$\Rightarrow 3x = 30 \Rightarrow \boxed{x=10}$

Put in I $x+3y-13=0$

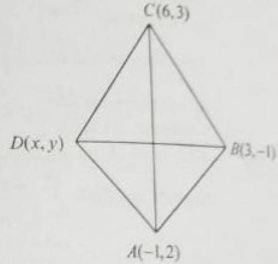
$10+3y-13=0 \Rightarrow 3y-3=0$

$3y=3 \Rightarrow y=1$

So required fourth pt is $D(10,1)$

7. **The points $A(-1, 2)$, $B(3, -1)$ and $C(6,3)$ are consecutive vertices of a rhombus. Find the fourth vertex and show that the diagonals of the rhombus are perpendicular to each other.**

Sol. **Suppose fourth vertex is $D(x, y)$ then**



Slope of $AB = \frac{-1-2}{3+1} = \frac{-3}{4}$

Slope of $BC = \frac{3+1}{6-3} = \frac{4}{3}$

Slope of $CD = \frac{y-3}{x-6}$

Slope of $DA = \frac{2-y}{-1-x}$

In rhombus opposite sides are parallel

So slope of AB = Slope of CD & Slope of BC = Slope of DA

$-\frac{3}{4} = \frac{y-3}{x-6}$ & $\frac{4}{3} = \frac{2-y}{-1-x}$

$\Rightarrow (-3)(x-6) = 4(y-3)$ & $4(-1-x) = 3(2-y)$

or $-3x+18 = 4y-12$ $\quad -4-4x = 6-3y$

or $3x+4y-12-18=0$ or $4x-3y+6+4=0$

or $3x+4y-30=0$ ___ I or $4x-3y+10=0$ ___ II

'×' I by 3, '×' II by 4

Adding I & II

$$9x+\cancel{12y}-90=0$$
$$16x-\cancel{12y}+40=0$$
$$25x-50=0$$

$\Rightarrow 25x = 50 \Rightarrow \boxed{x=2}$

Put value of x in I

$3(2)+4y-30=0$

$6+4y-30=0$

$4y-24=0 \Rightarrow \quad 4y=0$

$4y=24 \Rightarrow \boxed{y=6}$

Fourth vertex is $D(2,6)$

Now Slope of diagonal $AC = \frac{3-2}{6-(-1)} = \frac{1}{6+1} = \frac{1}{7}$

Slope of diagonal $BD = \frac{6-(-1)}{2-3} = \frac{6+1}{-1} = -7$

$x+2y=6 \longrightarrow I$

$x^2+y^2-2x-2y-39=0 \longrightarrow II$

From I $x+2y=6$

$x=6-2y$

Put value of x in II

$(6-2y)^2+y^2-2(6-2y)-2y-39=0$

$36+4y^2-24y+y^2-12+4y-2y-39=0$

$5y^2-22y-15=0$

$5y^2-25y+3y-15=0$

$5y(y-5)+3(y-5)=0$

$(y-5)(5y+3)=0$

$y-5=0 \quad or \quad 5y+3=0$

$y=5 \quad or \quad y=\frac{-3}{5}$

When $y=5$ then $x=6-2(5)=-4$

When $y=\frac{-3}{5}$ then $x=6-2\left(\frac{-3}{5}\right)$

$x=6+\frac{6}{5}=\frac{36}{5}$

Points of contacts are $p_1(-4,5)$ & $p_2\left(\frac{36}{5},\frac{-3}{5}\right)$

7. Find equation of Tangent to circle $x^2+y^2=2$

(i) Parallel to line $x-2y+1=0$ Sgd 2008, 09, 2014(L), Guj 2015(L), 2016, Lhr 2017, Bhwl 2016(L)

Sol: **Given circle is**

$x^2+y^2=2=(\sqrt{2})^2$

$r=a=\sqrt{2}$

Slope of line $=\frac{-a}{b}=\frac{-1}{-2}=\frac{1}{2}$

$m=\frac{1}{2}$ (Because Parallel)

Equation of tangent is

$y=mx\pm a\sqrt{1+m^2}$

$y=\frac{1}{2}x\pm\sqrt{2}\sqrt{1+\left(\frac{1}{2}\right)^2}$

$y=\frac{x}{2}\pm\sqrt{2}\sqrt{1+\frac{1}{4}}$

$\Rightarrow y=\frac{1}{2}x\pm\sqrt{4\left(\frac{1}{4}\right)+1}$

$y=\frac{1}{2}x\pm\sqrt{2} \Rightarrow 2y=x\pm 2\sqrt{2}$

$\Rightarrow x-2y\pm 2\sqrt{2}=0$

i.e., $x-2y+2\sqrt{2}=0$

$x-2y-2\sqrt{2}=0$

6. **Find equations of the tangents to the conic $9x^2-4y^2=36$ parallel to $5x-2y+7=0$.** Federal 2017

Sol. $9x^2-4y^2=36$

$\Rightarrow \frac{9x^2}{36}-\frac{4y^2}{36}=\frac{36}{36}$

$\Rightarrow \frac{x^2}{4}-\frac{y^2}{9}=1 \quad (1)$

$5x-2y+7=0 \quad (2)$

From (1) $a^2=4,\ b^2=9$

Slope of the line (2) $=\frac{5}{2}$

Now Slope of tangent parallel to (2) $=\frac{5}{2}$

Thus required equations of tangents to (1) and parallel to (2) are

$y=\frac{5}{2}x\pm\sqrt{4\left(\frac{25}{4}\right)-9}$

$y=\frac{5}{2}x\pm 4 \Rightarrow 2y=5x\pm 8$

$\Rightarrow 5x-2y\pm 8=0$

$\Rightarrow 5x-2y+8=0\ ,\ 5x-2y-8=0$

7. **Find equations of the common tangents to the given conics** Federal 2018

Sol. (i) $x^2=80y \quad (1)$

$x^2+y^2=81 \quad (2)$

Let $y=mx+c \quad (3)$

Be common tangent to (1) and (2)

Using (3) in (1) we have

$x^2=80(mx+c)$

$\Rightarrow x^2-80mx-80c=0 \quad (4)$

If (3) is tangent to (1) then (4) has equal roots.

$\Rightarrow$ Discriminant of (4) $=0$

$\Rightarrow (-80m)^2-4(1)-(-80c)=0$

$\Rightarrow 6400m^2+320c=0$

$\Rightarrow 320c=-6400m^2$

$\Rightarrow c=-\frac{6400m^2}{320}$

$\Rightarrow c=-20m^2 \quad (5)$

If (3) is tangent to (2) then

$c^2=81(1+m^2) \quad (6)$

Using (5) in (6) we have

$\Rightarrow 400m^2-81m^2-81=0 \quad (7)$

At D, $f(0,3)=2(0)+3=3$

So f is maximum at corner $B=(5,0)$

5. Maximize $f(x,y)=2x+3y$ (Sargodha 2011, Federal 2015)

$2x+y\le 8,\ \ x+2y\le 14,\ \ x\ge 0\ \&\ y\ge 0$

Sol. Let $2x+y\le 8 \longrightarrow I$

and $x+2y\le 14 \longrightarrow II$

Associated equation are

l_1 ; $2x+y=8 \longrightarrow III$

l_2 ; $x+2y=14 \longrightarrow IV$

$III \Rightarrow put\ x=0\ then\ y=8\ ;\ (0,8)$

$put\ y=0\ then\ x=4\ ;\ (4,0)$

$IV \Rightarrow put\ x=0\ then\ y=7\ ;\ (0,7)$

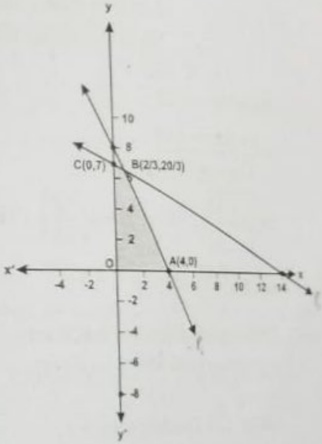
$put\ y=0\ then\ x=14\ ;\ (14,0)$

Put $(0,0)$ in eq. I & II

$I \Rightarrow 0<8(only\ take < value) \longrightarrow T$

$II \Rightarrow 0<14(only\ take < value) \longrightarrow T$

So the graph is.



Note: $Solve\ 2\times III-IV$

$$4x+\cancel{2y}=16$$
$$\frac{\pm x \pm \cancel{2y} = \pm 14}{3x\quad =2} \Rightarrow x=\frac{2}{3}$$

$$put\ x=\frac{2}{3}\ in\ III \Rightarrow 2\left(\frac{2}{3}\right)+y=8 \Rightarrow \boxed{y=\frac{20}{3}}$$

So, $B\left(\frac{2}{3},\frac{20}{3}\right)$

Corner points of feasible region are:

$O(0,0),\ A(4,0),\ B\left(\frac{2}{3},\frac{20}{3}\right),\ C(0,7),$

$At\ O,\ f(0,0)=2(0)+3(0)=0$

$At\ A,\ f(4,0)=2(4)+3(0)=8$

$At\ B,\ f\left(\frac{2}{3},\frac{20}{3}\right)=2\left(\frac{2}{3}\right)+3\left(\frac{20}{2}\right)=21.33$

$At\ C,\ f(0,7)=2(0)+3(7)=21$

So f is maximum at corner $B\left(\frac{2}{3},\frac{20}{3}\right)$

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Equation	$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1, a>b$ $c^2=a^2-b^2$	$\frac{x^2}{b^2}+\frac{y^2}{a^2}=1, a>b$ $c^2=a^2-b^2$
Foci	$(\pm c,0)$	$(0,\pm c)$
Directrices	$x=\pm\frac{c}{e^2}$	$y=\pm\frac{c}{e^2}$
Major axis	Y = 0	X = 0
Vertices	$(\pm a,0)$	$(0,\pm a)$
Covertices	$(0,\pm b)$	$(\pm b,0)$
Centre	(0,0)	(0,0)
Eccentricity	$e=\frac{c}{a}<1$	$e=\frac{c}{a}<1$
Length of Major axis = 2a, length of Minor axis = 2b, Length of latus rectum = $\frac{2b^2}{a}$		

1. Find an equation of the ellipse with given data and sketch its graph:

(i) Foci $(\pm 3,0)$ and minor axis of length 10 (Fed 2015, Sgd 2015(L), AJK 2016, Lhr 2017,18 Fsd 2018, Mul 2014, 2017(L),2018, Dgk 2018

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Solution: (i) Foci $(\pm 3,0)$

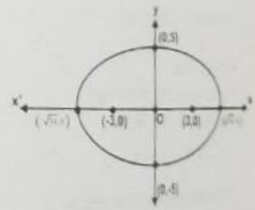
Length of Minor Axis = 10

Here $c=3$ *and* $2b=10 \Rightarrow b=5$

Now $c^2=a^2-b^2 \Rightarrow a=a^2-25 \Rightarrow a^2=34$

Thus requied equation of the

Ellipse is $\frac{x^2}{34}+\frac{y^2}{25}=1$



From (1)

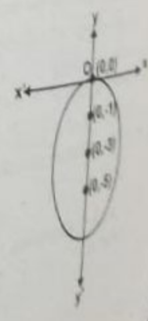
$a^2=34 \Rightarrow a=\pm\sqrt{34}$

$b^2=25 \Rightarrow b=\pm 5$

$\therefore$ Vertices of the ellipe on the x-axis are $(\pm\sqrt{34},0)$ and co-vertices are $(0,\pm 5)$ and graph of the ellipse

(ii) Foci (0, −1) and (0, −5) and major axis of length 6.

Sol. Foci $(0,-1)$, & $(0,5)$



Length of Major Axis = 6

Here centre of the ellipse

$=\left(\frac{0+0}{2},\frac{-1-5}{2}\right)=(0,-3)$ and c is the distance from the centre to each focus. So

$C=\sqrt{(0,0)^2+(-3+1)^2}=\sqrt{0+4}=2 \Rightarrow \boxed{c=2}$

Also given that $2a=6$

$\Rightarrow a=3$

Now using $c^2=a^2-b^2$

$\Rightarrow 4=9-b^2 \Rightarrow b^2 - 9 - 4 \Rightarrow b^2 = 5$