Assignment 2

1. Suppose $M = (Q, \Sigma, \delta, q_0, A)$ with extended transition function $\delta^\star : Q \times \Sigma^\star \to Q$. Prove that for any $q \in Q$ and $x, y \in \Sigma^\star$ that $\delta^\star(q, xy) = \delta^\star(\delta^\star(q, x), y)$. Hint: prove by induction on $|y|$.

2. Prove by induction on n that if L is a language and R is a regular expression such that $L = L(R)$ then there exists a regular expression R_n such that $L(R_n) = L^n$. Be sure to use the fact that if R_1 and R_2 are regular expressions then $L(R_1R_2) = L(R_1) \cdot L(R_2)$.

3. Every finite-sized language L has a regular expression that accepts it. Informally, the expression for $\{w, x, y\}$ is $w + x + y$. Prove this is true for all finite-sized langauges using induction on the number of words in L. You may need to use these definitions in your proof: (i) if w is a word then w is also a regular expression with $L(w) = \{w\}$, (ii) if R_1 and R_2 are regular expressions then $R_1 + R_2$ is a regular expression, and (iii) $L(R_1 + R_2) = L(R_1) \cup L(R_2)$.

4. Give a generic construction for a finite state machine that accepts a non-empty finite-size language $L = \{w_1, \ldots w_n\}$. Define all parts of the machine. (You do not need to prove it accepts the langauge). Hint: do this for a simple language like $\{\epsilon, a, aa, ba, ab\}$ and then generalize what you did so it can work for any finite language. Hint: if you make $Q \subseteq \Sigma^\star$ you can have an easy to describe construction.

5. Suppose L is a regular language, and $M = (Q, \Sigma, \delta, q_0, A)$ is a deterministic finite state machine such that $L(M) = L$. Prove that if $|Q| = 2$ then at least one of the following hold: (i) $L = \emptyset$ (ii) $\epsilon \in L$, or (iii) $\exists a \in \Sigma$ such that $a \in L$. Note that different M with $|Q| = 2$ may have a different property that holds. Hint: do a case analysis for configurations of A.