Assignment 4

1. Prove that the following language is not regular: $\{a^i b^j a^j b^i : i \leq j\}$.
2. Prove that the following language is not regular: $\{w \in \Sigma^\star : w \neq w^r\}$.
3. Consider the following "addition checker" language $\{a^i b^j c^{i+j} : i, j \geq 0\}$ where the number of c's is equal to the sum of the number of a's and b's. Provide a grammar for that language and formally prove that the grammar accepts the language. You may use without proof the fact that the grammar $S \to 0S1|\epsilon$ generates the language $\{0^i 1^i : i \geq 0\}$.
4. (Note: this question has two parts but it will be marked as though it were two separate questions.) In class we showed that the language $\texttt{prefix}(L)$ is closed for context-free languages, meaning that if L is context-free then $\texttt{prefix}(L)$ is also context free. This does not imply, however, that if L is *not regular* then $\texttt{prefix}(L)$ is *not regular*.

   (a) Give an example non-regular context-free language L_1 and *prove* that $\texttt{prefix}(L_1)$ is *regular*.

   (b) Give an example non-regular context-free language L_2 and *prove* that $\texttt{prefix}(L_2)$ is *not regular*.

   Note that you may specify L_1 or L_2 using math notation and give a grammar *without* doing a proof that the grammar generates the language. If you use a language that we have not shown was non-regular for L_1 or L_2, however, you must prove that the language you use is non-regular (i.e., with pumping lemma, etc.).