CPSC 331 HW1

Ali Akbari

TOTAL POINTS

27 / 30

QUESTION 1

Asymptotic Notations 10 pts

1.1 Summation is Theta(n^(k+1)) 3 / 3

✓ **- 0 pts** **Correct**

- 0.5 pts minor mistakes or missing

- 1 pts conceptually correct with some incorrect portions, or partial conceptual errors

- 2 pts important conceptual mistakes, some correct portions

- 2.5 pts mostly incorrect

- 3 pts No submission

- 1.5 pts either big O or big omega is missing in case proven by definition

1.2 Transitivity of little-o 2 / 3

✓ **+ 3 pts** **Correct**

+ 2.5 pts Mostly correct

+ 1 pts the proof is not convincing. some correct parts

- 0.5 pts in case definition used: constant c missing or some mistakes

✓ **- 0.5 pts** **in case definition used: mistake or missing in N or Ni**

+ 0.5 pts attempt

+ 0 pts not submitted or completely incorrect

- 1 pts in case of using definition: the main part of proof has some missing parts or mistakes

✓ **- 0.5 pts** **a mistake in the main part of proof**

1 wrong

2 not necessarily the same N as the previous one so needs a different notation

3 ?

1.3 Polynomials are Theta(n^k) 2 / 2

✓ **+ 2 pts** **Correct**

+ 1.5 pts minor mistake or missing - proof is correct in general

+ 1 pts partially correct or either big O or big omega is correct not both

+ 0.5 pts unconvincing attempt

+ 0 pts no submission or incorrect

1.4 Function examples 2 / 2

✓ **- 0 pts** **Correct**

- 2 pts Incorrect

- 1 pts Major issue

- 0.5 pts Major-ish issue

QUESTION 2

2 Loop Complexity 8 / 10

- 1 pts Big O instead of Theta

- 3 pts No summation/loop invarient

- 3 pts Not tracing the code

- 1 pts Loose upper bound

✓ **- 4 pts** **Underestimating the upper bound**

✓ **- 0 pts** **Correct**

- 4 pts Over estimating the tight bound

- 10 pts Not submitted

+ 2 Point adjustment

good effort, a slight error in the summations caused the final answer to be wrong

QUESTION 3

Horner's Rule 10 pts

3.1 Trace execution 2 / 2

✓ **- 0 pts** **Correct**

- 0.5 pts Minor issue

- **1 pts** Major issue
- **2 pts** DNS
- **0.25 pts** Micro issue

3.2 Show loop invariant 3 / 3

✓ **- 0 pts Correct**

- **0.5 pts** little problem in maintenance part.
- **1.5 pts** incomplete answer
- **1 pts** Problem in maintenance part.
- **2 pts** incorrect responses in both parts.
- **1 pts** initialization step is not well described.
- **0.5 pts** little problem in initialization part.
- **3 pts** answer not provided.
- **2.5 pts** maintenance step is not provided and initialization is not correct.
- **2 pts** maintenance step is not provided.

3.3 Partial correctness 3 / 3

✓ **- 0 pts Correct**

- **0.25 pts** precondition is not discussed
- **0.25 pts** Post-condition is not discussed
- **0.25 pts** precondition and post condition are not shown well.
- **2 pts** The answer is not explained well.
- **1 pts** termination is not discussed correctly.
- **3 pts** Answer is not provided.
- **2.5 pts** the answer is not including post and pre condtion and termination condition.
- **1.5 pts** no mathematical proof is provided. pre-condition and post-condition is ot discussed.

3.4 Complexity 2 / 2

✓ **+ 2 pts Correct**

- **2 pts** incorrect
- **1 pts** The solution is close but not precisely correct.

CPSC 331 Assignment #1
Ali Akbari
30010402

1) Asymptotic Notations

a) $Prove\ that\ \sum_{i=1}^{n} i^k = \Theta(n^{k+1})$

$Let\ f(n) = \sum_{i=1}^{n} i^k = 1^k + 2^k + 3^k \ldots\ldots + n^k$

$Let\ g(n) = n^{k+1}$

$f(n) = \Theta(g(n))\ if\ and\ only\ if\ f(n) = O(g(n))\ and\ f(n) = \Omega(g(n)).$

$By\ the\ definition\ of\ \Theta\ we\ have\ to\ prove\ that$

$c_1 * n^{k+1} \leq \sum_{i=1}^{n} i^k \leq c_2 * n^{k+1},\ where\ c_1\ \&\ c_2\ are\ positive\ constant,\ k \geq 1,$

$n > N$

Upper Bound:

$f(n) = O\,g(n),\ if\ f(n) \leq c * g(n),\ where\ c > 0\ \&\ n > N_2.$

$f(n) = \sum_{i=1}^{n} i^k = 1^k + 2^k + 3^k \ldots\ldots + n^k$

Assume that every term is as big as the last term,

then by maximizing the terms we find an upper bound.

$so,\ has\ to\ be\ f(n) = \sum_{i=1}^{n} i^k = 1^k + 2^k + 3^k \ldots\ldots + n^k \leq n^k + n^k + n^k \ldots\ldots + n^k$

$n^k + n^k + n^k \ldots\ldots + n^k\ has\ n\ terms,\ and\ can\ be\ generalized\ to$

$n * n^k = 1 * n^{k+1}$

$so\ this\ proves\ that\ f(n) \leq g(n),$

$\sum_{i=1}^{n} i^k = 1^k + 2^k + 3^k \ldots\ldots n^k \leq 1 * n^{k+1}$

$where\ c = 1$

$Thus,\ f(n) = O\,g(n)$

Lower Bound:

$$f(n) = \sum_{i=1}^{n} i^k = 1^k + 2^k + 3^k \,..... + (\tfrac{n}{2})^k + ... n^k$$

$$g(n) = n^{k+1}$$

$$f(n) = \Omega\, g(n), \ \ if\, f(n) \geq c * g(n), \ where\ c > 0\ \&\ n > N_1.$$

Since we know that at the halfway point it is $(\frac{n}{2})^k$ *and has a complexity of* n^k, *by taking the right half of* $f(n)$ *we are left with a truncated series :*

$$f(n) = \sum_{i=1}^{n} i^k \geq (\tfrac{n}{2})^k + (\tfrac{n}{2} + 1)^k + (\tfrac{n}{2} + 2)^k \,..... + n^k$$

Assume that every term is as small as the first term,
then by decreasing the terms we find a lower bound.

$$f(n) \geq (\tfrac{n}{2})^k + (\tfrac{n}{2})^k + (\tfrac{n}{2})^k \,..... + n^k$$

This has $\frac{n}{2}$ *terms (half of* $f(n)$*), and can be generalized to*

$$\frac{n}{2} * (\frac{n}{2})^k = (\frac{n^{k+1}}{2^{k+1}})$$

the $\frac{1}{2^{k+1}}$ *is just a constant so, it is the same as* $c * n^{k+1}$

so this proves that $f(n) \geq g(n)$,

$$\sum_{i=1}^{n} i^k = 1^k + 2^k + 3^k \,..... n^k \geq \frac{1}{2^{k+1}} * n^{k+1}$$

where $c = \frac{1}{2^{k+1}}$

$$f(n) = \Omega\, g(n)$$

Using both upper and lower bounds, it proves that $f(n) = O\, g(n)$ *and* $f(n) = \Omega\, g(n)$

$$\frac{1}{2^{k+1}} * n^{k+1} \leq \sum_{i=1}^{n} i^k \leq 1 * n^{k+1}, \ c_1 * g(n) \leq f(n) \leq c_2 * g(n)$$

For all $n > max(N1,\ N2)$ *where,* $c_1 = \frac{1}{2^{k+1}}, \ c_2 = 1$

$$\Rightarrow f(n) = \Theta(g(n))$$

1.1 Summation is Theta(n^(k+1)) **3 / 3**

✓ **- 0 pts** **Correct**

- **- 0.5 pts** minor mistakes or missing
- **- 1 pts** conceptually correct with some incorrect portions, or partial conceptual errors
- **- 2 pts** important conceptual mistakes, some correct portions
- **- 2.5 pts** mostly incorrect
- **- 3 pts** No submission
- **- 1.5 pts** either big O or big omega is missing in case proven by definition

gradescope

b) *Little o Transitive Proof*

Prove that little $-$ *o is transitive, ie. if* $f(n) = o(g(n))$ *and* $g(n) = o(h(n))$, *then* $f(n) = o(h(n))$.

By definition of little o :

if $f(n) = o(g(n)$ *then for any positive constant* c, *there exists an* N *such that* $f(n) < c * g(n)$ *for all* $n > N$.

As $g(n)$ *is a loose uper bound, it must grow at a faster rate*

Equivalently the limit is

$$\lim_{n\to\infty} \frac{f(n)}{g(n)} = \frac{f(n)}{\infty} = 0.$$

Similarily it follows that

if $g(n) = o(h(n)$ *then for any positive constant* c', *there exists an* N *such that* $g(n) < c' * h(n)$ *for all* $n > N$.

As $h(n)$ *is a loose uper bound, it must grow at a faster rate*

Equivalently the limit is

$$\lim_{n\to\infty} \frac{g(n)}{h(n)} = \frac{g(n)}{\infty} = 0.$$

Thus, it follows that

$f(n) < c * o(g(n)) < c'\, o(h(n))$

Since $h(n)$ *is loose upper bound of* $g(n)$ *then it is also a looser upper bound of* $f(n)$. $f(n) < c' o(h(n))$

Equivalently the limit is

$$\lim_{n\to\infty} \frac{f(n)}{h(n)} = \frac{f(n)}{\infty} = 0.$$

If $g(n)$ *has a loose upper bound on* $f(n)$ *and* $h(n)$ *has a loose upper bound on* $g(n)$*, then* $h(n)$ *also is a loose upper bound on* $f(n)$

example : $f(n) = n,\ g(n) = n^2,\ h(n) = n^n$

$f(n) < 1 * g(n) < 1 * h(n)$

$f(n) < 1 * h(n)$

Transivity rules hold for little $- o$.

c) *Show that for polynomial* $p(n) = \Theta(n^k)$.

Let $p(n) = a_0 + a_1 n + a_2 n^2 + \cdots + a_k n^k$ *where* $a_k > 0$. *Show that* $p(n) = \Theta(n^k)$.

By using the limit property of Θ *we prove this* :

Let $f(n) = p(n) = a_0 + a_1 n + a_2 n^2 + \cdots + a_{k-1} n^{k-1} + a_k n^k$

Let $g(n) = n^k$

$f(n) = \Theta\ g(n)$ *if* $\lim_{n\to\infty} \frac{f(n)}{g(n)} = 0 < c < \infty$ *so,*

$\lim_{n\to\infty} \frac{f(n)}{g(n)} = \frac{a_0 + a_1 n + a_2 n^2 + \cdots + a_k n^k}{n^k}$

by dividing out comon term

$numerator = a_k * n^k * \left(\frac{a_0}{a_k}\frac{1}{n^k} + \frac{a_1}{a_k}\frac{1}{n^{k-1}} + \frac{a_2}{a_k}\frac{1}{n^{k-2}} + \cdots + \frac{a_{k-1}}{a_k}\frac{1}{n} + 1 \right)$

As n approaches ∞

$numerator = a_k * n^k * \left(\frac{a_0}{a_k}\frac{1}{\infty} + \frac{a_1}{a_k}\frac{1}{\infty} + \frac{a_2}{a_k}\frac{1}{\infty} + \cdots + \frac{a_{k-1}}{a_k}\frac{1}{\infty} + 1 \right)$

$numerator = a_k * n^k * (0 + 0 + 0 + \cdots + 0 + 1)$

$numerator = a_k * n^k * (1)$

simplified limit, $\lim_{n\to\infty} \frac{f(n)}{g(n)} = \frac{a_k * n^k}{n^k} = a_k$

$\lim_{n\to\infty} \frac{f(n)}{g(n)} = a_k$, *so* $c = a_k$ *and* $f(n) = p(n) = \Theta(n^k)$.

1.2 Transitivity of little-o 2 / 3

- ✓ **+ 3 pts** Correct
- **+ 2.5 pts** Mostly correct
- **+ 1 pts** the proof is not convincing. some correct parts
- **- 0.5 pts** in case definition used: constant c missing or some mistakes
- ✓ **- 0.5 pts** in case definition used: mistake or missing in N or Ni
- **+ 0.5 pts** attempt
- **+ 0 pts** not submitted or completely incorrect
- **- 1 pts** in case of using definition: the main part of proof has some missing parts or mistakes
- ✓ **- 0.5 pts** a mistake in the main part of proof

1. wrong
2. not necessarily the same N as the previous one so needs a different notation
3. ?

gradescope

If $g(n)$ *has a loose upper bound on* $f(n)$ *and* $h(n)$ *has a loose upper bound on* $g(n)$*, then* $h(n)$ *also is a loose upper bound on* $f(n)$

example : $f(n) = n,\ g(n) = n^2,\ h(n) = n^n$

$f(n) < 1 * g(n) < 1 * h(n)$

$f(n) < 1 * h(n)$

Transivity rules hold for little − o.

c) *Show that for polynomial* $p(n) = \Theta(n^k)$.

Let $p(n) = a_0 + a_1 n + a_2 n^2 + \cdots + a_k n^k$ *where* $a_k > 0$. *Show that* $p(n) = \Theta(n^k)$.

By using the limit property of Θ *we prove this :*

Let $f(n) = p(n) = a_0 + a_1 n + a_2 n^2 + \cdots + a_{k-1} n^{k-1} + a_k n^k$

Let $g(n) = n^k$

$f(n) = \Theta\, g(n)$ *if* $\lim_{n\to\infty} \frac{f(n)}{g(n)} = 0 < c < \infty$ *so,*

$\lim_{n\to\infty} \frac{f(n)}{g(n)} = \frac{a_0 + a_1 n + a_2 n^2 + \cdots + a_k n^k}{n^k}$

by dividing out comon term

$numerator = a_k * n^k * \left(\frac{a_0}{a_k}\frac{1}{n^k} + \frac{a_1}{a_k}\frac{1}{n^{k-1}} + \frac{a_2}{a_k}\frac{1}{n^{k-2}} + \cdots + \frac{a_{k-1}}{a_k}\frac{1}{n} + 1 \right)$

As n approaches ∞

$numerator = a_k * n^k * \left(\frac{a_0}{a_k}\frac{1}{\infty} + \frac{a_1}{a_k}\frac{1}{\infty} + \frac{a_2}{a_k}\frac{1}{\infty} + \cdots + \frac{a_{k-1}}{a_k}\frac{1}{\infty} + 1 \right)$

$numerator = a_k * n^k * (0 + 0 + 0 + \cdots + 0 + 1)$

$numerator = a_k * n^k * (1)$

simplified limit, $\lim_{n\to\infty} \frac{f(n)}{g(n)} = \frac{a_k * n^k}{n^k} = a_k$

$\lim_{n\to\infty} \frac{f(n)}{g(n)} = a_k$ *, so* $c = a_k$ *and* $f(n) = p(n) = \Theta(n^k)$.

1.3 Polynomials are Theta(n^k) 2 / 2

✓ + **2 pts** **Correct**

+ **1.5 pts** minor mistake or missing - proof is correct in general
+ **1 pts** partially correct or either big O or big omega is correct not both
+ **0.5 pts** unconvincing attempt
+ **0 pts** no submission or incorrect

gradescope

d)

Find two positive valued functions $f(n)$ *and* $g(n)$ *such that*
Neither $f(n) = O(g(n))$ *nor* $g(n) = O(f(n))$.

Not all functions are asymptotically comparable as some do not grow at an orderly rate, some functions oscillate, some functions have conditions like piecewise functions.
For example we can not compare the following :
$(CLRS,\ p.\ 52)$
$f(n) = n$
$g(n) = n^{1 + sin(n)}$
The exponent of $g(n)$ *equates to* $(1 + \ -1)$ *or* $(1 + 1)$
oscillating between $0,\ 1$
Thus, $g(n)$ *oscillates between*
$g(n) = n^0 = 1$ *or* $g(n) = n^2$

Example #2
$f(n) = n$
$g(n) = \{1$ *if* n *is odd*
$\quad 0$ *if* n^2 *is even*

Neither $f(n) = O(g(n))$ *nor* $g(n) = O(f(n))$ *for both examples.*
They can not be compared using asymptotic notation.

2) Loop Invariant asymptotic complexity

```
sum = 0;
for( i = 1; i < n; i++ )
    for( j = 1; j < i * i; j++ )
        if( j % i == 0 )
            for( k = 0; k < j; k++ )
                sum ++;
```

1.4 Function examples 2 / 2

✓ **- 0 pts** **Correct**

- 2 pts Incorrect

- 1 pts Major issue

- 0.5 pts Major-ish issue

gradescope

d)

$Find\ two\ positive\ valued\ functions\ f(n)\ and\ g(n)\ such\ that$
$Neither\ f(n) = O(g(n))\ nor\ g(n) = O(f(n)).$

$Not\ all\ functions\ are\ asymptotically\ comparable\ as\ some\ do\ not\ grow\ at\ an$
$orderly\ rate,\ some\ functions\ oscillate,\ some\ functions\ have\ conditions\ like$
$piecewise\ functions.$
$For\ example\ we\ can\ not\ compare\ the\ following:$
$(CLRS,\ p.\ 52)$
$f(n)\ =\ n$
$g(n)\ =\ n^{1\ +\ sin(n)}$
$The\ exponent\ of\ g(n)\ equates\ to\ (1 +\ \ -1)\ or\ (1\ +\ 1)$
$oscillating\ between\ 0,\ 1$
$Thus,\ g(n)\ oscillates\ between$
$g(n)\ =\ n^0\ =\ 1\ or\ g(n)\ =\ n^2$

$Example\ \#2$
$f(n)\ =\ n$
$g(n)\ =\ \{1\ if\ n\ is\ odd$
$\quad\quad\quad 0\ if\ n^2\ is\ even$

$Neither\ f(n) = O(g(n))\ nor\ g(n) = O(f(n))\ \ for\ both\ examples.$
$They\ can\ not\ be\ compared\ using\ asymptotic\ notation.$

2) Loop Invariant asymptotic complexity

```
sum = 0;
for( i = 1; i < n; i++ )
    for( j = 1; j < i * i; j++ )
        if( j % i == 0 )
            for( k = 0; k < j; k++ )
                sum++;
```

i	j	$j \bmod i = 0$	sum
1	1	1	1
2	$1,\ 2,\ 3$	2	$1 + (1 + 1)\ = 3$
3	$1, 2, 3, 4, 5, 6, 7, 8$	$3,\ 6$	$3 + (1 + 1 + 1)\ = 6$ $6 + (1 + 1 + 1 + 1$ $+ 1 + 1) = 12$
4	$1, 2, 3, \ldots\ldots., 15$	$4,\ 8,\ 12$	$+=\ 4,\ 8,\ 12$
5	$1,\ 2,\ 3, \ldots.., \ 24$	$5,\ 10,\ 15,\ 20$	$+=\ 5,\ 10,\ 15,\ 20$
i	$1,\ 2,\ 3, \ldots..\ i^2\ - 1$	$i,\ 2i,\ 3i,\ \ldots., i * (i - 1)$	$+= i,\ 2i,\ 3i,\ \ldots., i * (i - 1)$

$sum\ is\ dependent\ on\ i\ and\ sum\ increments\ by\ i,\ 2i,\ 3i\ \ldots.\ until\ i * (i - 1)$

$sum\ =\ i\ +\ 2i\ +\ 3i\ \ldots.\ (i - 1) * i$

$Let\ x\ =\ sum,\ then\ we\ get\ \ this\ arithmetic\ series\ \sum_{x=1}^{i-1} = xi$

$and\ i\ is\ incremented\ upto\ n - 1\ from\ the\ outer\ loop,\ so\ we\ get\ another\ summation$

$arithmetic\ series\ \sum_{i=1}^{n-1} = i\ \ and\ by\ combing\ we\ get\ the\ asymptotic\ value\ of\ \ sum.$

$\sum_{i=1}^{n-1} = i * \sum_{x=1}^{i-1} = xi\ \ and\ by\ using\ the\ arithmetic\ series\ formula\ we\ get:$

$$s_n\ =\ \frac{n(a_1+a_n)}{2}$$

$$\sum_{i=1}^{n-1} * \sum_{x=1}^{i-1}\ =\ \frac{i\,(i(i-1)+i)}{2}\ =\ \frac{i\,(i^2-i+i)}{2}\ \ =\ \frac{(i^3)}{2}$$

$f(n)\ =\ \Theta\ g(n)\ if\ \lim_{n\to\infty} \frac{f(n)}{g(n)} = 0 < c\ < \infty\ \ \ so,$

$$Let\, f(n) \;=\; \frac{(n^3)}{2} \quad Let\; g(n) = n^3$$

$$\lim_{n\to\infty} \frac{f(n)}{g(n)} \;=\; \frac{\frac{(n^3)}{2}}{n^3} \Rightarrow \frac{1}{2} * \lim_{n\to\infty} \frac{f(n)}{g(n)} \;=\; \frac{n^3}{n^3} = 1 \Rightarrow 1 * \frac{1}{2} = \frac{1}{2}$$

Therefore sum has a precise asmyptotic characterization of $sum \;= \theta(n^3)$ *where* $c = \frac{1}{2}$.

3)

a) *Tracing polynomial using Horner's Rule at* $x = 3$

$$f(x) = \sum_{k=0}^{n-i} a_k\, x^k$$

$$\begin{aligned} f(x) \;&=\; 4x^4 \;+ 8x^3 + x + 2 \\ &=\; 4\,(3 * 3 * 3 * 3) \;+ 8(3 * 3 * 3) \;+\; (3) \;+ 2 \\ &= 545 \end{aligned}$$

$y \;= 0;$

$for\; i \;=\; n\; to\; 0\; do$

$\quad y \;= a_i + (x * y);$

$\quad i-\;-$

end

$i \;= n = degree\; of\; polynomial$

$a_i = coefficient\; of\; said\; term\; with\; exponent\; i$

i	y
4	$y \;= 4 + (3 * 0)$
3	$y \;= 8 + (3 * 4)$
2	$y \;= 0 + (3 * 20)$
1	$y \;= 1 + (3 * \;60)$
0	$y \;= 2 + (3 * 181) \;=\; 545$

2 Loop Complexity **8 / 10**

- **- 1 pts** Big O instead of Theta
- **- 3 pts** No summation/loop invarient
- **- 3 pts** Not tracing the code
- **- 1 pts** Loose upper bound
- ✓ **- 4 pts Underestimating the upper bound**
- ✓ **- 0 pts Correct**
- **- 4 pts** Over estimating the tight bound
- **- 10 pts** Not submitted
- **+ 2 Point adjustment**

good effort, a slight error in the summations caused the final answer to be wrong

gradescope

$Let\, f(n) = \frac{(n^3)}{2}$ $Let\, g(n) = n^3$

$\lim_{n\to\infty} \frac{f(n)}{g(n)} = \frac{\frac{(n^3)}{2}}{n^3} \Rightarrow \frac{1}{2} * \lim_{n\to\infty} \frac{f(n)}{g(n)} = \frac{n^3}{n^3} = 1 \Rightarrow 1 * \frac{1}{2} = \frac{1}{2}$

Therefore sum has a precise asmyptotic characterization of $sum = \theta(n^3)$ *where* $c = \frac{1}{2}$.

3)

a) *Tracing polynomial using Horner's Rule at* $x = 3$

$$f(x) = \sum_{k=0}^{n-i} a_k x^k$$

$$f(x) = 4x^4 + 8x^3 + x + 2$$
$$= 4\,(3 * 3 * 3 * 3) + 8(3 * 3 * 3) + (3) + 2$$
$$= 545$$

$y = 0;$
$for\ i = n\ to\ 0\ do$
$\quad y = a_i + (x * y);$
$\quad i - -$
end
$i = n = degree\ of\ polynomial$
$a_i = coefficient\ of\ said\ term\ with\ exponent\ i$

i	y
4	$y = 4 + (3 * 0)$
3	$y = 8 + (3 * 4)$
2	$y = 0 + (3 * 20)$
1	$y = 1 + (3 * 60)$
0	$y = 2 + (3 * 181) = 545$

3.1 Trace execution 2 / 2

✓ **- 0 pts** **Correct**

- 0.5 pts Minor issue

- 1 pts Major issue

- 2 pts DNS

- 0.25 pts Micro issue

gradescope

b)

Consider the following loop invariant :

At the start of each iteration of the for loop :

$$y = \sum_{k=0}^{n-(i+1)} a_{k+i+1}x^k$$

Show that this loop invariant holds by showing the initialization and maintenance steps. This can be shown by induction.

Base/ Initialization Case :

Let $i = n$ *then,*

$y = \sum_{k=0}^{-1} a_{k+n+1}\, x^k$ *since the summation would have no terms, i.e empty summation and based on given algorithm in part a* $y = 0$.

Induction/ Maintenance Step :

proof for $i > n$

Suppose the loop invariant holds for ith iteration, and the loop runs one more time.

$$y = \sum_{k=0}^{n-(i+1)} a_{k+i+1}x^k$$

$$y = \sum_{k=0}^{n-i-1} a_{k+i+1}x^k$$

From the above tracing and from the for loop algorithm we know that at some ith iteration we have :

$$y = a_i + (x * (\sum_{k=0}^{n-i-1} a_{k+i+1}x^k) - \textit{previous summation})$$

This is equivalent to $y = a_i + \sum_{k=0}^{n-i-1} a_{k+i+1}\, x^{k+1}$

where x was put back into the sum

If we shift or go to the next iteration as previously stated then we get :

$$y = a_i + \sum_{k=1}^{n-i} a_{k+i}x^k$$

Since a_i *is just part of the sum iteration when* x^0*, it can be put back into the summation.*

$$y = \sum_{k=0}^{n-i} a_{k+i}x^k$$

holds for the next iteration as $n-i-1$ *was incremented to* $n-1$*.*
Thus, the loop invariant holds for both initialization and maintenance.

c)

Partial Correctness
Assume the loop terminates, the loop also has the following conditions :
Pre − condition : $n \geq 0$ *and the coefficient sequence* $(a_0, a_1, ..., a_n)$ *is given.*
Post − condition : $y = \sum_{k=0}^{n} a_k x^k$ *− from part A*
Based on our maintenance from part B we got this summation

$$y = \sum_{k=0}^{n-i} a_{k+i}x^k$$

Based on the assumption that the loop terminates, and that it terminates at $i-1$ *it means that i has passed* 0 *so in the post condition* $i = 0$*.*

$$f(x) = y = \sum_{k=0}^{n-0} a_{k+0}x^k \Rightarrow f(x) = y = \sum_{k=0}^{n} a_k x^k$$

Thus, since termination is assumed and post − condition follows the loop is correct.

3.2 Show loop invariant **3 / 3**

- ✓ **- 0 pts** Correct
- **- 0.5 pts** little problem in maintenance part.
- **- 1.5 pts** incomplete answer
- **- 1 pts** Problem in maintenance part.
- **- 2 pts** incorrect responses in both parts.
- **- 1 pts** initialization step is not well described.
- **- 0.5 pts** little problem in initialization part.
- **- 3 pts** answer not provided.
- **- 2.5 pts** maintenance step is not provided and initialization is not correct.
- **- 2 pts** maintenance step is not provided.

gradescope

where x was put back into the sum

If we shift or go to the next iteration as previously stated then we get :

$$y = a_i + \sum_{k=1}^{n-i} a_{k+i}x^k$$

Since a_i *is just part of the sum iteration when* x^0, *it can be put back into the summation.*

$$y = \sum_{k=0}^{n-i} a_{k+i}x^k$$

holds for the next iteration as $n - i - 1$ *was incremented to* $n - 1$.

Thus, the loop invariant holds for both initialization and maintenance.

c)

Partial Correctness

Assume the loop terminates, the loop also has the following conditions :

Pre − condition : $n \geq 0$ *and the coefficient sequence* $(a_0,\ a_1, ..., \ a_n)$ *is given.*

Post − condition : $y = \sum_{k=0}^{n} a_k x^k$ *− from part A*

Based on our maintenance from part B we got this summation

$$y = \sum_{k=0}^{n-i} a_{k+i}x^k$$

Based on the assumption that the loop terminates, and that it terminates at $i - 1$ *it means that i has passed* 0 *so in the post condition* $i = 0$.

$$f(x) = y = \sum_{k=0}^{n-0} a_{k+0}x^k \Rightarrow f(x) = y = \sum_{k=0}^{n} a_k x^k$$

Thus, since termination is assumed and post − condition follows the loop is correct.

3.3 Partial correctness 3 / 3

✓ **- 0 pts** Correct

- 0.25 pts precondition is not discussed

- 0.25 pts Post-condition is not discussed

- 0.25 pts precondition and post condition are not shown well.

- 2 pts The answer is not explained well.

- 1 pts termination is not discussed correctly.

- 3 pts Answer is not provided.

- 2.5 pts the answer is not including post and pre condtion and termination condition.

- 1.5 pts no mathematical proof is provided. pre-condition and post-condition is ot discussed.

gradescope

d)

Find precise asymptotic notation of Horner's Rule

$y = 0;$ $\quad$ $Cost = 1$

$for\ i = n\ to\ 0\ do$ $\quad$ $Cost = 1$

$\quad y = a_i + (x * y);$ $\quad$ $Cost = 3$

$\quad i - -$ $\quad$ $Cost = 1$

end

//This algorithm runs for all elements $n + 1$, *so* $2 + 4(n + 1) = 6 + 6n$ *is the number of steps. Cost is in terms of operations and initialization.*

As the loop runs through all n degree $+ 1$ $(1\ for\ a_i x^0)$, *i.e* $i = n$ *and i is decremented to* -1. *From question* $1c$, *a polynomial* $P(n) = \Theta(n^k)$. *Substituting we get that our k is* 1, *and* $6n^0 + 6n^1 = \Theta(n^1)$.

The precise running time complexity of the Horner's rule is $\Theta(n)$.

3.4 Complexity 2 / 2

✓ **+ 2 pts** **Correct**

- 2 pts incorrect

- 1 pts The solution is close but not precisely correct.

gradescope