Modeling Statistical phenomena
Assignment 4

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Contents

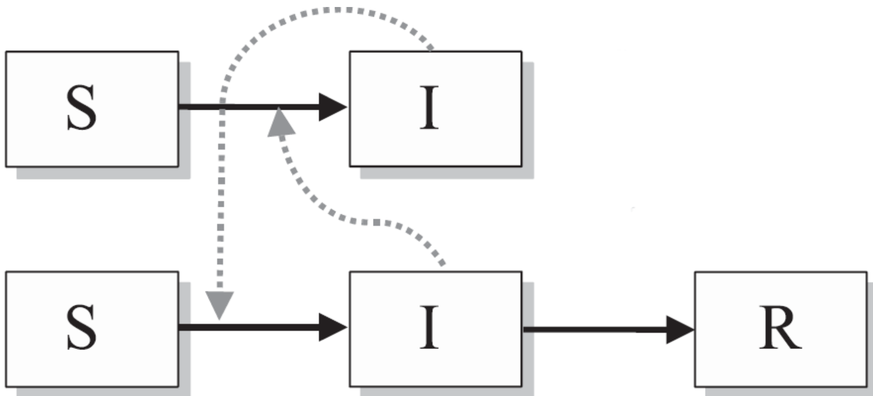


Figure 1: schematic of the model used in Ex1 [1]

1 Ex. 1

1.1 Part A

We choose $\omega = 2 * \pi$ and $\phi = 0$. The reason behind this choice is observing the fact that mosquitos bite more during the night and less during the time that an individual is awake. So, if we assume that each day begins at 12 am, we choose ϕ and ω such that $b(t)$ reach to its maximum at $t = 2k\pi$ or equivalently, at 12 midnight and its minimum at $t = \frac{3k\pi}{2}$ or at 12 noon. These conditions will be satisfied by choosing $\omega = 2 * \pi$ and $\phi = 0$.

$$\omega = 2\pi$$
$$\phi = 0$$

1.2 Part B

In fig 1 you see the schematic of the model that we use for this problem. In this figure, S, I, and R represent susceptible, infected, and recovered groups, respectively. In eq 1, you see the governing equations of this system.

$$\begin{aligned}
\frac{dX_H}{dt} &= \nu_H - rT_{HM}Y_M X_H - \mu_H X_H, \\
\frac{dY_H}{dt} &= rT_{HM}Y_M X_H - \mu_H Y_H - \gamma_H Y_H, \\
\frac{dX_M}{dt} &= \nu_M - rT_{MH}Y_H X_M - \mu_M X_M, \\
\frac{dY_M}{dt} &= rT_{MH}Y_H X_M - \mu_M Y_M,
\end{aligned} \tag{1}$$

1.3 Part C

You can see the numerical solution of eq 1 in fig 2 with the given initial condition and parameters in the question. As it is seen, There is a drop in the values of susceptible humans and mosquitoes in the first few days but then after this period of decreasing, both of these values reach to their equilibrium values. We see a similar behaviour for infected groups with this difference that first, there is a rise in the values of this two groups and then, they reach to their equilibrium values.

1.4 Part D

in fig 3, you see the final state of the system. As you can see, all of the state variables oscillate. To see this clearly, we plot the phase diagram of the system's final state (fig 4). Evidently, the phase diagram for human variables (X_h, Y_H, Z_H) has an elliptical shape. In other word the system final state is a stable cycle with period 1.

1.5 Bonus

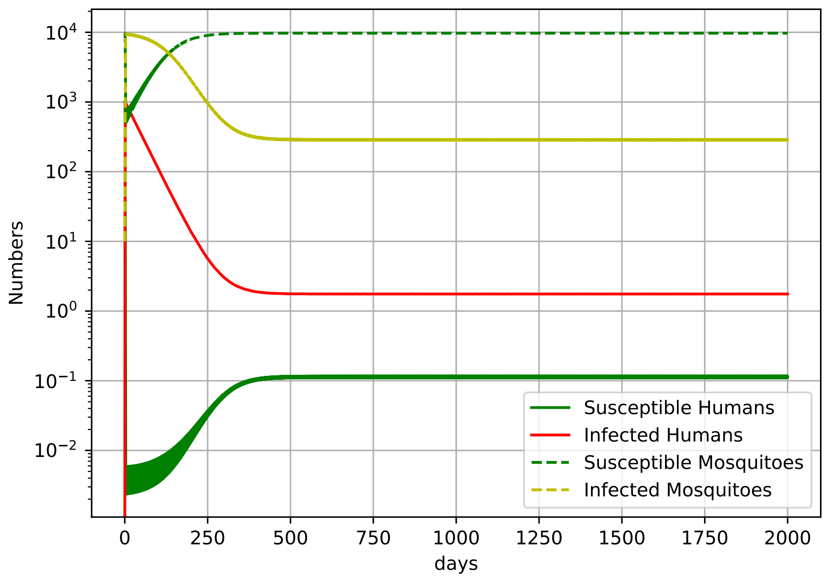
1.5.1 Part E

In this part, we let the probability of getting bitten be time-dependent. We choose a periodic function in the form of $A\,cos\,(\omega\,x + \phi) + B$ for both P_{MH} and P_{HM}. In equation 5 you see the exact function we used for these two parameters. The reason for choosing the period equal to one year is to consider the seasonal effects on the mosquito population. We also set $\phi = 0$ assuming t=0 in our simulation is the first day of spring in which there is a peak in mosquitoes' population. Note that we chose B as the same as the constant values of P_{MH} and P_{HM} and chose A in such a way to avoid negative values.

$$\begin{aligned} P_{MH} &= 0.3\,sin(\frac{2\pi}{360}\,t) + 0.4 \\ P_{HM} &= 0.6\,sin(\frac{2\pi}{360}\,t) + 0.8 \end{aligned} \tag{2}$$

1.5.2 Part F

In fig 5, you see the numerical solution of this system with time dependant transmission probabilities given in equ 1. As observed, the equilibrium state of the system consists of oscillations. In fig 5 you see the phase diagram for the equilibrium state of this system. We can see that the shape of the phase diagram is a closed-loop that shows the system's oscillatory behavior. Based on the equilibrium phase diagram (fig 6), we can say that the system's final state is a stable cycle of period 1.



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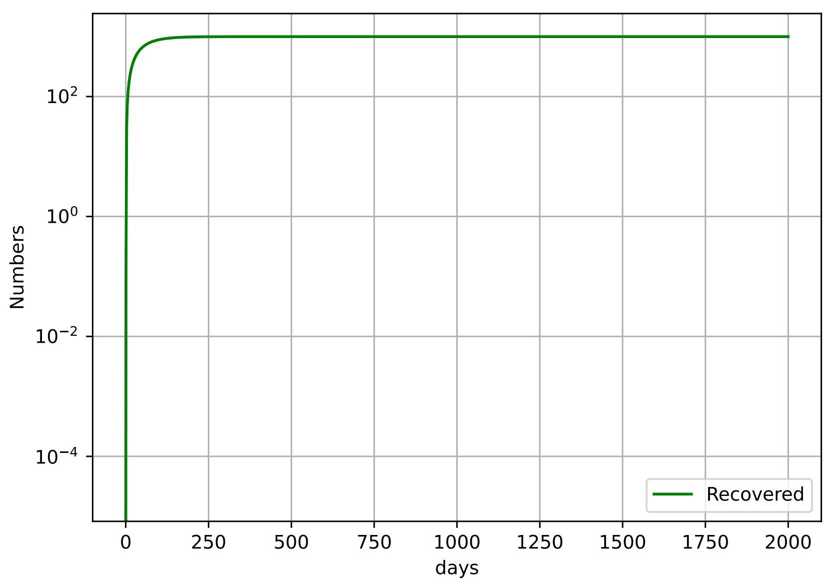
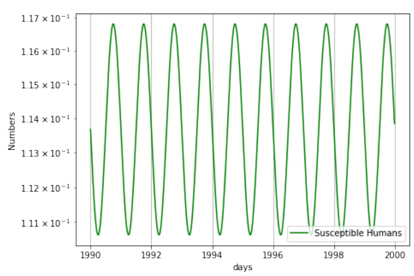


(b)

Figure 2: population of different groups versus time for equation 1 for total human population of $N_H = 1000$ and total mosquitoes population of $N_m = 10N_H$. The initial conditions are $X_H(0) = N_h$, $Y_H(0) = 0$, $Z_H(0) = 0$, $X_M(0) = N_m$, and $Y_M(0) = 10^{-3}N_H$. The parameters. The average life time of a human is 70 years for mosquitoes are 7 days. Other parameters are $P_{HM} = 0.3$, $P_{MH} = 0.6$, $\gamma = \frac{1}{45}$, $\omega = 2\pi$, $\phi = 0$, $b_0 = 4$ and $b_1 = 2$.The population of each group (human and mosquitoes) is constant.



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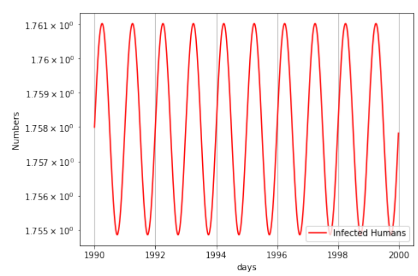

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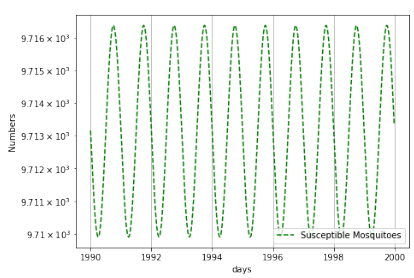

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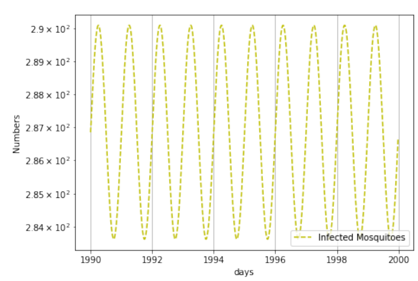


(d)

Figure 3: Final sate of the system of equations 1 with parameters given in fig 2. a) susceptible humans b)infected humans c) susceptible mosquitoes d) infected mosquitoes

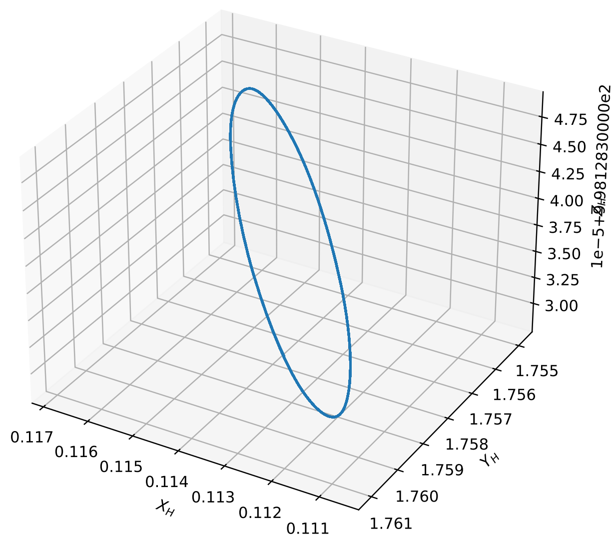


Figure 4: Phase diagram for final state of equation 1 with given parameters in fig 2. As you can see, the final state of the system is a stable cycle of period 1.

2 Ex. 2

2.1 Part A

In fig 7 you see the schematic of the model that we use for this problem. In this figure, S, I_1, I_2 and R represents susceptible, infected by wild, infected by resistant, and recovered group, respectively. In eq 3, you see the governing equations of this system.

$$
\begin{aligned}
\frac{dS}{dt} &= \nu - S(\beta_w + \beta_r I_r) - \mu S, \\
\frac{dI_w}{dt} &= \beta_w S I_w - (\gamma + T) I_w - \mu I_w, \\
\frac{I_r}{dt} &= \beta_r S I_r - \gamma I_r - \mu I_r, \\
\frac{dR}{dt} &= (\gamma + T) I_w + \gamma I_r - \mu R
\end{aligned}
\tag{3}
$$

2.2 Part B

In fig 8, you see the infected$_w(\infty)$ and infected$_r(\infty)$ versus $\eta = \frac{\beta_w \gamma}{(T+\gamma)\beta_r}$. For plotting this figure we keep all the parameters constant and change the value of T (the treatment rate from 0 to 0.15 with the step size of 0.003. As you can

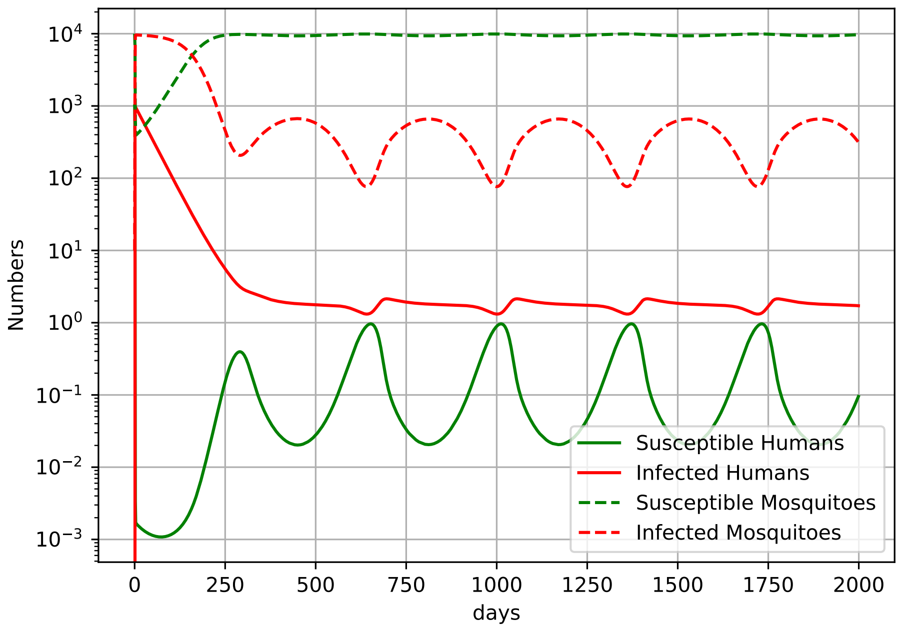


Figure 5: population of different groups versus time for equation 1 with time dependent transmission probabilities in equation 5 for total human population of $N_H = 1000$ and total mosquitoes population of $N_m = 10N_H$. The initial conditions are $X_H(0) = N_h$, $Y_H(0) = 0$, $Z_H(0) = 0$, $X_M(0) = N_m$, and $Y_M(0) = 10^{-3}N_H$. The parameters. The average life time of a human is 70 years for mosquitoes are 7 days. Other parameters are $P_{HM} = 0.3$, $P_{MH} = 0.6$, $\gamma = \frac{1}{45}$, $\omega = 2\pi$, $\phi = 0$, $b_0 = 4$ and $b_1 = 2$.

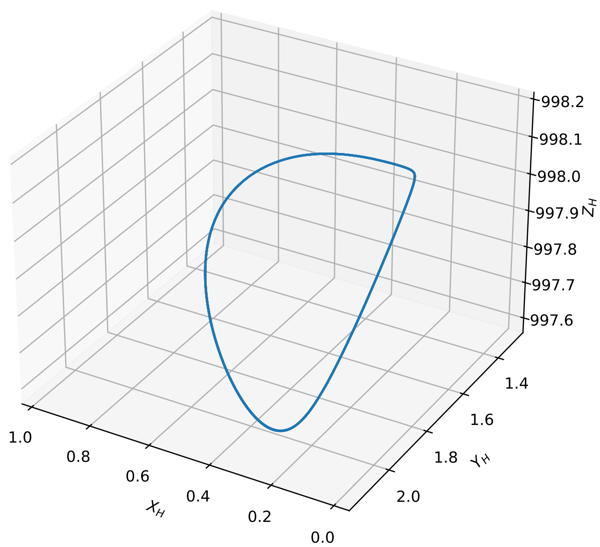


Figure 6: Phase diagram for final state of equation 1 with given parameters in fig 2 and time-dependent transmission probabilities given in 5. As you can see, the final state of the system is a stable cycle of period 1.

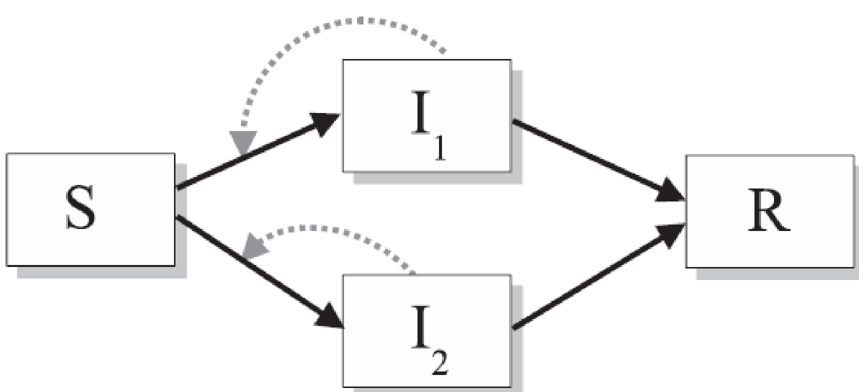
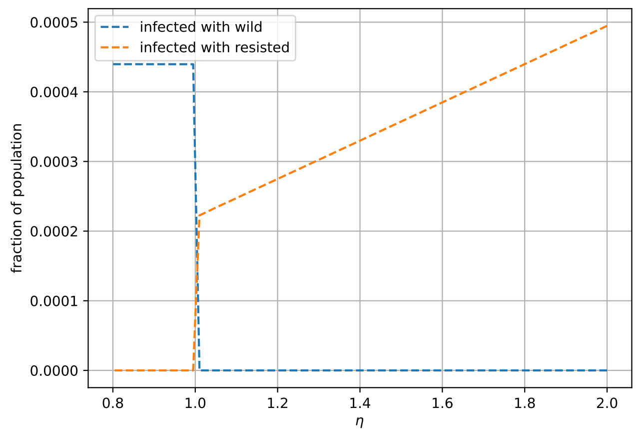


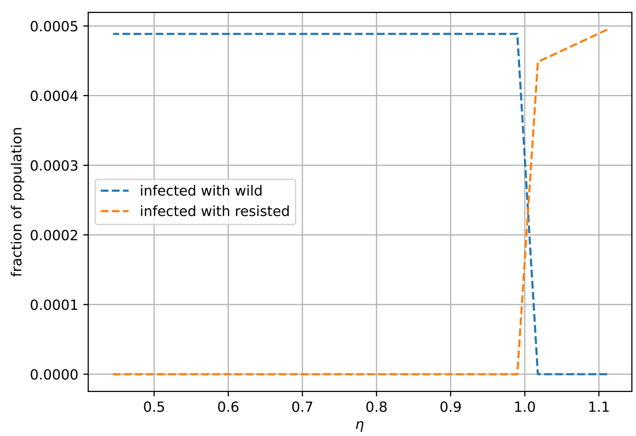
Figure 7: schematic of the model used in Ex.2 [1]

see, for the values of η less than 1, the resisted virus dominates the wild type, which is expected. In this regime, due to the resistance of the resisted virus to vaccination, it can survive. However, the wild type was wiped out from the system because of the low transmission and inability to resist vaccination. By increasing the value of η, we see a phase transition at $\eta = 1$. For the bigger values of η, the wild type dominates the resisted type.

$$\eta = \frac{\beta_w \gamma}{(T + \gamma)\beta_r}$$



(a)



(b)

Figure 8: I(∞) for infected with wild and resisted virus of equation 3 for a) $\beta_r = 0.5$ and b) $\beta_r = 0.9$. The initial values and the parameters are: $I_w(0) = 0.0001, I_r(0) = 0.0001$, and S(0)=1-$I_r(0)-I_w(0)$,$\gamma_w = 0.1, \gamma_r = 0.1$, and $\beta_w = 1$. As you can see there is a phase transition at $\eta = 1$.

References

1. Keeling, M. J. & Rohani, P. *Modeling Infectious Diseases in Humans and Animals* `https://doi.org/10.2307/j.ctvcm4gk0` (Princeton University Press, Sept. 2011).