mechanics chapter 4

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1 Chapter 4 : Work Power Energy

Differential :

$$du = \frac{\partial u}{\partial x}dx + \frac{\partial u}{\partial y}dy + \frac{\partial u}{\partial z}dz$$

Gradients :
In Cartesian Coordinates :

$$\vec{grad} = \vec{\nabla} = \frac{\partial}{\partial x}\vec{i} + \frac{\partial}{\partial y}\vec{j} + \frac{\partial}{\partial z}\vec{z}$$

In Cylindrical Coordinates :

$$\vec{grad} = \vec{\nabla} = \frac{\partial}{\partial \rho}\vec{\rho} + \frac{1}{\rho}\frac{\partial}{\partial \phi}\vec{\phi} + \frac{\partial}{\partial z}\vec{z}$$

In Spherical Coordinates :

$$\vec{grad} = \vec{\nabla} = \frac{\partial}{\partial r}\vec{r} + \frac{1}{r}\frac{\partial}{\partial \theta}\vec{\theta} + \frac{1}{r\sin(\theta)}\frac{\partial}{\partial \phi}\vec{\phi}$$

Divergence of a vector :
In Cartesian Coordinates :

$$div\,\vec{u} = \vec{\nabla}\cdot\vec{u} = \frac{\partial}{\partial x} + \frac{\partial}{\partial y} \ + \frac{\partial}{\partial z}$$

In Cylindrical Coordinates :

$$div\,\vec{u} = \frac{\partial}{\partial \rho} + \frac{1}{\rho}\frac{\partial}{\partial \phi} + \frac{\partial}{\partial z}$$

In Spherical Coordinates :

$$div\,\vec{u} = \frac{\partial}{\partial r} + \frac{1}{r}\frac{\partial}{\partial \theta} + \frac{1}{r\sin(\theta)}\frac{\partial}{\partial \phi}$$

Rotation of a Vector :

$$\vec{Rot}\,\vec{A} = \vec{\nabla} \wedge \vec{A} = \frac{1}{h_1 h_2 h_3} \begin{vmatrix} h_1\vec{e_1} & h_2\vec{e_2} & h_3\vec{e_3} \\ \frac{\partial}{\partial x_1} & \frac{\partial}{\partial x_2} & \frac{\partial}{\partial x_3} \\ h_1 A_1 & h_2 A_2 & h_3 A_3 \end{vmatrix}$$

In Cartesian Coordinates :

$$h_1 = h_2 = h_3 = 1$$

$$\vec{e_1} = \vec{e_x} \qquad \vec{e_2} = \vec{e_y} \qquad \vec{e_3} = \vec{e_z}$$

In Cylindrical Coordinates :

$$h_1 = 1 \qquad h_2 = \rho \qquad h_3 = 1$$

$$\vec{e_1} = \vec{e_\rho} \qquad \vec{e_2} = \vec{e_\phi} \qquad \vec{e_3} = \vec{e_z}$$

In Spherical Coordinates :

$$h_1 = 1 \qquad h_2 = r \qquad h_3 = r\sin(\theta)$$

$$\vec{e_1} = \vec{e_r} \qquad \vec{e_2} = \vec{e_\theta} \qquad \vec{e_3} = \vec{e_\phi}$$

Circulation of a Vector Field :

$$dC = \vec{u}d\vec{M}$$

(Where dC is elementary element of circulation)

$$C = \int_c \vec{u}d\vec{M}$$

In Cartesian Coordinates :

$$C = \int_{(c)} u_x dx + u_y dy + u_z dz$$

In Cylindrical Coordinates :

$$C = \int_{(c)} u_\rho d\rho + \rho u_\phi d\phi + u_z dz$$

In Spherical Coordinates :

$$C = \int_{(c)} u_r dr + r u_\theta d\theta + r\sin(\theta) u_\phi d\phi$$

Circulation of a Conservative Field :

$\vec{A}\,is\,said\,to\,be\,conservative\,if\,it\,is\,path\,independant$

$$\int_{(c)} \vec{A} d\vec{M}$$

$$has\, same\, value\, for\, any\, path\, taken$$

$$\Rightarrow \vec{A} = -\vec{grad}\, u$$

$$C = \int_{(c)} \vec{A} d\vec{M} = \int_{(c)} -\vec{grad}\, u\, d\vec{M} = -\int_{p}^{q} du = u_p - u_q$$

$$Conclusion\, :\, \vec{A} = -\vec{grad}\, u \iff \vec{A}\, is\, Conservative$$

Characteristics of a Conservative Field $\vec{A}$

$$\vec{Rot}\, \vec{A} = \vec{0} \iff \vec{A}\, is\, Conservative \iff \vec{A} = -\vec{grad}\, u$$

$$The\, Circulation\, of\, a\, conservative\, field\, along\, closed\, curve\, is\, zero$$

$$If\, \vec{A} = A(q)\vec{e_q} \Rightarrow \vec{Rot}\, \vec{A} = \vec{0}\, and\, A\, is\, Conservative$$

Work :

$$W = C = \int_{(c)} \vec{F}\, d\vec{M}$$

$$\vec{F} = \vec{F_\tau} + \vec{F_n}$$

$$W = \int \vec{F_\tau}\, d\vec{M} + \int \vec{F_n}\, d\vec{M}$$

Remark :

For a Conservative Vector Field $\vec{A}$:

$$\vec{A} = -\vec{grad}\, u$$

For a Conservative Field of Force $\vec{F}$:

$$\vec{F} = -\vec{grad}\, EP$$

Power :

$$dw = \vec{F}\, d\vec{M} = \vec{F}\, \frac{d\vec{M}}{dt}\, dt = \vec{F}\, \vec{V}\, dt$$

$$p = \frac{dw}{dt} = \vec{F}\, \vec{V}$$

Theorem of Kinetic Energy :

$$\vec{F} = \frac{d\vec{P}}{dt} = m\,\frac{d\vec{V}}{dt}$$

$$dw = \vec{F}\,d\vec{M} = \vec{F}\,\frac{dM}{dt}\,dt = \vec{F}\,\vec{V}\,dt = m\,\frac{d\vec{V}}{dt}\,\vec{V}\,dt = m\,\vec{V}\,d\vec{V}$$

$$W = \int_A^B dw = \int_{V_i}^{V_f} m\,\vec{V}\,d\vec{V} = \frac{1}{2}\,m\,V^2\,\Big|_{V_i}^{V_f}$$

$$\Rightarrow w = \Delta KE = \frac{1}{2}\,m\,(V_f^2 - V_i^2)$$