University: Sharif University of Technology

Department: Electrical Engineering

Course Name: Advanced Neuroscience

Homework 1 Report

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Integrate and Fire Neuron

a) For the Poisson process, we know that:

$$\lim_{\Delta t \to 0} \mathbb{E}\left[\frac{N(t+\Delta t, t)}{\Delta t}\right] = \lambda \longrightarrow \lim_{\Delta t \to 0} \mathbb{P}\left(\text{one spike in } [t, t+\Delta t]\right) = \lambda \Delta t$$

Hence, for generating our spike train we use *rand()* function and assume values less than $\lambda \Delta t$ as spikes. Figure 1 shows a 50 trial raster plot of generated spike train. In the following, we use 10,000 trials to get better accuracy.

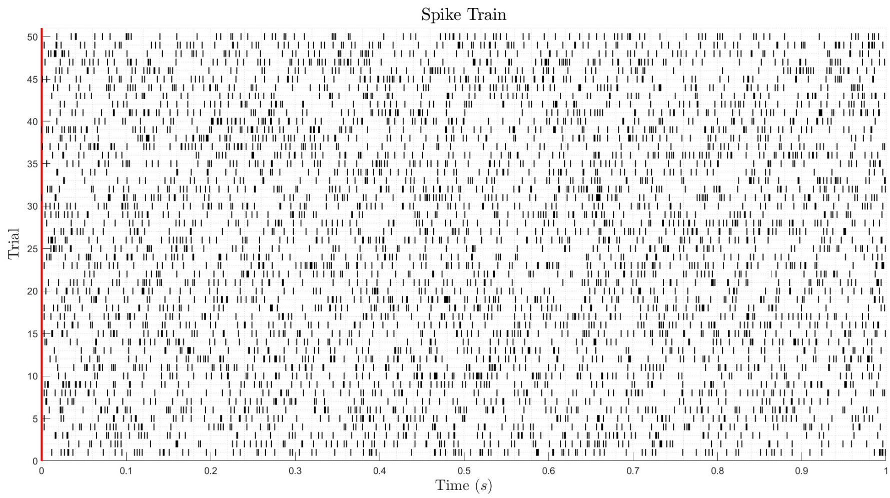

Figure 1: Raster plot of Poisson spike trains

b) If we count the number of spikes in each trial and plot the histogram, the results would be Figure 2 superimposed with PDF of distribution Poisson with parameter 100.

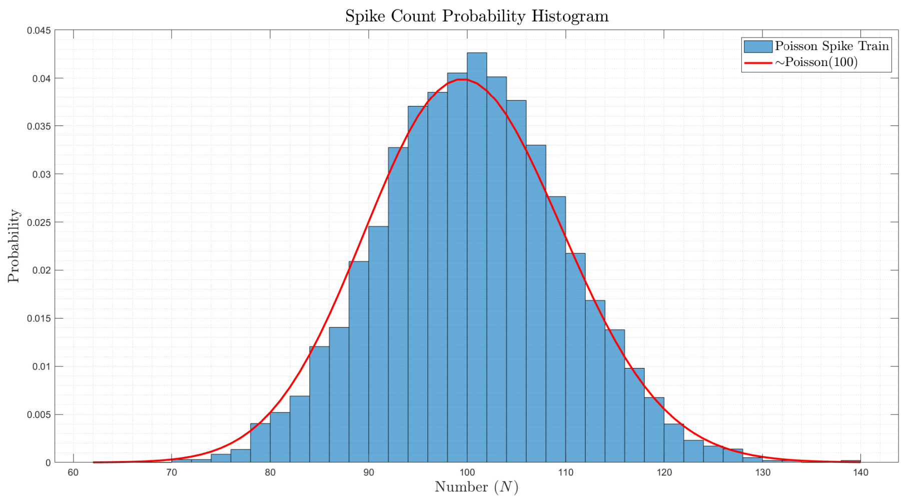

Figure 2: Spike count probability histogram

c) First, we imply that distribution of Inter-Spike Interval (ISI) for Poisson process:

$$
\begin{aligned}
f_\tau(\tau_0) &= \lim_{\Delta t \to 0} \frac{\mathbb{P}(N(\tau_0) = 0, N(\Delta t) = 1)}{\Delta t} \\
&= \lim_{\Delta t \to 0} \frac{\mathbb{P}(N(\tau_0) = 0)\mathbb{P}(N(\Delta t) = 1)}{\Delta t} \\
&= \lim_{\Delta t \to 0} \frac{e^{-\lambda \tau_0}(\lambda \Delta t)}{\Delta t} \\
&= \lambda e^{-\lambda \tau_0}
\end{aligned}
$$

That is the PDF of distribution Exponential with parameter λ. In this section, we calculate the ISI in all trials of spike train, using our own *findISI()* function. Histogram of ISI superimposed with distribution Exponential with parameter around 10, would be Figure 3. Note that the parameter of Exp distribution is calculated from *fitdist()* function of Matlab.

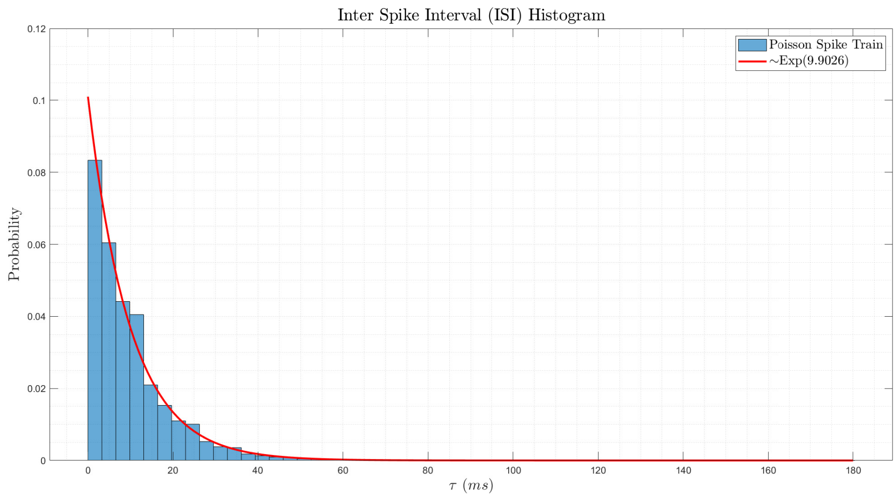

Figure 3: Inter-Spike Interval histogram

d) In this simplified model, if the post-synaptic neuron receives k spikes in its input, it will fire. So, if we use the spike train matrix generated before as the pre-synaptic firings, and select the kth spikes, we have actually integrated over post-synaptic input.

Now we repeat steps a-c with just selecting the kth spike:

- Using our own *kth_spike()* function, we generate spike train matrix of post-synaptic neuron.
- If we repeat part b for this new matrix, we have Figure 4. This histogram is just like Poisson histogram but has been shifted from 100 to around 20 and has been compressed in x-axis.
- For part c, we have Figure 5, which is no longer Exponential. It is best fitted on Gamma distribution with a and b parameters specified in the legend.

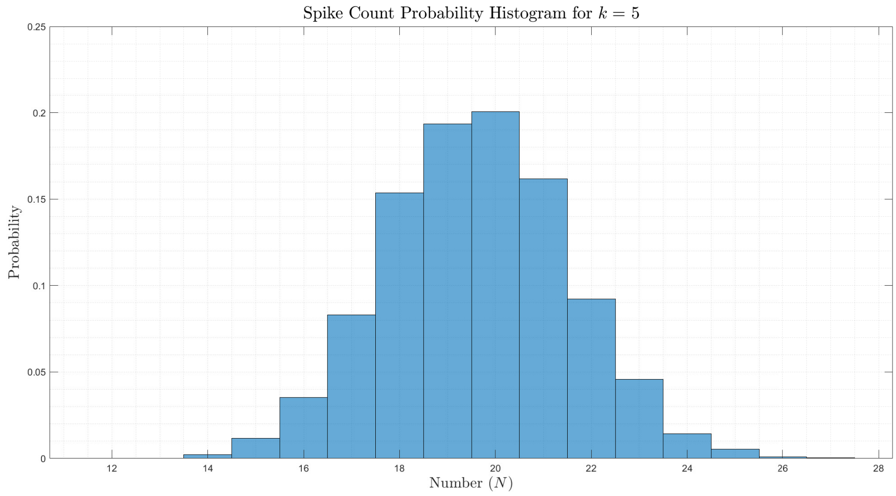

Figure 4: Spike count probability histogram

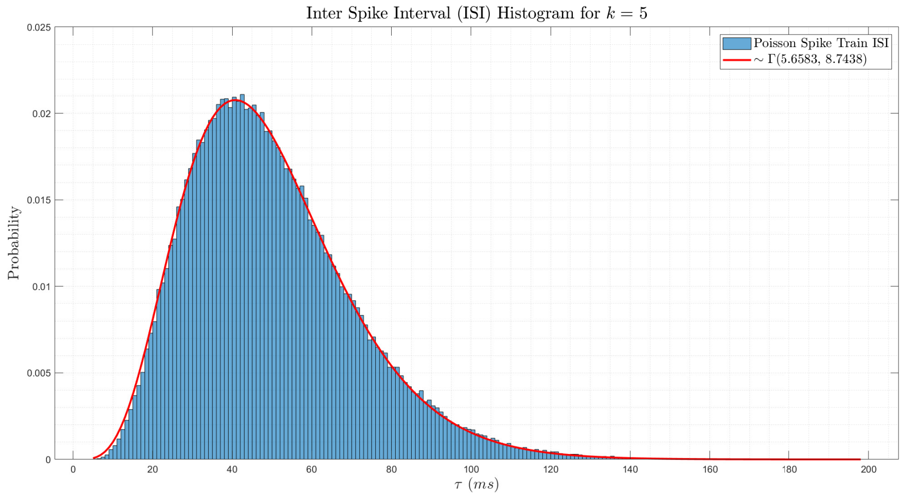

Figure 5: Inter-Spike Interval histogram

Now we use both data and calculate C_v. For Poisson process we have:

$$X \sim \text{Exponential}(\lambda) \begin{cases} \mathbb{E}\left[X\right] = \dfrac{1}{\lambda} \\ Var\left[X\right] = \dfrac{1}{\lambda^2} \end{cases} \implies \text{Coefficient of Variation} = \frac{std\left[X\right]}{\mathbb{E}\left[X\right]} = \frac{\frac{1}{\lambda}}{\frac{1}{\lambda}} = 1$$

And after calculation of C_v, it is given:

Cv of perfectly Exponential ISI: 1
Cv of spike train with Poisson process: 0.94807
Cv of spike train with kth spike: 0.42354

As we see in the above results, C_v of τ in first data is nearly equal to the perfectly Exponential. And when we choose every kth spike, the C_v drops significantly which the reason is discussed in the next section. But as a comparison:

$$\text{5-th spike ISI } C_v \approx \frac{1}{\sqrt{5}} \approx 0.4472$$

e) If we assume the ISI histogram, a Gamma function of order $N_{th} - 1$:

$$p(\Delta t) \sim (R\Delta t)^{N_{th}-1} \exp(-R\Delta t)$$
$$\overline{\Delta t} = \frac{\int_0^{+\infty} \Delta t p(\Delta t) d\Delta t}{\int_0^{+\infty} p(\Delta t) d\Delta t} = \frac{N_{th}}{R}$$
$$\sigma^2_{\Delta t} = \frac{\int_0^{+\infty} (\Delta t - \overline{\Delta t})^2 p(\Delta t) d\Delta t}{\int_0^{+\infty} p(\Delta t) d\Delta t} = \frac{N_{th}}{R^2}$$

So, C_v is:

$$C_v = \frac{\sigma_{\Delta t}}{\overline{\Delta t}} = \frac{\frac{\sqrt{N_{th}}}{R}}{\frac{N_{th}}{R}} = \frac{1}{\sqrt{N_{th}}} \tag{1}$$

f) Just like part b and c, we create 10,000 trials of length 1s with Poisson process and calculate ISI. Here we change the rate in a *for* loop and calculate C_v for each rate. Results are shown in Figure 6.

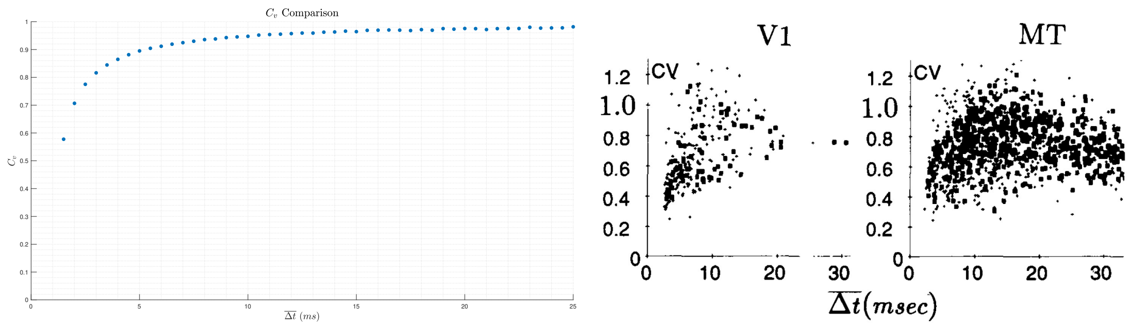

Figure 6: Comparison of C_v

As we can see, generated C_vs from Poisson process are not really matched to the real data-set. But it is similar that with increase of $\overline{\Delta t}$, C_v will increase too.

g) For implementing the refractory period, we should prevent consecutive spikes with ISI less than 1ms. Since we set $\Delta\tau = 1ms$, it will automatically produce ISIs greater than 1ms. But if we wanted to use refractory period greater than 1ms, we should have checked consecutive spikes whether the ISI is less or greater than refractory period time and delete the second one if it is less.

If we change the rate from 1,000 to 40 ($\overline{\Delta t} = 1ms$ and $\overline{\Delta t} = 25ms$ respectively), and calculate C_v for different N_{th}s, we can generate a plot similar to the figure 6 of paper (Figure 7 and Figure 8). Note that the upper bounds of C_v (no refractory period) are calculated from Equation 1.

The left figure is when we sweep on the rate, so we have high precision of x-axis in small values (good for transient part). The right figure is when we sweep on $\overline{\Delta t}$ with constant steps that gives us good precision of x-axis in greater values. Both figures are derived from 10,000 trials of each 3s length.

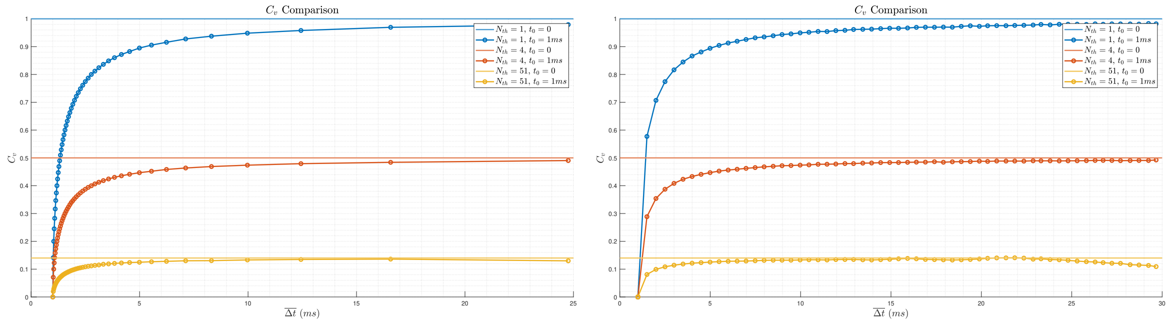

Figure 7: Comparison of C_v from integrator models, Refractory Period = 1ms

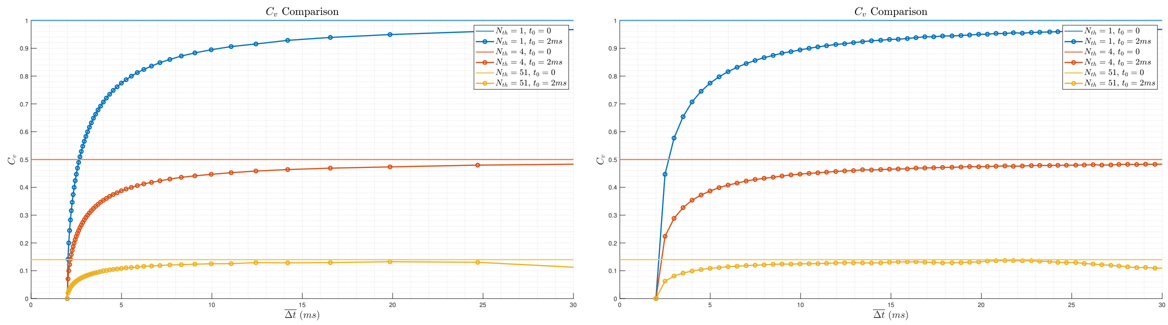

Figure 8: Comparison of C_v from integrator models, Refractory Period = 2ms

From the figures, we obtain that for high rates (low $\overline{\Delta t}$) the coefficient of variation is low because of the *Refractory Period*. But when rate decreases (higher $\overline{\Delta t}$), the C_v converges to the expected value for non-refractory period neuron.

The difference between Figure 7 and Figure 8 is refractory period. In the first figure, the curve crosses the x-axis at 1ms, but in the second one crosses at 2ms. It seems like the curve just shifts in x-axis with increasing of refractory period duration, crossing the x-axis at t_0.

This is quite similar to the figure 6 of paper but, at the great values of $\overline{\Delta t}$ for $N_{th} = 51$. In fact, when the $\overline{\Delta t}$ gets bigger, we have less spikes in each trial. For instance, for $\overline{\Delta t} = 25$ the rate is 40. So in each trial we have approximately 120 spikes, which gives us only one (or maybe zero or two) ISI value because we select one spike out of every 51. In that case we have much lesser data to get average and std of, thereupon, less accuracy is obtained. To solve this problem we can increase the length of trials which takes more time in simulation.

Leaky Integrate and Fire Neuron

The characteristic equation is as below:

$$\tau_m \frac{dv}{dt} = -v(t) + RI(t) \tag{2}$$

a) From Equation 2, we can drive a numerical computation:

$$\left.\begin{aligned} \tau_m.dv &= (-v(t) + RI(t))dt \\ dv &= \frac{1}{\tau_m}(-v(t) + RI(t))dt \end{aligned}\right\} \implies v(t+dt) = v(t) + dv$$

With a constant value for $RI(t)$ and a realistic value for membrane time constant of $\tau = 13ms$, we solve the numerical simulation derived above. The results are shown in Figure 9. White bars are representing spikes of neuron, when the membrane potential has reached the threshold. Refractory period is assumed 2ms.

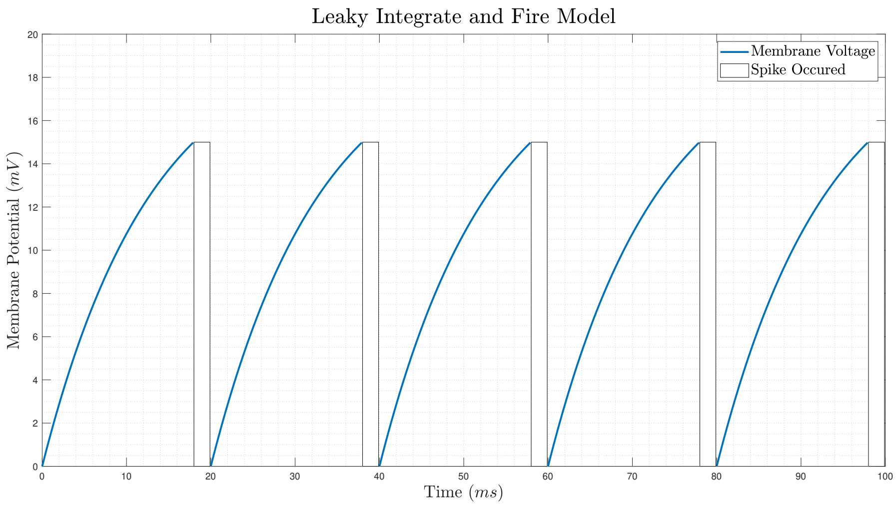

Figure 9: Time-course of the membrane potential

b) The answer of differential equation discussed in Equation 2, is:

$$v(t) = (V_r - RI)\exp(\frac{-t}{\tau_m}) + RI$$

As we can see in Figure 9, membrane potential increases until it reaches the threshold, so we can find the time it takes to reach the threshold with the answer above, it is:

$$
\begin{aligned}
v(t) = (V_r - RI)\exp(\frac{-t_{th}}{\tau_m}) + RI &= V_{th} \\
(V_r - RI)\exp(\frac{-t_{th}}{\tau_m}) &= V_{th} - RI \\
\exp(\frac{-t_{th}}{\tau_m}) &= \frac{V_{th} - RI}{V_r - RI} \\
\frac{-t_{th}}{\tau_m} &= \ln(\frac{V_{th} - RI}{V_r - RI}) \\
t_{th} &= -\tau_m \times \ln(\frac{V_{th} - RI}{V_r - RI})
\end{aligned}
$$

Therefore, it takes t_{th} to reach the threshold and Δt_r for the refractory period to reach to a single spike:

$$
t_s = t_{th} + \Delta t_r = -\tau_m \times \ln(\frac{V_{th} - RI}{V_r - RI}) + \Delta t_r
$$

Hence, mean firing rate of neuron at a constant input current I, with considering a refractory period of Δt_r is:

$$
\bar{r} = \frac{1}{t_s} = \frac{1}{\Delta t_r - \tau_m \times \ln(\frac{V_{th}-RI}{V_r-RI})}
$$

c) Using 5 Poisson process for 5 pre-synaptic neurons with firing rate 20, we can conclude following figures. Figure 10 shows the time-varying input current with the parameters described in legend. Generally we assumed that:

$$
I_s(t) = A \times t \times \exp(\frac{-t}{t_{peak}})
$$

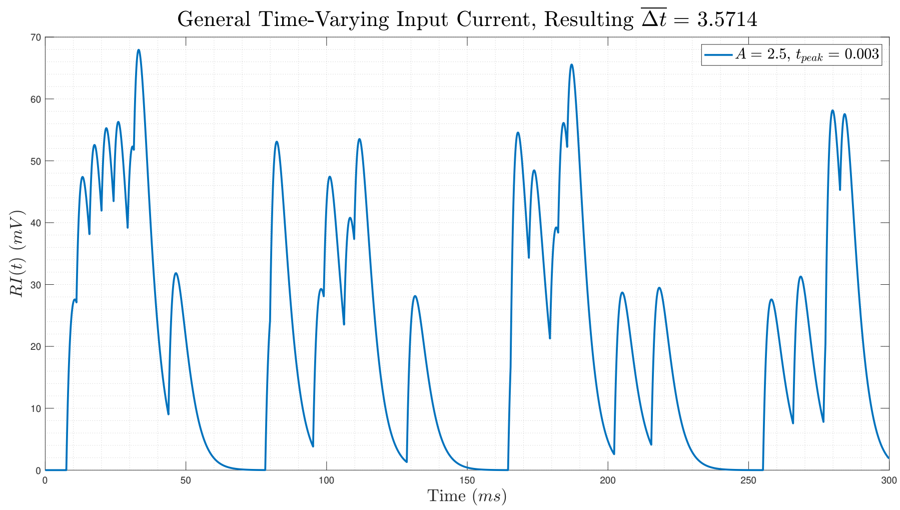

Figure 10: Time-varying input current derived from $I_s(t)$ convoluted with $\delta(t)$

- In Figure 11 you can see how membrane threshold can affect the C_v. By increasing the membrane threshold for firing, we can see a slight decrease in the C_v, because it needs more amplitude of current and become more random.

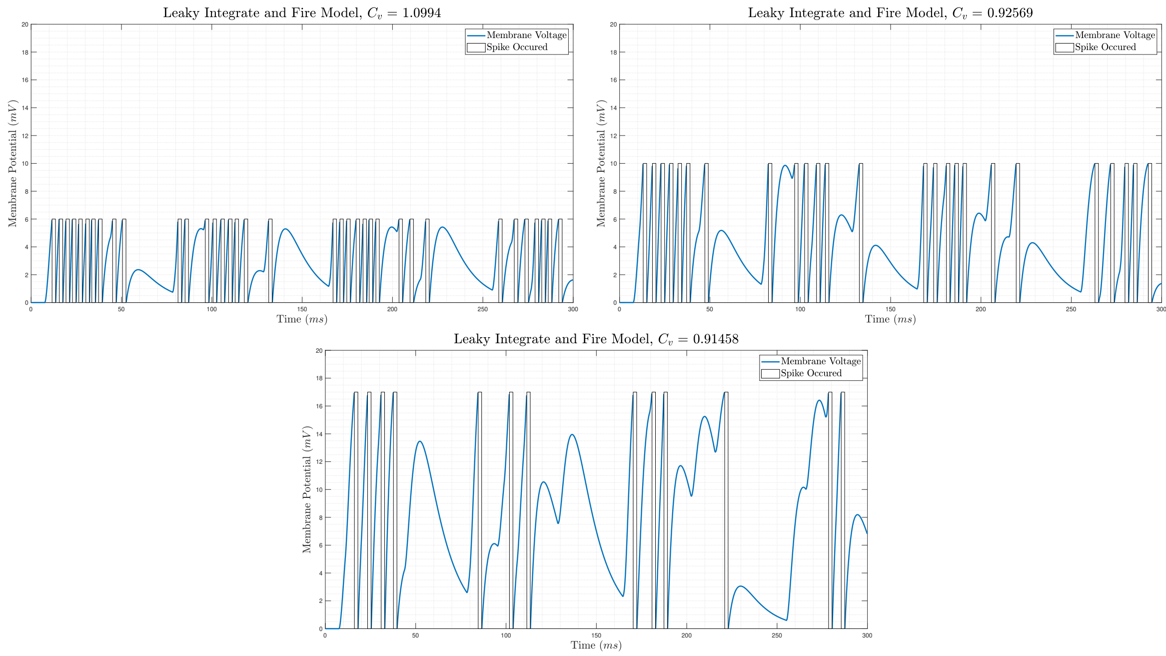

Figure 11: Comparison of threshold impact on C_v for the same input current

- If we change the amplitude of $I_s(t)$, we can observe Figure 12. By increasing A, we make spike occurrence more probable when there is enough input current. Hence, we are making it more random and C_v will be higher.

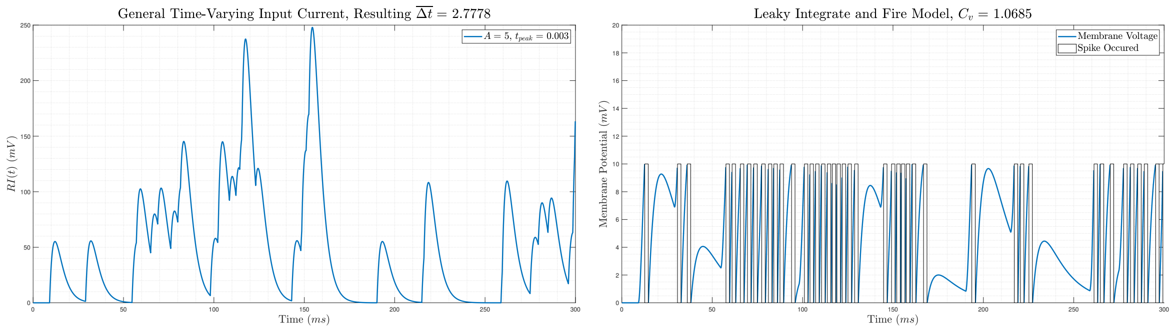

Figure 12: Time-varying input current and membrane voltage

- If we change the t_{peak} and make spike currents thinner, we can observe Figure 13. In this case, the probability of current super-positioning will decrease and the firing rate of post-synaptic neuron will decay. Thereupon, post-synaptic neuron will just integrate consecutive spike currents and fire after a while. It lowers the chance of variation and C_v will decrease by decreasing the t_{peak}.

d) In this part, we use the same spike train generated in previous section, and just select $n\%$ of it and make their current negative. For each percentage, we use *randperm()* function to randomly select IPSP. Figure 14 shows three different stages of IPSPs. As we can see, increasing the IPSPs will not always increase the C_v but should. Adding IPSPs will make the integral of current more random because it prevents a regular spike due to consecutive

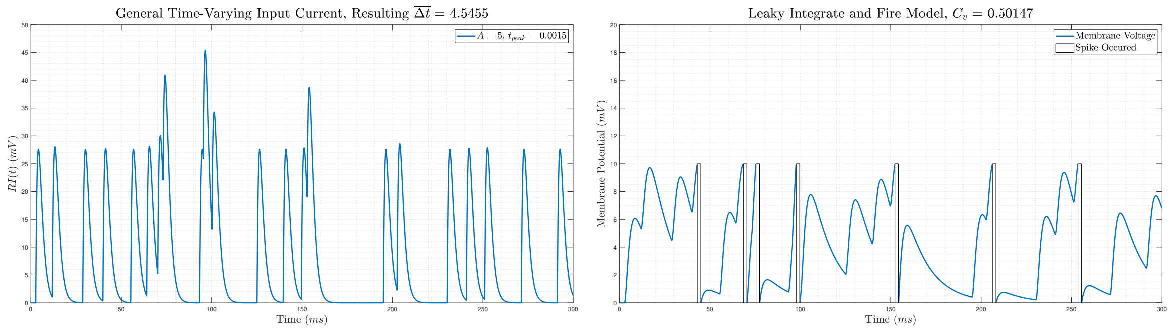

Figure 13: Time-varying input current and membrane voltage

spike currents. So generally, we expect increasing C_v and we can see that in the difference between 30% and 50% IPSPs.

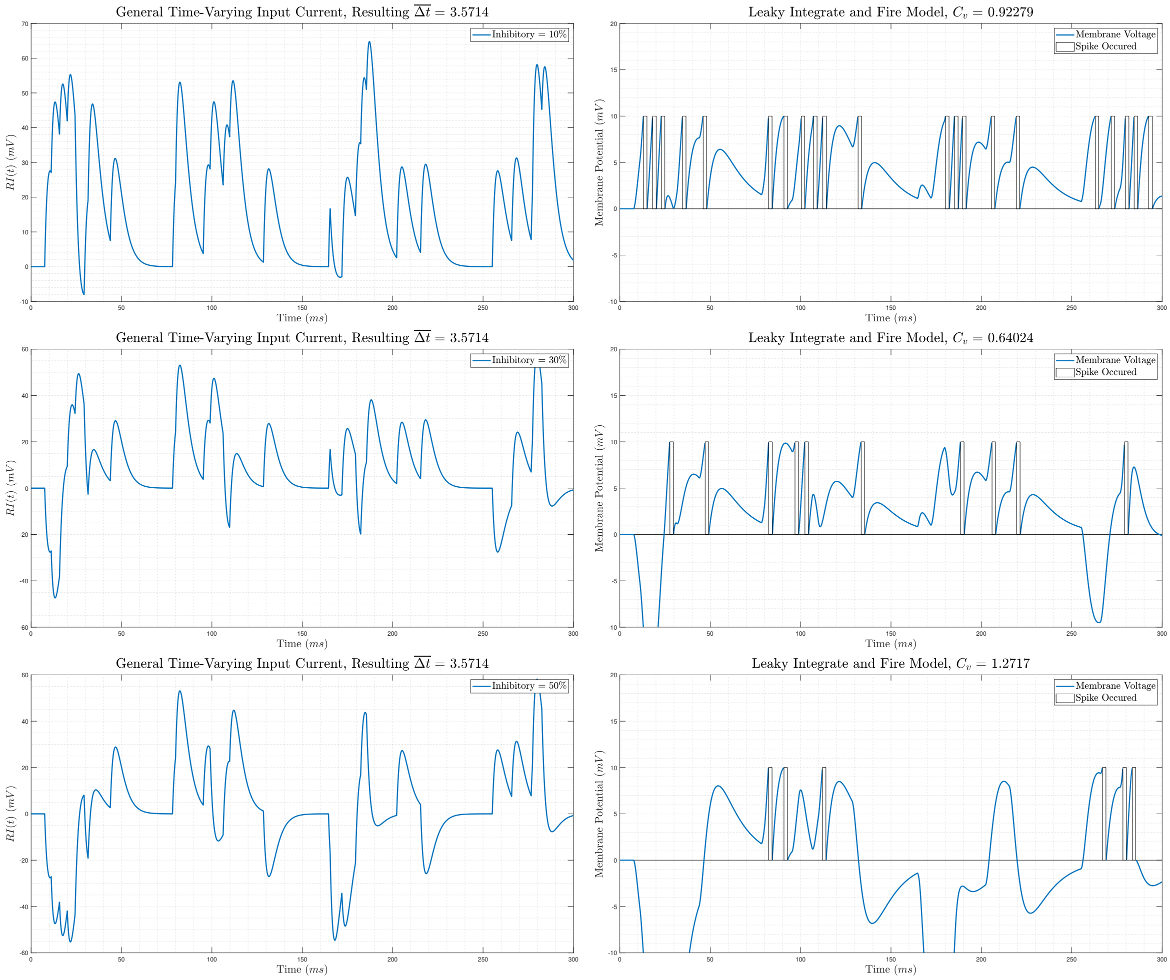

Figure 14: Time-varying input current and membrane voltage

e) In this part, we simulated time in 1s and $dt = 0.1ms$ to have more data and more accurate data. If we generate $M = 100$ processes and assume a window of length D, we can say whether there is more than N spikes in that window or not. In each calculation, we generate M process, add them together and slightly slide the window on it to get the number of spikes. We repeat this 5 times for each D and N and then report the average of it to lower the chance of noise.

- If we change the time window length (D), we observe:
  For D=7ms, M=100, N=15, CV is: 0.63468
  For D=9ms, M=100, N=15, CV is: 0.31792
  For D=12ms, M=100, N=15, CV is: 0.13993

  It shows that with increasing the time window length, the C_v is decreasing. Because we are getting and average of wider area and hence, lower the randomness.

- If we change the number of required spikes (N), we observe:
  For D=10ms, M=100, N=10, CV is: 0.10144
  For D=10ms, M=100, N=15, CV is: 0.16988
  For D=10ms, M=100, N=20, CV is: 0.54298

  It shows that with increasing the required spikes ($\frac{N}{M}$ ratio), the C_v is increasing too. Because it makes post-synaptic spikes more specialized. For instance, consider $N = 1$. In this case almost every time window would cause the post-synaptic neuron to spike and the coefficient of variation would be near 0. But if we increase N, more specific conditions is needed for post-synaptic neuron to spike. So C_v will increase.

f) In this part we use 30% of pre-synaptic firings as inhibitory.

- If we change the time window length (D), we observe:
  For D=10ms, M=100, N=15, CV is: 0.90989
  For D=12ms, M=100, N=15, CV is: 0.77617
  For D=17ms, M=100, N=15, CV is: 0.51822

  Again, with increasing the time window length, C_v decreases because of getting average of wider area. In addition, D values have increased a little compared to previous part, because IPSPs will neutralize some of EPSPs and we will need more time to reach the threshold.

- If we change the number of required spikes (N), we observe:
  For D=10ms, M=100, N=5, CV is: 0.27616
  For D=10ms, M=100, N=10, CV is: 0.66913
  For D=10ms, M=100, N=15, CV is: 0.84387

  Again, with increasing the $\frac{N}{M}$ ratio, C_v is increasing too. Because of the same reason in previous part. In addition, the range of N is lower because some of IPSPs would neutralize EPSPs and we will have a N_{net} lower than N_x.