Assignment 1

1. (10 marks) Prove with induction that $\forall n > 0\ \forall w \in \{a,b\}^\star : n_a(w^n) = n \cdot n_a(w)$. You may use without proof that $n_a(xy) = n_a(x) + n_a(y)$. Hint: the recursive definition of w^n is $w^0 = \epsilon$ and $w^{n+1} = w \cdot w^n$.

2. (10 marks) Prove that for any language $L \subseteq \Sigma^\star$ that $(L^r)^r = L$. You may use the fact that $(w^r)^r = w$ without proof. Hint: recall languages are sets, so this question relates to set equality.

3. (10 marks) Prove $(\forall i \in \mathbb{N} : w^i = \epsilon) \Leftrightarrow w = \epsilon$. Be sure to prove both directions of the implication. You may use without proof the fact that $\forall w \in \Sigma^\star \forall i \geq 0 : |w^i| = i \cdot |w|$.

4. (10 marks) Let L, L' languages over $\Sigma^\star$ with $0 < |L| \leq |L'| < n$ for some $n \in \mathbb{N}$. Prove formally using the definition of concatenation that $|L'| \leq |L \cdot L'| \leq |L| \cdot |L'|$. You may use the following lemma without proof: if w, x, y are words and $x \neq y$ then $wx \neq wy$.

5. (10 marks) Prove that $\forall L \subseteq \{0\}^\star \Rightarrow L = L^r$. You may use without prove the fact that $|w| = |w^r|$.