Mechanics

Ali Abdallah

June 2022

Chapter 3 : Dynamics

$$\vec{P} = m\vec{V}(M)$$
$$\vec{\sigma} = O\vec{M} \wedge \vec{P}$$
$$\vec{\sigma} = O\vec{M} \wedge m\vec{V}(M)$$

If mass is **Constant**:

$$\frac{d\vec{P}}{dt} = \vec{0}$$
$$\frac{d}{dt}(m\vec{V}(M)) = m\frac{d\vec{V}(M)}{dt} = m\vec{\Gamma}(M)$$
$$\vec{\Gamma}(M) = \vec{0} \Rightarrow \vec{V} = \vec{V_o} = \mathbf{cst}$$

In **General** case we have:

$$\frac{d\vec{P}}{dt} \neq \vec{0}$$
$$\Rightarrow \sum \vec{F}_{ext} \neq \vec{0}$$

In case where $\mathbf{m} \neq Constant$

$$\frac{d\vec{P}}{dt} = \sum \vec{F}_{ext}$$
$$\Rightarrow \frac{d}{dt}m\vec{V}(M) = \sum \vec{F}_{ext}$$
$$m\,\frac{d\vec{V}(M)}{dt} + \frac{dm}{dt}\,\vec{V}(M) = \sum \vec{F}_{ext}$$

Galilean Systems
passing from one Galilean system to another we have:

$$\vec{\Gamma}_a(M) = \vec{\Gamma}(M/R_o) = \vec{\Gamma}_r(M)$$

Its considered as rectilinear uniform translation motion

Non Galilean Systems

$$\sum \vec{F}_e = m\vec{\Gamma}_a(M)$$

$$\sum \vec{F}_e = m\vec{\Gamma}_r(M) + m\vec{\Gamma}_{tr}(M) + m\vec{\Gamma}_c(M)$$

$$\Rightarrow \sum \vec{F}_e - m\vec{\Gamma}_{tr}(M) - m\vec{\Gamma}_c(M) = m\vec{\Gamma}_r(M)$$

$$its\, as\, if\, we\, are\, adding\, two\, pseudo\, forces$$

$$\sum \vec{F}_e + \vec{F}_{ic} + \vec{F}_{itr} = m\vec{\Gamma}_r(M) = m\vec{\Gamma}(M/R_1)$$

$$such\, that:$$

$$\vec{F}_{ic} = -m\vec{\Gamma}_c(M)$$

$$\vec{F}_{itr} = -m\vec{\Gamma}_{tr}(M)$$

Angular Momentum Theorem:
In Galilean System:

$$\frac{d}{dt}\vec{\sigma}_o(M/R)|_R = \sum \vec{M}_o(\vec{F}_{ext})$$

In Non Galilean System:

$$\frac{d}{dt}\vec{\sigma}_o(M/R)|_{R1} = \sum \vec{M}_{01}(\vec{F}_{ext}) + \sum \vec{M}_{01}(\vec{F}_{itr}) + \sum \vec{M}_{01}(\vec{F}_{ic})$$

Differential Equations Solutions:

$$\ddot{x} - k^2 x = 0 \qquad where \qquad k > 0$$

$$Solution \;==>\; x = C_1 e^{kt} + C_2 e^{-kt}$$

$$\ddot{x} + k^2 x = 0 \qquad where \qquad k > 0$$

$$Solution \;==>\; x = A\cos(kt) + B\sin(kt)$$