	0.000	0.000	0.000	0.127	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	-1683.0
4	4	1		0.000	0.000	0.127	-0.661	0.127	0.000	0.000	0.000	0.127	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	-1683.0
5	5	1		0.000	0.000	0.000	0.127	-0.534	0.000	0.000	0.000	0.000	0.127	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	-1683.0
6	1	2		0.127	0.000	0.000	0.000	0.000	-0.661	0.127	0.000	0.000	0.000	0.127	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	-1683.0
7	2	2		0.000	0.127	0.000	0.000	0.000	0.127	-0.788	0.127	0.000	0.000	0.000	0.127	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	-1683.0
8	3	2		0.000	0.000	0.127	0.000	0.000	0.000	0.127	-0.788	0.127	0.000	0.000	0.000	0.127	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	-1683.0
9	4	2		0.000	0.000	0.000	0.127	0.000	0.000	0.000	0.127	-0.788	0.127	0.000	0.000	0.000	0.127	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	-1683.0
10	5	2		0.000	0.000	0.000	0.000	0.127	0.000	0.000	0.000	0.127	-0.661	0.000	0.000	0.000	0.000	0.127	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	-1683.0
11	1	3		0.000	0.000	0.000	0.000	0.000	0.127	0.000	0.000	0.000	0.000	-0.661	0.127	0.000	0.000	0.000	0.127	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	-1683.0
12	2	3		0.000	0.000	0.000	0.000	0.000	0.000	0.127	0.000	0.000	0.000	0.127	-0.788	0.127	0.000	0.000	0.000	0.127	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	-1683.0
13	3	3		0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.127	0.000	0.000	0.000	0.127	-0.788	0.127	0.000	0.000	0.000	0.127	0.000	0.000	0.000	0.000	0.000	0.000	0.000	-1783.0
14	4	3		0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.127	0.000	0.000	0.000	0.127	-0.788	0.127	0.000	0.000	0.000	0.127	0.000	0.000	0.000	0.000	0.000	0.000	-1683.0
15	5	3		0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.127	0.000	0.000	0.000	0.127	-0.661	0.000	0.000	0.000	0.000	0.127	0.000	0.000	0.000	0.000	0.000	-1683.0
16	1	4		0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.127	0.000	0.000	0.000	0.000	-0.661	0.127	0.000	0.000	0.000	0.127	0.000	0.000	0.000	0.000	-1683.0
17	2	4		0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.127	0.000	0.000	0.000	0.127	-0.788	0.127	0.000	0.000	0.000	0.127	0.000	0.000	0.000	-1683.0
18	3	4		0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.127	0.000	0.000	0.000	0.127	-0.788	0.127	0.000	0.000	0.000	0.127	0.000	0.000	-1683.0
19	4	4		0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.127	0.000	0.000	0.000	0.127	-0.788	0.127	0.000	0.000	0.000	0.127	0.000	-1683.0
20	5	4		0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.127	0.000	0.000	0.000	0.127	-0.661	0.000	0.000	0.000	0.000	0.127	-1683.0
21	1	5		0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.127	0.000	0.000	0.000	0.000	-0.534	0.127	0.000	0.000	0.000	-1683.0
22	2	5		0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.127	0.000	0.000	0.000	0.127	-0.661	0.127	0.000	0.000	-1683.0
23	3	5		0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.127	0.000	0.000	0.000	0.127	-0.661	0.127	0.000	-1683.0
24	4	5		0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.127	0.000	0.000	0.000	0.127	-0.661	0.127	-1683.0
25	5	5		0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.127	0.000	0.000	0.000	0.127	-0.534	-1683.0

Well Model for Horizontal Well

Well Model for Vertical Wells

$$q_{sc} = -J_w\left(\bar{p} - p_{wf}\right)$$

$$J_w = \frac{-2\,\pi\,\beta_c k_H h}{\mu\,B\left[\log_e\left(\frac{r_{eq}}{r_w}\right) + s - F\right]}$$

$$r_{eq} = 0.28\,\frac{\left\{\left[\left({}^{k_y}\!/_{k_x}\right)^{1/2}(\Delta x)^2\right] + \left[\left[\left({}^{k_x}\!/_{k_y}\right)^{1/2}(\Delta y)^2\right]\right]\right\}^{1/2}}{\left({}^{k_y}\!/_{k_x}\right)^{1/4} + \left({}^{k_x}\!/_{k_y}\right)^{1/4}}$$

- J_w *is different for horizontal wells*
  - k_h *which is average horizontal perm for vertical wells gets replaced with an average which includes perm in the z direction*
  - h, *which is the gridblock thickness for vertical wells, gets replaced with the length of the completion in the gridblock, i.e. gridblock length*
  - r_{eq} *which is the equivalent radius, must include perm in z direction*

Example

Calculate transmissibility

```
do i=1,nx
     do j=1,ny
          k=1
          m=i+(j-1)*nx+(k-1)*nx*ny
          .
          if (i .eq. 1 ) then
            tx_minus(m)=0
          else
            tx_minus(m)=betac*areax*permx/(visc*fvf*dx)
          endif

          if (i .eq. nx ) then
            tx_plus(m)=0
          else
            tx_plus(m)=betac*areax*permx/(visc*fvf*dx)
          endif

          if (j .eq. 1 ) then
            ty_minus(m)=0
          else
            ty_minus(m)=betac*areay*permy/(visc*fvf*dy)
          endif

          if (j .eq. ny ) then
            ty_plus(m)=0
          else
            ty_plus(m)=betac*areay(*permy/(visc*fvf*dy)
          endif

      enddo
   enddo
```

- 2 D
- Constant properties
- Calculate:
  - tx_minus,
  - tx_plus,
  - ty_minus,
  - ty_plus

Set up each row of Matrix

```
do ii=1,nx
    do jj=1,ny
          kk=1
          mm=ii+(jj-1)*nx+ (kk-1)*nx*ny

         if (ii .gt. 1) then
                amatrix(mm,mm-1)=tx_minus(mm)
         endif

         if (ii .lt. nx) then
                amatrix(mm,mm+1)=tx_plus(mm)
         endif

         if (jj .gt. 1) then
               amatrix(mm,mm-nx)=ty_minus(mm)
         endif

         if (jj .lt. ny) then
               amatrix(mm,mm+nx)=ty_plus(mm)
         endif

         amatrix(mm,mm)=-1.0*( (1/term1) + tx_minus(mm) +
         tx_plus(mm) + ty_minus(mm) + ty_plus(mm) )

     enddo
end do
```

```
term1=alphac*const_fvf*dt/(vb*por*compress)
```

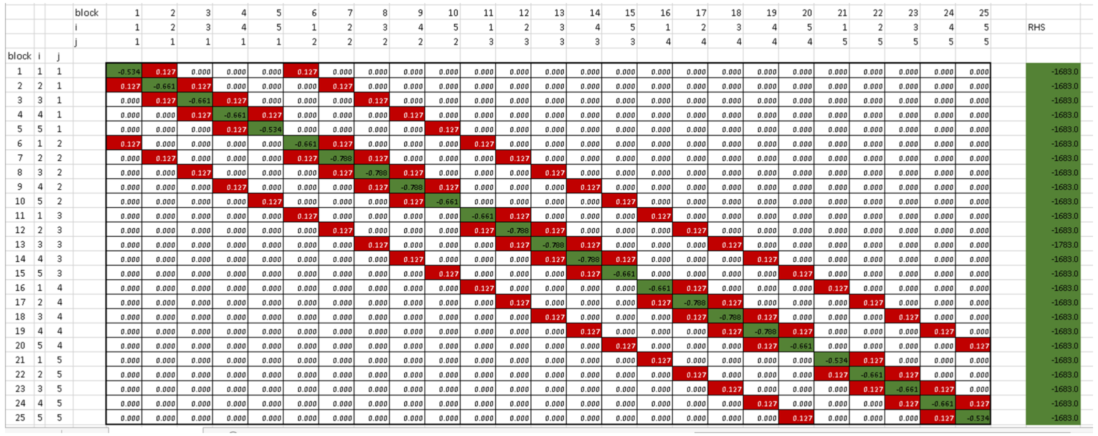
Set up Right Hand Side of Equation

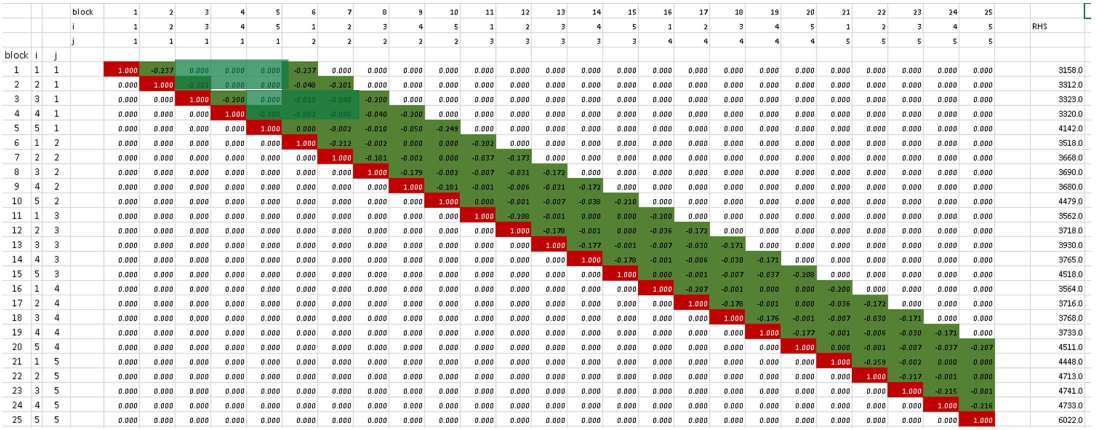
```
term1=alphac*const_fvf*dt/(vb*por*compress)
```

```
do i=1,nx
    do j=1,ny
        m=i+(J-1)*nx
        rhs(m)=-1.0* ( (pressure_old(m)/term1) +  rate(m) )
    end do
 end do
```

Linear Solver Step

- Call Linear solver
  - Pass to solver:
    - Dimensions of Matrix
    - Matrix Coefficients
    - RHS
  - Receive back from solver:
    - Updated Pressures





block	i	j		
1	1	1	Solution:	6022.2
2	2	1	Solution:	6032.4
3	3	1	Solution:	6044.2
4	4	1	Solution:	6032.4
5	5	1	Solution:	6022.2
6	1	2	Solution:	6032.4
7	2	2	Solution:	6056.6
8	3	2	Solution:	6098.5
9	4	2	Solution:	6056.6
10	5	2	Solution:	6032.4
11	1	3	Solution:	6044.2
12	2	3	Solution:	6098.5
13	3	3	Solution:	6280.7
14	4	3	Solution:	6098.5
15	5	3	Solution:	6044.2
16	1	4	Solution:	6032.4
17	2	4	Solution:	6056.6
18	3	4	Solution:	6098.5
19	4	4	Solution:	6056.6
20	5	4	Solution:	6032.4
21	1	5	Solution:	6022.2
22	2	5	Solution:	6032.4
23	3	5	Solution:	6044.2
24	4	5	Solution:	6032.4
25	5	5	Solution:	6022.2