Analysis of a Traffic Signal Cantilever Structure

Semester 6 PBL

Introduction to Finite Element Method

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Table of Contents

Objective

Problem Statement

Your task is to analyze a cantilever structure supporting a traffic signal. The weight of the traffic signal, which is approximately 15 kg, can impose significant loads on the mounting structure and ground supports.

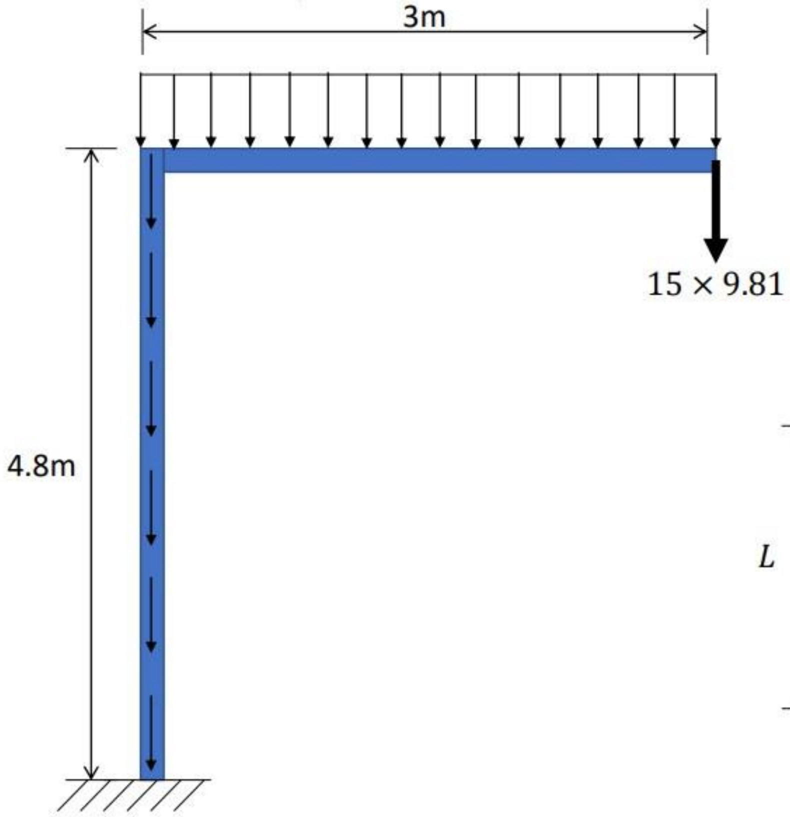
In the given simplified scenario, your objective is to determine the displacements and support reactions. To ensure that the maximum vertical displacement remains within the allowed limit of less than 5 cm, you will need to vary the cross section of the structure.

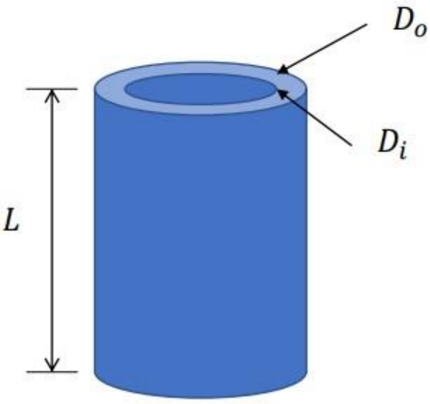
Requirement

1. What type of finite elements can you use to solve this problem?
2. Sketch a finite element model of the system
3. What loads and boundary conditions do you need to consider in this problem?
4. Starting with an outer diameter of 5cm with a fixed wall thickness of 2.5mm, determine the minimum outer diameter that will ensure that the vertical displacement of the signal is below 5cm
5. Study the effect of changing the structure material from Steel (207 GPa) to Aluminum (70 GPa)
    - Density of Steel: 7850 kg/m^3
    - Density of Aluminum: 2700 kg/m^3
6. Also report how the support reactions change with cross section and material

Idealization

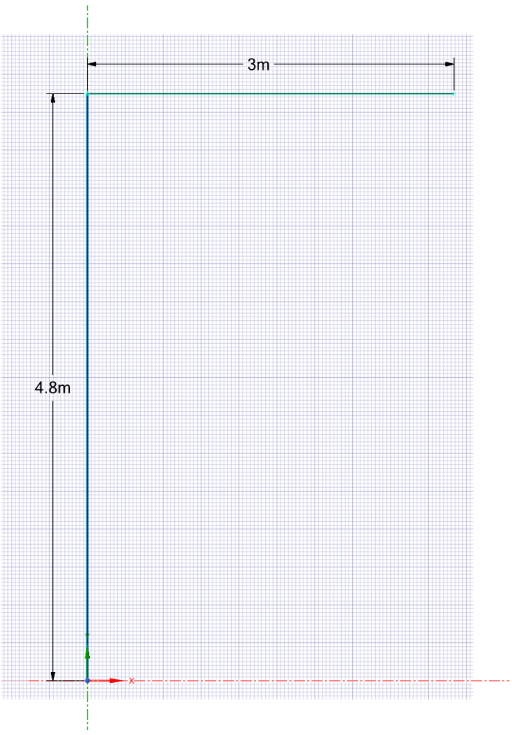
1. The concept can be realized using two 3D elements, but the initial design can be sketched in 2D (line body) and then transformed into a 3D model by applying the required conditions and constraints. To achieve practical solutions, the elements will be treated as Frame Elements and solved using the specified method
2. Here is a visual representation of the finite element method:



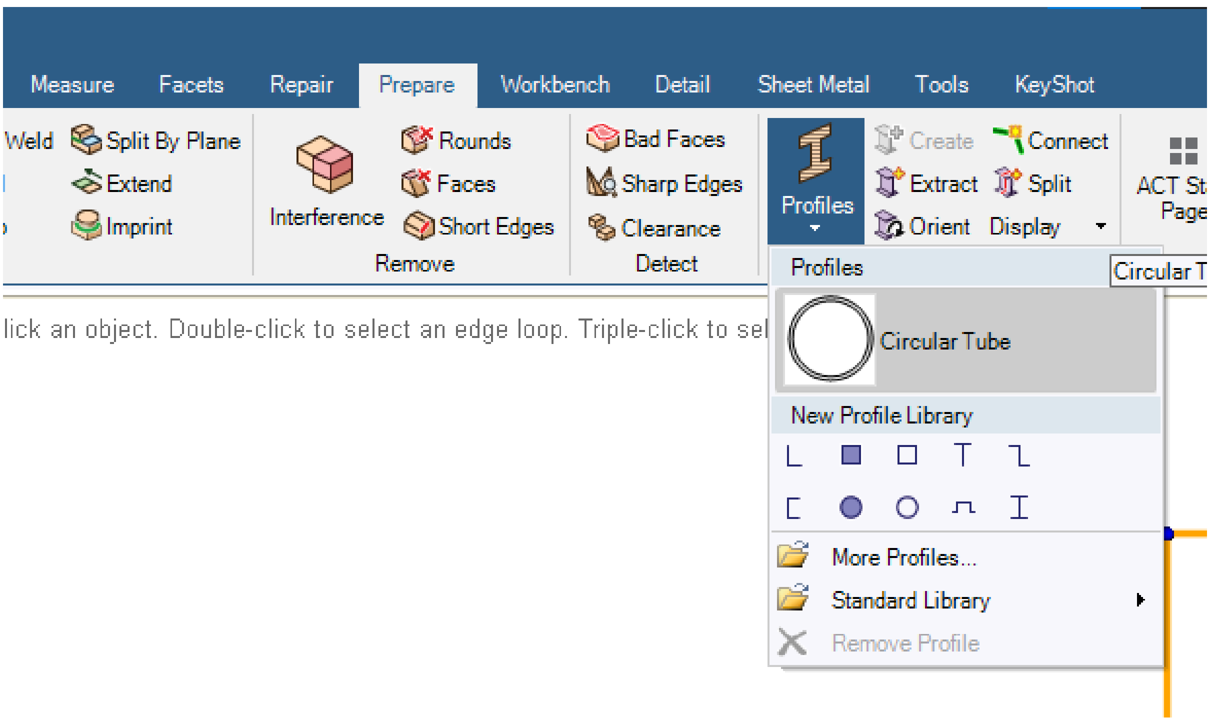


Geometry

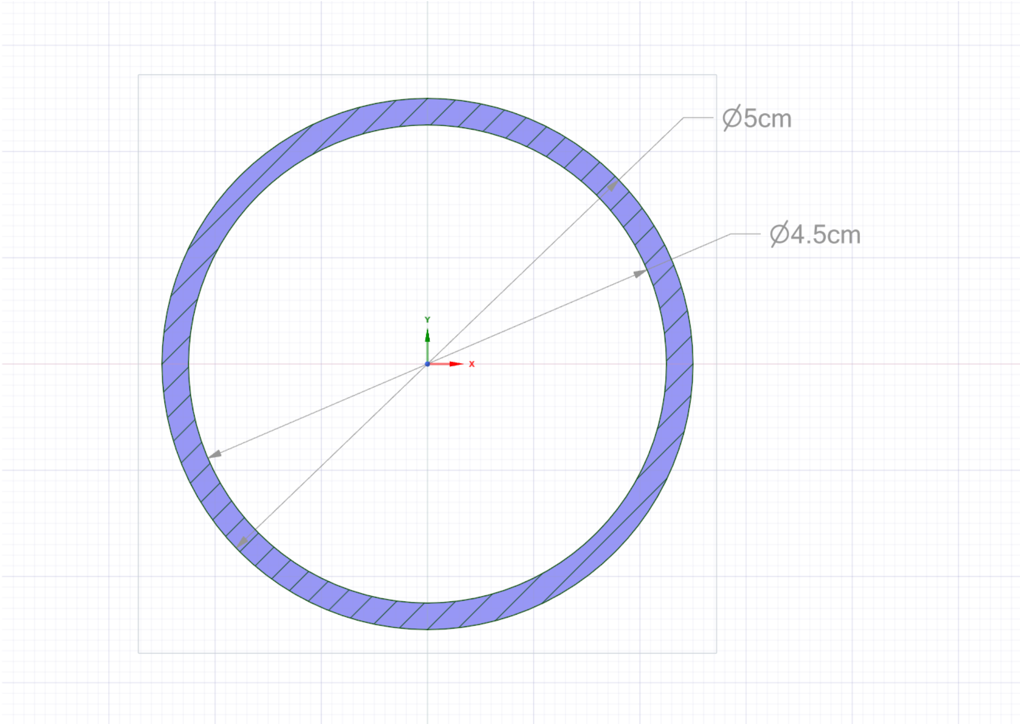
The elements were created in Ansys SpaceClaim as line bodies, with the vertical element having a dimension of 4.8 m and the horizontal element having a dimension of 3 m



Next, the Hollow Circular Tube beam profile was specified for both elements



The cross-sectional dimensions of the beam profile were defined as having a 5 cm outer diameter, 4.5 cm inner diameter, and a fixed thickness of 2.5 mm



Material Models

Steel

Steel was chosen as the material for the beam profile, with a density of 7850 kg/m³ and a Young's modulus of 207 GPa, as specified in the problem statement

Property	Value	Unit
Material Field Variables	Table	
Density	7850	kg m^-3
Isotropic Secant Coefficient of Thermal Expansion		
Coefficient of Thermal Expansion	1.2E-05	C^-1
Isotropic Elasticity		
Derive from	Young's Modulus and Poisson's Ratio	
Young's Modulus	207	GPa
Poisson's Ratio	0.3	
Bulk Modulus	1.725E+11	Pa
Shear Modulus	7.9615E+10	Pa
Strain-Life Parameters		
S-N Curve	Tabular	
Tensile Yield Strength	2.5E+08	Pa
Compressive Yield Strength	2.5E+08	Pa
Tensile Ultimate Strength	4.6E+08	Pa
Compressive Ultimate Strength	0	Pa

Aluminum

Aluminum was chosen as the material for the beam profile, with a density of 2700 kg/m³ and a Young's modulus of 70 GPa, as specified in the problem statement

Property	Value	Unit
Material Field Variables	Table	
Density	2700	kg m^-3
Isotropic Elasticity		
Derive from	Young's Modulus and Poisson's Ratio	
Young's Modulus	70	GPa
Poisson's Ratio	0.33	
Bulk Modulus	6.8627E+10	Pa
Shear Modulus	2.6316E+10	Pa
Bilinear Isotropic Hardening		
Active Table	Total	
Yield Strength	2.8E+08	Pa
Tangent Modulus	5E+08	Pa

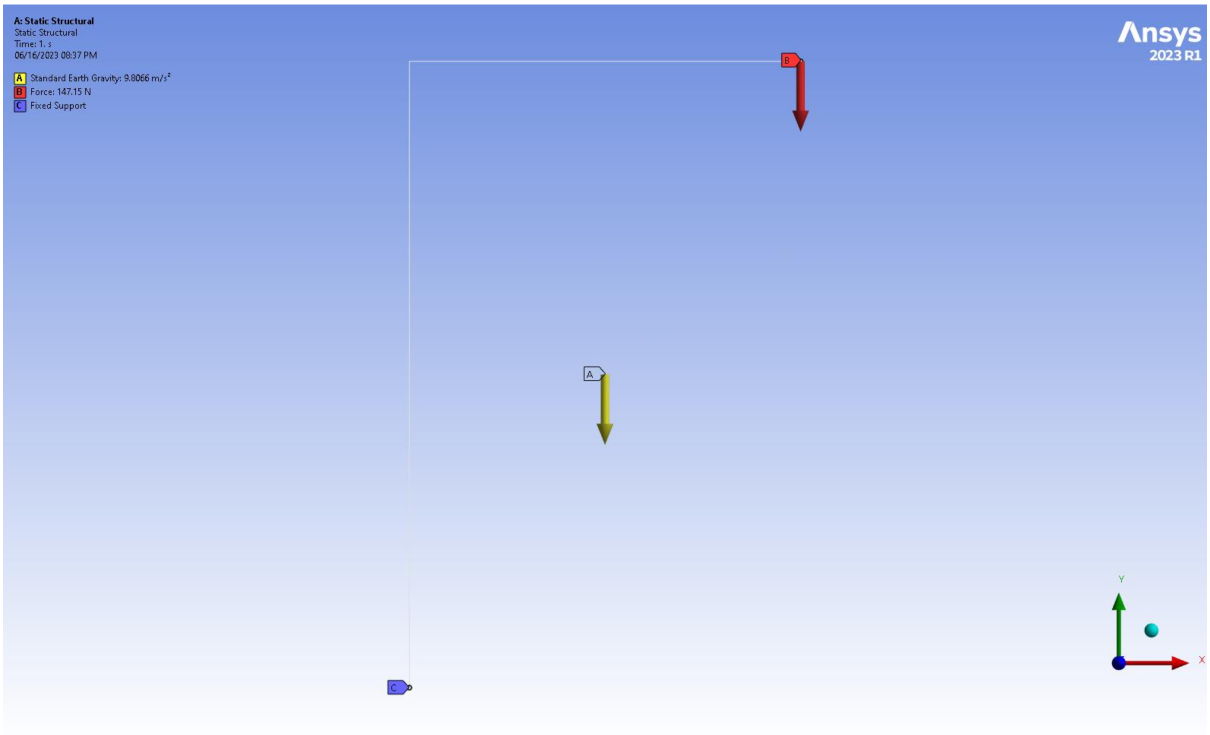
Loads and Boundary Conditions

3. The problem requires taking into account the following loads and boundary conditions:
   a. The weight of the traffic signal, which is 15 kg
   b. The weight of vertical element 1, applied through gravity
   c. The weight of horizontal element 2, applied through gravity
   d. A fixed support at node 1, resulting in $d_{1x} = d_{1y} = \Phi_1 = 0$ in

Force Vector	Displacement Vector
$\{F\} = \begin{Bmatrix} F_{1x} \\ F_{1y} \\ M_1 \\ F_{2x} \\ F_{2y} \\ M_2 \\ F_{3x} \\ F_{3y} \\ M_3 \end{Bmatrix} = \begin{Bmatrix} ? \\ ? \\ ? \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{Bmatrix}$ (Without beam distribution and signal weight)	$\{d\} = \begin{Bmatrix} d_{1x} \\ d_{1y} \\ \Phi_1 \\ d_{2x} \\ d_{2y} \\ \Phi_2 \\ d_{3x} \\ d_{3y} \\ \Phi_3 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ 0 \\ ? \\ ? \\ ? \\ ? \\ ? \\ ? \end{Bmatrix}$

Ansys Static Structural Setup (Steel and Aluminum)

The Ansys model was configured using the Static Structural project schematic. In the provided figure, the "A" arrow symbolizes the Standard Earth Gravity, directed downward along the negative y-axis, and is utilized to consider the weight of the elements in the calculations. The "B" arrow represents the downward weight of the signal. The "C" point represents the node where a Fixed Support is positioned, which in practical terms corresponds to the connection between the lamp structure and the Earth



Details of "Standard Earth Gravity"

Scope	
Geometry	All Bodies
Definition	
Coordinate System	Global Coordinate System
X Component	0. m/s^2 (ramped)
Y Component	-9.8066 m/s^2 (ramped)
Z Component	0. m/s^2 (ramped)
Suppressed	No
Direction	-Y Direction

Details of "Force"

Scope	
Scoping Method	Geometry Selection
Geometry	1 Vertex
Definition	
ID (Beta)	33
Type	Force
Define By	Components
Coordinate System	Global Coordinate System
X Component	0. N (ramped)
Y Component	-147.15 N (ramped)
Z Component	0. N (ramped)
Suppressed	No

Details of "Fixed Support"

Scope	
Scoping Method	Geometry Selection
Geometry	1 Vertex
Definition	
ID (Beta)	35
Type	Fixed Support
Suppressed	No

Solution

Calculations

	Element 01	Element 02
Modules of Elasticity (E)	207 GPa	207 GPa
Diameter Outer (Do)	5 cm	5 cm
Diameter Inner (Di)	4.5 cm	4.5 cm
Length (L_1 and L_2)	4.8 m	3 m
Density (ρ)	7850 kg/m^3	7850 kg/m^3
Gravity (g)	9.81 m/s^2	9.81 m/s^2
Theta (θ)	90	0

Element 01

$$\text{Area (A1)} = \frac{\pi}{4} \times (\text{Do}^2 - \text{Di}^2) = \frac{\pi}{4} \times (0.050^2 - 0.045^2) = 3.73 \times 10^{-4}\ \text{m}^2$$

$$\text{Inertia (I1)} = \frac{\pi}{64} \times (\text{Do}^4 - \text{Di}^4) = \frac{\pi}{64} \times (0.050^4 - 0.045^4) = 1.06 \times 10^{-7}\ \text{m}^4$$

$$\text{C} = \cos(90) = 0$$

$$\text{S} = \sin(90) = 1$$

$$\frac{12\text{I}}{\text{L}^2} = \frac{12 \times 1.06 \times 10^{-7}}{4.8^2} = 5.50 \times 10^{-8}$$

$$\frac{6\text{I}}{\text{L}} = \frac{6 \times 1.06 \times 10^{-7}}{4.8} = 1.32 \times 10^{-7}$$

$$\frac{\text{E}}{\text{L}} = \frac{207 \times 10^9}{4.8} = 4.31 \times 10^{10}$$

			d1x	d1y	Φ1	d2x	d2y	Φ2	
[k1]	=	4.31E+10	5.50E-08	0	-1.32E-07	-5.50E-08	0	-1.32E-07	d1x
			0	3.73E-04	0	0	-3.73E-04	0	d1y
			-1.32E-07	0	4.22E-07	1.32E-07	0	2.11E-07	Φ1
			-5.50E-08	0	1.32E-07	5.50E-08	0	1.32E-07	d2x
			0	-3.73E-04	0	0	3.73E-04	0	d2y
			-1.32E-07	0	2.11E-07	1.32E-07	0	4.22E-07	Φ2

Element 02

$$\text{Area (A2)} = \frac{\pi}{4} \times (\text{Do}^2 - \text{Di}^2) = \frac{\pi}{4} \times (0.050^2 - 0.045^2) = 3.73 \times 10^{-4}\ \text{m}^2$$

$$\text{Inertia (I2)} = \frac{\pi}{64} \times (\text{Do}^4 - \text{Di}^4) = \frac{\pi}{64} \times (0.050^4 - 0.045^4) = 1.06 \times 10^{-7}\ \text{m}^4$$

$$\text{C} = \cos(0) = 1$$

$$\text{S} = \sin(0) = 0$$

$$\frac{12\text{I}}{\text{L}^2} = \frac{12 \times 1.06 \times 10^{-7}}{3^2} = 1.41 \times 10^{-7}$$

$$\frac{6\text{I}}{\text{L}} = \frac{6 \times 1.06 \times 10^{-7}}{3} = 2.11 \times 10^{-7}$$

$$\frac{\text{E}}{\text{L}} = \frac{207 \times 10^9}{3} = 6.90 \times 10^{10}$$

			d2x	d2y	Φ2	d3x	d3y	Φ3	
[k2]	=	6.90E+10	3.73E-04	0	0	-3.73E-04	0	0	d2x
			0	1.41E-07	2.11E-07	0	-1.41E-07	2.11E-07	d2y
			0	2.11E-07	4.22E-07	0	-2.11E-07	2.11E-07	Φ2
			-3.73E-04	0	0	3.73E-04	0	0	d3x
			0	-1.41E-07	-2.11E-07	0	1.41E-07	-2.11E-07	d3y
			0	2.11E-07	2.11E-07	0	-2.11E-07	4.22E-07	Φ3

Global K Matrix

			d1x	d1y	Φ1	d2x	d2y	Φ2	
[k1]	=	1.00E+07	0.0002	0.0000	-0.0006	-0.0002	0.0000	-0.0006	d1x
			0.0000	1.6088	0.0000	0.0000	-1.6088	0.0000	d1y
			-0.0006	0.0000	0.0018	0.0006	0.0000	0.0009	Φ1
			-0.0002	0.0000	0.0006	0.0002	0.0000	0.0006	d2x
			0.0000	-1.6088	0.0000	0.0000	1.6088	0.0000	d2y
			-0.0006	0.0000	0.0009	0.0006	0.0000	0.0018	Φ2

			d2x	d2y	Φ2	d3x	d3y	Φ3	
[k2]	=	1.00E+07	2.5741	0.0000	0.0000	-2.5741	0.0000	0.0000	d2x
			0.0000	0.0010	0.0015	0.0000	-0.0010	0.0015	d2y
			0.0000	0.0015	0.0029	0.0000	-0.0015	0.0015	Φ2
			-2.5741	0.0000	0.0000	2.5741	0.0000	0.0000	d3x
			0.0000	-0.0010	-0.0015	0.0000	0.0010	-0.0015	d3y
			0.0000	0.0015	0.0015	0.0000	-0.0015	0.0029	Φ3

			d1x	d1y	Φ1	d2x	d2y	Φ2	d3x	d3y	Φ3	
[k]	=	1.00E+07	0.0002	0.0000	-0.0006	-0.0002	0.0000	-0.0006	0	0	0	d1x
			0.0000	1.6088	0.0000	0.0000	-1.6088	0.0000	0	0	0	d1y
			-0.0006	0.0000	0.0018	0.0006	0.0000	0.0009	0	0	0	Φ1
			-0.0002	0.0000	0.0006	2.5744	0.0000	0.0006	-2.5741	0.0000	0.0000	d2x
			0.0000	-1.6088	0.0000	0.0000	1.6098	0.0015	0.0000	-0.0010	0.0015	d2y
			-0.0006	0.0000	0.0009	0.0006	0.0015	0.0047	0.0000	-0.0015	0.0015	Φ2
			0	0	0	-2.5741	0.0000	0.0000	2.5741	0.0000	0.0000	d3x
			0	0	0	0.0000	-0.0010	-0.0015	0.0000	0.0010	-0.0015	d3y
			0	0	0	0.0000	0.0015	0.0015	0.0000	-0.0015	0.0029	Φ3

Force Vector and Displacement Vector

Distributed Beam Loading for Element 01

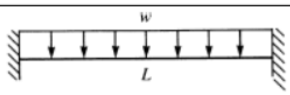
$$\text{Weight of Element (W)} = \text{Density} \times \text{Area} \times \text{Length} \times \text{Gravity}$$

$$= 7850 \times 3.73 \times 10^{-4} \times 4.8 \times 9.81 = 137.90\ \text{N}$$

F1y (a)	$= -\frac{W}{2} = -68.950$
F2y (b)	$= -\frac{W}{2} = -68.950$

Distributed Beam Loading for Element 02

Table D–1 Single element equivalent joint forces f_0 for different types of loads — Positive nodal force conventions

	f_{1y}	m_1	Loading case	f_{2y}	m_2
4.	$\frac{-wL}{2}$	$\frac{-wL^2}{12}$		$\frac{-wL}{2}$	$\frac{wL^2}{12}$

$$\text{Distributed Loading (w)} = \text{Density} \times \text{Area} \times \text{Gravity} = 7850 \times 3.73 \times 10^{-4} \times 9.81$$
$$= 28.729$$

F2y (c)	$= -\frac{wL_2}{2} = -43.094$
M2 (d)	$= -\frac{w(L_2)^2}{12} = -21.547$
F3y (e)	$= -\frac{wL_2}{2} = -43.094$
M3 (f)	$= \frac{w(L_2)^2}{12} = 21.547$

Loading due to Signal

Ws	$= 15 \times -9.81 = -147.15$

Force Vector	Displacement Vector
$\{F\} = \left\{\begin{matrix} F_{1x} \\ F_{1y} + a \\ M_1 \\ F_{2x} \\ F_{2y} + b + c \\ M_2 + d \\ F_{3x} \\ F_{3y} + e + Ws \\ M_3 + f \end{matrix}\right\}$	$\{d\} = \left\{\begin{matrix} d_{1x} \\ d_{1y} \\ \Phi_1 \\ d_{2x} \\ d_{2y} \\ \Phi_2 \\ d_{3x} \\ d_{3y} \\ \Phi_3 \end{matrix}\right\}$

Combined Equations

$$\{F\} = [k]\{d\}$$

$$\left\{\begin{matrix} F_{1x} \\ F_{1y} + a \\ M_1 \\ F_{2x} \\ F_{2y} + b + c \\ M_2 + d \\ F_{3x} \\ F_{3y} + e + Ws \\ M_3 + f \end{matrix}\right\} = [k]\left\{\begin{matrix} d_{1x} \\ d_{1y} \\ \Phi_1 \\ d_{2x} \\ d_{2y} \\ \Phi_2 \\ d_{3x} \\ d_{3y} \\ \Phi_3 \end{matrix}\right\}$$

$$\begin{Bmatrix} F_{1x} \\ F_{1y} - 68.950 \\ M_1 \\ 0 \\ 0 - 68.950 - 43.094 \\ 0 - 21.547 \\ 0 \\ 0 - 43.094 - 147.15 \\ 0 + 21.547 \end{Bmatrix} = 1.00 \times 10^7 \times \begin{bmatrix} 0.0002 & 0 & -0.0006 & -0.0002 & 0 & -0.0006 & 0 & 0 & 0 \\ 0 & 1.6088 & 0 & 0 & -1.6088 & 0 & 0 & 0 & 0 \\ -0.0006 & 0 & 0.0018 & 0.0006 & 0 & 0.0009 & 0 & 0 & 0 \\ -0.0002 & 0 & 0.0006 & 2.5744 & 0 & 0.0006 & -2.5741 & 0 & 0 \\ 0 & -1.6088 & 0 & 0 & 1.6098 & 0.0015 & 0 & -0.0010 & 0.0015 \\ -0.0006 & 0 & 0.0009 & 0.0006 & 0.0015 & 0.0047 & 0 & -0.0015 & 0.0015 \\ 0 & 0 & 0 & -2.5741 & 0 & 0 & 2.5741 & 0 & 0 \\ 0 & 0 & 0 & 0 & -0.0010 & -0.0015 & 0 & 0.0010 & -0.0015 \\ 0 & 0 & 0 & 0 & 0.0015 & 0.0015 & 0 & -0.0015 & 0.0029 \end{bmatrix} \times \begin{Bmatrix} 0 \\ 0 \\ 0 \\ d_{2x} \\ d_{2y} \\ \Phi_2 \\ d_{3x} \\ d_{3y} \\ \Phi_3 \end{Bmatrix}$$

$$\begin{Bmatrix} \cancel{F_{1x}} \\ \cancel{F_{1y} - 68.950} \\ \cancel{M_1} \\ 0 \\ -112.044 \\ -21.547 \\ 0 \\ -43.094 - 147.15 \\ 21.547 \end{Bmatrix} = 1.00 \times 10^7 \times \begin{bmatrix} \cancel{0.0002} & \cancel{0} & \cancel{-0.0006} & \cancel{-0.0002} & \cancel{0} & \cancel{-0.0006} & \cancel{0} & \cancel{0} & \cancel{0} \\ \cancel{0} & \cancel{1.6088} & \cancel{0} & \cancel{0} & \cancel{-1.6088} & \cancel{0} & \cancel{0} & \cancel{0} & \cancel{0} \\ \cancel{-0.0006} & \cancel{0} & \cancel{0.0018} & \cancel{0.0006} & \cancel{0} & \cancel{0.0009} & \cancel{0} & \cancel{0} & \cancel{0} \\ \cancel{-0.0002} & \cancel{0} & \cancel{0.0006} & 2.5744 & 0 & 0.0006 & -2.5741 & 0 & 0 \\ \cancel{0} & \cancel{-1.6088} & \cancel{0} & 0 & 1.6098 & 0.0015 & 0 & -0.0010 & 0.0015 \\ \cancel{-0.0006} & \cancel{0} & \cancel{0.0009} & 0.0006 & 0.0015 & 0.0047 & 0 & -0.0015 & 0.0015 \\ \cancel{0} & \cancel{0} & \cancel{0} & -2.5741 & 0 & 0 & 2.5741 & 0 & 0 \\ \cancel{0} & \cancel{0} & \cancel{0} & 0 & -0.0010 & -0.0015 & 0 & 0.0010 & -0.0015 \\ \cancel{0} & \cancel{0} & \cancel{0} & 0 & 0.0015 & 0.0015 & 0 & -0.0015 & 0.0029 \end{bmatrix} \times \begin{Bmatrix} \cancel{0} \\ \cancel{0} \\ \cancel{0} \\ d_{2x} \\ d_{2y} \\ \Phi_2 \\ d_{3x} \\ d_{3y} \\ \Phi_3 \end{Bmatrix}$$

After applying boundary conditions, the reduced combined

$$\begin{Bmatrix} 0 \\ -112.044 \\ -21.547 \\ 0 \\ -190.244 \\ 21.547 \end{Bmatrix} = 1.00 \times 10^7 \times \begin{bmatrix} 2.5744 & 0 & 0.0006 & -2.5741 & 0 & 0 \\ 0 & 1.6098 & 0.0015 & 0 & -0.0010 & 0.0015 \\ 0.0006 & 0.0015 & 0.0047 & 0 & -0.0015 & 0.0015 \\ -2.5741 & 0 & 0 & 2.5741 & 0 & 0 \\ 0 & -0.0010 & -0.0015 & 0 & 0.0010 & -0.0015 \\ 0 & 0.0015 & 0.0015 & 0 & -0.0015 & 0.0029 \end{bmatrix} \times \begin{Bmatrix} d_{2x} \\ d_{2y} \\ \Phi_2 \\ d_{3x} \\ d_{3y} \\ \Phi_3 \end{Bmatrix}$$

After solving 6 equations with 6 unknowns using MATLAB:

Solution.m

```
k_red = [2.57438     0.00000    0.00057   -2.57414    0.00000    0.00000
         0.00000     1.60981    0.00146    0.00000   -0.00097    0.00146
         0.00057     0.00146    0.00473    0.00000   -0.00146    0.00146
        -2.57414     0.00000    0.00000    2.57414    0.00000    0.00000
         0.00000    -0.00097   -0.00146    0.00000    0.00097   -0.00146
         0.00000     0.00146    0.00146    0.00000   -0.00146    0.00291];

k = 1e7*[k_red];

B = [0.00
    -112.044
    -21.547
     0.00
    -190.244
     21.547];

solution = inv(k)*B
```

Command Window

```
>> Solution
solution =
    0.3329
   -0.0000
   -0.1402
    0.3329
   -0.5049
   -0.1822
>>
```

Displacement Vector Solutions

$$\{d\} = \begin{Bmatrix} d_{1x} \\ d_{1y} \\ \Phi_1 \\ d_{2x} \\ d_{2y} \\ \Phi_2 \\ d_{3x} \\ d_{3y} \\ \Phi_3 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ 0 \\ 0.3329 \\ 0 \\ -0.1402 \\ 0.3329 \\ -0.5049 \\ -0.1822 \end{Bmatrix}$$

MATLAB Code

start.m | calc_frame.m

```
clc
clear
% Material Properties
E=207e9;
density = 7850;

% Element 1
L1=4.8;
th1=90; % deg
% Cross section of element 1
% Outer diameter
Do1=0.05; % in meters
% Inner Diameter
Di1=0.045; % in meters

A1=pi*(Do1^2-Di1^2)/4;
I1=pi*(Do1^4-Di1^4)/64;

k1=calc_frame(th1, A1, L1, E, I1);

% Element 2
L2=3; % in meters
th2=0; % deg

% Cross section of element 2
% Outer diameter
Do2=0.05; % in meters
% Inner Diameter
Di2=0.045; % in meters

A2=pi*(Do2^2-Di2^2)/4;
I2=pi*(Do2^4-Di2^4)/64;

k2=calc_frame(th2, A2, L2, E, I2);


% Expand k1 and k2 to system K matrix size ( 9x9 ) to apply superposition

k1_exp = [k1 zeros(6,3);
          zeros(3,9)];
```

```
Command Window
d =
         0
         0
         0
    0.3010
   -0.0000
   -0.1254
    0.3010
   -0.4503
   -0.1617
```

```
Command Window
 The maximum x displacement in the system is 0.30105 m
 The maximum y displacement in the system is -0.45028 m
 The maximum rotation in the system is -9.2633 deg
Force vector is
    0.0000
  371.2370
  570.7310
   -0.0000
    0.0000
    0.0000
    0.0000
 -147.1500
   -0.0000
>>
```

Comparing both displacements vectors

Displacement Vector	Displacement Vector	Percentage Error
$\{d\} = \begin{Bmatrix} 0 \\ 0 \\ 0 \\ 0.3329 \\ 0 \\ -0.1402 \\ 0.3329 \\ -0.5049 \\ -0.1822 \end{Bmatrix}$	$\{d\} = \begin{Bmatrix} 0 \\ 0 \\ 0 \\ 0.3010 \\ 0 \\ -0.1254 \\ 0.3010 \\ -0.4503 \\ -0.1617 \end{Bmatrix}$	$\%\text{Error} = \begin{Bmatrix} 0\% \\ 0\% \\ 0\% \\ 10.598\% \\ 0\% \\ 11.802\% \\ 10.598\% \\ 12.125\% \\ 12.678\% \end{Bmatrix}$

The percentage error above indicates the disparity between the results obtained from manual calculations and those derived from MATLAB code calculations

Model Validation

ANSYS Results

Steel

Name	P1 - Circular Tube Ri	P2 - Circular Tube Ro	P3 - Directional Deformation Minimum	P5 - Force Reaction Maximum Y Axis	P6 - Moment Reaction Maximum Z Axis
Units	cm	cm	cm	N	N m
DP 0 (Current)	2.25	2.5	-45.114	370.99	570.59
DP 1	4.75	5	-6.4339	606.6	706.52
DP 2	5.25	5.5	-4.9811	653.73	733.71
DP 3	5.5	5.75	-4.425	677.29	747.3
DP 4	5.75	6	-3.9531	700.85	760.89

4. By utilizing the Parameters feature and adjusting the values of the outer radius and inner radius, it becomes evident through visualization that the optimal range for the outer radius is approximately 5.5 to 5.75 cm. Within this range, the vertical displacement remains within the prescribed design constraint of 5 cm.

Aluminum

Name	P1 - Circular Tube Ri	P2 - Circular Tube Ro	P3 - Directional Deformation Minimum	P4 - Force Reaction Maximum Y Axis	P5 - Moment Reaction Maximum Z Axis
Units	cm	cm	cm	N	N m
DP 0 (Current)	2.25	2.5	-114.27	224.14	485.87
DP 1	6.5	6.75	-6.1119	361.91	565.35
DP 2	6.75	7	-5.5133	370.01	570.02
DP 3	7	7.25	-4.9929	378.11	574.7
DP 4	7.25	7.5	-4.5381	386.22	579.37

4. By utilizing the Parameters feature and adjusting the values of the outer radius and inner radius, it becomes evident through visualization that the optimal range for the outer radius is approximately 7.25 to 7.5 cm. Within this range, the vertical displacement remains within the prescribed design constraint of 5 cm.

MATLAB Results

Steel

```
% Element 1
L1=4.8;
th1=90; % deg
% Cross section of element 1
% Outer diameter
Do1=0.11; % in meters
% Inner Diameter
Di1=0.105; % in meters

A1=pi*(Do1^2-Di1^2)/4;
I1=pi*(Do1^4-Di1^4)/64;

k1=calc_frame(th1, A1, L1, E, I1);

% Element 2
L2=3; % in meters
th2=0; % deg

% Cross section of element 2
% Outer diameter
Do2=0.11; % in meters
% Inner Diameter
Di2=0.105; % in meters
```

Command Window

```
  The maximum x displacement in the system is 0.033476 m
  The maximum y displacement in the system is -0.049708 m
  The maximum rotation in the system is -1.0158 deg
Force vector is
     0.0000
   654.2944
   734.0333
    -0.0000
     0.0000
    -0.0000
          0
  -147.1500
    -0.0000
>>
```

Aluminum

```
% Element 1
L1=4.8;
th1=90; % deg
% Cross section of element 1
% Outer diameter
Do1=0.145; % in meters
% Inner Diameter
Di1=0.14; % in meters

A1=pi*(Do1^2-Di1^2)/4;
I1=pi*(Do1^4-Di1^4)/64;

k1=calc_frame(th1, A1, L1, E, I1);

% Element 2
L2=3; % in meters
th2=0; % deg

% Cross section of element 2
% Outer diameter
Do2=0.145; % in meters
% Inner Diameter
Di2=0.14; % in meters
```

```
Command Window
  The maximum x displacement in the system is 0.033291 m
  The maximum y displacement in the system is -0.049799 m
  The maximum rotation in the system is -1.0239 deg
Force vector is
     0.0000
   378.3736
   574.8482
          0
     0.0000
    -0.0000
          0
  -147.1500
     0.0000
>>
```

Upon executing the MATLAB script with varying radii or diameter, the following results have been obtained:

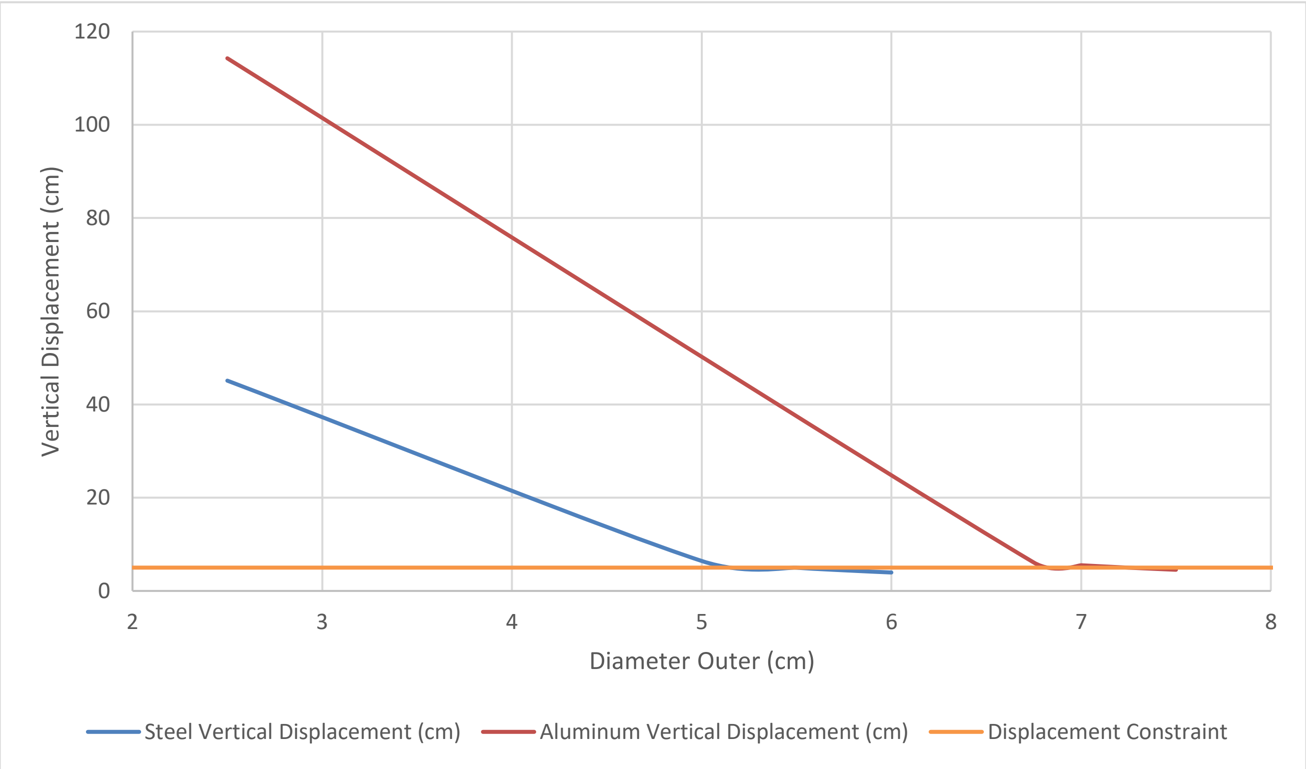
Steel		Aluminum	
Ro (cm)	Max y Displacement (cm)	Ro (cm)	Max y Displacement (cm)
2.5	-45.028	2.5	-114.03
5	-6.4209	6.75	-6.0966
5.5	-4.9708	7	-5.4993
5.75	-4.4157	7.25	-4.9799
6	-3.9446	7.5	-4.5261

5. The maximum vertical displacement values obtained from both the Ansys simulation and MATLAB script are nearly identical. This indicates a high level of consistency between the two methods in predicting the vertical displacement

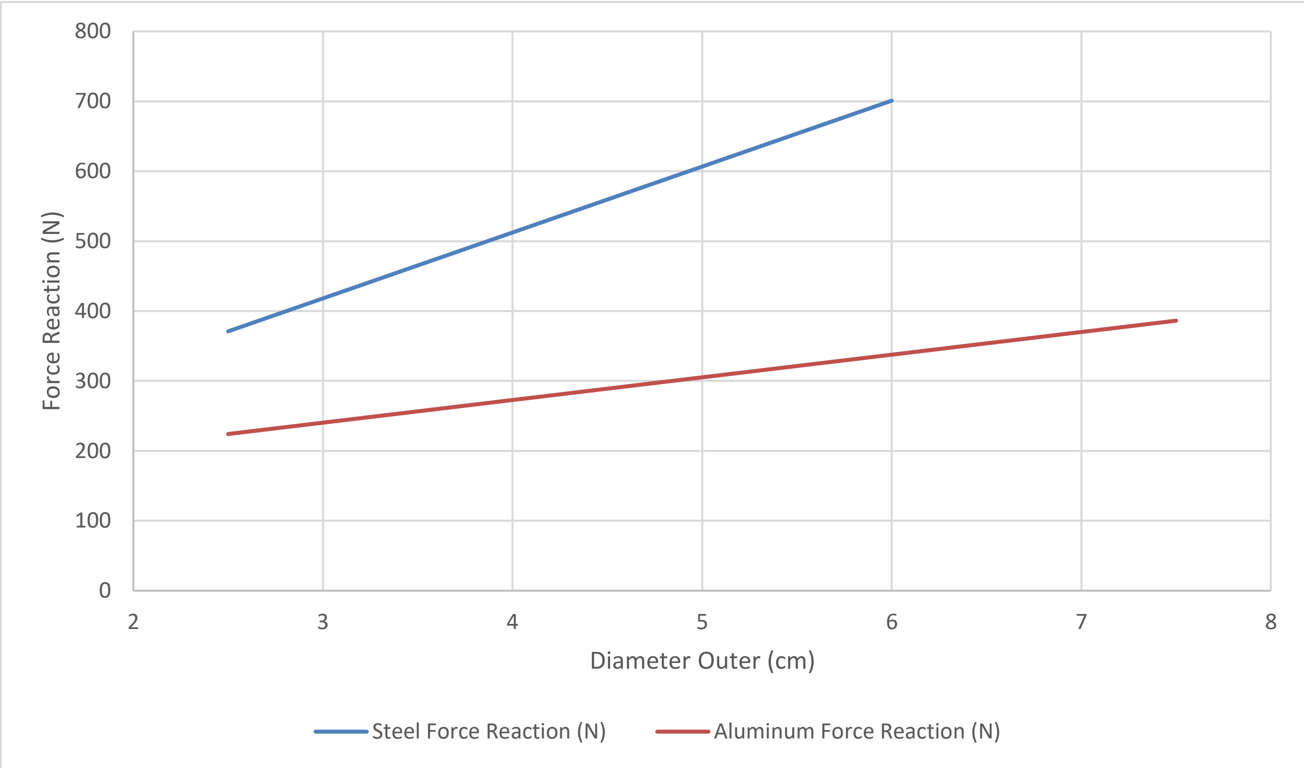
Comparison between Steel and Aluminum

Graphical

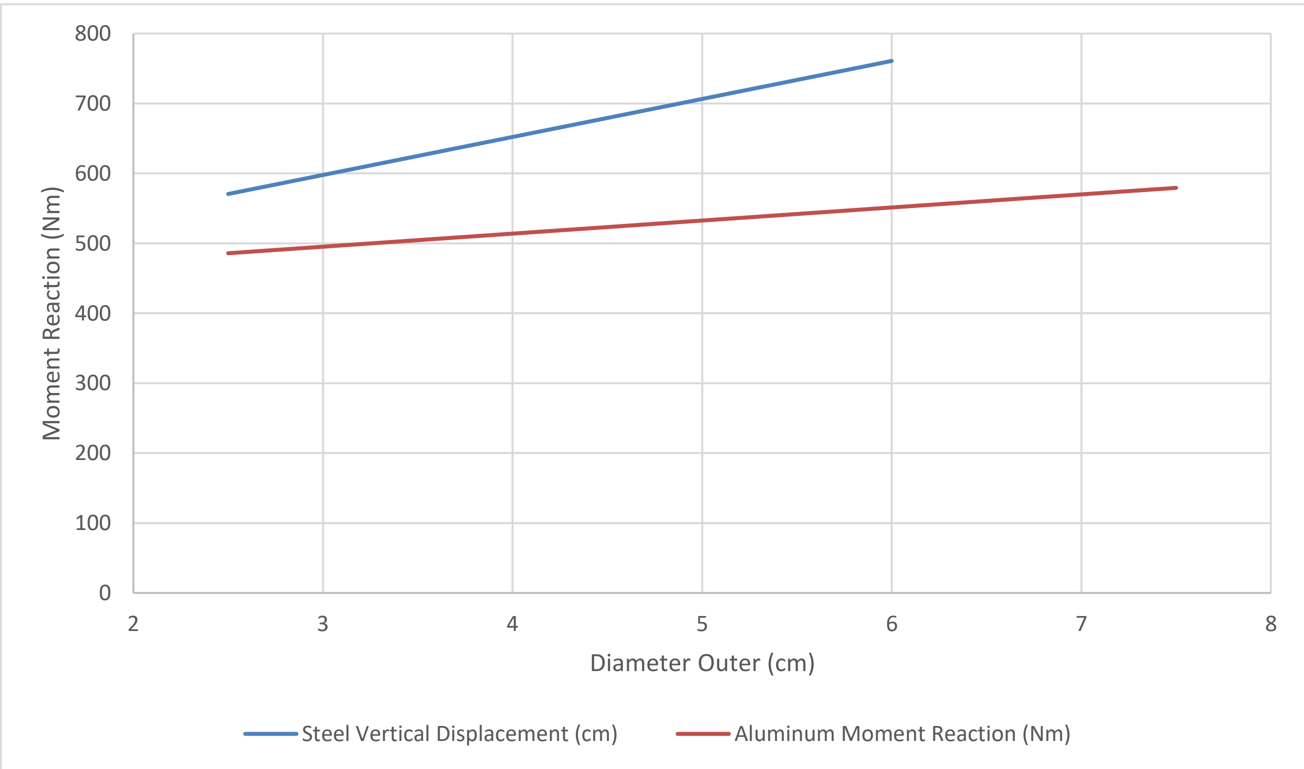
<u>Vertical Displacement</u>



Force Reaction



Moment Reaction



Numerical

Using the above graphs and Microsoft Excel:

	Steel	Aluminum	Steel	Aluminum	Steel	Aluminum
Ro (cm)	Vertical Displacement (cm)	Vertical Displacement (cm)	Force Reaction (N)	Force Reaction (N)	Moment Reaction (Nm)	Moment Reaction (Nm)
2.50	45.1140		370.99		570.59	
5.00	6.4339		606.6		706.52	
5.50	4.9811		653.73		733.71	
5.75	4.4250		677.29		747.3	
6.00	3.9531		700.85		760.89	
2.50		114.2700		224.14		485.87
6.75		6.1119		361.91		565.35
7.00		5.5133		370.01		570.02
7.25		4.9929		378.11		574.7
7.50		4.5381		386.22		579.37

Intersection		
Material	Steel	Aluminum
Optimal Outer Radius (cm)	5.59	7.15
Optimal Outer Diameter (cm)	11.19	14.31
Vertical Displacement Constraint (cm)	5.00	5.00
Optimal Reaction Force (N)	662.21	374.87
Optimal Moment Reaction (Nm)	738.59	572.83

Conclusion

6. By applying intersection formulae and interpolation formulae, it can be determined that the optimal diameter to achieve a vertical displacement of 5 cm is approximately 11.19 cm for steel and 14.31 cm for aluminum. At these diameters, the optimal force reaction is estimated to be 662.21 N for steel and 374.87 N for aluminum. Additionally, the optimal moment reaction is projected to be 738.59 Nm for steel and 572.83 Nm for aluminum. These calculations provide valuable insights for selecting the appropriate diameter and understanding the expected force and moment reactions for steel and aluminum materials