

# Computer Systems

## Exercise 6

# Agenda

- Bonus Task: First Deadline is next Thursday 01.11.18
- Feedback on Course
- Last Weeks Exercise (Scheduling / IO)
- Byzantine Agreement
- Shared Coin - Reliable Broadcast
- This weeks exercise

# Byzantine nodes

- Node which has arbitrary behavior
- So it can:
  - Decide not to send messages
  - Sending different messages to different nodes
  - Sending wrong messages
  - Lie about input value
- If an algorithm works with  $f$  byzantine nodes, it is  $f$ -resilient



# Different Validities

- Any-input validity:
  - The decision value must be input of any node
  - That includes byzantine nodes, might not make sense
- Correct-input validity:
  - The decision value must be input of a correct node
  - Difficult because byzantine node could behave like normal one just with different value
- All-same validity:
  - if all correct nodes start with the same value, the decision must be that value
- Median validity:
  - If input values are orderable, byzantine outliers can be prevented by agreeing on a value close to the median value of the correct nodes

# Byzantine agreement in the synchronous model

- Assumption: nodes operate in synchronous rounds. In each round, each node may send a message to each other node, receive the message by other nodes and do some computation.
  - -> runtime is easy, since it is only the number of rounds

# King Algorithm (synchronous byzantine agreement)

Idea:

If not all correct input nodes have the same value, decide on value of one correct input node. Ensure this by doing  $f+1$  rounds, since there must be at least one correct input node.

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**Algorithm 11.14** King Algorithm (for  $f < n/3$ )

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1:  $x = \text{my input value}$

2: **for** phase = 1 to  $f + 1$  **do** Do until at least one correct input node

*Round 1*

3: Broadcast value( $x$ ) Send out own value

*Round 2*

4: **if** some value( $y$ ) received at least  $n - f$  times **then**

5:     Broadcast propose( $y$ )

6: **end if**

7: **if** some propose( $z$ ) received more than  $f$  times **then**

8:      $x = z$

9: **end if**

*Round 3*

10: Let node  $v_i$  be the predefined king of this phase  $i$

11: The king  $v_i$  broadcasts its current value  $w$

12: **if** received strictly less than  $n - f$  propose( $y$ ) **then**

13:      $x = w$

14: **end if**

15: **end for**

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If some value received from all nodes but byzantine ones (or at least  $((n - f) - f)$  correct ones), propose that value

If some value proposed by at least one correct node, set your value to that value

King of this phase broadcasts its value

If didn't get propose from all nodes but byzantine ones (or at least  $((n - f) - f)$  correct ones), set your value to value of king

# King Algorithm (synchronous byzantine agreement)

## Why $f+1$ ?

- Because there are  $f$  byzantine nodes, at least one of the kings will be a correct node

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### Algorithm 11.14 King Algorithm (for $f < n/3$ )

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```
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2: for phase = 1 to  $f + 1$  do
    Round 1
3: Broadcast value( $x$ )
    Round 2
4: if some value( $y$ ) received at least  $n - f$  times then
5:   Broadcast propose( $y$ )
6: end if
7: if some propose( $z$ ) received more than  $f$  times then
8:    $x = z$ 
9: end if
    Round 3
10: Let node  $v_i$  be the predefined king of this phase  $i$ 
11: The king  $v_i$  broadcasts its current value  $w$ 
12: if received strictly less than  $n - f$  propose( $y$ ) then
13:    $x = w$ 
14: end if
15: end for
```

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# King Algorithm (synchronous byzantine agreement)

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**Algorithm 11.14** King Algorithm (for  $f < n/3$ )

---

1:  $x = \text{my input value}$   
2: **for** phase = 1 to  $f + 1$  **do**

*Round 1*

3: Broadcast value( $x$ )

*Round 2*

4: **if** some value( $y$ ) received at least  $n - f$  times **then**  
5:     Broadcast propose( $y$ )  
6: **end if**  
7: **if** some propose( $z$ ) received more than  $f$  times **then**  
8:      $x = z$   
9: **end if**

*Round 3*

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11: The king  $v_i$  broadcasts its current value  $w$   
12: **if** received strictly less than  $n - f$  propose( $y$ ) **then**  
13:      $x = w$   
14: **end if**  
15: **end for**

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# King Algorithm (synchronous byzantine agreement)

Why  $n-f$ ?

- Because if there are  $n-f$  correct nodes, so we can't wait for more. If we wait for less than  $f + 1$  nodes, all the input values could be fake. Because  $3f < n$ ,  $n - f > f$ .
- Ensures only one proposal: If one node sees  $n-f$  values  $v$ , then every other node sees at least  $n-2f$  times  $v$ . Because  $n - (n-2f) = 2f < n-f$ , there can be no proposal for another value.
- All same validity ensured here!

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Algorithm 11.14 King Algorithm (for  $f < n/3$ )

---

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11: The king  $v_i$  broadcasts its current value  $w$ 
12: if received strictly less than  $n - f$  propose( $y$ ) then
13:    $x = w$ 
14: end if
15: end for
```

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# King Algorithm (synchronous byzantine agreement)

Why more than  $f$ ?

- If we just waited for  $\leq f$  propose messages, they all could be byzantine.

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**Algorithm 11.14** King Algorithm (for  $f < n/3$ )

---

```
1:  $x = \text{my input value}$ 
2: for phase = 1 to  $f + 1$  do
    Round 1
3: Broadcast value( $x$ )
    Round 2
4: if some value( $y$ ) received at least  $n - f$  times then
5:   Broadcast propose( $y$ )
6: end if
7: if some propose( $z$ ) received more than  $f$  times then
8:    $x = z$ 
9: end if
    Round 3
10: Let node  $v_i$  be the predefined king of this phase  $i$ 
11: The king  $v_i$  broadcasts its current value  $w$ 
12: if received strictly less than  $n - f$  propose( $y$ ) then
13:    $x = w$ 
14: end if
15: end for
```

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# King Algorithm (synchronous byzantine agreement)

Why  $n-f$  propose messages?

- Similar as for  $n-f$  broadcast messages. We can wait for at most  $n-f$  ones because those are the correct nodes, and we have to wait for at least  $f+1$  ones.

After a correct king, the correct nodes will not change their values anymore! Why?

- If all of them have less than  $n-f$  propose messages, all correct nodes will have the king value and then “all same validity” holds. If one does not adapt, this means that it got  $n-f$  propose messages. This means, every other message got at least  $n-f-f > f$  propose messages, so it adapted its value to the propose. So the king also adapted its value and again all nodes have the same value.

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**Algorithm 11.14** King Algorithm (for  $f < n/3$ )

---

```
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2: for phase = 1 to  $f + 1$  do
    Round 1
3: Broadcast value( $x$ )
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4: if some value( $y$ ) received at least  $n - f$  times then
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7: if some propose( $z$ ) received more than  $f$  times then
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9: end if
    Round 3
10: Let node  $v_i$  be the predefined king of this phase  $i$ 
11: The king  $v_i$  broadcasts its current value  $w$ 
12: if received strictly less than  $n - f$  propose( $y$ ) then
13:    $x = w$ 
14: end if
15: end for
```

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# King Algorithm (synchronous byzantine agreement)

- Does it solve byzantine agreement?
  - Validity: All same validity!
  - Agreement: They agree at least after the first correct king.
  - Termination: After  $(f+1)$  phases

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**Algorithm 11.14** King Algorithm (for  $f < n/3$ )

---

1:  $x = \text{my input value}$

2: **for** phase = 1 to  $f + 1$  **do**

*Round 1*

3: Broadcast value( $x$ )

*Round 2*

4: **if** some value( $y$ ) received at least  $n - f$  times **then**

5:     Broadcast propose( $y$ )

6: **end if**

7: **if** some propose( $z$ ) received more than  $f$  times **then**

8:      $x = z$

9: **end if**

*Round 3*

10: Let node  $v_i$  be the predefined king of this phase  $i$

11: The king  $v_i$  broadcasts its current value  $w$

12: **if** received strictly less than  $n - f$  propose( $y$ ) **then**

13:      $x = w$

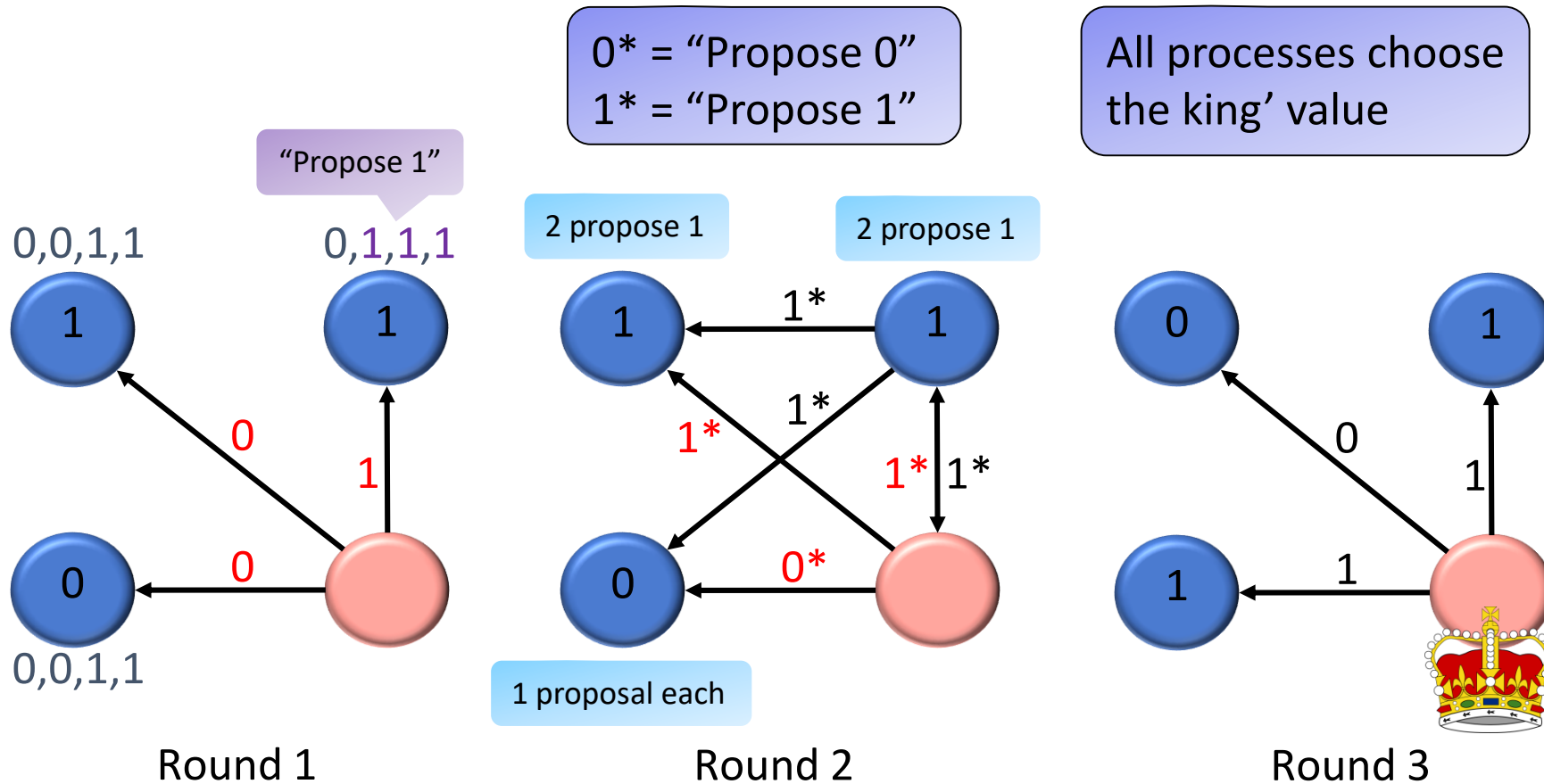
14: **end if**

15: **end for**

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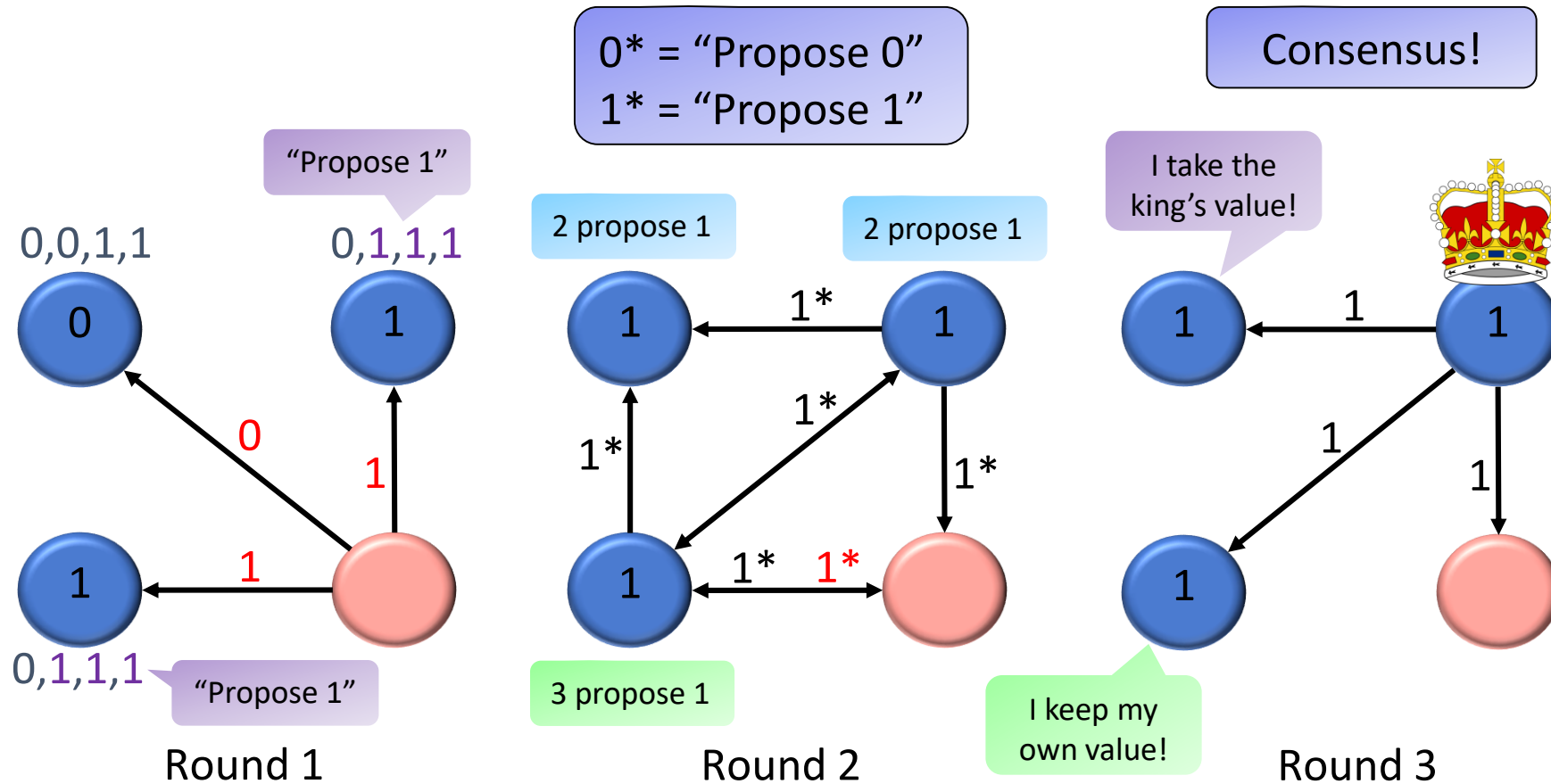
# The King Algorithm: Example

- Example:  $n = 4, f = 1$
- Phase 1:



# The King Algorithm: Example

- Example:  $n = 4, f = 1$
- Phase 2:



# Asynchronous Byzantine Agreement

- Assumption: Messages do not need to arrive at the same time anymore. They have variable delays.

-> Also works, but is a lot more complicated.

-> Algorithm in script is proof of concept, so don't worry about it too much.

->Asynchrony changes messages you have to wait for, but not principle

- Problem: slow! (exponential runtime)

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**Algorithm 11.21** Asynchronous Byzantine Agreement (Ben-Or, for  $f < n/10$ )

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1:	$x_i \in \{0, 1\}$	$\triangleleft$ input bit
2:	$r = 1$	$\triangleleft$ round
3:		
4:	Broadcast own value	
5:	Do until converged	
6:	Wait for enough messages	
7:	If big enough amount agrees, decide and terminate	
8:		
9:	If some agree, adapt your value but don't decide yet	
10:		
11:	If no popular value, decide randomly	
12:		
13:	Broadcast own value	
14:		
15:		
16:	<b>until</b> decided (see Line 8)	
17:	decision = $x_i$	

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# Asynchronous Byzantine Agreement with oracle

- Now, if no popular value, all correct nodes will decide on same oracle value.
- Constant runtime
- Problem: oracle does not exist

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**Algorithm 11.21** Asynchronous Byzantine Agreement (Ben-Or, for  $f < n/10$ )

---

```
1:  $x_i \in \{0, 1\}$             $\triangleleft$  input bit
2:  $r = 1$                   $\triangleleft$  round
3:  $\text{value} = x_i$ 
4: Broadcast own value
5: Do until converged
6: Wait for enough messages
7: If big enough amount agrees, decide and terminate
8: If some agree, adapt your value but don't decide yet
9:  $\text{value} = \text{value} \oplus \text{agreed\_value}$ 
10:  $\text{value} = \text{value} \oplus \text{agreed\_value}$ 
11: If no popular value, ask oracle
12:  $\text{value} = \text{value} \oplus \text{oracle\_value}$ 
13: Broadcast own value
14:  $\text{value} = \text{value} \oplus \text{agreed\_value}$ 
15:  $\text{value} = \text{value} \oplus \text{agreed\_value}$ 
16: until decided (see Line 8)
17: decision =  $x_i$ 
```

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# Asynchronous Byzantine Agreement with random bitstring

- New idea: generate a random bitstring and take next value of bitstring instead of asking oracle
- Problem: byzantine nodes know “random” value and can adapt their behavior

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**Algorithm 11.21** Asynchronous Byzantine Agreement (Ben-Or, for  $f < n/10$ )

---

```
1:  $x_i \in \{0, 1\}$             $\triangleleft$  input bit
2:  $r = 1$                   $\triangleleft$  round
3:  $\text{Broadcast own value}$ 
4:  $\text{Do until converged}$ 
5:  $\text{Wait for enough messages}$ 
6:  $\text{If big enough amount agrees, decide and terminate}$ 
7:  $\text{If some agree, adapt your value but don't decide yet}$ 
8:  $\text{If no popular value, ask look at bitstring}$ 
9:  $\text{Broadcast own value}$ 
10:  $\text{...}$ 
11:  $\text{...}$ 
12:  $\text{...}$ 
13:  $\text{...}$ 
14:  $\text{...}$ 
15:  $\text{...}$ 
16: until decided (see Line 8)
17:  $\text{decision} = x_i$ 
```

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# Asynchronous Byzantine Agreement with blackboard

- Back to the roots! – shared coin
- Implement it by writing values to a public blackboard, after seeing a certain amount of values nodes decide on coin value
- Constant probability that value is the same for all
- Similar to shared coin but works asynchronously
- Byzantine nodes don't know value of shared coin in advance

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**Algorithm 11.21** Asynchronous Byzantine Agreement (Ben-Or, for  $f < n/10$ )

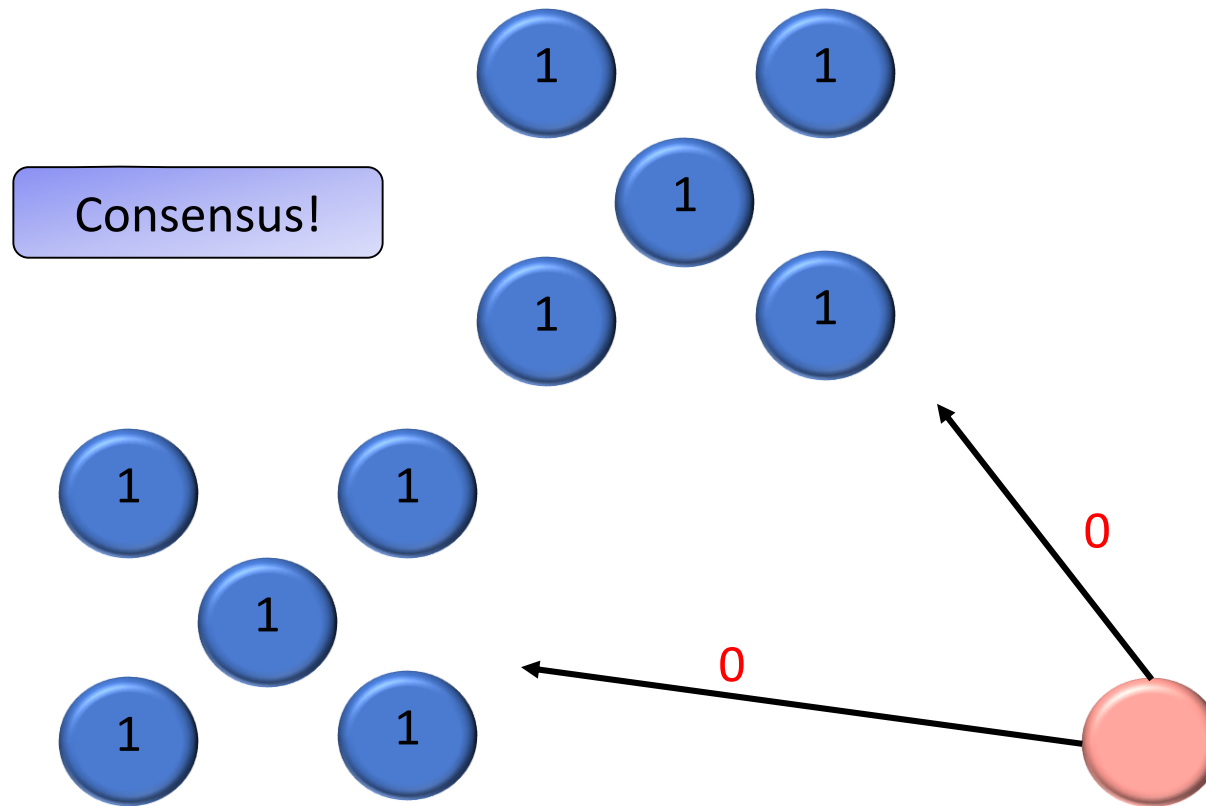
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```
1:  $x_i \in \{0, 1\}$             $\triangleleft$  input bit
2:  $r = 1$                   $\triangleleft$  round
3:  $\text{value} = x_i$ 
4: Broadcast own value
5: Do until converged
6: Wait for enough messages
7: If big enough amount agrees, decide and terminate
8: If some agree, adapt your value but don't decide yet
9:
10:
11: If no popular value, generate shared coin
12:
13: Broadcast own value
14:
15:  $r = r + 1$ 
16: until decided (see Line 8)
17: decision =  $x_i$ 
```

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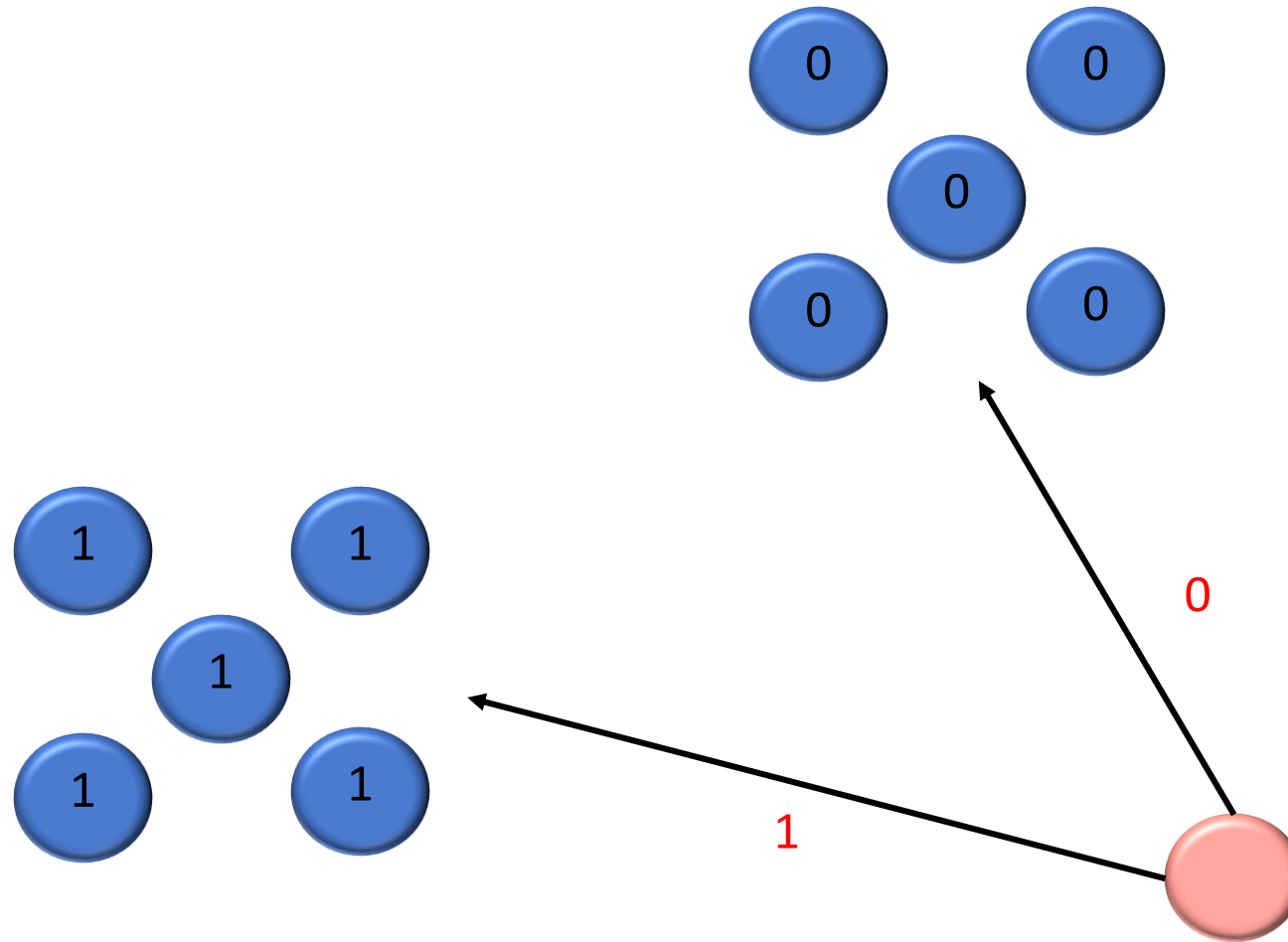
# Ben-Or Algorithm: – All-Same Validity

- Example:  $n = 11, f=1$
- Byzantine node has no power



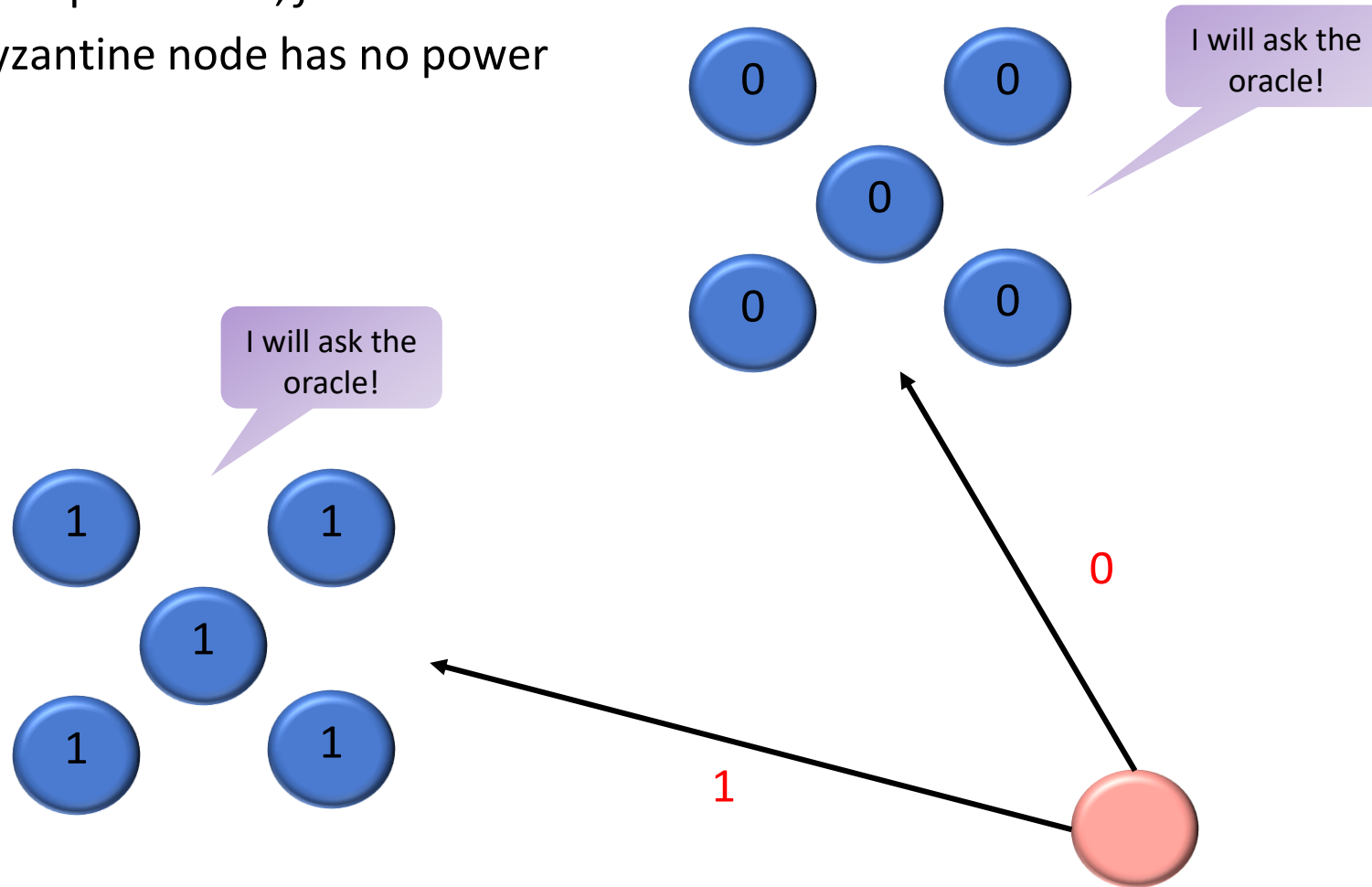
# Ben-Or Algorithm: Example – Shared Coin

- Example:  $n = 11, f = 1$



# Ben-Or Algorithm: Example – Shared Coin

- Example:  $n = 11, f = 1$
- Byzantine node has no power



# Reliable Broadcast

- **Best effort broadcast**

- Best effort broadcast ensures that a message that is **sent** from a correct node  $v$  to another correct node  $w$  will be received and accepted by  $w$

- **Reliable broadcast**

- Reliable broadcast ensures that the nodes eventually agree on all **accepted** messages. That is, if a correct node  $v$  considers message  $m$  as accepted, then every other node will eventually consider message  $m$  as accepted.

- **FIFO (reliable) broadcast**

- The FIFO (reliable) broadcast defines an order in which the messages are accepted in the system. If a node  $u$  **broadcasts message  $m_1$  before  $m_2$** , then any node  $v$  will accept the message  $m_1$  first.

- **Atomic broadcast**

- Atomic broadcast makes sure that all messages are always received in the same order. So for two random nodes  $u_1$  and  $u_2$  and **two random messages  $m_1$  and  $m_2$** , if  $u_1$  sees  $m_1$  first,  $u_2$  will also see  $m_1$  first.

# Reliable Broadcast

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**Algorithm 4.15** Asynchronous Reliable Broadcast (code for node  $u$ )

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1:	Broadcast own value
2:	If message received from node directly, broadcast it together with your own name
3:	
4:	
5:	If you do not get message from node directly, but from a reasonable amount of others also broadcast with own name
6:	
7:	
8:	If you get enough forwarded messages, accept message
9:	
10:	

---

# Reliable Broadcast

## Guarantees:

- If a node broadcasts a message reliably, all correct nodes will eventually accept that value
- If a correct node has not broadcast a message, it will not be accepted by any other correct node
- If a correct node accepts a message from a (byzantine) node, it will be eventually accepted by every correct node

## Problem:

- Does not terminate!
- Does only tolerate  $\leq n/5$  byzantine nodes
  - This is better if we use the FIFO assumption