Minimising total race time for an F1 car taking into account the fuel load effect, and tyre degradation

Mathematics Analysis and Approaches HL Internal Assessment

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Table of Contents

Introduction 1
Rationale 1
Aim 2
Background information 2
Finding the fuel effect of Vettel's car 3
Fuel correcting the lap times 5
Modelling each tyre compound's degradation 7
Linear model graphs (lap times) 8
Quadratic model graphs (lap times) 9
Forming an equation for total race time using the models 10
Testing the models 11
Choosing the best model 11
Finding the optimal number of laps for each tyre 12
Conclusion 17

Introduction

Formula one is considered the pinnacle of motorsport. Where you have the most technically advanced race cars with prestigious talent such as Lewis Hamilton and Sebastian Vettel and youngsters such as Max Verstappen drive these cars to the limit. Throughout the history of these races, the sole focus of the whole sport has mainly been on either the driver or the car. No doubt that these two play a vital role in determining how well a team performs over the course of a race, but there is also another factor that isn't given as much attention which is team strategy. The strategy of a team during a race can consist of many different aspects. One aspect can be which power modes will be implemented during the race, which can determine the fuel consumption rate of the car and also the acceleration and top speed of the car. Another aspect can be the wing setup that the team uses. This can range from a setup that encourages high downforce (where the speed in turns can be higher but top speed is reduced) or straight-line speed (where a car can go faster in the straight parts of the track). The third aspect, which is one of the most important, is the tyre strategy. This entails things such as what tyres to use, and when to conduct a pitstop during the race (so as to change those tyres).

Rationale

The strategy of a team can in some cases overcome and even topple variables such as the superiority of the car and also the skill of the driver. If the team uses a strategy that is superior to other teams, then the chances of them winning the race can increase significantly. However, if a team somehow chooses and executes the wrong strategy, it can completely ruin the whole race. For example, in the 2016 Spanish Grand Prix, Daniel Ricciardo and Max Verstappen (who both raced in the same team and had the same car) were approximately 44 seconds apart (almost half a lap) with Ricciardo coming a distant fourth while Max Verstappen won the race. The main difference between both of their strategies was that Ricciardo had done 3 pitstops while Verstappen had done 2. This difference in tyre strategy is what created such a stark difference in the outcome. That is why it is important that the tyre strategy and its effect be investigated, so that not only can I further my interest and understanding of such a complex sport, but also shed some light on the importance of this part of the strategy which is often undermined when compared with things such as driver skill and car performance.

Aim

F1 is a sport which I watched since I was 6 years old, and hearing and seeing the supremacy of the Scuderia Ferrari team and their consecutive world titles made me a big fan of 'the car in red'. However, with the recent decline in performance of Ferrari, and how they have been beaten by competing teams, I want to look into the team's tyre strategy and what possibly could have gone wrong. That is why I will be looking to explore how I can minimise Sebastian Vettel's total race time in the Ferrari, taking into account the fuel load effect and tyre degradation. Since I have personally visited the 2019 Abu Dhabi grand Prix, I will be looking at the lap times of Vettel's Ferrari during the 2019 Abu Dhabi grand Prix.

Background information

The theory this investigation is based around is that firstly during a race a car consumes fuel, burns it, and then releases the exhaust gases. This consumption and release means that as the car progresses in a race, the weight of the total fuel in the tank goes down. Since $F_{net} = ma$ (Newton's second law, where m is the mass, a is the acceleration, and F_{net} is the net force) which states that the net force (in this case the net force moving the car) is equal to the product of the mass and the acceleration. Then, if the mass decreases over time, and the force the engine is providing remains more or less constant, then with the same engine power output, the car will be able to accelerate faster, hence decreasing lap times.

The second part of the investigation revolves around tyre degradation. In an F1 race, teams get an option to choose from 3 different tyre compounds; Soft, Medium and Hard. Soft tyres provide more grip and are initially faster, but degrade quick. Hard tyres don't provide as much speed, but last much longer than the softs and mediums. Medium tyres are initially faster than the hard compound but slower than the softs, but last longer than the softs, and shorter than the hards. When the tyre degrades to a point where the car becomes much slower, then the team carries out a pitstop where the tyres are changed to a fresher tyre. However this comes with a catch, and this is that the time for the team to change the tyres counts to the total race time. So teams would like to minimize the number of pitstops they make because each one costs them around 22 seconds. Nonetheless, if they do not make a pitstop at all, then the tyre keeps on degrading so much so that the driver will become much slower than his competitors, so teams would also like to make sure that this doesn't happen. Finding the right balance and optimizing this strategy is what can lead to a race win, and hence every single team has a dedicated team of race strategists that try their best to come up with the best strategy to use during the race.

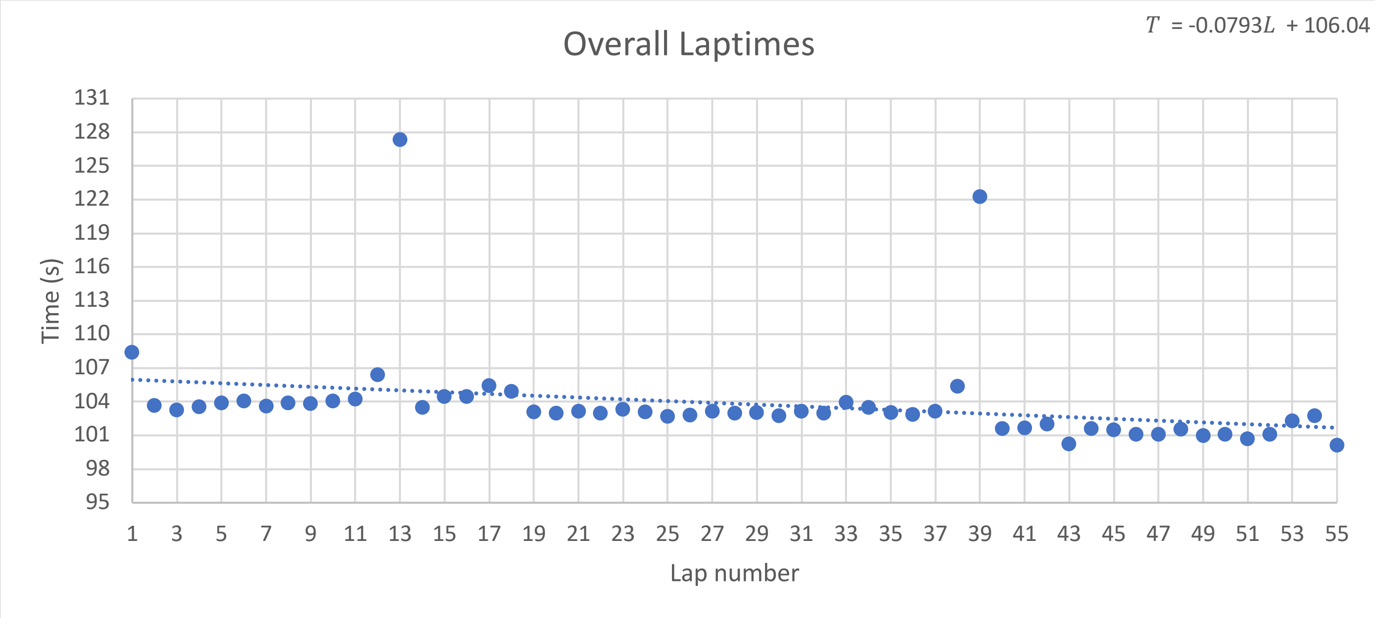
Finding the fuel effect of Vettel's car

Lap times of Sebastian Vettel in the 2019 Abu Dhabi Grand prix[1]

Lap	Time (seconds)
1	108.361
2	103.643
3	103.234
4	103.513
5	103.888
6	104.065
7	103.575
8	103.900
9	103.821
10	104.061
11	104.195
12	106.406
13	127.307
14	103.496
15	104.467
16	104.461
17	105.414

[1] www.fia.com. 2021. No page title. [ONLINE] Available at: https://www.fia.com/sites/default/files/2019_21_uae_f1_r0_timing_racelapanalysis_v01.pdf. [Accessed 10 January 2021].

18	104.900
19	103.060
20	102.989
21	103.123
22	102.995
23	103.303
24	103.091
25	102.707
26	102.802
27	103.129
28	102.965
29	103.048
30	102.760
31	103.143
32	102.976
33	103.921
34	103.476
35	103.011
36	102.847
37	103.143
38	105.352
39	122.236
40	101.569
41	101.645
42	102.017
43	100.246
44	101.605
45	101.455
46	101.104
47	101.074
48	101.561
49	100.989
50	101.099
51	100.657
52	101.105
53	102.301
54	102.733
55	100.128
Total time (seconds)	5710.072



[Race lap analysis of the 2019 Abu Dhabi Grand prix from the F1 governing body: the 'FIA']

These are the overall lap times of Vettel in the 2019 Abu Dhabi Grand Prix (where T is the time taken to complete a lap in seconds and L is the lap number). What can be seen here is that the first derivative of the line of best fit (with an equation of $\mathrm{T} = -0.0793\mathrm{L} + 106.04$) can give us the magnitude of the fuel effect per lap which is $\frac{dT}{dL} = -0.0793 \approx -0.08$ (by using the differentiation rule $y = x^n + c \rightarrow \frac{dy}{dx} = nx^{n-1}$). This means that the car goes 0.08 seconds faster every lap due to the fuel being consumed and the weight being reduced.

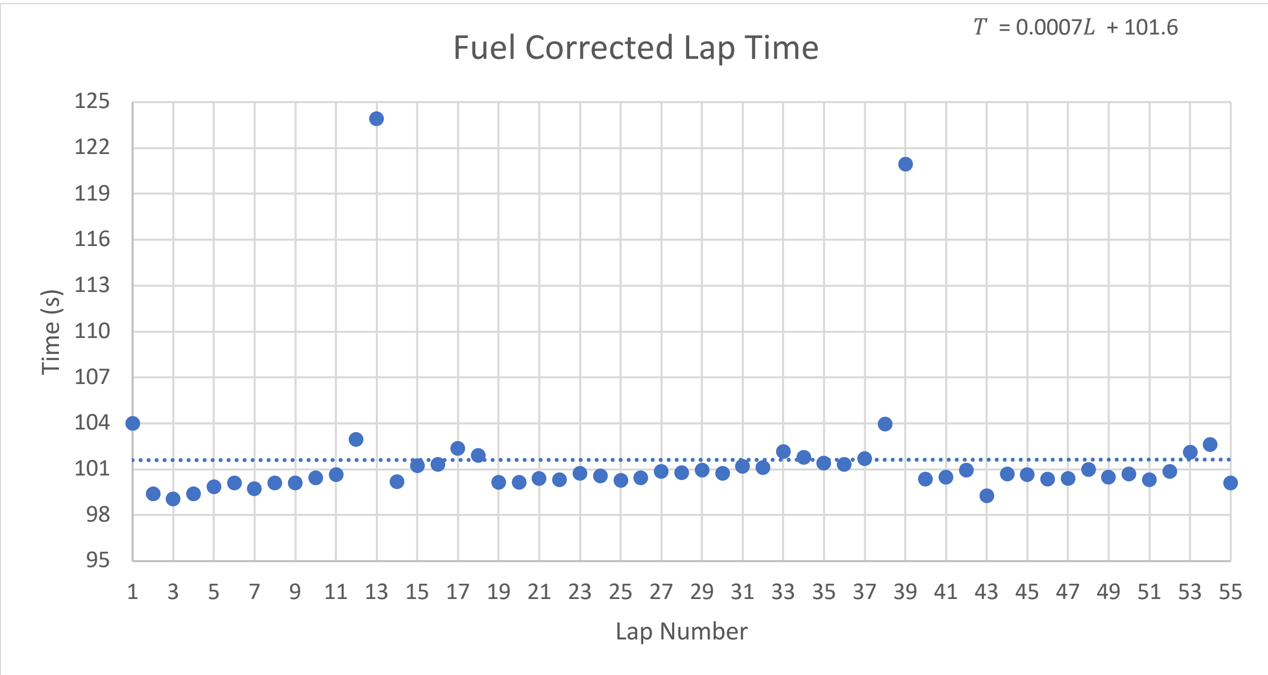
Fuel correcting the lap times

Fuel correction is where the lap times are corrected so that they show the lap times if fuel was not a variable, therefore the fuel effect is mitigated so that when analysing, only the tyre variables are being analysed.

You can fuel correct by calculating the average specific fuel effect of a certain lap, and then subtracting the original time with the average specific fuel effect. The average specific fuel effect can be calculated by taking the fuel effect per lap (in this scenario it is 0.08 seconds, which was how much faster a car goes each lap) and multiplying it by the average fuel left in the tank in terms of laps.

Sample table (of first 5 laps) for fuel correction

Lap	Original time (seconds)	Average fuel left in the tank in terms of laps	Average specific Fuel effect (seconds)	Lap	Fuel corrected lap time (seconds)
1	108.361	54.5	4.36	1	104.001
2	103.643	53.5	4.28	2	99.363
3	103.234	52.5	4.2	3	99.034
4	103.513	51.5	4.12	4	99.393
5	103.888	50.5	4.04	5	99.848



This graph shows that the fuel effect of each lap has been significantly mitigated, as the line of best fit has a gradient of approximately zero. Now we can separate these fuel corrected lap times into 3 stints (a stint is the section of the race between the start of the race to the next pitstop, the section between one pitstop and another pitstop, or the section between one pitstop to the end of the race. In one stint, only one set of tyres is used.), to see the tyre degradation of each tyre compound.

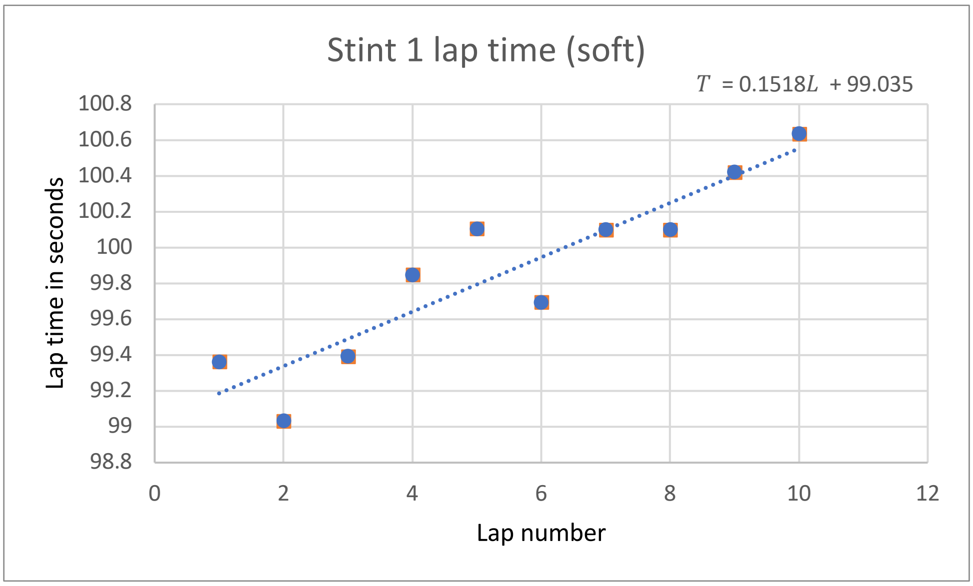
Modelling each tyre compound’s degradation

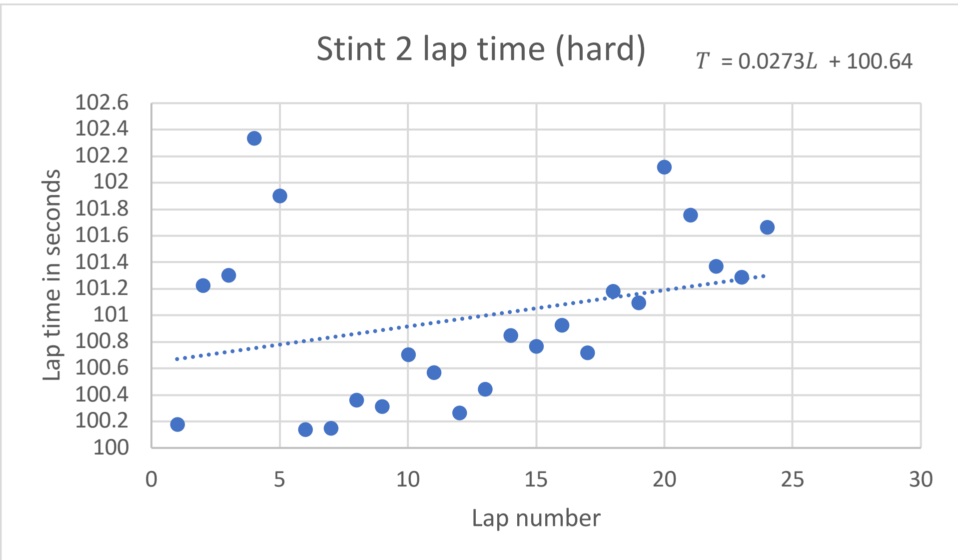
To model the degradation of each tyre compound, it first needs to be set what types of models are going to be created. This means that it needs to be decided what will the lines of best fit for the models look like. For this, the linear and the quadratic model are going to be used as these two are the most logical choice for displaying tyre degradation. That is, either the tyre wears out at a constant rate, or it wears out more and more as time progresses. For example, using a cubic model wouldn’t make sense because the tyres wouldn’t start to degrade fast, slow-down in degradation and then speed up again, as this doesn’t follow the logic and the theory behind tyre degradation.

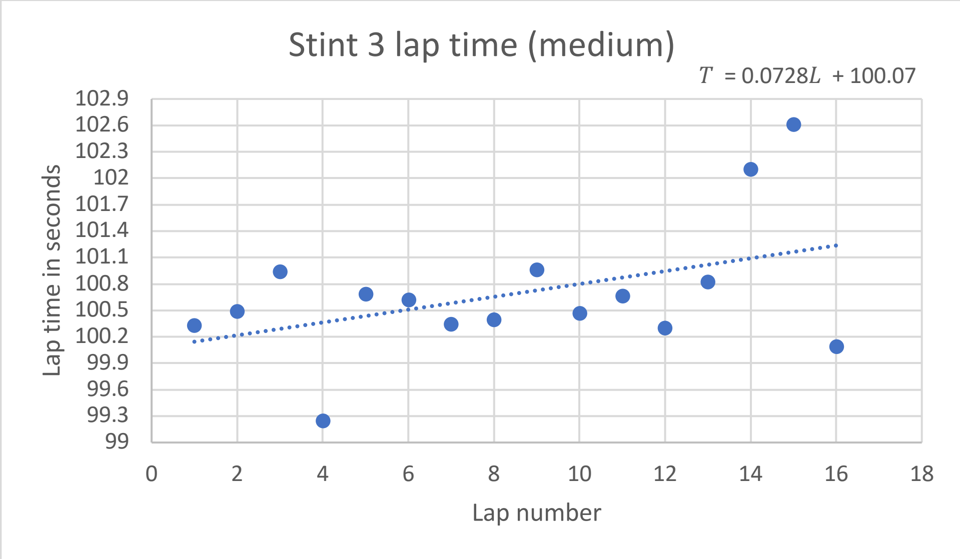
The second thing to consider is that we have to remove all outliers such as the first lap, as the cars are significantly slower at that time (due to them starting the race from a standstill, and there being other cars around them which means they cannot drive as fast as they possibly could). The other outliers which have to be removed are the pitstop laps and the lap before the pitstops. The pitstop laps have a significant increase due to the car having to stop and change tyres, and then being limited to 100 km/h when in the pit lane. The lap before the pitstop is also considered an outlier as the drivers have to slow down an incredible amount before they actually enter the pitlane (due to the speed limit) and hence that would be an outlier as well. Another outlier would be any laps in which the driver was under a safety car (a car which comes onto the track, if there is an accident on the track, to limit how fast the race cars move so that track marshals can clean up the crash debris and make sure the track is safe to race on), if there was a blue flag (when car ‘A’ laps car ‘B’ and needs to get in front of car ‘B’ so that car ‘A’s race is not affected) or there was an unusual amount of cars near each other (which slows down all the cars involved).

Linear model graphs (lap times)

The following graphs are the tyre models for different compounds of tyre, and this data has been taken from the fuel corrected lap times sheet. All of the following tyre models have a linear line of best fit, which implies that the tyre degrades in a linear fashion. In Stint 1, the 'soft' tyre compound was being used. In Stint 2, the 'hard' tyre compound was being used. In Stint 3, the 'medium' tyre compound was being used.

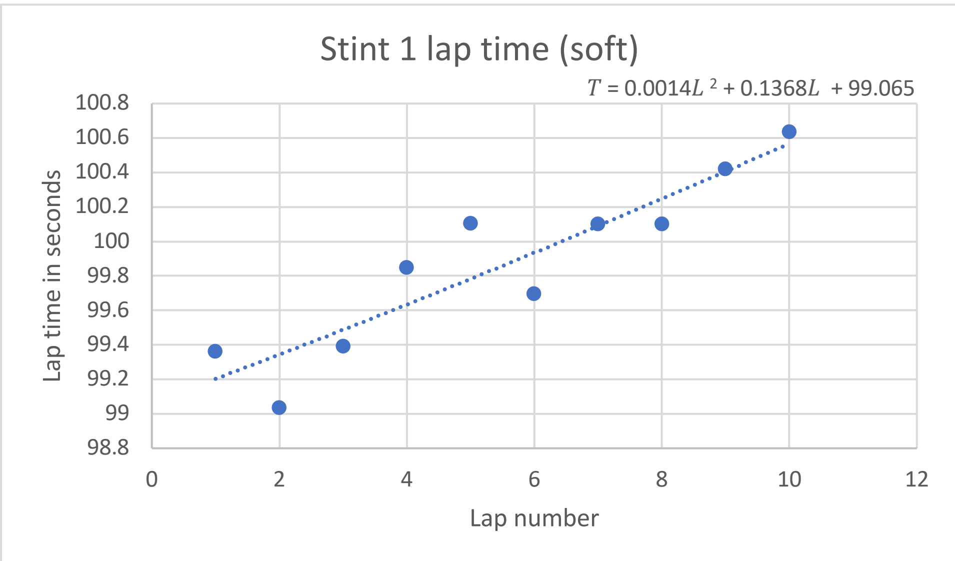


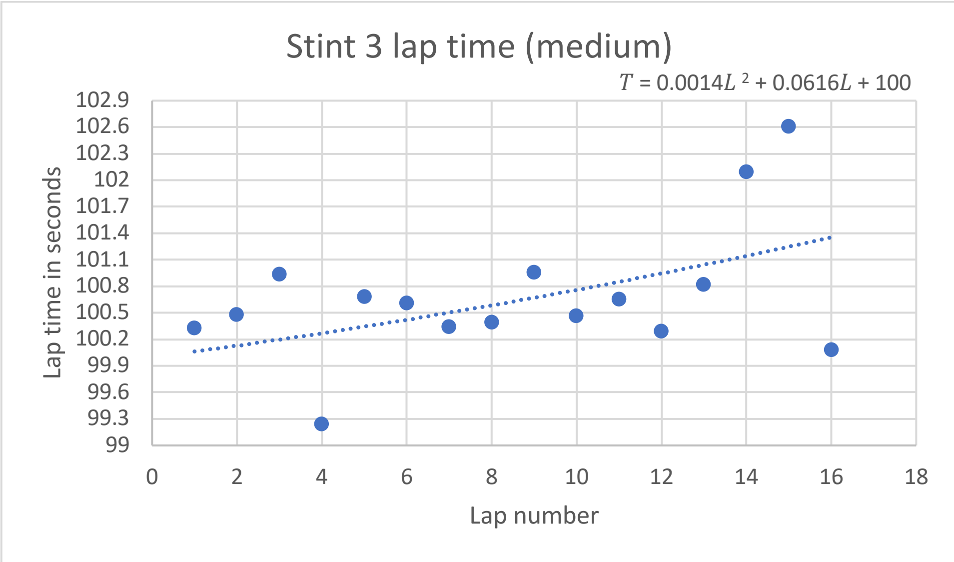


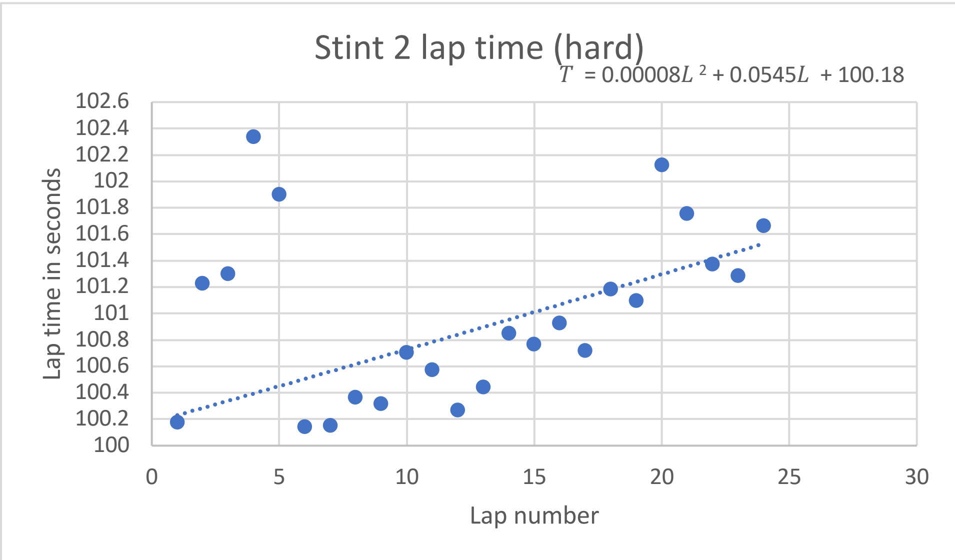


Quadratic model graphs (lap times)

The following graphs are the tyre models for different compounds of tyre, and this data has been taken from the fuel corrected lap times sheet. All of the following tyre models have a quadratic line of best fit, which implies that the tyre degradation rate increases as time passes. In Stint 1, the ‘soft’ tyre compound was being used. In Stint 2, the ‘hard’ tyre compound was being used. In Stint 3, the ‘medium’ tyre compound was being used.







Forming an equation for total race time using the models

By finding the definite integrals of all the equations for the line of best fit for all stints, with the lower bound being 1 and the higher bound being the laps spent on the specific tyre compound (l_H being the laps completed on the hard tyre, l_M being the laps completed on the medium tyre, and l_S being the laps completed on the soft tyre), and adding in the number of pitstops in the race, we can get the total race time T_{total}. For the purpose of this investigation, the number of pitstops ‘p’ will be set to a constant value of 2. This was done so as to minimize the total number of changing variables at play, and that usually, especially in the Abu Dhabi Grand Prix, teams mostly run two pitstops for a driver. Not only that, but the driver which this model is based off of, only took part in 2 pit stops during the Abu Dhabi Grand Prix. This means that only the best lap to pit will be determined, and not the number of pitstops. This ensures not only the simplicity of the investigation, but also allows a direct comparison to be made to Sebastian Vettel's tyre strategy.

Linear model (total race time)

$$T_{total} = \int_1^{l_H} (0.0273L + 100.64)\, dL + \int_1^{l_M} (0.0728L + 100.07) dL + \int_1^{l_S} 0.1518L + 99.035\, dL + 22p$$

Quadratic model (total race time)

$$T_{total} = \int_1^{l_H} (8 \times 10^{-5} L^2 + 0.0545L + 100.1)\, dL + \int_1^{l_M} (0.0014L^2 + 0.0616L + 100) dL + \int_1^{l_S} (0.0014L^2 + 0.1368L + 99.065)\, dL + 22p$$

Testing the models

Substituting in the values for l_H, l_M, and l_S with the values from the actual race data ($l_H = 26,\ l_M = 17$ and $l_S = 12$) :

Linear model

$$T_{total} = \int_1^{26} (0.0273L + 100.64)\, dL + \int_1^{17} (0.0728L + 100.07) dL + \int_1^{12} 0.1518L + 99.035\, dL + 22(2) \quad = 5281.05565\ seconds$$

Approximately 6% discrepancy between modelled time and actual time of 5589.072 seconds (total race time from fuel corrected data).

Quadratic model

$$T_{total} = \int_1^{26} (8 \times 10^{-5} L^2 + 0.0545L + 100.1)\, dL + \int_1^{17} (0.0014L^2 + 0.0616L + 100) dL + \int_1^{12} (0.0014L^2 + 0.1368L + 99.065)\, dL + 22(2) \quad = 5276.82722\ seconds$$

The quadratic model also has approximately 6% discrepancy between modelled time and actual time of 5589.072 seconds (total race time from fuel corrected data).

Choosing the best model

Due to the linear model being closer to the actual time of the total race time, the linear model will be implemented. The linear model is also much simpler to utilize and interpret, as there is a fixed rate of change. This can allow for a better analysis later in this investigation. Also, in addition to this, the linear model has been shown to be used much more by other strategists to showcase tyre wear profiles due to the uniform and homogenous nature of the Pirelli tyres that are currently being used in all races. This shows that the linear model is not only superior in that it is more closer to the actual value, but it is also more reliable and can easily provide more understanding into the topic.

Finding the optimal number of laps for each tyre

Now that the linear model is chosen, optimal number of laps for each tyre has to be determined. This can be done by first getting all the definite integrals present in the linear model, and then minimizing the total race time by using multivariable calculus.

Finding the definite integral of the linear tyre model (by using the integration rule $\int_p^q x^n dx = \left[\frac{x^{n+1}}{n+1}\right]_p^q$)

$$T_{total} = \left[\frac{0.0273L^2}{2} + 100.64L\right]_1^{l_H} + \left[\frac{0.0728L^2}{2} + 100.07L\right]_1^{l_M} + \left[\frac{0.1518L^2}{2} + 99.035L\right]_1^{l_S} + 22p$$

Substituting upper and lower bounds:

$$T_{total} = \frac{0.0273{l_H}^2}{2} + 100.64l_H - (\frac{0.0273}{2} + 100.64) + \frac{0.0728{l_M}^2}{2} + 100.07l_M - (\frac{0.0728}{2} + 100.07) + \frac{0.1518{l_S}^2}{2} + 99.035l_S - (\frac{0.1518}{2} + 99.035) + 22p$$

$$T_{total} = \frac{0.0273{l_H}^2}{2} + 100.64l_H + \frac{0.0728{l_M}^2}{2} + 100.07l_M + \frac{0.1518{l_S}^2}{2} + 99.035l_S - 343.871$$

Before we start minimizing the total race time, certain constraints have to be put in place. The main constraint is that all of the laps for each tyre compound should add up to 55. This is because that there 55 laps in the Abu Dhabi Grand prix, and that the sum of all tyres should give the number of laps for the whole race.

Hence the constraint is:

$$l_H + l_M + l_S = 55$$

Now, we can utilize multivariable calculus techniques to find the optimal laps for each tyre compound. However, due to the constraint present, we have to include in that in our equation. This can be done by substituting l_S with $55 - l_M - l_H$.

Substituting l_S with $55 - l_M - l_H$

$$T_{total} = \frac{0.0273{l_H}^2}{2} + 100.64l_H + \frac{0.0728{l_M}^2}{2} + 100.07l_M + \frac{0.1518(55 - l_M - l_H)^2}{2} + 99.035(55 - l_M - l_H) - 343.871$$

Simplifying the expression

$$T_{total} = 0.08955{l_H}^2 + 0.1518l_Hl_M - 6.744l_H + 0.1123{l_M}^2 - 7.314l_M + 6020.3955$$

Using this equation, we can now find partial derivatives of T_{total} with respect to l_H and l_M. This would be done just like finding a normal derivative of a function, except when differentiating T_{total} with respect to (for example) l_H, then any term without l_H would be considered a constant. For example, if we have $z = x^2 + xy + y^2$, $then\ \frac{\partial z}{\partial x} = 2x + x, and\ \frac{\partial z}{\partial y} = y + 2y$. In this situation, we are differentiating z with respect to x (which gives $\frac{\partial z}{\partial x}$) and with respect to y (which gives $\frac{\partial z}{\partial y}$).

$$\frac{\partial T_{total}}{\partial l_H} = 0.1791l_H + 0.1518l_M - 6.744$$

$$\frac{\partial T_{total}}{\partial l_M} = 0.1518l_H + 0.2246l_M - 7.314$$

Now that the partial derivatives are found, we can equate them both to zero and make l_H the subject

$$\frac{\partial T_{total}}{\partial l_H} = 0 \rightarrow l_H = \frac{-0.1518l_M + 6.744}{0.1791} = -0.8476l_M + 37.65$$

$$\frac{\partial T_{total}}{\partial l_M} = 0 \rightarrow l_H = \frac{-0.2246 l_M + 7.314}{0.1518} = -1.4796 l_M + 48.18$$

Since both equations have l_H as the subject, we can equate them both to solve for l_M

$$-0.8476 l_M + 37.65 = -1.4796 l_M + 48.18$$
$$l_M \approx 16.66 \approx 17$$

Now that we have l_M, we can substitute the value back into one of the previous equations to get the value for l_H

$$l_H = -0.8476(16.66) + 37.65$$
$$l_H \approx 23.53 \approx 23$$

Now that we have l_M and l_H, we can substitute the values back into the equation for the constraint to get the value for l_S

$$(55 - l_M - l_H) = l_S$$
$$l_S = 14.81 \approx 15$$

The value for l_H is rounded down so that the sum of all lap numbers can equal 55, which is the total laps in the race.

When $l_H = 23,\ l_M = 17$ and $l_S = 15$, T_{total} is a minimum with a value of $5280.382\ seconds$. This is less by 0.67365 seconds from the original fuel corrected modelled lap time (which used the linear model) which utilized $l_H = 26,\ l_M = 17$ and $l_S = 12$.

To make sure T_{total} has been correctly optimized, another method will be used to verify the previous answer. The method which will be used will utilize LaGrange multipliers.

Firstly, we need to take the original equation and subtract it with the product of λ (the Lagrangian multiplier) and the constraint.

$$\frac{0.0273 {l_H}^2}{2} + 100.64 l_H + \frac{0.0728 {l_M}^2}{2} + 100.07 l_M + \frac{0.1518 {l_S}^2}{2} + 99.035 l_S - 343.871 - \lambda(l_H + l_M + l_S - 55)$$

Expanding the equation.

$$\frac{0.0273{l_H}^2}{2} + 100.64l_H + \frac{0.0728{l_M}^2}{2} + 100.07l_M + \frac{0.1518{l_S}^2}{2} + 99.035l_S - 343.871 - \lambda l_H - \lambda l_M - \lambda l_S + \lambda 55)$$

Then find the partial derivates with respect to each variable ($l_H,\ l_M,\ l_S\ and\ \lambda$)

$$\frac{\partial T_{total}}{\partial l_H} = 0.0273l_H + 100.64 - \lambda$$

$$\frac{\partial T_{total}}{\partial l_M} = 0.0728l_M + 100.07 - \lambda$$

$$\frac{\partial T_{total}}{\partial l_S} = 0.1518l_M + 99.035 - \lambda$$

$$\frac{\partial T_{total}}{\partial \lambda} = -l_H - l_M - l_S + 55$$

Then equate all equations to zero and make the lap number the subject for $\frac{\partial T_{total}}{\partial l_H}, \frac{\partial T_{total}}{\partial l_M}\ and\ \frac{\partial T_{total}}{\partial l_S}$

$$\frac{\partial T_{total}}{\partial l_H} = 0.0273l_H + 100.64 - \lambda = 0 \rightarrow l_H = \frac{\lambda - 100.64}{0.0273}$$

$$\frac{\partial T_{total}}{\partial l_M} = 0.0728l_M + 100.07 - \lambda = 0 \rightarrow l_M = \frac{\lambda - 100.07}{0.0728}$$

$$\frac{\partial T_{total}}{\partial l_S} = 0.1518l_M + 99.035 - \lambda = 0 \rightarrow l_S = \frac{\lambda - 99.035}{0.1518}$$

$$\frac{\partial T_{total}}{\partial \lambda} = -l_H - l_M - l_S + 55 = 0$$

Substitute in the values for $l_H,\ l_M\ and\ \ l_S$

$$\frac{\partial T_{total}}{\partial \lambda} = -\frac{\lambda - 100.64}{0.0273} - \frac{\lambda - 100.07}{0.0728} - \frac{\lambda - 99.035}{0.1518} + 55 = 0$$

Solve for λ

$$\lambda \approx 101.28257$$

Plug the value of λ back in to the equations formed for $l_H,\ l_M\ and\ \ l_S$ to get their values

$$l_H = \frac{101.28257 - 100.64}{0.0273} \approx 23.537 \approx 23$$

$$l_M = \frac{101.28257 - 100.07}{0.0728} \approx 16.656 \approx 17$$

$$l_S = \frac{101.28257 - 99.035}{0.1518} \approx 14.806 \approx 15$$

The same values have been achieved as before, hence when $l_H = 23,\ \ l_M = 17$ and $l_S = 15$, T_{total} is certainly a minimum.

If we compare the minimum time achieved by optimizing the variables $l_H,\ l_M\ and\ \ l_S$ (which was 5280.382 seconds) and the actual time achieved by using the linear model and the values of the laps for each compound, it can be seen that there is only a 0.67365 seconds difference. In the actual race, Ferrari pitted in laps 12 and 38 achieve a fuel corrected total race time of 5589.072 seconds. Using the model created, which postulates that the tyres degrade in such a way that the total time taken to complete a lap linearly increases, the theoretical race time (fuel corrected) using the original Ferrari strategy would be 5281.05565 seconds. The approximate discrepancy taken into account, if we adjust the minimum race time value, then it can be analyzed and compared with the real value. Since we achieved a minimum race time of 5280.382 seconds, if this is multiplied by 1.05832, to get an approximate 6% increase then the potential fuel corrected race time would be 5588.33 seconds. This still doesn't seem like a lot, but it has to be taken into account that Ferrari have been in the sport the longest compared to any other team and they have the experience and skill to choose the best possible strategy. This shows that Ferrari was already trying to use the best team strategy, but was off by 7 tenths of a second. Usually, this wouldn't be such a huge difference, but in a sport like F1, where there is always a neck-to-neck battle with drivers, the 7 tenths could have been the difference between a podium position and 4^{th} place.

Conclusion

By looking at the outcome of this investigation, it can be seen that to minimize the lap times of an F1 car, you have to take into account the fuel load effect, and that you need to have previous race data of the driver on the same circuit. This is due to the fact that the driving style of each driver can be widely different and that each track requires different characteristics from a driver which can heavily influence tyre degradation. By employing many techniques and skills used in calculus, I was able to model not only the fuel load effect but also the tyre degradation rate of each compound that was used in the 2019 Abu Dhabi Grand Prix. Using the data that was acquired, the optimal strategy for Ferrari was to start with the soft tyres first, and then after 15 laps into the race they should have made a pitstop. After that, they should have put on the hard tyres for a further 23 laps, and then after that they should have pitted again. Then lastly they should have gone the remaining 17 laps with the medium tyres. There were some assumptions made regarding the number of pitstops, and the choice of tyres that the drivers could use, such as there will only be two pitstops during the whole race for one driver, and that the driver will have to use all three tyre compounds throughout the course of the race. A limitation of this investigation is that it doesn't take into account any random error that could occur during a race, such as a safety car (which basically forces the cars to go slower and follow a car so that track marshals have enough time to clean up any debris that may have occurred due to a crash). Also, another limitation is that this model is very specific and is only applicable to Sebastian Vettel in the Abu Dhabi Grand Prix. If one wanted to use this model for any other driver or track location then it wouldn't be accurate. An extension to this exploration would be to come up with driver profiles of significant drivers in the grid such as Lewis Hamilton and Max Verstappen. This would allow me to mathematically analyze and see the correlation between their tyre preservation and their overall success. By creating such driver profiles, I would then be able to use base data available for a track on the circuit, and then predict the best strategy that the team should use before the race starts, thanks to the tools provided by multivariable calculus.

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