cpsc 313 assignment 1

Ali Akbari

TOTAL POINTS

45 / 50

QUESTION 1

1 question 1 6 / 10

- **0 pts** Correct

Base Case

- **1 pts** Minor mistake in the base case

- **3 pts** No/Wrong Base Case

Inductive hypothesis

- **1 pts** Minor Mistake

✓ - **2 pts** **No/Wrong Inductive Hypothesis**

Inductive case

- **1 pts** Minor sloppiness

✓ - **2 pts** **Sloppiness in proof**

- **4 pts** No/Wrong induction case proof

- **6 pts** Not using induction

- **10 pts** Incorrect/no answer

QUESTION 2

2 question 2 9 / 10

- **0 pts** Correct

✓ - **1 pts** **Minor issue**

- **2 pts** Sloppy proof in either direction

- **4 pts** Major logical jumps/mistakes/Sloppiness in either direction

- **5 pts** Missing either L ⊆ (L□)□ or (L□)□ ⊆ L

- **6 pts** Major sloppiness/mistakes in both directions

- **8 pts** Mostly Incorrect (something is correct)

- **10 pts** Incorrect/No Answer

- **0 pts** rev

1 number 3 do not show that directly

2 you can use this

QUESTION 3

3 question 3 10 / 10

✓ - **0 pts** Correct

- **2 pts** Minor sloppiness in either proof

- **0 pts** Logical jump (since e^i = e is given in the lecture, no marks are deducted)

- **4 pts** Mostly wrong proof of either side

- **5 pts** Missing/Wrong w□ = epsilon => w = epsilon

- **5 pts** Missing/Wrong w = epsilon => w□ = epsilon

- **6 pts** Major Sloppiness and mistakes in both directions

- **8 pts** Mostly Incorrect

- **10 pts** Incorrect/No answer

QUESTION 4

4 question 4 10 / 10

✓ - **0 pts** Correct

- **1 pts** Minor Issue

- **3 pts** Not using the definition of concatenation

- **3 pts** Major issues

- **5 pts** Incomplete proof - Half proof is missing

- **10 pts** Incorrect

- **10 pts** Missing

3 examples are not a proof

QUESTION 5

5 question 5 10 / 10

✓ - **0 pts** Correct

- **2 pts** Minor mistakes

- **5 pts** Major Logical jump

- **10 pts** Incorrect

- **10 pts** Missing

💬 the key part that you need to show is that any word in the language is equal to its reverse

4 this is the key bit that you need to show

1) Prove with induction that $\forall n > 0 \ \forall w \in \{a, b\}^* : n_a(w^n) = n \cdot n_a(w)$. You may use without proof that $n_a(xy) = n_a(x) + n_a(y)$. Hint: the recursive definition of w^n is $w^0 = \varepsilon$ and $w^{n+1} = w \cdot w^n$.

Base Case:
$n = 1$
$n_a(w^1) = 1 \cdot n_a(w) =$
$n_a(w) = n_a(w)$
This holds true for the base case, now prove for $n + 1$.

Induction Step:
$n_a(w^{n+1}) = n_a(w^n) + n_a(w^1)$
$n_a(w^{n+1}) = n_a(w^n) + n_a(w) - From \ base \ case$
Let $x = n_a(w^n)$ $- we \ know \ that \ w \ is \ the \ same \ word \ concatenated \ with \ self \ n \ time \ so,$
$x = n_a(w) + n_a(w) + n_a(w) + n_a(w) + n_a(w)... \ goes \ on \ for \ n \ times$
Since the word is the same $n_a(w)$ equates to the same value thus, this can be simiplified as $= n \cdot n_a(w)$
When we plug this back to the above equation :
$n_a(w^{n+1}) = n \cdot n_a(w) + n_a(w)$
And with some simplification of dividing out common term we get :
$n_a(w^{n+1}) = n \cdot n_a(w) + n_a(w) \ ->$
$n_a(w^{n+1}) = (n + 1) \cdot n_a(w)$
Thus, this proves that $n_a(w^n) = n \cdot n_a(w)$ hold for $n + 1$.
QED.

2) Prove that for any language $L \subseteq \Sigma^*$ that $(L^r)^r = L$. You may use the fact that $(w^r)^r = w$ without proof. Hint: recall languages are sets, so this question relates to set equality.

Using the formal definition of set equality :
$\forall L \forall (L^r)^r [\forall w \ (w \in L \Leftrightarrow w \in (L^r)^r \Rightarrow L = (L^r)^r$
To prove equivalence we have to prove that each is a subset/($\subseteq$) of each other.
Let :
1) $L = \{w \ : \ w \in L\}$
2) $L^r = \{w^r \ : \ w \in L\}$
3) $(L^r)^r = \{(w^r)^r \ : \ w^r \in L^r\}$

First prove that $(L^r)^r \subseteq L$:
$L^r = \{w^r \ : \ w \in L\}$ then,
$(L^r)^r = \{(w^r)^r \ : \ w^r \in L^r\}$

1 question 1 6 / 10

- **0 pts** Correct

Base Case
- **1 pts** Minor mistake in the base case
- **3 pts** No/Wrong Base Case

Inductive hypothesis
- **1 pts** Minor Mistake
✓ - **2 pts** No/Wrong Inductive Hypothesis

Inductive case
- **1 pts** Minor sloppiness
✓ - **2 pts** Sloppiness in proof
- **4 pts** No/Wrong induction case proof

- **6 pts** Not using induction
- **10 pts** Incorrect/no answer

1) Prove with induction that $\forall n > 0 \; \forall w \in \{a, b\}^* : n_a(w^n) = n \cdot n_a(w)$. You may use without proof that $n_a(xy) = n_a(x) + n_a(y)$. Hint: the recursive definition of w^n is $w^0 = \varepsilon$ and $w^{n+1} = w \cdot w^n$.

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Induction Step:

$n_a(w^{n+1}) = n_a(w^n) + n_a(w^1)$

$n_a(w^{n+1}) = n_a(w^n) + n_a(w) - From\ base\ case$

$Let\ x = n_a(w^n) \qquad\qquad - we\ know\ that\ w\ is\ the\ same\ word\ concatenated\ with\ self\ n\ time\ so,$

$x = n_a(w) + n_a(w) + n_a(w) + n_a(w) + n_a(w)... \ goes\ on\ for\ n\ times$

Since the word is the same $n_a(w)$ equates to the same value thus, this can be simiplified as $= n \cdot n_a(w)$

When we plug this back to the above equation :

$n_a(w^{n+1}) = n \cdot n_a(w) + n_a(w)$

And with some simplification of dividing out common term we get :

$n_a(w^{n+1}) = n \cdot n_a(w) + n_a(w) \rightarrow$

$n_a(w^{n+1}) = (n+1) \cdot n_a(w)$

Thus, this proves that $n_a(w^n) = n \cdot n_a(w)$ hold for $n+1$.

QED.

2) Prove that for any language $L \subseteq \Sigma^*$ that $(L^r)^r = L$. You may use the fact that $(w^r)^r = w$ without proof. Hint: recall languages are sets, so this question relates to set equality.

Using the formal definition of set equality :

$\forall L \forall (L^r)^r [\forall w \, (w \in L \Leftrightarrow w \in (L^r)^r \Rightarrow L = (L^r)^r$

To prove equivalence we have to prove that each is a subset/($\subseteq$) of each other.

Let :

1) $L = \{w : w \in L\}$

2) $L^r = \{w^r : w \in L\}$

3) $(L^r)^r = \{(w^r)^r : w^r \in L^r\}$

First prove that $(L^r)^r \subseteq L$:

$L^r = \{w^r : w \in L\}$ *then,*

$(L^r)^r = \{(w^r)^r : w^r \in L^r\}$

and by the fact that $(w^r)^r = w$, we can use this operation for all the words in the
language $(L^r)^r = \{(w^r)^r = w, (w^r)^r = w, (w^r)^r = w, (w^r)^r = w, (w^r)^r = w \ldots\}$
This means that all elements of $\boxed{(L^r)^r = w \text{ which is } \{w : w \in L\}.}$
Since all element of $(L^r)^r$ exist within the L from number 1 this proves that $(L^r)^r \subseteq L$.

2

Now prove $L \subseteq (L^r)^r$:
$L = \{w : w \in L\}$
$L^r = \{w^r : w \in L\}$ — *Reverse once*
$(L^r)^r = \{(w^r)^r : w^r \in L^r\}$ — *Reverse twice*
We can do this because of the fact that $(w^r)^r = w$, we can use this operation for all the words in the
L, and this operation of double reversing should not change the words.
$L = \{w = (w^r)^r, w = (w^r)^r, w = (w^r)^r, w = (w^r)^r, w = (w^r)^r \ldots\}$
This means that all elements of $L = (w^r)^r$ which is $\{(w^r)^r : w^r \in L^r\}$.

1

Since all element of L exist within the $(L^r)^r$ from number 3 this proves that $L \subseteq (L^r)^r$.
By the definition of set equality, and since both are L and $(L^r)^r$ are subsets of each other,
$L \subseteq (L^r)^r$ and $(L^r)^r \subseteq L$
then $(L^r)^r = L$.

3) Prove $(\forall i \in N : w^i = \varepsilon) \Leftrightarrow w = \varepsilon$. Be sure to prove both directions of the implication.
You may use without proof the fact that $\forall w \in \Sigma^* \; \forall i \geq 0 : |w^i| = i \cdot |w|$.

Using Biconditional proof $p \Leftrightarrow q$:
$p \Rightarrow (\forall i \in N : w^i = \varepsilon)$
$q \Rightarrow w = \varepsilon$

LHS $(p \rightarrow q)$:
Based on p assume $(\forall i \in N : w^i = \varepsilon)$.
$(\forall i \in N : w^i = \varepsilon)$ by using the above fact that $|w^i| = i \cdot |w|$
No matter what value i posses $\forall i \geq 0$ which is the natural numbers, and no
matter what the word is, the cardinality is, $\forall w \in \Sigma^* : |w^i| = i \cdot |w|$. In this case the word is,
based on q, $w = \varepsilon$, $|\varepsilon^i| = i \cdot |\varepsilon| \rightarrow$
$|\varepsilon \cdot \varepsilon \cdot \varepsilon \cdot \varepsilon \ldots (i \text{ times})| = i \cdot |\varepsilon|$
since $|\varepsilon|$ is $= 0 \rightarrow$
$0 = i \cdot 0 = 0$
This means that no matter what value of i is, if the word is of size 0 which is
epsilon, ε. Then the length will be 0 for any i (multiple concatenation of epsilon $=$ epsilon).
$w^i = w = \varepsilon$.
Thus $w = \varepsilon$, as p implies q $(p \rightarrow q)$.

2 question 2 **9 / 10**

- **0 pts** Correct

✓ **- 1 pts** Minor issue

- **2 pts** Sloppy proof in either direction

- **4 pts** Major logical jumps/mistakes/Sloppiness in either direction

- **5 pts** Missing either L ⬚ (L⬚)⬚ or (L⬚)⬚ ⬚ L

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1 number 3 do not show that directly

2 you can use this

and by the fact that $(w^r)^r = w$, we can use this operation for all the words in the
language $(L^r)^r = \{(w^r)^r = w, (w^r)^r = w, (w^r)^r = w, (w^r)^r = w, (w^r)^r = w ....\}$
This means that all elements of $(L^r)^r = w$ which is $\{w : w \in L\}$.
Since all element of $(L^r)^r$ exist within the L from number 1 this proves that $(L^r)^r \subseteq L$.

Now prove $L \subseteq (L^r)^r$:

$L = \{w : w \in L\}$
$L^r = \{w^r : w \in L\}$ *− Reverse once*
$(L^r)^r = \{(w^r)^r : w^r \in L^r\}$ *− Reverse twice*
We can do this because of the fact that $(w^r)^r = w$, we can use this operation for all the words in the
L, and this operation of double reversing should not change the words.
$L = \{w = (w^r)^r, w = (w^r)^r, w = (w^r)^r, w = (w^r)^r, w = (w^r)^r .....\}$
This means that all elements of $L = (w^r)^r$ which is $\{(w^r)^r : w^r \in L^r\}$.
Since all element of L exist within the $(L^r)^r$ from number 1 this proves that $L \subseteq (L^r)^r$.
By the definition of set equality, and since both are L and $(L^r)^r$ are subsets of each other,
$L \subseteq (L^r)^r$ and $(L^r)^r \subseteq L$
then $(L^r)^r = L$.

3) Prove $(\forall i \in N : w^i = \varepsilon) \Leftrightarrow w = \varepsilon$. Be sure to prove both directions of the implication. You may use without proof the fact that $\forall w \in \Sigma^* \; \forall i \geq 0 : |w^i| = i \cdot |w|$.

Using Biconditional proof $p \Leftrightarrow q$:
$p \Rightarrow (\forall i \in N : w^i = \varepsilon)$
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LHS $(p \rightarrow q)$:
Based on p assume $(\forall i \in N : w^i = \varepsilon)$.
$(\forall i \in N : w^i = \varepsilon)$ by using the above fact that $|w^i| = i \cdot |w|$
No matter what value i posses $\forall i \geq 0$ which is the natural numbers, and no
matter what the word is, the cardinality is, $\forall w \in \Sigma^* : |w^i| = i \cdot |w|$. In this case the word is,
based on q, $w = \varepsilon$, $|\varepsilon^i| = i \cdot |\varepsilon| \rightarrow$
$|\varepsilon \cdot \varepsilon \cdot \varepsilon \cdot \varepsilon.... (i \; times)| = i \cdot |\varepsilon|$
since $|\varepsilon|$ is $= 0 \rightarrow$
$0 = i \cdot 0 = 0$
This means that no matter what value of i is, if the word is of size 0 which is
epsilon, ε. Then the length will be 0 for any i (multiple concatenation of epsilon $=$ epsilon).
$w^i = w = \varepsilon$.
Thus $w = \varepsilon$, as p implies q ($p \rightarrow q$).

This can be proved by contradiction as well :

Suppose $(\forall i \in N : w^i ! = \varepsilon) \Leftrightarrow w = \varepsilon$

Let $i = 2$, *then :*

$(\forall i \in N : w^i ! = \varepsilon) \Leftrightarrow w = \varepsilon$, $w = \varepsilon$ *based on q and if we raise the word to the second power*
$w^2 = \varepsilon \cdot \varepsilon = \varepsilon$

This poses a contradiction on the above assumption $w^i ! = \varepsilon \Leftrightarrow w = \varepsilon$. *As* w^2 *does* $= \varepsilon$.

Thus, $(\forall i \in N : w^i = \varepsilon) \Leftrightarrow w = \varepsilon$.

RHS $(q \rightarrow p)$:

Based on q assume $w = \varepsilon$

Suppose we raise the word w to an arbitrary i, $\forall i \in N$.

$w^i = \varepsilon \cdot \varepsilon \cdot \varepsilon \cdot \varepsilon.... i$ *times.*

By using the fact that $\forall w \in \Sigma^* \ \forall i \geq 0 : |w^i| = i \cdot |w|$,

we get that $|\varepsilon^i| = i \cdot |\varepsilon| \rightarrow |\varepsilon \cdot \varepsilon \cdot \varepsilon \cdot \varepsilon....(i \ times)| = i \cdot |\varepsilon|$, *since length of empty string is* 0,

and using the fact that $\varepsilon \cdot \varepsilon \cdot \varepsilon \cdot \varepsilon....(i \ times) = \varepsilon$. *The cardinality equates to* $0 = i \cdot 0 = 0$.

This means that no matter what value of i is, if the word is of size 0 *which is*

epsilon, ε. *Then the length will be* 0 *for any i (multiple concatenation of epsilon* $=$ *epsilon).*

$w = w^i = \varepsilon$.

Thus $(\forall i \in N : w^i = \varepsilon)$, *as q implies p* $(q \rightarrow p)$.

This can be proved by contradiction as well :

Suppose $(\forall i \in N : w^i = \varepsilon) \Leftrightarrow w! = \varepsilon$

Let $i = 3$, *then :*

$(\forall i \in N : w^i = \varepsilon) \Leftrightarrow w ! = \varepsilon$, *by using the fact that* $\forall w \in \Sigma^* \ \forall i \geq 0 : |w^i| = i \cdot |w|$, *and that*
the length will be 0 *for any i (multiple concatenation of epsilon* $=$ *epsilon),* $w^3 = \varepsilon \cdot \varepsilon \cdot \varepsilon = \varepsilon$
$w = \varepsilon$, *even if the word is raised to the third power*

This poses a contradiction on the above assumption $w^i = \varepsilon \Leftrightarrow w ! = \varepsilon$. *As* w^3 *does* $= \varepsilon$.

Thus, $(\forall i \in N : w^i = \varepsilon) \Leftrightarrow w = \varepsilon$.

Since LHS $=$ *RHS, and by the definition of biconditional proof, we see that either*
p & q are true or false.

QED.

3 question 3 10 / 10

✓ **- 0 pts** Correct

    - 2 pts Minor sloppiness in either proof

    - 0 pts Logical jump (since e^i = e is given in the lecture, no marks are deducted)

    - 4 pts Mostly wrong proof of either side

    - 5 pts Missing/Wrong w▯ = epsilon => w = epsilon

    - 5 pts Missing/Wrong w = epsilon => w▯ = epsilon

    - 6 pts Major Sloppiness and mistakes in both directions

    - 8 pts Mostly Incorrect

    - 10 pts Incorrect/No answer

ılı gradescope

4) Let L, L' languages over Σ^* with $0 < |L| \leq |L'| < n$ for some $n \in N$. Prove formally using the definition of concatenation that $|L'| \leq |L \cdot L'| \leq |L| \cdot |L'|$. You may use the following lemma without proof: if w, x, y are words and $x \neq y$ then $wx \neq wy$.

Definition of concatention of $L \cdot L' = \{xy \mid x \varepsilon L \; ; \; y \varepsilon L'\}$

Prove the lower bound that $|L'| \leq |L \cdot L'|$:
Let L, L' be languages over Σ^* with $0 < |L| \leq |L'| < n$

There are two cases :
Case 1 : L consist of one element $|L'| = |L \cdot L'|$.
$|L| = 1$
The maximum number of words after concatenation of L and L' is
$|L \cdot L'|$, since L has one word x and it is concatenated once to every word y of L' ,
this makes a new word in each instance xy. From the definition of concatenation.
Since a word is made for every word in L', the number of new words is equal to the number
element in L'. Thus $|L \cdot L'| = |L'|$.
For example :
$L = \{x\}$
$L' = \{y, z\}$
$|L \cdot L'| = |\{xy, xz\}| = 2$ and $|L'| = |\{y, z\}| = 2$
$|L'| = |L \cdot L'|$

Case 2 : L contains multiple elements $|L'| < |L \cdot L'|$.
$|L| = n$
Intuitively if the cardinality of $|L'| = |L \cdot L'|$ when $|L| = 1$ from the above case then with multiple
words in L the cardinality of $|L'| < |L \cdot L'|$. From the definition of concatenation we see that a
word x in L and a word y in L' make a unique word for every word in L, thus the cardinality is $|L \cdot L'|$
and that has to be bigger as the cardinality of $|L'|$
For example :
$L = \{y, yx\}$
$L' = \{y, xy\}$ ③
$|L \cdot L'| = |\{yy, yxy, yxxy\}| = 3$ and $|L'| = |\{y, yx\}| = 2 \implies |L'| < |L \cdot L'|$

Prove the upper bound that $|L \cdot L'| \leq |L| \cdot |L'|$:
Let L, L' be languages over Σ^* with $0 < |L| \leq |L'| < n$

There are two cases :

Case 1 : No words are repeated and $|L \cdot L'| = |L| \cdot |L'|$.

The maximum number of words with no repeats after concatenation of L and L' is
$|L| \cdot |L'|$, *since for every word x of L, it is concatenated once to every word y of L' when the two languages are concatenated. This makes a new word in each instance xy. From the definition of concatenation. Thus* $|L \cdot L'| = |L| \cdot |L'|$, *for example :*

$L = \{x, w\}$
$L' = \{y, z\}$
$|L \cdot L'| = |\{xy, xz, wy, wz\}| = 4 \text{ and } |L| \cdot |L'| = |\{x, w\}| \cdot |\{y, z\}| = 2 \cdot 2 = 4$

Case 2 : Words are repeated and $|L \cdot L'| < |L| \cdot |L'|$.

Intuitively if the maximum words of $|L \cdot L'|$ *is* $|L| \cdot |L'|$ *from the above case then with repeated word it would be that those word(s) can cancel out and the length of* $|L \cdot L'|$ *will become shorter.*

So, $|L \cdot L'| < |L| \cdot |L'|$

For example :

$L = \{y, yx\}$
$L' = \{y, xy\}$
$|L \cdot L'| = |\{yy, yxy, yxxy\}| = 3 \text{ and } |L| \cdot |L'| = |\{y, yx\}| \cdot |\{y, xy\}| = 2 \cdot 2 = 4$

5) Prove that $\forall L \subseteq \{0\}^* \Rightarrow L = L^r$. You may use without prove the fact that $|w| = |w^r|$.

Intuitively :

Assume w is an arbituary element from the unary set $\{0\}^*$, *then* $w = 0^i$, $\forall i \, \varepsilon \, N$.
And from the given fact that w^r *must equal the same length/ cardinality as w, and since it is a unary set and the form of the word and length of the word is the same reversed, then it must mean that for the languages containing those words, it is also the same in reverse. Since w & w^r, are element of L L is a Palindrome language, which mean* $L = L^r$.

First using Induction I will prove that $\forall w, x$ *in L where* $\forall L \subseteq \{0\}^*$ *if* $|w| = |x|$ *then* $w = x$, *where* $n = n_0(w)$, *as lemma.*

Base Case (n = 1):

Let $n = n_0(w)$, *the number unary unit in the word.*
Choose $n = 1$, *so* $w = 0$ *then,*
$|w| = |x| \Rightarrow |0| = |x| \Rightarrow 1 = |x|$, *length of x must equal 1, therefore*
$x = 0$, $w = x$, *w & x in unary* $= 0$.
Thus this holds true for the base case.

Induction Step(n + 1):

Suppose one more unary unit is added so $n + 1 = n_0(w)$,
$|w| = |x| \Rightarrow n + 1 = |x| \Rightarrow n + 1 = |x|$, *length of x must equal n + 1, therefore*

4 question 4 **10 / 10**

   ✓ **- 0 pts** Correct

     - 1 pts Minor Issue

     - 3 pts Not using the definition of concatination

     - 3 pts Major issues

     - 5 pts Incomplete proof - Half proof is missing

     - 10 pts Incorrect

     - 10 pts Missing

   3 examples are not a proof

gradescope

There are two cases :

Case 1 : No words are repeated and $|L \cdot L'| = |L| \cdot |L'|$.
The maximum number of words with no repeats after concatenation of L and L' is
$|L| \cdot |L'|$, since for every word x of L, it is concatenated once to every word y of L' when
the two languages are concatenated. This makes a new word in each instance xy. From the
definition of concatenation. Thus $|L \cdot L'| = |L| \cdot |L'|$, for example :
$L = \{x, w\}$
$L' = \{y, z\}$
$|L \cdot L'| = |\{xy, xz, wy, wz\}| = 4$ and $|L| \cdot |L'| = |\{x, w\}| \cdot |\{y, z\}| = 2 \cdot 2 = 4$

Case 2 : Words are repeated and $|L \cdot L'| < |L| \cdot |L'|$.
Intuitively if the maximum words of $|L \cdot L'|$ is $|L| \cdot |L'|$ from the above case then with repeated word
it would be that those word(s) can cancel out and the length of $|L \cdot L'|$ will become shorter.
So, $|L \cdot L'| < |L| \cdot |L'|$
For example :
$L = \{y, yx\}$
$L' = \{y, xy\}$
$|L \cdot L'| = |\{yy, yxy, yxxy\}| = 3$ and $|L| \cdot |L'| = |\{y, yx\}| \cdot |\{y, xy\}| = 2 \cdot 2 = 4$

5) Prove that $\forall L \subseteq \{0\}^* \Rightarrow L = L^r$. You may use without prove the fact that $|w| = |w^r|$.

Intuitively :
Assume w is an arbituary element from the unary set $\{0\}^*$, then $w = 0^i$, $\forall i \in N$.
And from the given fact that w^r must equal the same length/ cardinality as w, and since it is a unary
set and the form of the word and length of the word is the same reversed, then it must mean that for
the languages containing those words, it is also the same in reverse. Since w & w^r, are element of L
L is a Palindrome language, which mean $L = L^r$.

First using Induction I will prove that $\forall w, x$ in L where $\forall L \subseteq \{0\}^*$ if $|w| = |x|$ then $w = x$,
where $n = n_0(w)$, as lemma.
Base Case (n = 1):
Let $n = n_0(w)$, the number unary unit in the word.
Choose $n = 1$, so $w = 0$ then,
$|w| = |x| \Rightarrow |0| = |x| \Rightarrow 1 = |x|$, length of x must equal 1, therefore
$x = 0$, $w = x$, w & x in unary $= 0$.
Thus this holds true for the base case.

Induction Step(n + 1):
Suppose one more unary unit is added so $n + 1 = n_0(w)$,
$|w| = |x| \Rightarrow n + 1 = |x| \Rightarrow n + 1 = |x|$, length of x must equal $n + 1$, therefore

$x = n + 1,\ w = x,\ w\ \&\ x\ in\ unary = n + 1\ unary\ units.$

This means that if two words of the unary set $\{0\}^*$ have the same length, they must be the same word.

Thus this holds true for the induction step.

Then by the given fact above for the any set in this case $\{0\}^*$, we know that the length of $|w| = |w^r|$, then w must equal w^r based on the above lemma.

$\forall L \subseteq \{0\}^* \Rightarrow L = L^r$ *can be proven as set equality.*

To prove equivalence we have to prove that each is a subset/($\subseteq$) of each other.

1) $L = \{w\ :\ w \in L\,\}$

2) $L^r = \{w^r\ :\ w \in L\,\}$

First prove that $L \subseteq L^r$:

$L = \{w\ :\ w \in L\,\}$

By the cardinality $|w| = |w^r|$, and by our lemma $w = w^r$ for all words in $\forall L \subseteq \{0\}^*$.

And since all w^r words are elements of L^r from number 2, it must be that $L \subseteq L^r$.

Now prove $L^r \subseteq L$:

$L^r = \{w^r\ :\ w \in L\,\}$

By the cardinality $|w^r| = |w|$, and by our lemma $w^r = w$ for all words in $\forall L^r \subseteq \{0\}^*$.

And since all w words are elements of L, it must be that $L^r \subseteq L$.

By the definition of set equality, and since both are L and L^r are subsets of each other,

$L \subseteq L^r\ and\ L \subseteq L^r$

then $L = L^r$.

5 question 5 **10 / 10**

✓ **- 0 pts** **Correct**

   - 2 pts Minor mistakes

   - 5 pts Major Logical jump

   - 10 pts Incorrect

   - 10 pts Missing

  💬 the key part that you need to show is that any word in the language is equal to its reverse

4 this is the key bit that you need to show

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