University: Sharif University of Technology

Department: Electrical Engineering

Course Name: Advanced Neuroscience

Homework 7 Report

Evidence Accumulation

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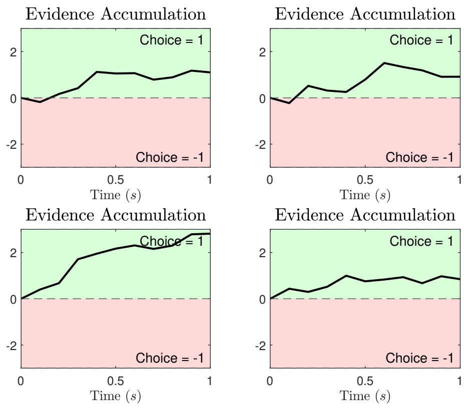
Part 1

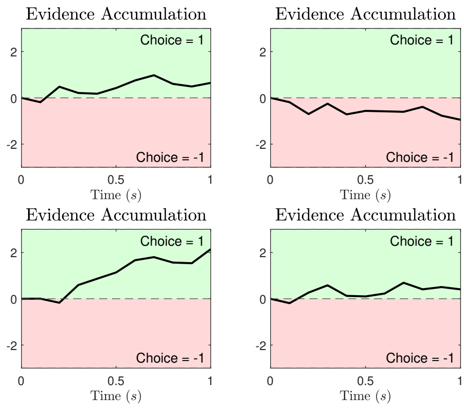
1, 2) A simple implementation of model presented in (Ratcliff et al. 2008), is described below:

$$dX = B\,dt + \sigma\,dW$$

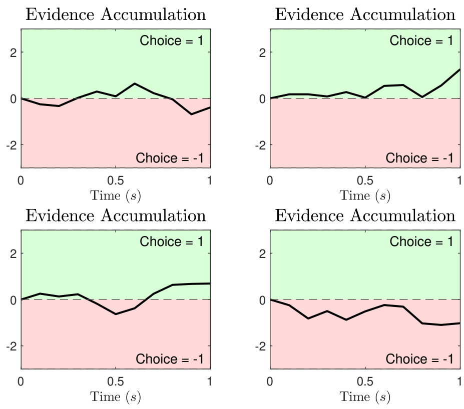
Generating 20 trials with parameters: $B = 1,\ \sigma = 1,\ dt = 0.1$



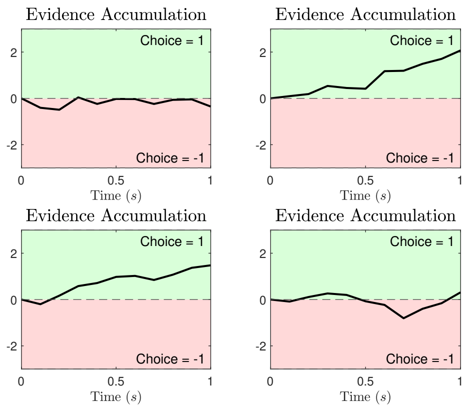
(a) Trial 1-4



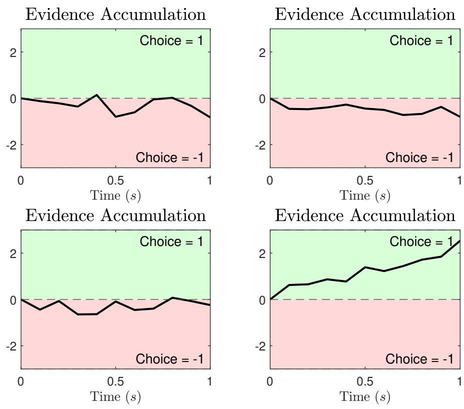
(b) Trial 5-8



(c) Trial 9-12



(d) Trial 13-16



(e) Trial 17-20

Figure 1: Simple DDM Model

If we plot the histogram of choices, we will achieve a binomial distribution due to binary values of choices (Figure 2).

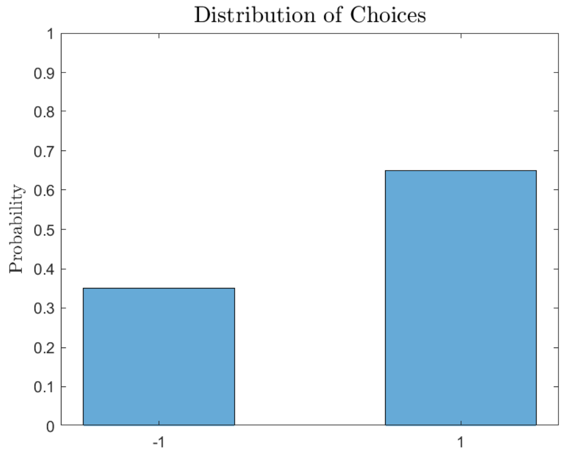


Figure 2: Go / No Go Task; Histogram of Choices

We can run the function 100 times and get *mean* and *std* of returned $X(t)$ for different values of B. Figure 3 shows this difference. The higher the value of B, the greater slope of evidence accumulation. Note that these lines are smoothed because of averaging, but as a trial variability would be noticeable.

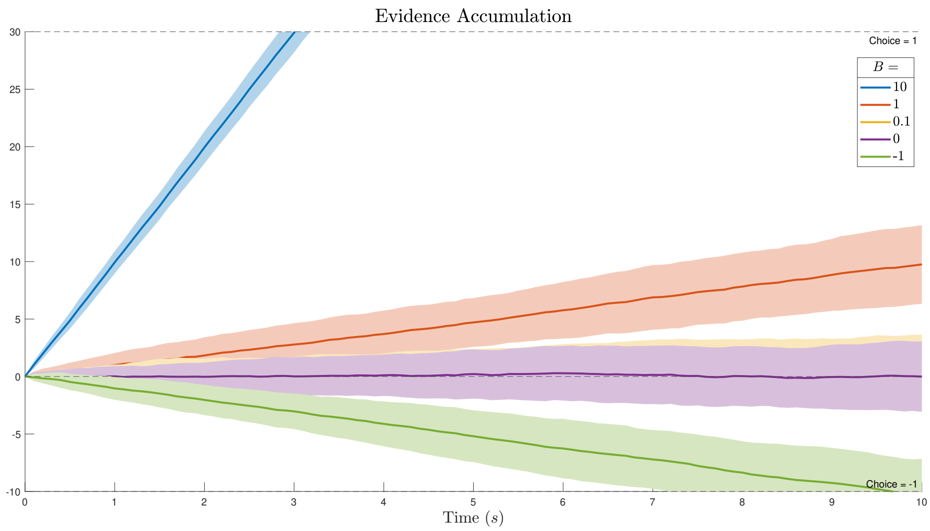
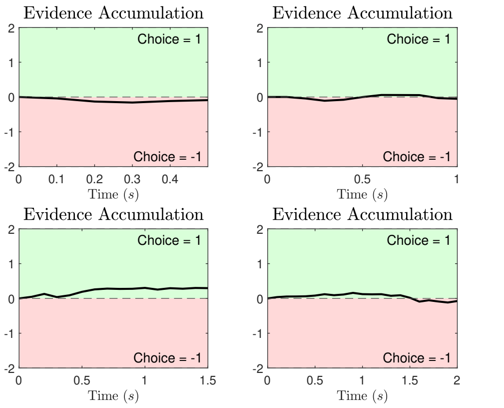
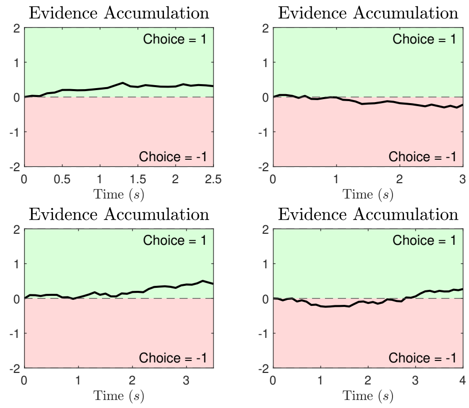


Figure 3: Go / No Go Task; Different Values of B

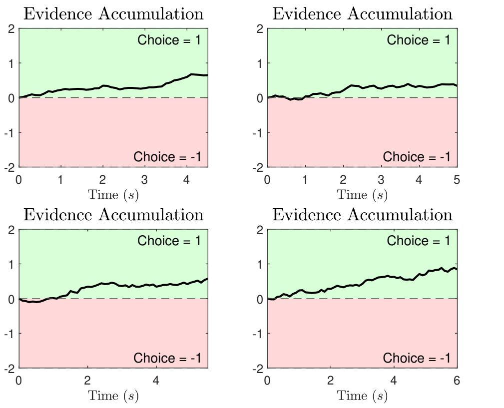
3) Now we generate 50 trials for each time interval and get *mean* of them and plot as an example. Parameters used are: $B = 0.1$, $\sigma = 1$, $dt = 0.1$



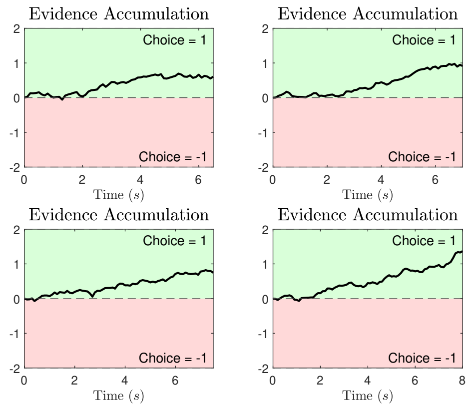
(a) Duration 0.5-2 sec



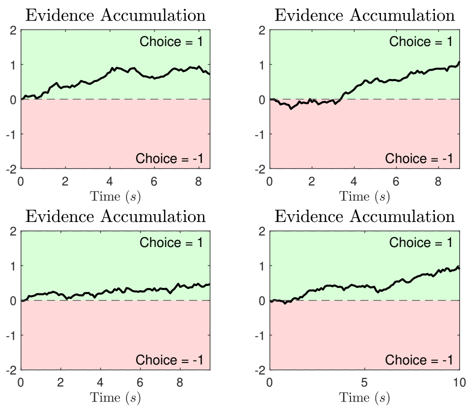
(b) Duration 2.5-4 sec



(c) Duration 4.5-6 sec



(d) Duration 6.5-8 sec



(e) Duration 8.5-10 sec

Figure 4: Simple DDM Model

Now we generate 1000 trials for time intervals [0.5 30] sec. Error rate is defined as ratio of incorrect choices number over all trials. Figure 5 shows influence of time interval on the error rate. As time interval increases, sudden and chancy variations are restrained and error rate decreases slowly.

Parameters are set to: $B = 0.1,\ \sigma = 1,\ dt = 0.1$

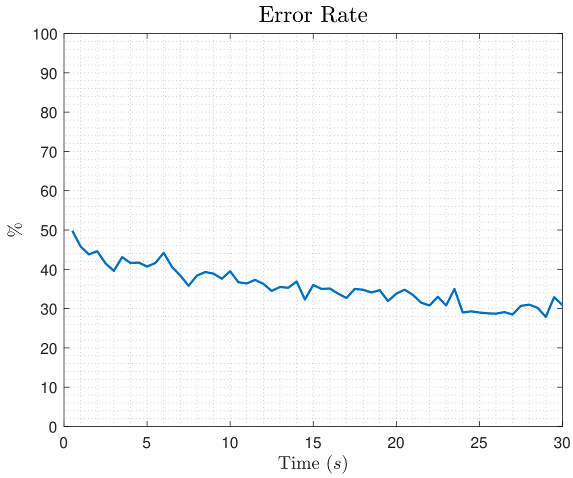


Figure 5: Simple DDM Model; Error Rate

4) From the (Ratcliff et al. 2008), discrete version of diffusion drift model (DDM) for evidence accumulation is:

$$
\begin{gathered}
dX = B\,dt + \sigma\,dW \\
\int_0^t dx = B\int_0^t dt + \sigma\int_0^t dW \\
X(t) - X(0) = Bt + \sigma(W(t) - W(0)) \\
X(t) = Bt + \sigma W(t)
\end{gathered}
$$

Where W(t) is $\mathcal{N}(0,t)$. Hence, the expected value of $X(t)$:

$$
\begin{aligned}
\mathbb{E}[X(t)] &= \mathbb{E}[Bt] + \mathbb{E}[\sigma W(t)] \\
&= Bt + \sigma\mathbb{E}[W(t)] \\
&= Bt
\end{aligned}
$$

And the variance of $X(t)$ would be:

$$
\begin{aligned}
\mathrm{Var}(X(t)) &= \mathrm{Var}(Bt + \sigma W(t)) \\
&= |\sigma|\mathrm{Var}(W(t)) \\
&= |\sigma|t
\end{aligned}
$$

Summing up, we obtain:

$$X(t) \sim \mathcal{N}(Bt, |\sigma|t)$$

Now that we have the distribution of $X(t)$ we can plot Figure 6 to see the existence probability of $X(t)$ in different locations in time.

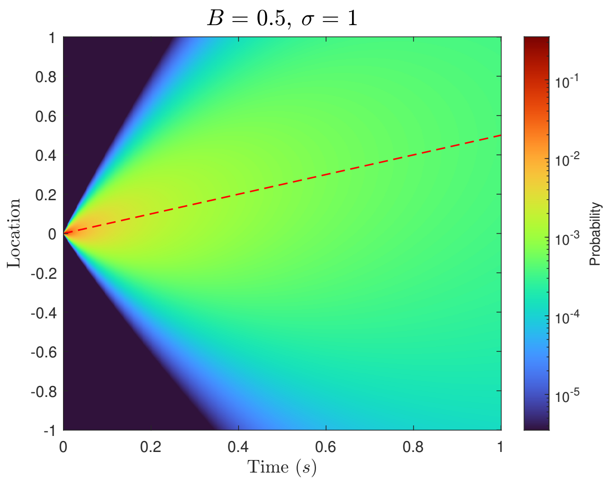


Figure 6: Simple DDM Model; Existence Probability of $X(t)$ in Plane

Now if we run the simulation for 50 trials, with parameters set to: $B = 0.1, \sigma = 1, dt = 0.1$, we would obtain Figure 7 for all trajectories and *mean* and *std*. Figure 8 also shows comparison of expected value and variance between theory calculations and measurements.

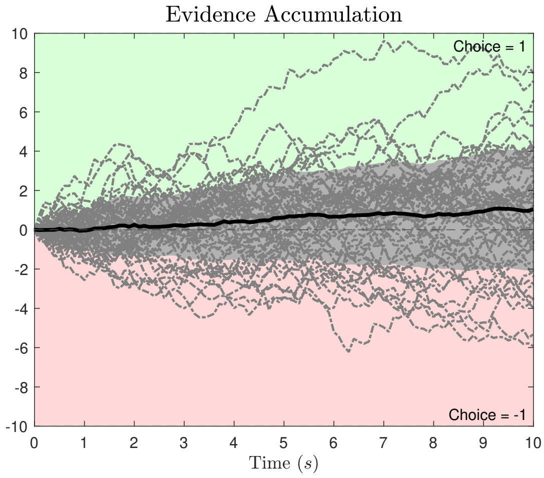


Figure 7: Simple DDM Model; Trajectories of $X(t)$

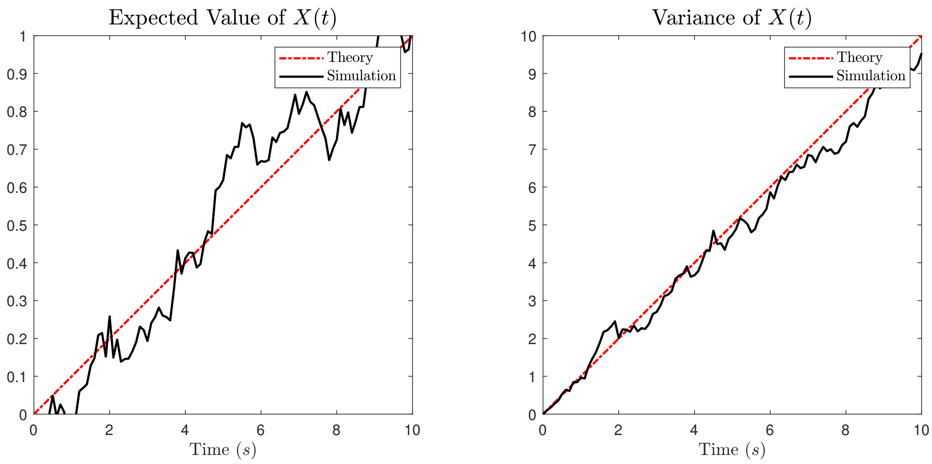


Figure 8: Simple DDM Model; Expected Value and Variance of $X(t)$

5) Using a simple function as described below, first we calculate *mean* and *std* of choice with respect to the function inputs, then we calculate the probability of being above the start point and return it.

```
function prob = simple_model2(x0, B, sigma, trial_time)
    mean_choice = B*trial_time;
    std_choice = sqrt(sigma*trial_time);
    prob = normcdf(x0, mean_choice, std_choice);
end
```

We can run the function for different values of $X(0)$ and *time interval*. Figure 9 shows the probability of reaching the correct choice with respect to the starting point. If we start from higher locations, we have a better chance to reach the top threshold and choose correct choice.

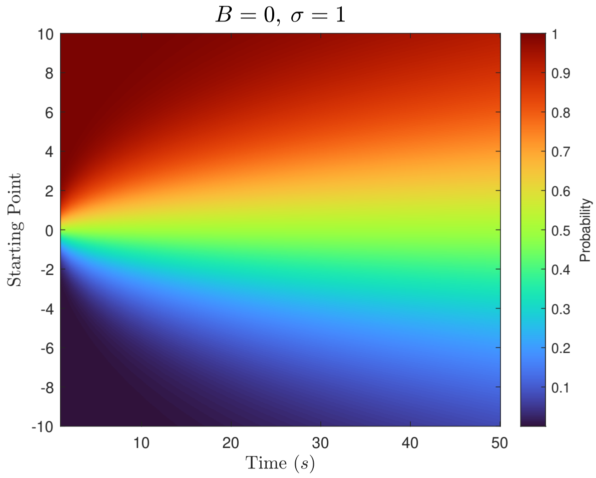


Figure 9: Simple DDM Model; Probability of Reaching Correct Choice

6) Here we set a threshold for choice 1 and 2. This task is time free and continues until the variable reaches a threshold. If we run this model 10^5 times with parameters set to $B = 0.1$, $\sigma = 1$, $dt = 0.01$, we obtain Figure 10. This figure shows the distribution of reaction time for correct trials and incorrect trials. As we can see, both distributions are the same and are likely to be *Inverse Gaussian* as discribed in (Ratcliff et al. 2008).

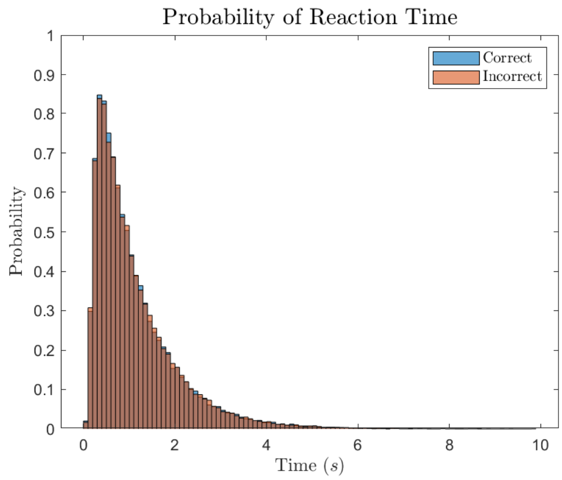


Figure 10: Two-Choice Model; Distribution of Reaction Time

7) Function *race_trial()* accepts two $X(0)$, two Bias (B), two σ, dt and two thresholds for each variable as input, and returns the number of the winner. We can check the effect of each parameter on win rate. The effect of $X(0)$ and B has been discussed before, here we change the value of σ. Figure 11 shows this effect. When the value of σ for racer 1 decreases and value of σ for racer 2 increases, winning chance of racer 1 increases.

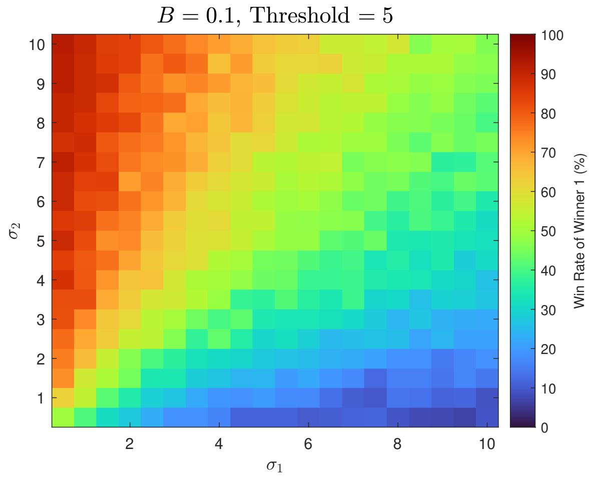


Figure 11: Race Model; Effect of σ on Winning Rate

8) Function *race_trial2()* accepts also a time interval. If the racers do not reach the threshold before the time interval, racer with higher value will win. Figure 12 shows that if we stop the race before the actual time, winner would be a bit random.

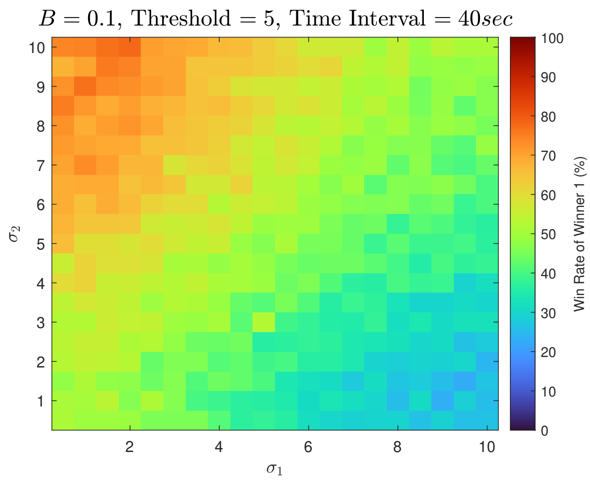


Figure 12: Race Model 2; Effect of σ on Winning Rate

Part 2

1) Using below parameters, we simulate the model proposed by (Shadlen et al. 2001) for the interaction between area MT and LIP. Figure 13 shows raster plot of results.

$$\begin{aligned}
\text{MT P Values } &= \begin{bmatrix}0.05 & 0.025\end{bmatrix}\\
\text{LIP Weights } &= \begin{bmatrix}0.1 & -0.2\end{bmatrix}\\
\text{LIP Threshold } &= 50\\
\text{Evidence Threshold } &= 0.5
\end{aligned}$$

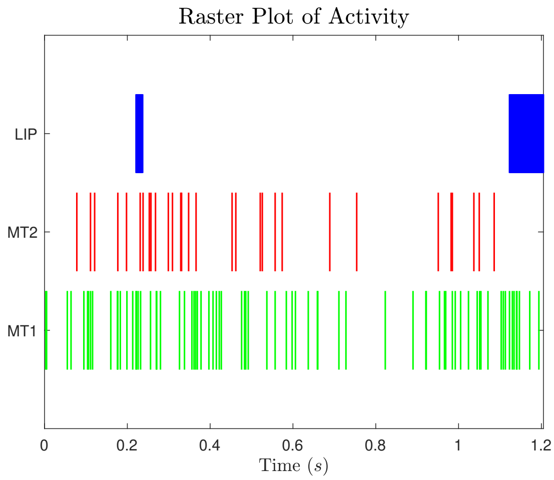


Figure 13: LIP Activity Model; Raster Plot

As can be seen in Figure 13, when MT1 is activated and MT2 is not, LIP will get the chance to fire. The actual action behind this model is that LIP neuron is integrating from MT1 with positive sign and MT2 with negative sign.

2) First we make tuning curves for MT neurons. We assume that stimulus is randomly selected from Figure 14 interval. For each value of stimulus, we have firing rate for MT1 and MT2. Then in the main function, the value of stimulus is randomly selected, response of each MT neuron is calculated, and LIP inputs will set according to weights described:

$$\text{LIP Weights} = \begin{bmatrix} 0.1 & -0.1 \\ -0.1 & 0.1 \end{bmatrix}$$

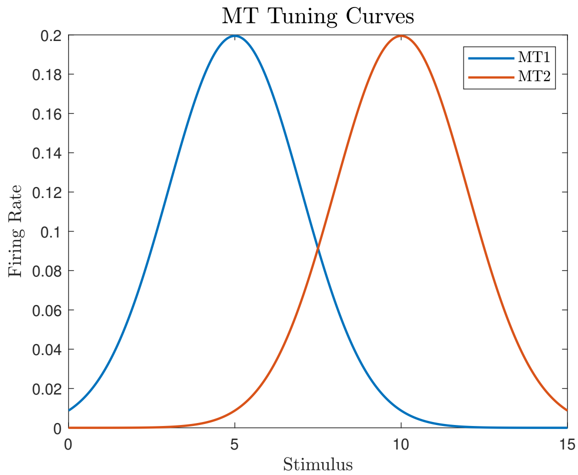


Figure 14: LIP Activity Model; Tuning Curves

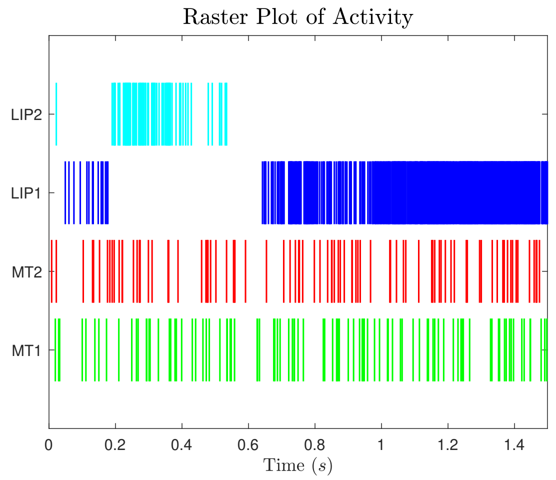


Figure 15: LIP Activity Model; Raster Plot

As can be seen in Figure 15, because MT1 excites LIP1 and MT2 inhibits LIP1 (and vice versa), there is a flip-flop behaviour observed in activities of LIP1 and LIP2.

References

Ratcliff, Roger and Gail McKoon (Apr. 2008). “The Diffusion Decision Model: Theory and Data for Two-Choice Decision Tasks”. In: *Neural Computation* 20.4, pp. 873–922. DOI: 10.1162/neco.2008.12-06-420. URL: https://doi.org/10.1162/neco.2008.12-06-420.

Shadlen, Michael N and William T Newsome (2001). “Neural basis of a perceptual decision in the parietal cortex (area LIP) of the rhesus monkey”. In: *Journal of neurophysiology* 86.4, pp. 1916–1936.